The cyclotron frequency shift for electrons localized at the surface of liquid helium

V. B. Shikin

Institute of Solid State Physics, Academy of Sciences USSR
(Submitted 21 March 1979)
Zh. Eksp. Teor. Fiz. 77, 717–732 (August, 1979)

It is shown that for electrons localized at the surface of liquid helium the cyclotron resonance shift $\Delta\omega$ is of twofold origin: a dynamic shift $\Delta\omega^d$ due to interaction between electrons and thermal oscillations of the helium surface, and a static shift $\Delta\omega^s$ due to the self-consistent deformations of the helium surface under the localized electron that exerts pressure on the free surface of the helium, and also due to the reaction of the deformation on the electron motion. The relative role of $\Delta\omega^d$ and $\Delta\omega^s$ in the limiting case of a low surface electron density n_s is studied. It was found that in the limiting quantum case $T \ll \hbar\omega_H$, where ω_H is the cyclotron frequency, the static shift $\Delta\omega^s$ plays the dominant role. However, in the intermediate temperature range $T \lesssim \hbar\omega_H$, the dynamic shift $\Delta\omega^d$ begins to compete with $\Delta\omega^s$ and even turns out to be favored. The problem of the effect exerted on $\Delta\omega$ by a finite density n_s of surface electrons is discussed. The concept is introduced of the characteristic density n_s^*, beginning with which the electron-electron interaction turns out to have a significant effect on the mean square displacement of the electrons from the equilibrium position and, consequently, on the value of the static shift $\Delta\omega^s$. The theoretical results describe correctly the existing experimental data for $\Delta\omega$ in the case of electrons localized on a He^4 surface over a wide range of applied fields $E_\perp$ and electron densities. However, there are also some discrepancies which require additional investigation, both experimentally and theoretically.

PACS numbers: 67.40.Fd

INTRODUCTION

The phenomenon of cyclotron resonance for free electrons localized on the surface of liquid helium has been studied in detail in the experiments of Édel'man.[1,2] One of the interesting effects there proved to be the decrease in the cyclotron mass of the surface electrons as a function of the intensity of the external clamping electric field. A discussion of the possible reasons for such an effect is carried out in the present work.

The effect of the applied field on the dispersion law of a surface electron in helium is not unexpected. It was noted in due course by the author and Monarkha[3] that, in the presence of interaction of the electron with surface vibrations of the form

$$H_{int}=eE_\perp\xi(\mathbf{r}), \tag{1}$$

(ξ is the amplitude of the surface vibrations, $\mathbf{r}$ is the coordinate of the electron on the surface) the increment $\delta\varepsilon(\mathbf{k})$ to the dispersion law of the free surface electron, calculated in second order perturbation theory at $T=0$, has the form

$$\delta\varepsilon(\mathbf{k})=\gamma\frac{e^2E_\perp{}^2m}{2\pi\hbar(2\alpha\rho k)^{1/2}}, \quad \gamma\approx 1, \tag{2}$$

ρ and α are the density and coefficient of surface tension of liquid helium, $\mathbf{k}$ is the wave number of the electron, and $k=|\mathbf{k}|$. However, this change in the spectrum was discussed in Ref. 3 only in connection with its diverging character as $k\to 0$ for illustration of the reasons leading to localization of the surface electrons and to the generation of surface anions. As will be shown below, the renormalization of the electron spectrum due to its interaction with thermal ripplons can have a bearing on the change in the cyclotron mass of the surface electrons.

Still another, qualitatively different, possibility of change of the cyclotron mass of the electron can occur if we are dealing with surface electrons localized in the vapor–liquid plane. Such a localization always arises upon increase of the applied field and decrease of the temperature. The surface of helium at the place of localization of the electron is deformed here in self-consistent fashion. In turn, the resultant deformation of the surface serves as a potential well for the localized electron. The process of localization of the surface electrons is materially facilitated by the initiation of a strong magnetic field perpendicular to helium surface, because the wave function of the electron here is already localized on the helium surface in the limits of the Larmour radius. Consequently, the effect of the field $E_\perp$ on a charged spot of limited dimensions on the helium surface will automatically lead to the deformation of the surface. A self-consistent calculation of the parameters of the emerging complex of localized electron and local deformation of the helium, carried out in the limits of low temperatures in Ref. 4, gives the following finite result:

$$\begin{aligned}&\langle u^2\rangle={}^1/_2\langle u^2\rangle_\xi^{-1}+({}^1/_4\langle u\rangle_\xi^{-2}+\langle u^2\rangle_H^{-2})^{1/2},\\ &\langle u^2\rangle_\xi=2\pi\alpha\hbar^2/me^2E_\perp{}^2, \quad \langle u^2\rangle_H=2c\hbar/eH.\end{aligned} \tag{3}$$

