account is taken of the repulsion forces).[3] Since $\omega_0 \sim 10$ eV and $\omega_p \sim 10$ eV, which corresponds to $\lambda_0 \sim 10^3$ Å, we can neglect retardation at least for distances ~10–100 Å.

It must be remembered, however, that the spectrum of complex atoms has in fact, besides the main contribution of the outer electrons with low ionization energy, also deep lying levels with energies 100–1000 eV, corresponding to $\lambda_0 \sim 10-100$ Å. The total contribution of such levels to the potential $U(r)$ is estimated at several per cent to several dozen per cent. A manifestation of such a contribution is apparently the deviation from the d^{-3} law for the chemical potential of helium films on fluorite crystals, starting with film thicknesses ~100 Å.[9] We suggest therefore that for an accurate calculation of the interaction potential of molecules with spherical and cylindrical particles and with cavities of diameter 10–100 Å it is necessary in the general case to use our formulas.

The authors thank L. P. Pitaevskiĭ for interest in the work.

[1)]Equation (14) of Schmeits and Lucas is in error: a factor 2 was left out in front of the expression for $2l+1$ under the summation sign [see Eq. (47) of Ref. 4].

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Relaxation processes in ferromagnetic metals

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The effect of the s–d exchange interaction on magnon and conduction-electron relaxation in ferromagnetic metals is investigated. It is shown that magnon scattering by electrons is the dominant relaxation process for long-wave magnons with wave vectors $k < k_0$ (k_0 is the threshold wave-vector value in the single-magnon interaction process). The expression obtained for the magnon damping allows the elucidation of the temperature-independent contribution to the ferromagnetic-resonance (FMR) line width at relatively high temperatures and the increase of the line width with decreasing temperature. The conductivity of a ferromagnetic metal at low temperatures, when electron scattering by two magnons is the dominant electron-relaxation process, is considered. It is also shown that the nonequilibrium long-wave magnons that arise under FMR conditions make an additional contribution to the electron and magnon relaxation that can be experimentally detected at attainable resonance-field amplitudes.

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1. INTRODUCTION

In ferromagnetic metals, the spin waves corresponding to the spatial oscillations of the d-electron magnetization interact most strongly with the conduction electrons (the s electrons). This interaction leads to relaxation in both the electron and magnon systems. In experiment, the relaxation in the electron system is reflected in the temperature dependence of the conductivity and other kinetic characteristics of the metal. The relaxation of the long-wave magnons is investigated in experiments by the method of ferromagnetic resonance (FMR), while the relaxation of the short-wave magnons are investigated in experiments with the aid of neutron scattering. The relaxation of the electron subsystem in ferromagnetic metals has been fairly fully investigated both theoretically and experimentally,[1] something which cannot be said about the relaxation of the long-wave magnons.

Extensive experimental data have thus far been accumulated on the FMR line width in different metals.[2-8] It has been found that the line width is determined by two main factors: inhomogeneous line broadening, which arises as a result of the finite depth of penetration of the high-frequency field into the metal (the "inhomogeneous exchange" mechanism)[9,10] and the relaxation contribution.[2,6] Experimentally, these two contributions reliably split up, the relaxation contribution predominating at relatively high frequencies. But the attempts that have been made to theoretically interpret the relaxation contribution cannot be considered to be satisfactory.[11-13] The relaxation channels that have been considered either have an incorrect temperature dependence, or yield for the FMR line width a value that is much smaller than the experimentally observed value.

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The long-wave magnons with wave-vectors $k < k_0 = 2JS/v_F$ (J is the exchange integral, v_F is the electron velocity at the Fermi surface, and S is the spin of the magnetic atom) cannot attenuate in single-magnon processes, but it is precisely such magnons that are excited in FMR experiments. The problem for theory thus consists in the determination of the principal relaxation channel for the long-wave magnons in the higher orders of perturbation theory, since the first order is forbidden by the conservation laws.

In the present paper we show that the dominant relaxation processes for the long-wave magnons in ferromagnetic metals are the processes of magnon scattering by electrons (two-magnon processes) that occur without electron-spin flipping. The expression obtained on the basis of this process for the magnon-damping constant describes the temperature dependence of the FMR line width, and has the correct order of magnitude. We also investigate the resistance of ferromagnetic metals at low temperatures and the processes of electron and magnon relaxation under FMR conditions, when the nonequilibrium long-wave magnon occupation numbers are high.

