$$\Delta f=(2\pi)^{-1}[\omega_0(F=3)-\omega_0(F=4)]=2\mu_I H_0/I\hbar \quad (21)$$

where μ_I is the magnetic moment of the nucleus of the ^{133}Cs atom and I is the nuclear spin. In an $H_0=0.486$-Oe field, in which the signal shown in Fig. 3b was obtained, $\Delta f=546$ Hz, which coincides with the value of the modulation frequency determined experimentally from Fig. 3b.

Thus, we have observed and investigated nonstationary RF coherence (free precession, nutation, spin echo) signals during the impulsive excitation of the HFS transitions of optically oriented cesium atoms. The RF-coherence spin-echo signals were obtained with the aid of either two successive microwave pulses, or one microwave pulse with the subsequent inversion of the magnetic-field gradient. We have obtained for the nonstationary coherence signals theoretical expressions which have allowed us to explain the characteristic SHF-pulse-duration dependence for the nutation signal and the appearance of subsidiary spin-echo signals, as well as to determine the optimal conditions for echo-signal excitation for two methods of realizing it. We have also explained the behavior of the nonstationary signals excited by an unmodulated microwave-radiation pulse. There is good agreement between the experimental results and the theory.

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Relativistic dynamical invariants and the Hamiltonian formalism of charged particles in plane waves

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The longitudinal integral of the motion of a charged particle in a plane wave, an integral which is equivalent to conserved particle energy or conserved longitudinal particle momentum in reference systems in which the wave is either stationary or uniform, can be treated as a one-dimensional Hamiltonian if the quantity kt-ωz is regarded as the time. The validity of the adiabatic invariant that follows from this Hamiltonian is investigated on the basis of the exact equation. Adiabatic invariants that are valid in the particular cases when the above-mentioned adiabatic invariant is not conserved are constructed.

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The integrals of, and the adiabatic invariants associated with, the motion of charged particles located in plane periodic electromagnetic fields enable us to investigate such problems as the acceleration of trapped particles when the wave velocity increases and the attendant damping or monochromatization of the waves,[1-4] the autoresonant motion and radiation of electrons,[5-8] stationary nonlinear[9] and slowly evolving waves in a plasma,[10] semiclassical quantization,[11] etc. This incomplete list shows that the dynamical invariants are fairly widely used, and it is therefore useful to generalize this approach, which is the object of the present communication.

Let us consider a plane electromagnetic field that has the nature of a wave, and is described by the potentials

$$\mathbf{A},\ \varphi=\mathbf{A},\ \varphi(\psi),\quad \psi=\omega t-kz, \quad (1)$$

where the potentials, the frequency ω, and the wave number k can be slowly varying functions of the longitudinal coordinate z and the time t. Everywhere lengths and velocities are divided by c; the momentum p, by mc; the energy γ, by mc^2; the potentials, by mc^2/e, where m and e are the rest mass and the charge of the particle. Because of the cyclicity of the transverse coordinates of a charged particle in the field (1), the generalized transverse momentum

$$\mathbf{p}_\perp+\mathbf{A}_\perp=\mathbf{C}=\text{const} \qquad (2)$$

is conserved. If the potentials A and φ, besides being dependent on the phase ψ, also depend explicitly on the variables z and t, and ω and k also depend on these variables, so that they are now determined by the equalities

$$\omega=\partial\psi/\partial t, \quad k=-\partial\psi/\partial z, \qquad d\psi/dt=\omega-ku_z, \qquad (3)$$

where u is the particle velocity, then the equations for p_z and γ can be written in the form

$$\frac{dp_z}{dt}=k\frac{\partial\varphi}{\partial\psi}-\omega\frac{\partial A_z}{\partial\psi}-\frac{\partial\varphi}{\partial z}-\frac{\partial A_z}{\partial t}+\mathbf{u}_\perp\left(\frac{\partial\mathbf{A}_\perp}{\partial z}-k\frac{\partial\mathbf{A}_\perp}{\partial\psi}\right), \qquad (4)$$

$$\frac{d\gamma}{dt}=u_z\left(k\frac{\partial\varphi}{\partial\psi}-\omega\frac{\partial A_z}{\partial\psi}-\frac{\partial\varphi}{\partial z}-\frac{\partial A_z}{\partial t}\right)-\mathbf{u}_\perp\left(\frac{\partial\mathbf{A}_\perp}{\partial t}+\omega\frac{\partial\mathbf{A}_\perp}{\partial\psi}\right). \qquad (5)$$

