[1)]If the power of wave $c_0\mathscr{E}_0^*$ is higher than that of wave $e_0\mathscr{E}_0$, as can be done in the case of two cells by placing an amplifier between them, then the reflection of the weak component $e_1\mathscr{E}_1$ can be accompanied by amplification.

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Modulation oscillations of a light wave in the Stark-pulse technique

A. I. Alekseev and A. M. Basharov

Moscow Engineering Physics Institute

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A theoretical investigation is made of the modulation oscillations of the intensity of a light wave in a gas medium perturbed by two Stark pulses of a constant electric field. The echo phenomena that occur both during the time of action of the second Stark pulse and after this pulse are investigated. New photon-echo manifestation, in the form of induction echo and nutation echo, are established. The first is formed on the basis of free optical induction, and the second on the basis of optical nutation. A new modification of the edge echo that appears after the action of only one perturbing Stark pulse is also investigated. The new echo phenomena, just as those previously observed, make it possible to determine the dipole moments of atomic transitions, the time of irreversible relaxation in a medium, as well as the Stark level shifts.

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Brewer and Shoemaker[1,2] have proposed an original procedure for the investigation of optical nutation, of free optical induction, and of photon echo. In this procedure, the gas medium is located for a long time in the field of a monochromatic wave whose frequency ω is close to the frequency of some atomic transition. As a result of the Doppler effect the frequencies of this transition in the gas atoms are scattered about the principal value ω_0. Inside the Doppler contour there are therefore two groups of atoms, one at resonance with the monochromatic wave, and the other not. All the atoms are subjected for a short time τ to the action of a strong constant electric field (a Stark pulse). The Stark effect shifts the energy levels, so that for the resonant atoms the application of the Stark pulse is equivalent to the vanishing of the monochromatic wave. At the same time the nonresonant atom become resonant with the monochromatic wave after the application of the Stark pulse and react thus as if they were exposed to a light pulse of frequency ω and duration τ_1. The reradiation of the photons produces modulation oscillations of the monochromatic-wave intensity, and a second application of a Stark pulse of duration τ_2 is accompanied by a photon echo. This Stark-pulse technique was subsequently employed in many experiments (see, e.g., Refs. 3–7).

The optical nutation and the free induction produced by a single Stark pulse were considered by Hopf, Shea, and Scully.[8] Yet no theoretical investigation has been made so far of the modulation oscillations of the light wave under the influence of two successive Stark pulses. For a qualitative explanation of photon echo, use was made[1,3,5,7] of the physical concepts previously employed in the problem of the passage of two short light pulses through a gas.[9,10] This explanation, however, must be substantiated.

We present below a unified description of optical nutation, free optical induction, and photon and edge echo, which are observed in the Stark-pulse technique. In addition to the traditional pulse echo produced after the second Stark pulse, we describe new nonlinear effects that evolve within the time interval that the second Stark pulse is on and after the pulse. The approach proposed has made it possible to establish that only in the experiment of Glorieux *et al.*[5] does the formation of the photon echo under certain conditions proceeed in the same manner as when the medium is excited by two light pulses.[9,10] In both cases (Ref. 5 and Refs. 9 and 10) is the echo formed on the basis of free optical induction. Yet in other experiments[1,3,7] the produced echo is of different origin and is differently described, since it is formed on the basis of optical nutation.

1. CALCULATION METHOD AND BASIC FORMULAS

Assume that a light wave

$$\mathbf{E}=\mathbf{l}\mathscr{E}e^{i(kz-\omega t)}+\text{c.c.} \tag{1}$$

propagates through a gas medium along the Z axis. Here $\mathbf{E}$ is the electric field intensity, $\mathscr{E}=\mathscr{E}(z,t)$ is a slowly varying amplitude, and $\mathbf{l}$ is a unit vector perpendicular to the Z axis. The carrier frequency $\omega = kc$ is close to the frequency ω_0 of the atomic transition $|\omega-\omega_0| \ll \omega_0$. Together with (1) there pass through the gas, in succession, two rectangular electric-field Stark pulses

$$\mathbf{E}_{\text{st}}=\begin{cases}\mathbf{E}_0, & 0\leqslant t-z/c\leqslant\tau_1,\\ 0, & t-z/c>\tau_1,\\ \mathbf{E}_0, & \tau+\tau_1\leqslant t-z/c\leqslant\tau+\tau_1+\tau_2\\ 0, & t-z/c>\tau+\tau_1+\tau_2\end{cases}, \tag{2}$$