Here $\langle u^2\rangle$, $\langle u^2\rangle_H$, $\langle u^2\rangle_\xi$ are the resultant, magnetic, and deformation localization lengths of the electron, respectively, H is the external magnetic field. According to the definition (3), the resultant length $\langle u^2\rangle$ of localization of the electron in a magnetic field on the deformed helium surface is somewhat smaller (because $\langle u^2\rangle_\xi<\infty$) than for a magnetized electron in a vacuum. Experimentally, the decrease in the effective localization length, and consequently the increase in the resonance frequency at fixed H, must be interpreted as a decrease in the effective mass of the electron under the action of the applied electric field. Consequently, the observed decrease in the effective mass of the electron in the

 0038-5646/79/080361-09$02.40

quantizing magnetic field under the action of the applied electric field can be related to the phenomenon of self-consistent deformation of the helium surface under the action of the localized electron.

The classical determination of the shift in the cyclotron frequency of deformation origin in the harmonic approximation, having the meaning of an upper estimate, can easily be obtained on the basis of the results of a number of works.[3-5] Subsequently, Cheng and Platzman[6] showed that in the presence of a strong magnetic field, the problem of the effect of the self-consistent deformation of the surface of the helium on the value of the cyclotron frequency of the surface electrons can be solved more accurately without the use of the harmonic approximation. However, the final quantitative results in that work[6] were obtained only numerically. The corresponding analytic solution of the problem will be given explicitly below.

With increase in the density of surface electrons, when the Coulomb interaction between electrons begins to exceed the kinetic and Zeeman energies of a single electron in the magnetic field, the one-dimensional approximation loses meaning and the problem of the shift of the cyclotron frequency must be solved again in the appropriate multiparticle terms.

The considerations just given allow us to rough out the program of action with regard to the elucidation of the problem as to the nature of the field dependence of the cyclotron mass of the electrons on the surface of the helium. The theory of the phenomenon should contain contributions from both channels of change of the effective mass of the electron. Under the conditions of a small self-consistent deformation of the helium surface and linearity of the equations that describe the vibrations of the helium surface, the problems of the static (due to self-consistent deformation of the helium surface) and dynamic (due to the thermal fluctuations of the helium surface) shifts of the cyclotron frequency are not coupled and can be solved independently. The total change in the effective mass of the electron is the simple sum of the two independent effects.

DYNAMIC RENORMALIZATION OF THE CYCLOTRON MASS IN THE SINGLE-ELECTRON APPROXIMATION

The effect of the electron–phonon interaction on the spectrum of electrons in metals and semiconductors has been the object of numerous investigations. At the present time, there exist various methods for the solution of this problem, which enable us to investigate the problem over a wide ranges of constants of the electron-phonon interaction. Nevertheless, we shall limit ourselves in the present work to the simplest possibility of calculation of the dynamic renormalization of the cyclotron mass of the surface electrons, considered in the second order perturbation theory.

1. We calculate first the increment to the dispersion law of the surface electrons due to their interaction (1) with thermal ripplons in the limiting case of sufficiently high temperatures $T > \hbar\omega_H$. The starting expression for $\delta\varepsilon(\mathbf{k})$ in second order perturbation theory has the form

$$\delta\varepsilon(\mathbf{k}) = -e^2E_\perp^2 \sum_{\mathbf{q}} \frac{|\xi_{\mathbf{q}}|^2}{\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}}},$$
$$\varepsilon_{\mathbf{k}} = \hbar^2k^2/2m,\ \omega_{\mathbf{q}}^2 = \alpha q^3/\rho,\ \omega_H = eH/mc,$$
$$|\xi|_{\mathbf{q}}^2 = S^{-1}Q_{\mathbf{q}}^2(2n_{\mathbf{q}}+1),\quad Q_{\mathbf{q}}^2 = \hbar q/2\rho\omega_{\mathbf{q}},\quad q = |\mathbf{q}|, \tag{4}$$

where $\mathbf{k}$ and $\mathbf{q}$ are the wave numbers of the electron and ripplon, respectively, m is the mass of the free electron, n_q is the Bose distribution of the ripplons, and S is the area of the surface of the helium.