2. THE HAMILTONIAN AND THE EFFECTIVE VERTICES

Let us write the Hamiltonian of the electron-phonon system in the representation of second quantization:

$$\mathscr{H}_{e-m}=\sum_{\mathbf{k}}\omega_{\mathbf{k}}a_{\mathbf{k}}^{+}a_{\mathbf{k}}+\sum_{\mathbf{p}\sigma}\varepsilon_{\mathbf{p}\sigma}c_{\mathbf{p}\sigma}^{+}c_{\mathbf{p}\sigma}-J\left(\frac{2S}{N}\right)^{1/2}\sum_{\mathbf{kp}}(a_{\mathbf{k}}c_{\mathbf{p}\uparrow}\overset{+}{c}_{\mathbf{k}+\mathbf{p}\downarrow}+\text{H. c.})$$
$$+\frac{J}{N}\sum_{\substack{\mathbf{kk}'\\ \mathbf{pp}'}}a_{\mathbf{k}'}^{+}a_{\mathbf{k}}(\overset{+}{c}_{\mathbf{p}'\uparrow}c_{\mathbf{p}\uparrow}-\overset{+}{c}_{\mathbf{p}'\downarrow}c_{\mathbf{p}\downarrow})\Delta(\mathbf{k}'+\mathbf{p}'-\mathbf{k}-\mathbf{p}), \qquad (1)$$

where $a_{\mathbf{k}}^{+}$ is the magnon-creation operator; the $c_{\mathbf{p}\sigma}(c_{\mathbf{p}\uparrow}^{+}, c_{\mathbf{p}\downarrow}^{+})$ are the creation operators for the conduction electrons of the two subbands; $\omega_{\mathbf{k}} = \mu H + Dk^2$ is the spin-wave spectrum, $\mu = g\mu_0$, μ_0 is the Bohr magneton, g the spectroscopic factor, H the external magnetic field; $D = J^2/24\pi^2\varepsilon_F p_F^2$ is the coefficient of spin-wave dispersion,[1] ε_F is the Fermi energy, p_F the Fermi momentum;

$$\varepsilon_{\mathbf{p}\uparrow}=p^2/2m-SJ, \qquad \varepsilon_{\mathbf{p}\downarrow}=p^2/2m+SJ$$

are the spectrum of the electrons of the two subbands with allowance for the splitting in the spin direction, m is the effective electron mass; N is the number of magnetic atoms.

To the Hamiltonian $\mathscr{H}_{e-m}$, (1), correspond vertices describing the interaction of an electron with one magnon, $\Gamma^{(1)}$, and with two magnons, $\Gamma^{(2)}$:

$$\Gamma_{\uparrow\downarrow}^{(1)}=\Gamma_{\downarrow\uparrow}^{(1)}=\Gamma^{(1)}=(2S)^{1/2}J, \qquad \Gamma_{\uparrow\uparrow}^{(2)}=-\Gamma_{\downarrow\downarrow}^{(2)}=J.$$

Let us investigate the problem of the renormalization of the magnon-electron vertices. Because the electron spin is flipped in the single-magnon process, one magnon Green function and two $\Gamma^{(1)}$ vertices do not give rise to corrections to the single-magnon vertex. The nonvanishing lowest-order renormalizations are shown in Fig. 1. Since the second diagram contains two magnon Green functions, it depends significantly on the magnon occupation numbers, and, consequently, on the

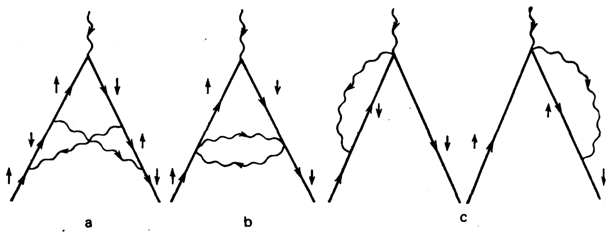

FIG. 1. Lowest-order corrections to the single-magnon vertex.

temperature:

$$\frac{\delta\Gamma_a^{(1)}}{\Gamma^{(1)}}\sim\left(\frac{J}{\varepsilon_F}\right)^4\frac{k}{p_F}, \quad k>k_0, \qquad \frac{\delta\Gamma_a^{(1)}}{\Gamma^{(1)}}\sim\left(\frac{J}{\varepsilon_F}\right)^3\left(\frac{k}{p_F}\right)^2, \quad k<k_0;$$

$$\frac{\delta\Gamma_b^{(1)}}{\Gamma^{(1)}}\sim\left(\frac{J}{\varepsilon_F}\right)^2\frac{k}{p_F}\frac{T}{\Theta_c}, \quad k>k_0, \qquad \frac{\delta\Gamma_b^{(1)}}{\Gamma^{(1)}}\sim\frac{J}{\varepsilon_F}\frac{T}{\Theta_c}\left(\frac{k}{p_F}\right)^2, \quad k<k_0.$$