where $\mathbf{E}_0$ is a constant vector, τ_1 and τ_2 are the durations of the Stark pulses, τ is the time interval between them, and $z=0$ is the point of entry into the medium. The Stark pulses split and shift the energy levels, making one of the split atomic transitions resonant to the light wave (or, contrariwise, upset the strict resonance). For simplicity we neglect level degeneracy. The basic laws determined below remain in force, in general outline, also if degeneracy is taken into account. We emphasize also that in some cases the degeneracy in the Stark technique is of no importance, for example for atomic transitions with change of total angular momentum

$$0\rightleftarrows 1 \text{ and } {}^1/_2\to{}^1/_2.$$

We describe the interaction of the fields (1) and (2) with the two-level atoms with the aid of the d'Alambert equation

$$\left(\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\int\mathbf{P}\,dv \tag{3}$$

and the quantum-mechanical equation for the density matrix

$$i\hbar\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial z}+\hat{\Gamma}\right)\rho=H\rho-\rho H+(\mathbf{E}+\mathbf{E}_{\text{st}})(\rho\mathbf{d}-\mathbf{d}\rho). \tag{4}$$

Here $\mathbf{P}=\mathrm{Tr}\,\rho\mathbf{d}$ is the polarization of the medium, $\mathbf{d}$ is the dipole-moment operator, H is the Hamiltonian of the free atom in its c. m. s., and v is the projection of the atom velocity on the Z axis. The relaxation operator $\hat{\Gamma}$ takes into account the spontaneous transition of the atom from the excited to the ground state in accord with the formulas

$$(\hat{\Gamma}\rho)_{22}=\gamma\rho_{22},\ (\hat{\Gamma}\rho)_{11}=-\gamma\rho_{22},\ (\hat{\Gamma}\rho)_{21}={}^1/_2\gamma\rho_{21},$$

where γ is the probability of the spontaneous emission of the atom, and the matrix elements are taken over the wave functions of the unperturbed Hamiltonian H. The subscripts 1 and 2 correspond to the ground and excited states of the atoms with energies E and $E_2(E_2-E_1=\hbar\omega_0)$. The total number of the two-level atoms remains constant

$$\rho_{11}+\rho_{22}=n_0 f(v),\quad f(v)=\frac{1}{\pi^{1/2}u}\exp\left(-\frac{v^2}{u^2}\right),$$

where n_0 is the density of these atoms, $f(v)$ is the Maxwell distribution function, and u is the mean thermal velocity.

We take into account the level shift in (4) during the time of passage of the Stark pulses (2) by perturbation theory; this reduces to the replacement

$$\mathbf{E}_{\text{st}}(\rho\mathbf{d}-\mathbf{d}\rho)_{mn}\to(-1)^m(1-\delta_{mn})\rho_{mn}\delta E,$$

where $\delta E=\delta E_2-\delta E_1$, and δE_1 and δE_2 are the shifts of the first and second levels of the atom in the constant field $\mathbf{E}_0$. The subscripts m and n run through the values from 1 to 2.

In addition, we separate the rapidly oscillating factor of the polarization of the medium

$$\mathbf{lP}=pe^{i(kz-\omega t)}+\text{c.c.},$$

where $p=\rho_{21}(\mathbf{l}\cdot\mathbf{d})_{12}^{*}e^{i(\omega t-kz)}$ is a slow function. We then obtain from (3) and (4) in the resonance approximation the basic equations for the slow functions:

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\mathscr{E}=i\,2\pi\int p\,d\eta, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\gamma}{2}+i(\eta-\Delta_s)\right)p=i\frac{|d|^2}{\hbar}\mathscr{E}N, \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\gamma\right)N=\frac{2i}{\hbar}(\mathscr{E}^*p-\mathscr{E}p^*)+\gamma n_0 f(v). \tag{7}$$

Here $d=(\mathbf{l}\cdot\mathbf{d})_{12}$, $\eta=kv$, $N=\rho_{11}-\rho_{22}$ is the excess population density, and the parameters Δ_s takes into account the shift of the levels under the influence of the Stark field (2):

$$\Delta_s=\begin{cases}\omega-\omega_0-\delta E/\hbar=\Delta_0 \text{ in the field } \mathbf{E}_0, & s=0,\\ \omega-\omega_0=\Delta_1 \quad \text{without the field } \mathbf{E}_0, & s=1.\end{cases} \tag{8}$$