The separation of the explicit dependence on z and t evidently makes sense only when this dependence is much slower than the explicit dependence on ψ. Multiplying (5) by k and (4) by ω, and subtracting, we obtain an exact equation containing only derivatives with respect to the slow variables:

$$\frac{dY}{dt}=(\gamma+\varphi)\frac{dk}{dt}-(p_z+A_z)\frac{d\omega}{dt}+k\frac{\partial\varphi}{\partial t}+\omega\frac{\partial\varphi}{\partial z}-\mathbf{u}\left(k\frac{\partial\mathbf{A}}{\partial t}+\omega\frac{\partial\mathbf{A}}{\partial z}\right), \qquad (6)$$

$$Y=k(\gamma+\varphi)-\omega(p_z+A_z). \qquad (7)$$

It follows from (6) that, in the absence of the indicated dependence on the slow variables z and t, the quantity Y is an integral of the motion. This integral of the motion was apparently first obtained by Gilinskiĭ,[12] and is used in many of the papers cited above, as well as in other papers. Let us show that it is equivalent to quantities that are conserved in special reference systems.

1. If $\omega < k$, then there exists a reference system K_1 moving relative to the laboratory system along the z axis with velocity ω/k, in which the dependence of the potentials on time disappears, i.e., in which

$$\psi=-z'(k^2-\omega^2)^{1/2}=-k'z' \qquad (8)$$

and therefore the total energy is conserved:

$$\gamma'+\varphi'=(p_z'^2+(\mathbf{C}-\mathbf{A}_\perp)^2+1)^{1/2}+\varphi'=\text{const}. \qquad (9)$$

2. For $\omega > k$, the dependence on z disappears in the system K_2 moving with velocity k/ω:

$$\psi=t'(\omega^2-k^2)^{1/2}=\omega't' \qquad (10)$$

and the generalized longitudinal momentum is conserved:

$$p_z'+A_z'=(\gamma'^2-(\mathbf{C}-\mathbf{A}_\perp)^2-1)^{1/2}+A_z'=\text{const}. \qquad (11)$$

The transformation of the expressions (9) and (11) into the laboratory reference system yields the integral of the motion Y divided by the Lorentz-invariant factor $(k^2-\omega^2)^{1/2}$.

Notice that for the integral, Y, of the motion to exist, it is only necessary that the variables z and t enter into the potentials in the combination $\omega t - kz$, no restriction being imposed on the x and y dependence of the potentials. This significantly broadens the range of applicability of the integral, Y, of the motion: it includes fields in waveguides, the wave can be superposed on constant electromagnetic fields, etc.

Let us show further that, for charged particles in the field (1), we can introduce a one-dimensional canonical formalism with the conserved Hamiltonian

$$H=Y/(k^2-\omega^2)^{1/2}, \qquad (12)$$

with the role of time played by the variable τ:

$$\tau=kt-\omega z, \qquad (13)$$

and the coordinate ψ having as its canonical conjugate the momentum

$$\Pi=\frac{\omega(\gamma+\varphi)-k(p_z+A_z)}{(k^2-\omega^2)^{1/2}}. \qquad (14)$$

We can, without loss of generality, impose on the potentials the Lorentz-invariant gauge

$$\omega\frac{d\varphi}{d\psi}=k\frac{dA_z}{d\psi}, \quad \omega\varphi=kA_z, \qquad (15)$$

and then

$$\Pi=(\omega\gamma-kp_z)/(k^2-\omega^2)^{1/2}. \qquad (16)$$

The canonical equations

$$d\Pi/d\tau=-\partial H/\partial\psi, \quad d\psi/d\tau=\partial H/\partial\Pi \qquad (17)$$

which follow from the Hamiltonian (12),

$$H=\frac{k\varphi-\omega A_z}{(k^2-\omega^2)^{1/2}}+\text{sign}(k-\omega u_z)(\Pi^2+(\mathbf{C}-\mathbf{A}_\perp)^2+1)^{1/2} \qquad (18)$$

are equivalent to Eqs. (3)–(5) (in the absence of any explicit dependence on the slow variables z and t).