Transforming in (4) from summation over $\mathbf{q}$ to the corresponding integration, and carrying out the explicit integration over the angle between the vectors $\mathbf{k}$ and $\mathbf{q}$, we find from (4)

$$\delta\varepsilon(\mathbf{k}) = -\frac{e^2E_\perp^2 m}{2\pi\hbar(2\rho\alpha k)^{1/2}} \int_1^\infty \frac{dx \operatorname{cth}(\lambda x^{3/2})}{x(x^2-1)^{1/2}}$$
$$\lambda = \hbar T^{-1}(2k)^{3/2}(\alpha/\rho)^{1/2}. \tag{5}$$

In the limit $T \to 0$, the expression (5) reduces to (2). If $T \neq 0$, then the integral over x cannot be obtained in general form. Because of this, it is necessary to use additional considerations, namely, that the energy of the electrons in which we shall be interested in the following, i.e., the electrons excited by the resonant high-frequency field, have a scale not less than the temperature of the medium:

$$\hbar^2k^2/2m \geqslant T. \tag{5a}$$

Setting $T \approx 0.1 - 1$ K, which is of most interest in the applications, and taking into account the scale of k from (5a), it is not difficult to obtain for the parameter λ in (5) the estimate $\lambda \approx 10^{-2} - 10^{-3}$. Consequently, the argument of the hyperbolic tangent in (5) becomes greater than unity only in the region of very large $x \gg 1$, where the numerical value of the integrand is already sufficiently small. Thus, we conclude that we can use in the calculation of the integral, under the conditions (5a), the asymptotic form of coth $\lambda x^{1/2}$ at small values of the argument: coth $\lambda x^{3/2} \approx \lambda^{-1}x^{-3/2}$. As a result,

$$\delta\varepsilon(p) = -\frac{e^2E_\perp^2}{4\pi\alpha}\frac{mT}{p^2},\quad p = \hbar k. \tag{6}$$

The transition from (6) to (2) takes place at temperatures $T \approx 10^{-3} - 10^{-4}$ K,

Thus, the refined dispersion law of surface electrons, interacting through (1) with the capillary waves at finite temperatures and fairly large values of the momentum of the electrons taken with account of (6), takes the following form

$$\varepsilon(p) = \frac{p^2}{2m} - \frac{e^2E_\perp^2}{4\pi\alpha}\frac{mT}{p^2}. \tag{7}$$

The resultant spectrum $\varepsilon(p)$ is essentially nonparabolic. For this reason, it is natural, for the determination of the cyclotron mass of the electron, to use the general rules usually employed in work with an arbitrary dispersion law for the electrons.[7] That is, we set

$$\varepsilon(p) = \text{const} = \varepsilon. \tag{8}$$

Equation (8) describes a closed trajectory in momentum space, having a definite area $S(\varepsilon)$. According to Ref. 7, the definition of the cyclotron mass in terms of $S(\varepsilon)$ is

$$m^* = \frac{1}{2\pi}\frac{\partial S}{\partial \varepsilon}. \qquad (9)$$

Using the given definitions and the concrete form of $\varepsilon(p)$ from (7), it is not difficult to obtain the following expression for m^*[8]:

$$m^* = m\left(1 - \frac{e^2E_\perp^2}{8\pi\alpha}\frac{T}{\varepsilon^2}\right), \quad \Delta\omega = \omega_H \frac{e^2E_\perp^2}{8\pi\alpha}\frac{T}{\varepsilon^2}. \qquad (10)$$

The expression (10) enables us to conclude that the ripplon renormalization of the cyclotron mass actually exists and has a sign corresponding to a decrease in the cyclotron mass in the presence of an applied field.

Under equilibrium (or quasiequilibrium) conditions, the quantity ε is approximately equal to the mean thermal energy of the electron. Setting $T = 0.5$ K, $E_\perp = 300$ W/cm, $\alpha = 0.36$ erg/cm^2 and $\varepsilon \approx T$, we find from (10) the estimate $(m^* - m)m^{-1} \approx 10^{-3}$.