With the aid of (5)–(7) we obtain the stationary regime that settles in the gas medium when a monochromatic wave of type (1) with constant real amplitude a is applied at the input. The stationary values of the polarization p_0 and of the excess population N_0 are

$$p_0=\frac{n_0|d|^2}{\hbar}f(v)\mathscr{E}_0\frac{\eta-\Delta_s+i\gamma/2}{(\eta-\Delta_s)^2+\gamma^2/4+2|\mathscr{E}_0 d/\hbar|^2}, \tag{9}$$

$$N_0=n_0 f(v)\left(1-\frac{2|\mathscr{E}_0 d/\hbar|^2}{(\eta-\Delta_s)^2+\gamma^2/4+2|\mathscr{E}_0 d/\hbar|^2}\right). \tag{10}$$

Here $\mathscr{E}_0=\mathscr{E}_0(z)$ is the stationary value of the amplitude of the wave (1), and is a solution of the transcendental equation

$$\frac{d}{dz}\mathscr{E}_0=2i\pi\int p_0\,d\eta$$

with boundary condition $\mathscr{E}_0(0)=a$. If we introduce the real quantities b_1 and b_2 defined by

$$2i\pi\int p_0\,d\eta=(-b_1+ib_2)\mathscr{E}_0,\quad b_1>0,$$

then we see that at $\Delta_s\neq 0$ the quantity $\mathscr{E}_0$ is complex, $\mathscr{E}_0=|\mathscr{E}_0|e^{i\varphi}$. The stationary intensity $I_0=c|\mathscr{E}_0(z)|^2/2\pi$ decreases monotonically with increasing z, owing to the nonresonant losses, in accord with the equation

$$dI_0/dz=-2b_1 I_0.$$

At the same time the stationary phase φ increase in the medium like

$$\varphi=\int\limits_0^z b_2\,dz.$$

We consider now modulation oscillations produced by a short-duration perturbation in the amplitude of the wave (1) near the stationary regime. We seek the solution of Eqs. (5)–(7) in the form

$$\mathscr{E}=\mathscr{E}_0+\varepsilon,\ p=p_0+q,\ N=N_0+n.$$

We assume furthermore that the deviation, described by the quantities ε, q, and n, from the stationary regime is small. This is possible under the condition

$$Ln_0T_0|d|^2\omega/\hbar c\ll 1, \tag{11}$$

where L is the length of the gas medium and $T_0=1/ku$ is the time of the reversible Doppler relaxation. If (11) is satisfied the stationary intensity I_0 likewise changes little over the length L, and the change of the phase φ in this problem is insignificant.

In this case the modulation oscillations are described by the equations

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\varepsilon=2i\pi\int q\,d\eta, \tag{12}$$

$$\left(\frac{\partial}{\partial t}+\frac{\gamma}{2}+i(\eta-\Delta_0)\right)q=\frac{ia|d|^2n}{\hbar}, \tag{13}$$

$$\left(\frac{\partial}{\partial t}+\gamma\right)n=\frac{2ia(q-q^*)}{\hbar}. \tag{14}$$

The first of them can be solved independently:

$$\varepsilon=2i\pi z\int q\,d\eta, \tag{15}$$

and the general solution of the two other, (13) and (14), was investigated before.[11,12]

The intensity I of the wave (1) after emergence from the medium is

$$I=\frac{c}{2\pi}|\mathscr{E}_0(L)|^2\left(1+\frac{2\varepsilon'}{a}\right), \tag{16}$$

where ε' is the real part of the amplitude (15) at the point $z=L$.

The quantities (16), (9), and (10), together with the general solution of the system (13) and (14), make it easy to determine the modulation oscillations of the intensity of the wave (1) following the passage of the Stark pulses. We shall henceforth be interested mainly in the nonstationary oscillations which are attenuated by the Doppler dephasing of the emitters within a time interval that is small compared with the time $1/\gamma$ of the irreversible relaxation:

$$T_0\ll 1/\gamma,\ \hbar/a|d|\ll 1/\gamma. \tag{17}$$

2. MODULATION OSCILLATIONS OF A NONRESONANT WAVE

To make the physical phenomena in the Stark-pulse technique stand out better, we consider first a possible experiment wherein all the atoms inside the Doppler contour are off resonance with the stationary wave prior to the application of the Stark pulses (2), and some of them enter into resonance after the pulses are applied:

$$|\Delta_1|\gg 1/T_0>|\Delta_0|,\ |\Delta_1|\gg a|d|/\hbar. \tag{18}$$

In the absence of a Stark pulse, the quantities

$$\frac{\mathscr{E}_0|d|(\eta-\Delta_1+i\gamma/2)/\hbar}{(\eta-\Delta_1)^2+\gamma^2/4+2|\mathscr{E}_0d/\hbar|^2}\qquad\frac{2|\mathscr{E}_0d/\hbar|^2}{(\eta-\Delta_1)^2+\gamma^2/4+2|\mathscr{E}_0d/\hbar|^2}$$

in (9) and (19) are then negligibly small and the atoms behave as if they were free. After application of the Stark field, however, some of the atoms inside the Doppler contour becomes resonant with the stationary wave (1), as is seen from (9) and (10). For these atoms, application of the Stark field for a time τ_1 is equivalent to passage of a light pulse of the type (1) and of duration τ_1.

During the time of action of the Stark field the nonstationary polarization q and the excess population n are the solution of Eqs. (13) and (14) with initial conditions $q(0)=\tilde{p}_0-p_0$ and $n(0)=\tilde{N}_0-N_0$, where the tilde labels the quantities in the absence of the Stark field, $\tilde{p}_0=0$ and $\tilde{N}_0=n_0f(v)$. Therefore in the interval $0\leq t\leq\tau_1$ the intensity (16) executes nutation oscillations, just as in the case of passage of a steplike light pulse.[13,14] If the resonant levels are degenerate, the optical nutation attenuates much more rapidly in the Stark-pulse technique, for the same reason as in Refs. 15 and 16.

The atoms react tothe turning off of the Stark pulse as the instant $t=\tau_1$ in the same manner as on the turning off of the light pulse. Thereforein the region $\tau_1\leq t\leq\tau_1+\tau$ there are observed the same induction oscillations as in the problem of the passage of a single light pulse.[17]

After the second Stark pulse is turned on at the instant $t=\tau_1+\tau$, nutation oscillations are produced again; these have heretofore not been investigated. They are described by the function

$$\varepsilon_1'(t)=-\varepsilon_0\int\limits_{-\infty}^{\infty}\exp[-(\eta T_0)^2](A\sin\Omega_0\tilde{t}+B\cos\Omega_0\tilde{t})\frac{d\eta}{\Omega_0}, \tag{19}$$

$$\varepsilon_0=2\pi^{1/2}n_0LT_0a|d|^2\omega/\hbar c,\qquad \Omega_0^2=(\eta-\Delta_0)^2+\Omega^2.\qquad \Omega=2a|d|/\hbar,$$

$$A=\cos\Omega_0\tau_1-\frac{\eta-\Delta_0}{\Omega_0}\sin\Omega_0\tau_1\sin(\eta-\Delta_1)\tau$$
$$+\left(\frac{\eta-\Delta_0}{\Omega_0}\right)^2(1-\cos\Omega_0\tau_1)(1-\cos(\eta-\Delta_1)\tau),$$

$$B=\sin\Omega_0\tau_1\cos(\eta-\Delta_1)\tau-\frac{\eta-\Delta_0}{\Omega_0}(1-\cos\Omega_0\tau_1)\sin(\eta-\Delta_1)\tau,$$

where $\tilde{t}=t-\tau_1-\tau$, and the term z/c, which takes into account the delay of the electromagnetic signal, has been omitted.

The nutation oscillations (19) have at $\Delta_0=0$ a period $2\pi/\Omega$, and this makes it possible in practice to determine the dipole moment of the atomic transition. To this end it is necessary to choose the quantity $|d|$ in such a way that the theoretical curves of the nutational oscillations coincide with the experimental ones.

The damping of the oscillations (19) is due to the thermal motion of the atoms and depends substantially on the value of the parameter ΩT_0. The latter gives an idea of the fraction of the atoms inside the Doppler contour that interact resonantly with the wave (1). In particular, at $\Omega T_0 > 1$ the damping time is of the order of ΩT_0^2, while in the region $\Omega T_0 \ll 1$, $|\Delta_0| T_0 \ll 1$ the oscillations are damped after the lapse of several periods, the presence of a resonance defect $\Delta_0 \neq 0$ enhances the damping.