The developed Hamiltonian formalism is Lorentz invariant, and can be rewritten in a covariant form. In the reference system K_1, in which $\omega'=0$, the Hamiltonian (18) coincides with the total energy, $\Pi=-p_z'$, and $\tau=k't'$. In the case when $\omega > k$, H and Π are imaginary, and in the reference system K_2

$$p_z'+A_z'=iH, \quad \gamma'=i\Pi, \quad \tau=-\omega'z', \quad \psi=\omega't'. \qquad (19)$$

It follows from the canonical formalism that there exists for a field, (1), that is periodic in ψ, and dependent on the slow variables z and t the adiabatic invariant

$$J=\oint\Pi\,d\psi=-\oint\sigma\left(\frac{(Y-k\varphi+\omega A_z)^2}{k^2-\omega^2}-(\mathbf{C}-\mathbf{A}_\perp)^2-1\right)^{1/2}d\psi, \qquad \sigma=\text{sign}(ku_z-\omega). \qquad (20)$$

The adiabatic invariant (20) describes well the behavior of the particles during the slow variation of the field parameters, except for transit particles in the case when the phase velocity varies, i.e., when $\omega/k \neq \text{const}$. Therefore, let us investigate the applicability of the adiabatic invariant (20) when the only varying parameters of the wave are ω and k. In this case Eq. (6) with allowance for the expressions

$$\gamma=k(Y-k\varphi+\omega A_z)/(k^2-\omega^2)$$
$$+\omega\sigma\{(Y-k\varphi+\omega A_z)^2-(k^2-\omega^2)[(\mathbf{C}-\mathbf{A}_\perp)^2+1]\}^{1/2}/(k^2-\omega^2), \tag{21}$$

$$p_z=\omega(Y-k\varphi+\omega A_z)/(k^2-\omega^2)$$
$$+k\sigma\{(Y-k\varphi+\omega A_z)^2-(k^2-\omega^2)[(\mathbf{C}-\mathbf{A}_\perp)^2+1]\}^{1/2}/(k^2-\omega^2), \tag{22}$$

which follow from (2) and (7), assumes, after being averaged over ψ, the form

$$\frac{dY}{dt}=\frac{Y}{2(k^2-\omega^2)}\frac{d(k^2-\omega^2)}{dt}$$
$$+\frac{k^2}{k^2-\omega^2}\frac{dV}{dt}\langle\sigma\{(Y-k\varphi+\omega A_z)^2-(k^2-\omega^2)[(\mathbf{C}-\mathbf{A}_\perp)^2+1]\}^{1/2}\rangle, \tag{23}$$

where the angular brackets denote the mean value and $V=\omega/k$. When

$$V=\mathrm{const} \tag{24}$$

Eq. (23) is integrable:

$$Y/(k^2-\omega^2)^{1/2}=\mathrm{const}, \tag{25}$$

which coincides with the adiabatic invariant (20) when ω and k are the only varying field parameters.

Let us also note that the result (25) is valid for trapped particles even when the requirement (24) is not fulfilled, since for such particles the expression in the angular brackets in (23) vanishes on account of the fact that σ assumes the values +1 and −1 equally frequently for each value of ψ. The foregoing corresponds to the fact that, for $V\to 1$, the expression (25) implies unlimited growth of the energy, which is valid for trapped particles, but physically absurd for untrapped particles.

Let us discuss the difference between the situations for the transit and trapped particles in greater detail. If $\omega/k\neq\mathrm{const}$, then the reference systems K_1 and K_2 are noninertial, and a quasigravitational inertial-force field exists in them. This field can be neglected in the construction of the adiabatic invariant for trapped particles, since the motion of these particles is bounded; for transit particles the quasigravitational potential changes significantly, and allowance for this circumstance in the construction of the adiabatic invariant is necessary. The physical meaning of the foregoing becomes clear when we consider the case $\omega/k<1$. In going over into the K_1 reference system, we use the ω/k value corresponding to some phase ψ_0, i.e., to definite values of z and t. The dependence of the potentials on t disappears in the neighborhood of this phase, and the entire developed theory is valid for trapped particle, which remain close to the phase ψ_0. Untrapped particles get away from the phase ψ_0, but far from this phase the value of ω/k does not coincide with the value at the point ψ_0, the field becomes a running field, the Hamiltonian $\gamma'+\varphi'$ is not conserved, and the quantity J is not an adiabatic invariant. For untrapped particles the quantity $\gamma'+\varphi'+\varphi'_g$, where φ'_g is the gravitational potential in the K_1 system, is conserved. Therefore, the adiabatic invariant (20) is not conserved for untrapped particles in the case when $\omega/k\neq\mathrm{const}$.