Only an expression for $\delta\Gamma_a^{(1)}$ for $k \to p_F$ has been obtained in the analysis of the renormalizations.[14] The third and fourth diagrams make a temperature-independent contribution when $\omega_k \ll \varepsilon \ll \Theta_c$:

$$\delta\Gamma_c^{(1)}/\Gamma^{(1)}=(2S)^{-1/2}\left[\Sigma_{\downarrow}(\mathbf{p}+\mathbf{k}, \varepsilon+\omega_{\mathbf{k}})-\Sigma_{\uparrow}(\mathbf{p}, \varepsilon)\right]=(2S)^{-1/2}\omega_{\mathbf{k}}/J,$$

where Σ_σ are the self-energy parts, computed in, for example, Ref. 14, of the electrons of the two subbands.

Thus, the corrections to the single-magnon vertex are small in the entire temperature range and for all values of the magnon wave vector.

Let us now consider the four-particle interaction processes involving magnons, phonons, and electrons.

In view of the fact that the single- and two-magnon bare vertices $\Gamma^{(1)}$ and $\Gamma^{(2)}$ contain the same power J, there arise the following effective vertices, $\Phi_{2m-2e}^{\downarrow\downarrow}$ and $\Phi_{2m-2e}^{\uparrow\uparrow}$ (see Fig. 2), for magnon scattering by electrons:

$$\Phi_{2m-2e}^{\downarrow\downarrow}=\Gamma_{\downarrow\downarrow}^{(2)}+\frac{|\Gamma^{(1)}|^2}{\varepsilon_{\mathbf{p}\downarrow}-\omega_{\mathbf{k}}-\varepsilon_{\mathbf{p}-\mathbf{k}\uparrow}}, \qquad (2)$$

$$\Phi_{2m-2e}^{\uparrow\uparrow}=\Gamma_{\uparrow\uparrow}^{(2)}+\frac{|\Gamma^{(1)}|^2}{\varepsilon_{\mathbf{p}\uparrow}+\omega_{\mathbf{k}}-\varepsilon_{\mathbf{p}+\mathbf{k}\downarrow}}. \qquad (3)$$

The expression (3) is derived in Refs. 15 and 16 for ferromagnetic semiconductors, in which only the band with spins directed along the magnetization is filled by electrons. For relatively small magnon wave vectors satisfying the condition $k \ll k_0$ we find from (2) and (3) that

$$\Phi_{2m-2e}^{\downarrow\downarrow}=-\Phi_{2m-2e}^{\uparrow\uparrow}=-\mathbf{pk}/2Sm. \qquad (4)$$

An analysis shows that the corrections to the two-magnon vertices, one of which is shown in Fig. 3, can be neglected for the same reasons that the corrections to the single-magnon vertices can be neglected.

At relatively high temperatures, besides the two-

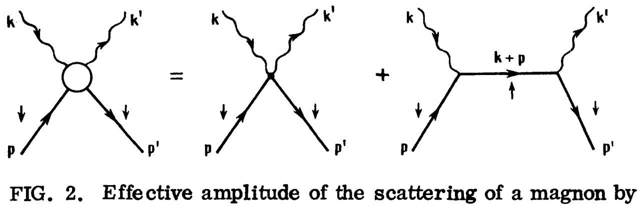

FIG. 2. Effective amplitude of the scattering of a magnon by an electron.

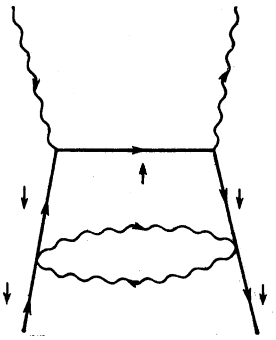

FIG. 3.

magnon process, the magnon-electron and phonon-electron interaction processes (see Fig. 4) can contribute to the long-wave magnon attenuation. The amplitude of this process has the form

$$\Phi^{\uparrow\downarrow}_{m-ph-2e}=\Gamma^{(1)}V_{e-ph}\left[\frac{1}{\omega_{\mathbf{k}}+\varepsilon_{\mathbf{p}\uparrow}-\varepsilon_{\mathbf{p}+\mathbf{k}\downarrow}}+\frac{1}{\varepsilon_{\mathbf{p}\uparrow}+\Omega_{\mathbf{q}}-\varepsilon_{\mathbf{p}+\mathbf{q}\uparrow}}\right], \tag{5}$$