2. In the opposite limiting case $\hbar\omega_H \gg T$ the calculation of the shift $\Delta\omega$ due to the interaction (1) consists of the following. Using the set of wave functions for the electron in the magnetic field, we can calculate the ground (ΔE_0) and the first excited Landau level (ΔE_1) in second order perturbation theory. The sought effect is $\hbar\Delta\omega = \Delta E_1 - \Delta E_0$. Using the representation of $\xi(r)$ in terms of the creation and annihilation operators of the ripplons, in analogy with (4), and the set of wave functions of the electron in a magnetic field, which arise in the calibration of the vector potential in the Landau form, we have the following expressions for ΔE_0 and ΔE_1 (see the similar calculations in Ref. 9):

$$\Delta E_0 = -\frac{e^2E_\perp^2}{2(\rho\alpha)^{1/2}S}\sum_q e^{-x}q^{-1}\sum_{n=1}^{\infty}\frac{x^n}{n!(n\omega_H+\omega_q)}, \qquad (11)$$

$$\Delta E_1 = \frac{e^2E_\perp^2}{2(\rho\alpha)^{1/2}S}\sum_q e^{-x}q^{-1}\left\{\frac{x^2}{\omega_H+\omega_q} - \sum_{n=2}^{\infty}\frac{x^{n-1}(n-x)^2}{n![(n-1)\omega_H+\omega_q]}\right\}, \qquad (12)$$

$x = \hbar q^2/m\omega_H$, (ω_H and ω_q are taken from (4)).

The investigation of the sums (11) and (12) in the general case is rather complicated. However, in the region of sufficiently strong magnetic fields, where

$$\omega_H \gtrsim \omega_q^{max} = (\alpha/\rho)^{1/2}(q^{max})^{3/2}, \quad q^{max} \lesssim 10^8 \text{ cm}^{-1}, \qquad (13)$$

the situation simplifies (actually, the region $\omega_H \gtrsim \omega_q^{max}$ corresponds magnetic fields $H \gtrsim 10^4$ Oe). Neglecting the quantity ω_q in the denominators of (11) and (12) in this limiting case, in comparison with ω_H, transforming from summation over q to the corresponding two-dimensional integral and completing the resultant integration over dq we get from (11) and (12):

$$\Delta E_0 = -\gamma\sum_{n=1}^{\infty}\frac{\Gamma(n+3/4)}{n!n}, \quad \gamma = \frac{e^2E_\perp^2}{8\pi(\alpha\rho)^{1/2}\omega_H}\left(\frac{m\omega_H}{\hbar}\right)^{3/4}, \qquad (14)$$

$$\Delta E_1 = \gamma\left\{\Gamma\left(3+\frac{3}{4}\right) - \sum_{n=1}^{\infty}\frac{\Gamma(n+3/4)(n+1)}{n!n} - \frac{\Gamma(n+1+3/4)}{n!n} + \frac{\Gamma(n+2+3/4)}{(n+1)!n}\right\} = \gamma\left\{\Gamma\left(3+\frac{3}{4}\right) - \sum_{n=1}^{\infty}\frac{\Gamma(n+3/4)}{n!} \times\left[-\frac{1}{2n} + \frac{1,5}{n+1} + \frac{21}{16(n+1)n}\right]\right\}; \qquad (14a)$$

Here $\Gamma(x)$ is the gamma function.

The principal value of the sums over n in the expressions cited is determined by large n. Therefore, for an estimate of the convergence of these series, we must use the asymptotic representation

$$\Gamma(x)|_{x\gg1} \approx \left(\frac{2\pi}{x}\right)^{1/2} e^{-x}x^x\left(1 + \frac{1}{12x} + \frac{1}{288x^2} + \ldots\right).$$

As a result, it is clear that the ratio $\Gamma(n+3/4)/n! \equiv \Gamma(n+3/4)/\Gamma(n+1)$ behaves in the following manner at large n:

$$\frac{\Gamma(n+3/4)}{\Gamma(n+1)}\Big|_{n\gg1} \approx \frac{e^{1/4}}{n^{1/4}}.$$

Thus, at large n, the series (14) and (14a) turn out to be identical with the series for the Riemann ζ functions, i.e., they converge. Replacing the ratio $\Gamma(n+3/4)n!$ by $e^{1/4}n^{-1/4}$ for all n, we can reduce the expressions for ΔE_0 and ΔE_1 to the following final form:

$$\Delta E_0 \approx -e^{1/4}\zeta(3/4)\gamma \approx -6.1\gamma, \quad \Delta E_1 \approx 0.67\gamma;$$
$$\Delta\omega = \hbar^{-1}(\Delta E_1 - \Delta E_0) \approx 6.77\gamma\hbar^{-1}, \quad \gamma = \frac{e^2E_\perp^2}{8\pi\alpha^{1/2}\rho^{1/2}\omega_H}\left(\frac{m\omega_H}{\hbar}\right)^{3/4}. \qquad (15)$$

The accuracy of the numerical coefficients in these expressions is limited by the value of the principal significant figures. For a more accurate calculation, it is necessary to sum the series (14) and (14a) numerically.