Next, let us derive the adiabatic invariants for a varying wave phase velocity, assuming the remaining wave parameters to be constants. We do this for the cases when either k, or ω, varies.

1. Let $\omega=\mathrm{const}$ and $k=k(z)$. This case corresponds to a stationary wave in an inhomogeneous medium. Equation (6) reduces to the form

$$dY/dk=\gamma+\varphi. \tag{26}$$

The adiabatic invariant for trapped particles in a circularly-polarized wave is obtained in Ref. 2 by going over into a reference system moving with the phase velocity of the wave, while the invariant for untrapped particles is obtained in Ref. 13 by direct integration of Eq. (26) after being averaged over z. Also constructed there is a Hamiltonian which allows us to generalize the obtained result to transverse waves of any mode of polarization.

It is not difficult to extend the results of Refs. 2 and 13 to the case of waves of the type (1) that also have a longitudinal component. In this case there exists the conserved Hamiltonian:

$$H=\frac{Y^2+\omega^2}{2\omega^2}=-\frac{1}{2}\Big[(n^2-1)(\gamma+\varphi)^2+2Yn\frac{\gamma+\varphi}{\omega}$$
$$+(\mathbf{C}-\mathbf{A}_\perp)^2+\frac{2YA_z}{\omega}+A_z^2-\varphi^2\Big],\quad n=\frac{k}{\omega}, \tag{27}$$

in which the momentum $\gamma+\varphi$ and the coordinate ψ should be regarded as the canonically conjugate variables.

The canonical equations that follow from (27) coincide with the equations of motion if as the time we use the proper time of the particle:

$$\eta=\int\omega\frac{dt}{\gamma}. \tag{28}$$

Thus, the adiabatic invariant with respect to the proper time is the expression

$$J'=\oint(\gamma+\varphi)d\psi. \tag{29}$$

2. Let $k=\mathrm{const}$ and $\omega=\omega(t)$. This is the case of a non-stationary uniform wave. The equation for Y assumes the form

$$dY/d\omega=-(p_z+A_z). \tag{30}$$

This case has hitherto not been considered. Let us, therefore, investigate it in somewhat greater detail. Let us first consider the circularly polarized wave

$$A_x=A\cos\psi,\quad A_y=-A\sin\psi,\quad \psi=\int\omega(t)dt-kz,$$
$$A=\mathrm{const},\quad A_z=\varphi=0,$$

and then generalize the obtained result after constructing the corresponding Hamiltonian.

Let $\omega<k$. In this case there exist trapped particles, the period of whose motion is given by

$$T=\frac{8k}{k^2-\omega^2}\left\{\beta+\frac{(C-A)^2+1}{4CA}\right\}^{1/2}K(\beta), \tag{31}$$

where

$$\beta=\frac{Y^2-[(C-A)^2+1](k^2-\omega^2)}{4CA(k^2-\omega^2)} \tag{32}$$

is a positive parameter, which, for trapped particles, is less then unity and $K(\beta)$ is the complete elliptic integral of the first kind. Averaging Eq. (30) with respect to t over the period (31), we arrive at the equation

$$K(\beta)\,d\beta/\beta B(\beta)=d(k^2-\omega^2)/(k^2-\omega^2),\qquad \beta B(\beta)=E(\beta)-(1-\beta)K(\beta), \tag{33}$$

where $E(\beta)$ is the complete elliptic integral of the second kind. Integrating (33), we obtain the adiabatic invariant

$$J=\beta B(\beta)/(1-V^2)^{1/2}=\text{const.} \tag{34}$$

Similar calculations for untrapped particles lead to the expressions

$$T=\frac{4}{k^2-\omega^2}\left[\frac{\pi\sigma\omega}{2}+k\left(1+\frac{(C-A)^2+1}{4CA\beta}\right)^{1/2}K\left(\frac{1}{\beta}\right)\right], \tag{35}$$

$$\left[K(x)+\frac{\pi\sigma V}{2(1+ax)^{1/2}}\right]\frac{dx}{x^{3/2}(1-V^2)^{1/2}}-[2VE(x)+\pi\sigma(1+ax)^{1/2}]\frac{dV}{x^{1/2}(1-V^2)^{3/2}}=0;$$

$$a=\frac{(C-A)^2+1}{4CA},\quad x=\frac{1}{\beta}<1. \tag{36}$$

The left member of (36) is a total differential, and therefore

$$J=\frac{2E(x)+\pi\sigma V(1+ax)^{1/2}}{x^{1/2}(1-V^2)^{1/2}}=\text{const.} \tag{37}$$