where $V_{e\text{-}ph}=\tilde{\varepsilon}(\Omega_{\mathbf{q}}/2Mu^2)^{1/2}$ is the electron-phonon interaction amplitude, $\tilde{\varepsilon}\sim\varepsilon_F$ is the deformation potential, $\Omega_{\mathbf{q}}=uq$ is the phonon energy, u is the speed of sound, and M is the ion mass. With allowance for the conservation laws, $\omega_{\mathbf{k}}+\Omega_{\mathbf{q}}+\varepsilon_{\mathbf{p}\uparrow}=\varepsilon_{\mathbf{p}+\mathbf{k}+\mathbf{q}\downarrow}$, we have for the process in question when $k\ll k_0$

$$\Phi^{\uparrow\downarrow}_{m-ph-2e}=-\frac{1}{4S^{3/2}}\frac{\tilde{\varepsilon}}{J}\left(\frac{\Omega_{\mathbf{q}}}{Mu^2}\right)^{1/2}\frac{\mathbf{kq}}{m}. \tag{6}$$

Besides the phonon-absorption process, there also occurs the phonon-emission process, whose amplitude has a similar form.

In principle, the long-wave magnon attenuation can be governed by the third- and fourth-order intrinsic anharmonicities of the magnon subsystem. The contribution of such processes are, however, small.[17]

Generally speaking, because of the relatively large magnitude of the saturation magnetization of ferromagnetic metals (thus, $4\pi M_0=6000$ G and 17000 G respectively in Ni and Co), it is necessary to take the dipole-dipole interaction into consideration in the study of the kinetic phenomena governed by the long-wave magnons. Its allowance is accomplished by means of the canonical transformation $\tilde{a}_{\mathbf{k}}=ua_{\mathbf{k}}+va^{+}_{-\mathbf{k}}$ to new spin-wave amplitudes,[17] which modifies the amplitudes, Φ_{2m-2e} and $\Phi_{m-ph-2e}$, of the processes, gives rise to new processes, and also renormalizes the spin-wave spectrum, which assumes the form

$$\tilde{\omega}_{\mathbf{k}}^2=(\mu H+Dk^2)(\mu H+Dk^2+{}^1/_2\omega_M\sin^2\theta_{\mathbf{k}}),\qquad \omega_M=4\pi\mu M_0, \tag{7}$$

where $\theta_{\mathbf{k}}$ is the angle between the wave vector $\mathbf{k}$ and the direction of the magnetization.

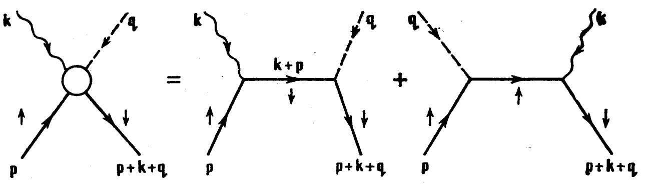

FIG. 4. Effective amplitude of the magnon-phonon-electron interaction.

In order not to encumber the exposition, we shall compute the long-wave magnon attenuation without allowance for the dipole-dipole interaction ($u=1, v=0$) and allow for it only in the final expressions. Because of the condition $Dk_0^2\gg\omega_M$, the dipole-dipole interaction does not play a role in the computation of short-wave magnon attenuation. As to electron relaxation, the dipole-dipole interaction should be taken into consideration at low temperatures, where the electrons relax on the long-wave magnons.

3. RELAXATION OF THE MAGNONS

The single-magnon processes do not contribute to the attenuation of the long-wave magnons; therefore, let us consider the processes introduced in the preceding section. Let us first consider the process of magnon scattering by electrons. From the kinetic equation for the two-magnon processes,

$$\dot{n}_{\mathbf{k}}=2\pi v_0^2\sum_\sigma\iint\frac{d\mathbf{p}\,d\mathbf{k}_1}{(2\pi)^6}|\Phi^{\sigma\sigma}_{2m-2e}|^2[n(\omega_{\mathbf{k}})(n(\omega_{\mathbf{k}_1})+1)f(\varepsilon_{\mathbf{p}})(1-f(\varepsilon_{\mathbf{p}'}))$$
$$-(n(\omega_{\mathbf{k}})+1)n(\omega_{\mathbf{k}_1})(1-f(\varepsilon_{\mathbf{p}}))f(\varepsilon_{\mathbf{p}'})]\delta(\omega_{\mathbf{k}}+\varepsilon_{\mathbf{p}}-\omega_{\mathbf{k}_1}-\varepsilon_{\mathbf{p}'}),\qquad \mathbf{p}'=\mathbf{p}+\mathbf{k}-\mathbf{k}_1 \tag{8}$$

where v_0 is the volume of the unit cell, we obtain for the magnon-damping constant the expression