If the inequalities

$$1/\Omega \ll \tau_1 < \tau_2, \; T_0 \ll \tau_1 \tag{20}$$

are satisfied, then the amplitude of the nutation oscillations (19) breaks up into two terms $\varepsilon_1'(t) = \varepsilon_{1nu}(t) + \varepsilon_{1nue}(t)$. One of them $\varepsilon_{1nu}(t)$ is damped as indicated above, while the second $\varepsilon_{1nu}(t)$ increases gradually up to the instant of time $t = 2\tau_1 + \tau$, and then again decreases. The maximum of the second term is reached at an instant of time when the first has already attenuated. The term $\varepsilon_{1nue}(t)$ with the anomalous behavior near the instant $t = 2\tau_1 + \tau$ describes a new echo phenomenon, which can naturally be called the nutation photon echo:

$$\varepsilon_{nue}(t) = -\frac{\varepsilon_0}{2} \int_{-\infty}^{\infty} \exp[-(\eta T_0)^2] \left(\frac{\Omega}{\Omega_0}\right)^2 \times (1-\cos(\eta-\Delta_1)\tau) \sin \Omega_0 (\tilde{t}-\tau_1) \frac{d\eta}{\Omega_0}. \tag{21}$$

The nutation echo (21) is connected with reradiation of photons by resonant atoms that were taken at the preceding instant of time out of the state of dyanmic equilibrium with the wave (1) by the Stark pulses. This echo evolves in the time interval when the second Stark pulse is still on and manifests itself particularly strongly at $T_0 \ll 1/\Omega \ll \tau_1 \lesssim 1/\gamma$. The presence of level degeneracy and of a detuning $|\Delta_0| > \Omega$ improves the conditions for the the observation of the nutational echo.

After the second Stark pulse is turned off at the instant $t = \tau_1 + \tau_2 + \tau$, there appear free induction oscillations in the form of two terms $\varepsilon_1'(t) = \varepsilon_{11}(t) + \varepsilon_{1e}(t)$. The first $\varepsilon_{11}(t)$ has a rather unwidely form and attenuates rapidly within a time of the order of T_0 on account of the Doppler dephasing of the emitters. The second term $\varepsilon_{1e}(t)$ increases, first slowly and then rapidly, and reaches a maximum at the approximate instant $t = \tau_1 + \tau_2 + 2\tau$. As a result, the photon echo typical of the Stark-pulse technique is produced against the background of the stationary wave

$$\varepsilon_{1e}(t) = \frac{\varepsilon_0}{2} \int_{-\infty}^{\infty} \exp[-(\eta T_0)^2] \left(\frac{\Omega}{\Omega_0}\right)^2 (1-\cos\Omega_0\tau_2) \left[\sin\Omega_0\tau_1 \cos(\eta-\Delta_1) t_e + \frac{\eta-\Delta_0}{\Omega_0} (1-\cos\Omega_0\tau_1) \sin(\eta-\Delta_1) t_e \right] \frac{d\eta}{\Omega_0}, \tag{22}$$

$$t_e = t - \tau_1 - \tau_2 - 2\tau.$$

In the resonance case $\Delta_0 = 0$ Eq. (22) takes the form

$$\varepsilon_{1e}(t) = F(t_e) \cos \Delta_1 t_e, \tag{23}$$

$$F(t_e) = \varepsilon_0 \int_0^{\infty} \exp[-(\eta T_0)^2] \left(\frac{\Omega}{\Omega_0}\right)^2 (1-\cos\Omega_0\tau_2) \times \left[\sin\Omega_0\tau_1 \cos\eta t_e + \frac{\eta}{\Omega_0}(1-\cos\Omega_0\tau_1) \sin\eta t_e \right] \frac{d\eta}{\Omega_0},$$

where $F(t_e)$ is the amplitude of the electric field of the photon echo produced by two light pulses.[18] For a narrow spectral line

$$\Omega T_0 \gg 1, \; T_0 \gg \tau_1, \; T_0 \gg \tau_2.$$

the function $F(t_e)$ coincides with the results of Refs. 9 and 10.

The amplitude (23), just as the initial (22), is modulated at a frequency Δ_1. From the inequality $|\Delta_1| \gg |\Delta_0|$ it follows that $\Delta_1 = \delta E/\hbar$, so that from the frequency of the modulations (22) and (23) we can determine the Stark shift of the levels.

If we add the photon echo amplitude[9,10,18] from the two exciting resonant light pulses with the wave (1), then the combined intensity takes the form (16), in which

$$\varepsilon'(t) = F(t_e) \cos(\Delta_1 t + \varphi_1 - 2\varphi_2),$$

where φ_1 and φ_2 are the constant phase shifts of the first and second light pulses. Thus, the formation of the photon echo in the Stark-pulse technique proceeds in the same manner as in the problem of the passage of two light pulses, but in a narrow region (18) at parallel propagation of the Stark pulses (2).