It is interesting to note that the ground level is shifted more than the first excited level. Formally, this is connected with the fact that all the terms of the sum (11) are negative, while in the expression (12) for ΔE_1 there are terms of different signs. In the final analysis, the difference $\Delta E_1 - \Delta E_0$ proves to be positive, i.e., the interaction with the zero vibrations of the surface of the liquid helium shifts the resonance cyclotron frequency toward higher frequencies.

In the finite-temperature region, the quantity $\Delta\omega$ (15) begins to depend on T. This dependence is contained in the general definitions of ΔE_0 and ΔE_1 in the form of the factor $2n_q + 1$, taken to be unity in (11) and (12) (n_q is the Bose distribution of the ripplons). Account of the temperature factor can be taken comparatively simply by use of the following approximation:

$$1 + 2n_q \approx 1 + 2T/\hbar\omega_q,$$

which describes correctly the properties of the factor $2n_q + 1$ in the region of large and small ω_q. Using this approximation for $2n_q + 1$ and performing calculations similar to the above, we can find the temperature dependence of $\Delta\omega$[1]:

$$\hbar\Delta\omega = \gamma[6.77 + 4T/\hbar\omega(q_H)], \quad \omega(q_H) = (\alpha/\rho)^{1/2}q_H^{3/2}, \quad q_H^2 = m\omega_H/\hbar. \qquad (16)$$

According to (16), the condition determining the smallness of the temperature effect on $\Delta\omega$ is not the inequality $T \ll \hbar\omega_H$, but the somewhat more complicated combination

$$T/\hbar\omega(q_H) \ll 1, \qquad (16a)$$

which contains the ratio of the temperature to the ener-

gy of the ripplon with wave number q_H, which has the scale of the reciprocal magnetic length. Thanks to satisfaction of the inequality $\omega(q_H) \ll \omega_H$, the requirement on the temperature (16a) proves to be much more stringent than $T \ll \hbar\omega_H$. For example, in the case $H \approx 10^4$ Oe we have $\hbar\omega_H \approx 1$ K and $\hbar\omega(q_H) \approx 10^{-3}$ K. As a result, the inequality (16a) begins to be satisfied starting with temperatures $T \lesssim 10^{-3}$ K. In the region $T \gtrsim 10^{-3}$ K, the thermal ripplons turn out to have a significant effect on the value of $\Delta\omega$.

Upon satisfaction of the inequality $T \gg \hbar(q_H)$, which is the inverse of (16a), the quantity $\Delta\omega$ from (16) can be written in the form

$$\Delta\omega \approx \frac{e^2E_\perp^2}{2\pi\alpha\hbar}\frac{T}{\hbar\omega_H}. \tag{16b}$$

It then follows that the thermal part of $\Delta\omega$ is identical with the classical expression for $\Delta\omega$ (10) if we replace ε by $\hbar\omega_H/2$ in the latter expression. This coincidence has a clear physical meaning and attests to the correctness of the given calculations.

STATIC RENORMALIZATION OF THE CYCLOTRON FREQUENCY IN THE SINGLE-ELECTRON APPROXIMATION

In contrast to the investigated dynamic shift of the cyclotron frequency of the surface electrons, which takes place both for weakly and for strongly excited electrons, a static shift of the cyclotron frequency is possible only under conditions of weak excitation of the electron system, when each of the electrons is in the ground state for the greater part of the time. The reason for this limitation on the action of this mechanism is the necessity of the existence of the local pressure exerted by the localized electron on the surface of the helium in the clamping field. It is obvious that with increase in the degree of excitation of the electrons, the value of the electron pressure on the surface of the helium decreases, and the effect of the static shift of the cyclotron frequency vanishes. In this connection, the calculation of the static effect should contain information on the value of the shift and on the conditions under which a similar shift can be accomplished.