$$\gamma_{2m-2e}=2\pi v_0^2\sum_\sigma\iint\frac{d\mathbf{p}\,d\mathbf{k}_1}{(2\pi)^6}|\Phi^{\sigma\sigma}_{2m-2e}|^2(f(\varepsilon_{\mathbf{p}})-f(\varepsilon_{\mathbf{p}+\mathbf{k}-\mathbf{k}_1}))$$
$$\times(1+n(\omega_{\mathbf{k}_1})+n(\omega_{\mathbf{k}}-\omega_{\mathbf{k}_1}))\delta(\omega_{\mathbf{k}}+\varepsilon_{\mathbf{p}}-\omega_{\mathbf{k}_1}-\varepsilon_{\mathbf{p}+\mathbf{k}-\mathbf{k}_1}), \tag{9}$$

where $f(\varepsilon_{\mathbf{p}})$ and $n(\omega_{\mathbf{k}_1})$ are the equilibrium distribution functions for the electrons and magnons. Substituting the formulas (4) into (9), we obtain the following expression for the decrement of the long-wave magnons in the two-magnon processes:

$$\gamma_{2m-2e}(\mathbf{k})=\frac{3\pi}{2^6S^2}\omega_k\left(\frac{k}{p_F}\right)^2\frac{T}{\Theta_c}\ln\frac{T}{\omega_k},\qquad \Theta_c=Dp_F^2. \tag{10}$$

Let us consider the magnon-phonon-electron processes. We obtain the expression for the damping constant $\gamma_{m\text{-}ph\text{-}2e}$ from the corresponding kinetic equation:

$$\gamma_{m-ph-2e}=2\pi v_0^2\iint\frac{d\mathbf{p}\,d\mathbf{q}}{(2\pi)^6}|\Phi^{\uparrow\downarrow}_{m-ph-2e}|^2\{(f(\varepsilon_{\mathbf{p}\uparrow})-f(\varepsilon_{\mathbf{p}\downarrow}+\omega_{\mathbf{k}}+\Omega_{\mathbf{q}}))$$
$$\times(N(\Omega_{\mathbf{q}})-N(\Omega_{\mathbf{q}}+\omega_{\mathbf{k}}))\delta(\omega_{\mathbf{k}}+\Omega_{\mathbf{q}}+\varepsilon_{\mathbf{p}\uparrow}-\varepsilon_{\mathbf{p}+\mathbf{k}+\mathbf{q}\downarrow})$$
$$+(f(\varepsilon_{\mathbf{p}\uparrow})-f(\varepsilon_{\mathbf{p}\downarrow}+\omega_{\mathbf{k}}-\Omega_{\mathbf{q}}))(N(\Omega_{\mathbf{q}})-N(\Omega_{\mathbf{q}}-\omega_{\mathbf{k}}))\delta(\omega_{\mathbf{k}}+\varepsilon_{\mathbf{p}\uparrow}-\Omega_{\mathbf{q}}-\varepsilon_{\mathbf{p}+\mathbf{k}-\mathbf{q}\downarrow})\}, \tag{11}$$

where $N(\Omega_{\mathbf{q}})$ is the equilibrium distribution function for the phonons.

Let us emphasize that the process in which the thermal phonon participates proceeds with electron-spin flipping, and, consequently, the energy threshold $2JS$ can be resolved only at sufficiently high temperatures $T>2JSu/v_F$. At lower temperatures, the magnons interact with the epithermal phonons, whose occupation numbers are exponentially small, owing to which the magnon-damping constant for such processes is also exponentially small.

As a result of calculations in different temperature regions where $\omega_{\mathbf{k}}\ll T$, we obtain

$$\gamma_{m-ph-2e}=\frac{45\pi}{2^3S^3}\omega_{\mathbf{k}}\left(\frac{k}{p_F}\right)^2\frac{T^5}{Mu^2\omega_D^4}\left(\frac{\tilde{\varepsilon}}{J}\right)^2,\qquad T\ll\omega_D,$$
$$\gamma_{m-ph-2e}=\frac{3\pi}{2^6S^3}\omega_{\mathbf{k}}\left(\frac{k}{p_F}\right)^2\frac{T}{Mu^2}\left(\frac{\tilde{\varepsilon}}{J}\right)^2,\qquad T>\omega_D. \tag{12}$$

In these formulas $\omega_D = up_F$. Let us note that similar formulas without numerical factors are derived by another method in Ref. 13.