3. MODULATION OSCILLATIONS OF RESONANT WAVE

We now investigate another experimental possibility, wherein most atoms inside the Doppler contour are at resonance with the wave (1) prior to application of the Stark pulses, and all atoms go off resonance after application of the pulse

$$|\Delta_0| \gg 1/T_0 > |\Delta_1|, \; |\Delta_0| \gg \Omega. \tag{24}$$

In the presence of a Stark field in the region $0 \leq t \leq \tau_1$ the quantities p_0 and $N_0 - n_0 f(v)$ in (9) and (10) can be neglected, but now for another reason than in the case (18). This approximation is equivalent to completely turning off the wave (1) for a time $0 \leq t \leq \tau_1$. The intensity (16) executes then free induction oscillations that coincide with the result of Ref. 8, when the longitudinal and transverse relaxations are inessential. No investigations of the modulation oscillations in the successive instants of time have been made so far, although many interesting phenomena occur in this region.

In the time interval $\tau_1 \leq t \leq \tau_1 + \tau$ following the turning off of the Stark field the atoms again interact resonantly with the wave (1), and the intensity of the summary wave undergoes nutational oscillations

$$\varepsilon_2'(t) = -\varepsilon_0 \int_{-\infty}^{\infty} \exp[-(\eta T_0)^2] \left[\frac{\eta-\Delta_1}{\Omega_1} (1-\cos(\eta-\Delta_0)\tau_1) \sin\Omega_1 (t-\tau_1) - \sin(\eta-\Delta_0)\tau_1 \cos\Omega_1 (t-\tau_1) \right] \frac{(\eta-\Delta_1) d\eta}{(\eta-\Delta_1)^2 + \Omega^2/2}, \tag{25}$$

$$\Omega_1^2 = (\eta-\Delta_1)^2 + \Omega^2.$$

We note that turning off the resonant wave (1) for a short time $\tau_1 \gg \Omega^{-1}$ is accompanied by an echo of the edge-echo type. It consists of an anomalous behavior of the damped oscillations of the optical nutation (25).

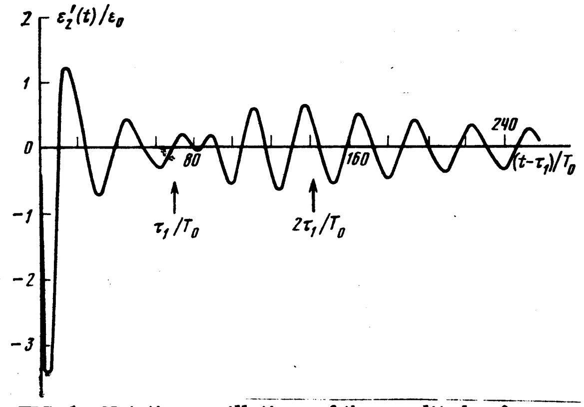

FIG. 1. Nutation oscillations of the amplitude of a resonant wave following the action of a Stark pulse of duration τ_1. The collapse of the oscillations takes place near $t=2\tau_2$, and the final damping of the amplitude $\varepsilon_2'(t)$ begins with the approximate instant of time $t=3\tau_1$. We have assumed $\tau_1=70.8\,T_0$, $T_0 a|d|/\hbar=0.1$, $\Delta_0 T_0=4$, $\Delta_1=0$, $\gamma\tau_1\ll 1$.

The anomaly consists in a collapse of the period and in a certain decrease, followed by an increase, of the amplitude near the instant of time $t=2\tau_1<\tau_1+\tau$ (see the figure). This new edge echo appears against the background of the nutational oscillations, in contrast to the known edge echo[17] which is formed on a base of free optical induction.

Interesting phenomena occur during the time $0\le t-\tau_1-\tau\le\tau_2$ of the repeated action of the Stark field. The produced free optical induction oscillations consist of two characteristic parts $\varepsilon_2'(t)=\varepsilon_{2i}(t)+\varepsilon_{2e}(t)$. The first $\varepsilon_{2i}(t)$ attenuates irreversibly with time. Yet the second $\varepsilon_{2e}(t)$ increases, first slowly and then rapidly, and reaches a maximum near the instant of time $t=2\tau_1+\tau$. This amplitude burst of the electric-field amplitude constitutes a unique induction photon echo

$$\varepsilon_{2e}(t)=\frac{\varepsilon_0}{2}\int_{-\infty}^{\infty}\exp[-(\eta T_0)^2]\frac{\Omega^2(\eta-\Delta_1)(1-\cos\Omega_1\tau)}{\Omega_1^2[(\eta-\Delta_1)^2+\Omega^2/2]}\sin(\eta-\Delta_0)(\tilde{t}-\tau_1)\,d\eta. \tag{26}$$