The adiabatic invariants (34) and (37) pertain to the case $\omega<k$. Let us write out the expressions for the opposite case, $\omega>k$:

$$T=\frac{4}{\omega^2-k^2}\left[\frac{\pi\omega}{2}-k\sigma'\left(1-\frac{(C+A)^2+1}{4CA\beta}\right)^{1/2}K\left(\frac{1}{\beta}\right)\right],\qquad \beta=\frac{Y_1^2+(\omega^2-k^2)[(C+A)^2+1]}{4CA(\omega^2-k^2)}>1. \tag{38}$$

For $\omega<k$, the quantity Y is always positive, while for $\omega>k$, it can also be negative. In (38) σ' is the sign of Y:

$$\sigma'=\text{sign}\,Y. \tag{39}$$

Equation (30) for $\omega>k$ assumes the form

$$\left[K(x)-\frac{\pi\sigma' V}{2(1-ax)^{1/2}}\right]\frac{dx}{x^{3/2}(V^2-1)^{1/2}}+[2VE(x)-\pi\sigma'(1-ax)^{1/2}]\frac{dV}{x^{1/2}(V^2-1)^{3/2}}=0. \tag{40}$$

The left-hand side of (40) is a total differential, and therefore

$$J=\frac{2E(x)+\pi\sigma' V(1-ax)^{1/2}}{x^{1/2}(V^2-1)^{1/2}}=\text{const.} \tag{41}$$

The adiabatic invariants (34), (37), and (41) coincide up to constant factors with the expression

$$J'=\oint p_z\,d\psi. \tag{42}$$

This is not accidental, because for $k=\text{const}$ and $\omega=\omega(t)$, we can construct the conserved Hamiltonian

$$H=\frac{k^2-Y^2}{2k^2}=-\frac{1}{2}\left[(1-V^2)(p_z+A_z)^2-2YV\frac{p_z+A_z}{k}+(C-A_\perp)^2+2Y\frac{\varphi}{k}+A_z^2-\varphi^2\right] \tag{43}$$

with the canonically conjugate momentum p_z+A_z and coordinate ψ and the time

$$\xi=\int k\frac{dt}{\gamma}, \tag{44}$$

the canonical equations that follow from (43) being the equations of motion of the particle in the wave.

Thus, the quantity

$$J'=\oint(p_z+A_z)\,d\psi \tag{45}$$

is an adiabatic invariant. Notice that the Hamiltonians (27) and (43) and the corresponding adiabatic invariants (29) and (45) are not Lorentz invariant. This is due to the fact that the reference systems in which $\omega=\text{const}$ and $k=k(z)$ or $k=\text{const}$ and $\omega=\omega(t)$ are, on the basis of these requirements, preferred systems.

From the above-expounded theory we can draw the following practical conclusion: in the case when the phase velocity of the wave is invariable, we should use the adiabatic invariant (20); in the cases, however, when $\omega=\text{const}$, $k=k(z)$ and $k=\text{const}$, $\omega=\omega(t)$ (29) and (45) are the adiabatically invariant expressions.

In conclusion, let us note that one of the most important consequences of the canonical formalism constructed here is, in our opinion, the possibility of writing down the equilibrium Gibbs distribution for particles in a strong wave:

$$F(\Pi,\psi)=\text{const}\cdot\exp(-H(\Pi,\psi)/\Theta).$$

Another possibility that arises from the above-developed approach consists in the construction of transverse adiabatic invariants:

$$J_x=\oint C_x\,dx,\quad J_y=\oint C_y\,dy,$$

which describe the slow dependence on the transverse coordinates, thereby removing the assumption that the wave is an ideal plane wave, an assumption which greatly limits the generality of the method.

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