Let us compare the expressions (10) and (12). Assuming that $Mu^2 \sim \tilde{\varepsilon} \sim \varepsilon_F$, we obtain

$$\frac{\gamma_{m-ph-2e}}{\gamma_{2m-2e}} = \begin{cases} \dfrac{20}{\pi^2 S}\left(\dfrac{T}{\omega_D}\right)^4 \ln^{-1}\left(\dfrac{T}{\omega_k}\right), & T \ll \omega_D \\ \left(24\pi^2 S \ln\dfrac{T}{\omega_k}\right)^{-1}, & T > \omega_D. \end{cases} \tag{13}$$

Consequently, the contribution of the two-magnon processes to the attenuation of the magnons exceeds the contribution of the processes in which phonons participate in the entire temperature range. Thus, the principal relaxation channel for the long-wave magnons with $k < k_0$ in metals is magnon scattering by electrons. Allowance for the dipole-dipole interaction leads to the replacement of the magnon energy by ω_M in the expression (10).

Let us note an important property that is peculiar to the relaxation channel in question: the marked dependence of the damping constant on the magnon wave vector. This fact will be used by us in the description of the FMR line width.

Taking account of the fact that the attenuation of the short-wave magnons with $k > k_0$ is governed by the single-magnon processes,[1] i.e., that

$$\gamma_{m-2e}(k) = \frac{8\pi}{3}\omega_k\left(\frac{J}{\varepsilon_F}\right)^2 \frac{p_F}{k},$$

we can schematically depict the total wave-vector dependence of the magnon-damping constant as shown in Fig. 5.

4. ELECTRON RELAXATION (CONDUCTIVITY)

It is well known that the conductivity of ferromagnetic metals is largely determined by the electron-magnon interaction.[1] Our aim is to take into consideration the effect of the dipole-dipole interaction and the two-magnon processes on the conductivity of ferromagnetic metals.

Since the characteristic values of the wave vector of the thermal magnon are determined from the condition $\omega_k \sim T$, we can neglect the effect of the dipole-dipole interaction when $T > \omega_M$, and restrict ourselves to the consideration of the single-magnon processes when $T > T_0$ $(T_0 \approx \omega(k_0))$. In this case for the electron-relaxation transport time τ_p, which determines the temperature dependence of the conductivity,[1] and the characteristic time of the relaxation in terms of energy, τ_ε, we obtain

$$\frac{1}{\tau_p^{(1)}} = \frac{3\pi^5}{2} S \frac{T^2}{\Theta_c}, \tag{14}$$

$$\frac{1}{\tau_\varepsilon^{(1)}} = 36\pi^3 ST \ln\frac{T}{\omega(k_0)}. \tag{15}$$

If the condition $T_0 < \omega_M$ is fulfilled in the ferromagnetic metal, then the dipole-dipole interaction should be taken into account in the single-magnon processes. It is easy to see that, for $\mu H \ll \omega_M$, the dominant contribution to the conductivity is made by magnons for which $Dk^2 \sim T^2/\omega_M$, as a result of which the relaxation transport time turns out to be proportional to the third power of the temperature:

$$\frac{1}{\tau_\varepsilon^{(1)}} = 2^5\pi^4 S\frac{T^3}{\omega_M \Theta_c}. \tag{16}$$

When the temperature is lowered further, i.e., when $T < \Delta \ll \omega_M$, where $\Delta = (\mu H \omega_M)^{1/2}$, the gap in the magnon spectrum counts, and

$$\frac{1}{\tau_\varepsilon^{(1)}} \propto T^2 \exp\left(-\frac{\Delta}{T}\right). \tag{17}$$

For real ferromagnetic metals, there exists a temperature range, $\omega_M < T < T_0$, in which the single-magnon processes are ineffective, owing to the exponential smallness of the occupation numbers for the magnons with $k > k_0$. In this case it is necessary to take the two-magnon processes into consideration, and then the electron-relaxation transport time will be given by the expression

$$\frac{1}{\tau_{p\sigma}^{(2)}} = 2\pi v_0^2 \iint \frac{d\mathbf{k}_1\, d\mathbf{k}_2}{(2\pi)^6} |\Phi_{2m-2e}^{\sigma\sigma}|^2 (1-\cos\theta)[n(\omega_{\mathbf{k}_1})(1+n(\omega_{\mathbf{k}_2})) + f(\omega_{\mathbf{k}_1}-\omega_{\mathbf{k}_2})(n(\omega_{\mathbf{k}_2})-n(\omega_{\mathbf{k}_1}))]\delta(\varepsilon_{\mathbf{p}\sigma}+\omega_{\mathbf{k}_1}-\omega_{\mathbf{k}_2}-\varepsilon_{\mathbf{p}+\mathbf{k}_1-\mathbf{k}_2\sigma}), \tag{18}$$

where θ is the angle between the initial, and final, direction of the electron momentum. Using the explicit form of the amplitude $\Phi_{2m-2e}^{\sigma\sigma}$, (4), we obtain

$$\frac{1}{\tau_{p\uparrow}^{(2)}} \approx \frac{1}{\tau_{p\downarrow}^{(2)}} \approx A\frac{\varepsilon_F}{S}\left(\frac{T}{\Theta_c}\right)^{9/2}, \tag{19}$$

where $A \sim 10^2$ is a numerical factor.