Just as in (22), the induction-echo amplitude (26) is modulated at a frequency $|\Delta_0|=\delta E/\hbar$; this confirms that these phenomena are produced by the same mechansim. In the limiting case $\Omega T_0\gg 1$ the expression (26) simplifies to

$$\varepsilon_{2e}(t)=\frac{2\pi^{1/2}\varepsilon_0}{(\Omega T_0)^2}\sin^2\frac{\Omega\tau}{2}\cos(\Delta_0(\tilde{t}-\tau_1))\frac{\tilde{t}-\tau_1}{T_0}\exp\left[-\left(\frac{\tilde{t}-\tau_1}{2T_0}\right)^2\right]. \tag{27}$$

When the second Stark pulse is turned on, the nutation oscillations reappear and consist, as in case (19) of two characteristic terms, $\varepsilon_2'(t)=\varepsilon_{2e}(t)+\varepsilon_{2nu}(t)$. The first attenuates in acord with the law that governs optical nutation, while the second increases gradually and is subject to anamalously large oscillations, near the instant of time $t=2\tau+\tau_1+\tau_2$, in the form of nutation photon echo

$$\varepsilon_{2nu}(t)=-\frac{\varepsilon_0}{2}\int_{-\infty}^{\infty}\exp[-(\eta T_0)^2]\left(\frac{\Omega}{\Omega_1}\right)^2 \times(1-\cos(\eta-\Delta_0)\tau_2)\Big[\sin(\eta-\Delta_0)\tau_1\cos\Omega_1 t_e + \frac{\eta-\Delta_1}{\Omega_1}(1-\cos(\eta-\Delta_0)\tau_1)\sin\Omega_1 t_e\Big]\frac{(\eta-\Delta_1)\,d\eta}{(\eta-\Delta_1)^2+\Omega^2/2}. \tag{28}$$

The nutation echo (28) is analogous in its character to the echo (21) considered above.

When the duration of the echo-observation experiment is comparable with $1/\gamma$ and the inequalities (17) are satisfied, the amplitudes (22) and (26) must be multiplied by an additional factor $e^{-\gamma t/2}$, that takes into account the irreversible relaxation. In the case of nutational photon echoes the amplitudes (21) and (28) are multiplied by the same factor if in addition the condition $\Omega T_0\ll 1$ is satisfied. This conclusion follows from the solution of Eqs. (12–14) with the parameter γ taken into account.

4. DISCUSSION OF RESULTS

At some times the Stark pulses are transmitted not in the parallel direction (2) but perpendicular to the motion of the stationary wave (1), i.e., across the axis of a thin tube of length L in which the investigated gas is located. The calculations for this case are similar, and in the final expressions $\varepsilon'(t)$ for the field amplitude (19), (21–23), and (25–28) we must make the substitutions

$$\sin\hat{\Omega}(t-t_0)\,d\eta\to\frac{\sin\hat{\Omega}L/2c}{\hat{\Omega}L/2c}\sin\hat{\Omega}\left(t-t_0+\frac{L}{2c}\right)d\eta,$$

$$\cos\hat{\Omega}(t-t_0)\,d\eta\to\frac{\sin\hat{\Omega}L/2c}{\hat{\Omega}L/2c}\cos\hat{\Omega}\left(t-t_0+\frac{L}{2c}\right)d\eta,$$

where, depending on the concrete formula for $\varepsilon'(t)$, we have

$$\hat{\Omega}=(\Omega_0,\ \Omega_1,\ \eta-\Delta_0,\ \eta-\Delta_1),$$
$$t_0=(\tau_1,\ \tau_1+\tau,\ 2\tau_1+\tau,\ \tau_1+\tau_2+\tau,\ \tau_1+\tau_2+2\tau).$$

It is seen that at small linear parameters of the volume in question, $\hat{\Omega}L/2c\ll 1$, the delay effects inside the volume can be neglected, and the final expression for the amplitude $\varepsilon'(t)$ does not depend on the propagation direction of the Stark pulses. On the contrary, for $\hat{\Omega}L/2c\gtrsim 1$ the presence of the additional factor $(2c/\hat{\Omega}L)\sin(\hat{\Omega}L/2c)$ in the integrand becomes quite essential, since it decreases the numerical value of (t) and accelerates its attenuation. In the case (23), the analogy with photon echo produced by two light pulses[9,10] no longer holds.