1. The simplest possibility of estimating the renormalization of the cyclotron frequency for surface electrons under conditions of the existence of a self-consistent deformation of the helium surface and a weak heating of the localized electrons is connected with the use of the classical harmonic approximation. In this case, the approximation discussed in Refs. 3–5, the actual potential well $eE_\perp\xi(\mathbf{r})$ under a localized electron is replaced by a parabolic approximation, which is identical with the initial well in the region of the minimum. It is evident that such an approximation describes the effect of the self-consistent deformation on the different observed characteristics of the surface electrons in helium and can therefore be used only as an upper estimate of these characteristics. Taking into account what has been said, and using the results of Refs. 3–5, we have the following classical equation of motion of the magnetized surface electron in an alternating field $E_\parallel$ parallel to the surface of the helium:

$$\ddot{\eta}+\omega_t^2\eta+i\omega_H\dot{\eta}+\tau^{-1}\dot{\eta}=\frac{eE_\parallel}{m}e^{-i\omega t},$$
$$\eta=x+iy, \quad \dot{\eta}=d\eta/dt, \tag{17}$$
$$\eta=\frac{eE_\parallel e^{-i\omega t}}{m(\omega_t^2-\omega^2+\omega_H\omega+i\omega/\tau)}, \quad \omega_t^2=\frac{e^2E_\perp^2}{2\pi\alpha m\langle u^2\rangle}.$$

Here $\langle u^2\rangle$ is taken from (3), and τ is the characteristic relaxation time.

The resonance frequency of Eq. (17) is shifted in the limit $\omega_H \gg \omega_t$ relative to the cyclotron frequency ω_H in the direction of higher frequencies, by an amount

$$\Delta\omega=\omega_t^2/\omega_H. \tag{18}$$

Taking it into account that here $\langle u^2\rangle \approx \langle u^2\rangle_H = 2c\hbar/eH$, we have, finally,

$$\Delta\omega=e^2E_\perp^2/4\pi\alpha\hbar. \tag{18a}$$

Under resonance conditions, the stationary classical orbit of the electron has the dimensions

$$R \approx eE_\parallel\tau/\omega_H. \tag{19}$$

The weakly-excited-electron approximation corresponds to a situation in which

$$R^2 \ll \langle u^2\rangle_H. \tag{19a}$$

Using the definition of R (19) and the connection between ω_H and $\langle u^2\rangle_H$, $\omega_H=\hbar/m\langle u^2\rangle_H$, it is not difficult to establish the fact that the inequality (19a) is equivalent to the requirement

$$\tau_\parallel=\hbar e^{-1}E_\parallel^{-1}\langle u^2\rangle_H^{-1/2} \gg \tau. \tag{19b}$$

The inequalities (19a) and (19b) ensure that the electron is predominantly in the ground state and, consequently, the effective action of the electron pressure on the surface of the liquid helium. Thus, these inequalities determine the region of existence of the static shift of the cyclotron frequency. In quantum terms, the time $\tau_\parallel$ corresponds to the time necessary for the transition of the electron from the ground state to the first excited state.

Any deviation from the approximation (19a) in this case, if the static shift of the cyclotron frequency is fundamental, is easily observed experimentally, because the value of $\Delta\omega$ begins to depend on the field intensity $E_\parallel$ in this case.

2. If $\omega_H \gg \omega_t$ the possibility appears of determining the shift $\Delta\omega$ without use of the harmonic approximation. This fact was first observed in Ref. 6. We are speaking of a perturbation theory, the zeroth approximation of which is the set of states of the electron in a magnetic field above a plane surface of helium. In a cylindrical system, which corresponds to the symmetry of the problem, these states are described by well-known expressions for the spectrum and wave functions of the electron. The ground (ψ_{00}) and the first excited (ψ_{0-1}) states, which are important for the dipole excitation, take the following form here (see Ref. 10, p. 495 of original)[2)]:

$$\psi_{00}=\pi^{-1/4}r_H^{-1}\exp(-r^2/2r_H^2),$$
$$\psi_{0-1}=\Pi^{-1/4}rr_H^{-2}\exp(-r^2/2r_H^2)e^{i\varphi}, \tag{20}$$
$$r_H^2\equiv\langle u^2\rangle_H, \quad E_{nl}=\hbar\omega_H\left(n+\frac{|l|-l+1}{2}\right).$$

Here φ is the angle in the cylindrical system of coordinates connected with the center of symmetry of the wave function, $r_H^{-2} = eH/2c\hbar$, while n and l are the principal and azimuthal quantum numbers.