Notice that the contribution of the two-magnon processes exceeds the contribution, $1/\tau_{e\text{-}ph} \sim T^5/\omega_D^4$, from the electron-phonon processes by a factor equal to the parameter $(T\Theta_c/\varepsilon_F)^{1/2}(\Theta_c/\omega_D)^4$. Schwerer and Silcox[18] have found that in pure Ni single crystals the resistance $\rho \propto T^4$, which is close to the $T^{9/2}$ dependence dictated by the two-magnon processes. On lowering the temperature further, so that $T < \omega_M < T_0$, because of the gapped character of the magnon spectrum, the factor $e^{-\Delta/T}$ appears in the expression for $1/\tau_p^{(2)}$, whose contribution then decreases.

The relaxation time $\tau_p^{(2)}$ in the two-magnon processes is calculated in Refs. 19 and 20, but because of the fact that the expression for the amplitude $\Gamma_{\sigma\sigma}^{(2)}$ is used

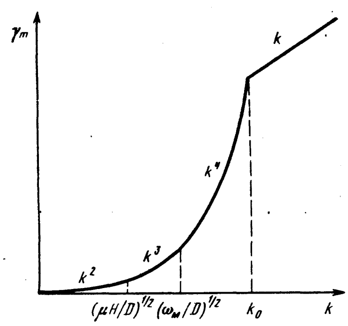

FIG. 5. Dependence, stemming from the electron-magnon interaction, of the magnon damping on the magnitude of the wave vector.

instead of the expression for the total amplitude $\Phi^{\sigma\sigma}_{2m-2e}$, the result obtained ($\rho \propto T^{7/2}$) turns out to be incorrect. Furthermore, the effect of the dipole-dipole interaction on the resistance is not taken into account in these papers.

The two-magnon process contributes to the conductivity even when $T > T_0$. The interaction amplitude has in this case a resonance singularity, for the removal of which we should take account of the electronic damping, γ_e, in the intermediate state. In a sufficiently pure metal, γ_e contains contributions from the electron-phonon, γ_{e-ph}, and electron-magnon, $\gamma_{e-m} = 1/\tau_{\varepsilon}^{(1)}$, interactions. A calculation similar to the one carried out in Ref. 21 shows that the additional contribution to the conductivity from the two-magnon process when $T > T_0$ has the same temperature dependence as the single-magnon contribution, but is an order-of-magnitude smaller in value. Allowance for the two-magnon processes in the relaxation of the short-wave magnons with $k > k_0$ leads to similar results.

5. THE FMR LINE WIDTH

Recently, in FMR experiments on ferromagnetic metals, a contribution in the line width was separated out which was of a relaxational nature[2,6]: $\Delta\omega \propto \lambda$ (λ is the relaxation parameter entering into the Landau-Lifshitz equation). It is important to emphasize that this contribution is temperature independent in a broad range of temperatures. It is also observed that $\Delta\omega$ increases with decreasing temperature, for example, in Ni when $T < 150$ K.[3,6]

The expression, (10), obtained in the third section of the present paper for the magnon-damping constant enables us to explain such a behavior of the FMR line width. As has already been noted, γ_{2m-2e} is proportional to k^2, and the characteristic value of the wave vector of the magnons excited under FMR conditions is given by the relation[11]

$$k^2 \sim \delta^{-2} = \frac{4\pi\sigma\omega\mu_2(\omega)}{c^2}, \tag{20}$$

where σ is the conductivity, ω is the FMR frequency, c is the velocity of light, and μ_2 is the imaginary part of the magnetic susceptibility. Under FMR conditions, when $H \ll 4\pi M$, we have $\mu_2 = \omega_M^2/2\omega\Delta\omega$, where $\Delta\omega$ is the total line width. Then from (10) and (20) we obtain the following equation:

$$\Delta\omega \approx \gamma_{2m-2e}(k\sim\delta^{-1}) \approx \frac{3\pi^2}{2^5 S^2}\frac{\sigma\omega_M^3}{\Delta\omega c^2 p_F^2}\frac{T}{\Theta_c}\ln\frac{T}{\omega_M}. \tag{21}$$

The conductivity $\sigma = ne^2\tau_e/m$ is determined by both the electron-magnon and the electron-phonon interaction ($\tau_e^{-1} = \tau_{em}^{-1} + \tau_{eph}^{-1}$); in pure metals, the impurity scattering is important only at low temperatures. As shown in Sec. 4, τ_{em} for $T > T_0$ is given by the expression (14), and since the electron-phonon interaction makes a smaller contribution, the conductivity $\sigma \propto T^2$ for $T < \omega_D$. For $T > \omega_D$, we find $\tau_{eph}^{-1} \sim T$ and is by and large of the same order of magnitude as τ_{em}^{-1}, but, as analysis of the experimental data given in Ref. 1 shows, the resistance of many ferromagnetic metals at sufficiently high temperatures (for Ni, for example, at $T > 300$ K) is well approximated by a linear function.