The conditions (18) were satisfied in an experiment[5] with CH_3F gas and with the following parameters: $\Delta_1=5.65\cdot 10^6\ \mathrm{sec}^{-1}$, $1/T_0=0.78\times 10^6\ \mathrm{sec}^{-1}$, $\Delta_0=0$, $\Omega=1.13\cdot 10^6\ \mathrm{sec}^{-1}$, $2\tau_1=\tau_2=3.28\cdot 10^{-6}$ sec, $=7.8\cdot 10^{-6}$ sec, and $\gamma=0.1\cdot 10^6\ \mathrm{sec}^{-1}$. Since 90- and 180-degree Stark pulses were used in Ref. 5, the nutational oscillations in the regions $0\le t\le\tau_1$ and $0\le t-\tau_1-t\le\tau_2$ were cut off at the quarter-period and half-period, respectively, and the induction oscillations and echo (23) were clearly pronounced. The experimental values of the period of the modulation oscillations, of the time to reach the maximum echo, and of the echo duration were respectively $1.07\cdot 10^{-6}$, $21.6\cdot 10^{-6}$ and $5.6\cdot 10^{-6}$ sec. The same quantities calculated from Eq. (23) are equal to $1.11\cdot 10^{-6}$, $21.8\cdot 10^{-6}$, and $6\cdot 10^{-6}$, respectively. The ratio of the maximal amplitudes of the phonon echo and of the optical nutation in the region $0\le t\le\tau_1$ obtained in experiment was equal to 0.05. The same ratio calculated theoretically with account taken of the irreversible pulsation in (23) agrees with the experimental value at $\gamma = 0.2\cdot 10^6$. The parameters of the experiment[5] did not correspond to the inequalities (20), so that no nutation echo (21) was produced.

In the other experiments[1-4,6,7] the conditions (18) and (24) were not satisfied, so that the formulas derived above are not valid. The situation realized in Refs. 1–4, 6, and 7 was such that part of the atoms inside the broad Doppler contour was at resonance with the stationary wave (1), while the remainder was not, and the Stark field (2) shifted the Doppler contour insignificantly

$$1/T_0 \gg |\Delta_0-\Delta_1|,\ 1/T_0 \gg |\Delta_1|,\ \Omega T_0 \ll 1. \qquad (29)$$

The analytic solution for this general case takes a rather cumbersome form that differs substantially from the result in the case of passage of the exciting light pulses.[9,10,18]

In the simpler situation

$$1/T_0 \gg |\Delta_0-\Delta_1| \gg \Omega,\ 1/T_0 \gg |\Delta_1|, \qquad (30)$$

The quantities (9) and (10) are sharp functions of the variable kv of approximate width Ω. This means that in the absence of a Stark field there exists a group of atoms with velocities $|kv-\Delta_1| \lesssim \Omega$, and at resonance with the wave (1). When the Stark field is turned on these atoms go off resonance, just as in the case (24) considered above. Another group of atoms with velocities $|kv-\Delta_0| \lesssim \Omega$, on the contrary, is not at resonance with the wave (1) without the Stark field. Yet in the presence of a Stark field these atoms turn out to be at resonance with the wave (1), just as in the case (18). Therefore the polarization, the excess population, and the field amplitude in the region (3) are sums of the two terms previously calculated for the cases (18) and (24). The echo phenomenon that appears after the second Stark pulse is a superposition of the two echo signals (22) and (28), in which the exponential factor $\exp[-(\eta T_0)^2]$ under the integral sign must be replaced by $\exp[-(\eta-\Delta_0)^2T_0^2]$ and $\exp[-(\eta-\Delta_1)^2T_0^2]$, respectively. The period of the oscillations (28) is large compared with the period in (22). Therefore the combined echo signal constitutes the smooth nutation echo contour (28), on which are superimposed the fast oscillations of the usual induction echo (22).

In the experiments on Refs. 1 and 3 the condition $|\Delta_0-\Delta_1| \approx \Omega$, obtained, so that the second inequality of (30) was violated. It is therefore incorrect to subdivide the molecules into two groups in (18) and (24), and the echo produced in this case is some modification of photon echo on the base of optical nutation. The exact echo-signal curve can be obtained only by numerical methods.

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