In accordance with the foregoing, $\Delta\omega$ is virtually temperature independent when $T > \omega_D$, and increases with decreasing temperature according to the law $\Delta\omega \propto T^{-1/2}$ when $T < \omega_D$:

$$\Delta\omega = \frac{3^{1/2}\pi}{2^{5/2}S}\omega_M\left(\tau_e T\ln\frac{T}{\omega_M}\right)^{1/2}\left(\frac{\omega_M}{\Theta_c}\right)^{1/2}\left(\frac{\varepsilon_F}{mc^2}\right)^{1/2}. \tag{22}$$

Substituting the typical values $\omega_M \sim 1$ K, $\Theta_c \sim 500$ K into this expression, we find that, for $T > \omega_D$, $\Delta\omega \sim 10^8$ sec^{-1}, which is in accord with the experimentally observed values.[2,6]

Thus, we can assert that the expression (22), which is determined by the two-magnon processes, coincides in order of magnitude with the relaxation parameter, λ, that figures in the Landau-Lifshitz equation. For a full comparison with experiment to have been possible, we ought to have substituted into the formula (22) the conductivity value measured on the same sample on which the FMR line width is being studied.

6. THE RELAXATION PROCESSES UNDER FMR CONDITIONS

The action of a high-frequency field on a ferromagnetic metal leads to the appearance in the sample of nonequilibrium long-wave magnons, whose total number, N, in the volume v_0 can be estimated from the following relation:

$$\omega\chi'' h^2 v_0 = \omega_k \tilde{N}/\tau_k, \tag{23}$$

where χ'' is the susceptibility at the frequency ω, h is the amplitude of the variable field, $\omega_k \sim \omega$ is the mean energy of the excited magnons, and τ_k is the lifetime of the excited magnons, which is given by the expression (10).

Using (23), and substituting into it the following characteristic values: $\chi'' = 10^{-2}$; $h = 10$ Oe; $\tau_k = 10^{-8}$ sec; $T = 300$ K, we obtain $\tilde{N} \sim 10^{-4}$, which corresponds to a nonequilibrium-magnon concentration of 10^{19} cm^{-3}. The presence of such a high concentration of nonequilibrium magnons leads to an additional contribution to the relaxation of the electrons and magnons.

Since the nonequilibrium magnons are long-wave magnons, the conservation laws allow only the process of interaction of an electron with two nonequilibrium magnons, as well as with one nonequilibrium, and one thermal, magnon. However, on account of the fact that the transport time for the electron relaxation contains the factor $(q/p_F)^2$, where q is the momentum transfer, it is clear that the second process is the dominant process.

Writing down the kinetic equation for an electron scattered by two magnons, one thermal and one nonequilibrium, we find

$$\Delta\tau_e^{-1} \approx 3\pi^5\frac{T^2}{\Theta_c}\tilde{N}. \tag{24}$$

Comparing the obtained expression with the formula (19), which gives the resistance of a ferromagnetic metal at low temperatures, we obtain

$$\Delta\rho/\rho\sim\bar{N}\frac{\Theta_c}{\varepsilon_F}\left(\frac{\Theta_c}{T}\right)^{1/2}. \quad (25)$$

At a temperature of the order of several degrees, $\Delta\rho/\rho\sim 1$, and, consequently, the experimental observation of $\Delta\rho$ is entirely possible. Notice also the dependence of $\Delta\rho$ on the amplitude of the variable field. Experimentally, it is desirable to study the contribution of the nonequilibrium magnons to the resistance on films of thickness smaller than the penetration depth, so as to obviate the skin effect.

The magnon damping in a two-magnon process in which a nonequilibrium magnon participates is computed in similar fashion:

$$\Delta\gamma_m\sim\omega_k\frac{k}{p_F}\bar{N}, \quad (26)$$

with

$$\frac{\Delta\gamma_m}{\gamma_m}\sim\bar{N}\frac{p_F}{k}\frac{\Theta_c}{T}. \quad (27)$$

Notice that the contribution of the nonequilibrium magnons to the total relaxation increases with decreasing temperature.

The experimental study of the electron- and magnon-relaxation processes under conditions of ferromagnetic resonance and a high rate of pumping by the high-frequency field seems to us to be of great interest.

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