Taking into account the assumption that the electron is predominantly in the ground state, it is not difficult to conclude that the pressure exerted on the helium surface by the electron is determined by the expression

$$p(r) = \frac{eE_\perp}{\pi r_H^2} \exp\left(-\frac{r^2}{r_H^2}\right). \tag{21}$$

The idea behind the further calculation is the following. We write out the equation (21) for the static deformation of the surface under the action of the pressure $p(r)$:

$$\Delta_r \xi - \varkappa^2 \xi = \alpha^{-1} p(r), \tag{22}$$

$\varkappa$ is the capillary constant of the liquid helium, and Δ_r is the Laplace operator. Defining $\xi(r)$ in terms of $p(r)$, we can then seek the corrections to the Landau levels for the perturbation energy $eE_\perp \xi(r)$ from the unperturbed wave functions of an electron in a magnetic field (20). The solution (22), obtained in Ref. 3, has the form

$$\xi(r) = \alpha^{-1} \int_0^\infty G(q) J_0(qr) q\, dq,$$
$$G(q) = \frac{eE_\perp}{2\pi(q^2+\varkappa^2)} \exp\left(-\frac{q^2 r_H^2}{4}\right), \tag{22a}$$

where $J_0(x)$ is a Bessel function of order zero.

This shift of a given level as a result of the interaction $eE_\perp \xi$ is determined by the expression

$$\Delta E_{nl} = \int |\psi_{nl}|^2 eE_\perp \xi(r) 2\pi r\, dr. \tag{23}$$

It must be noted that, generally speaking, the solutions (20) are degenerate in the azimuthal quantum number l. Therefore, the perturbation theory should be constructed with account of the possible lifting of the degeneracy in this quantum number. However, the perturbation $eE_\perp \xi(r)$ has cylindrical symmetry (it does not depend on the angle φ) and therefore non-vanishing matrix elements arise only for states with identical n and l. As a result, the general perturbation theory for degenerate systems is simplified in the given case. In particular, there is no need to determine the correct superpositions of the zeroth-order wave functions, and the matrix elements (23) can be computed from the wave functions (20).

The perturbation $eE_\perp \xi$ lowers the energy of the electron, i.e., each of the corrections ΔE_{nl} has a negative sign. Therefore, a result of correct sign is obtained for the shift of the resonance frequency if we define it by the expression

$$\hbar \Delta\omega_\xi = (\Delta E_{0-1} - \Delta E_{00}) = \int_r (|\psi_{00}|^2 - |\psi_{0-1}|^2) eE_\perp |\xi| 2\pi r\, dr. \tag{24}$$

The integrated expressions (24) in r give the following result (Ref. 11, p. 735):

$$\hbar \Delta\omega = \frac{eE_\perp}{\alpha} \int G(q) e^{-x} [1 - L_1(x)] q\, dq,$$
$$x = {}^1/_4 q^2 r_H^2. \tag{25}$$

Here $L_1(x)$ is the Laguerre polynomial: $L_1 = 1 - x$.

The expression in the square brackets in the integrand of (25) begins with a term proportional to x. For this reason, in the integration over q in (25) we can simplify the expression $G(q)$, setting the capillary constant in it equal to zero. This procedure has a simple meaning. So long as we are dealing with the calculation of $\xi(r)$, a finite expression is obtained for it only at finite values of the parameter $\varkappa$ (in the opposite case, the integral (22a) diverges logarithmically at small $q < \varkappa$). If we are interested in the difference $\Delta E_{0-1} - \Delta E_{00}$, then the logarithmically large components, which diverge as $q \to 0$, cancel each other and the remainder does not depend on $\varkappa$. Taking into account what has been said and integrating over q in (25), we obtain finally

$$\Delta\omega = e^2 E_\perp^2 / 8\pi\alpha\hbar. \tag{26}$$

This result for $\Delta\omega$ has a structure similar to the classical estimate (18a) of $\Delta\omega$ in the harmonic approximation, but has only half the numerical coefficient. The value of $\Delta\omega$ in (26) virtually coincides with the result for $\Delta\omega$ from Ref. 6.

3. Having determined the dynamic $\Delta\omega^d$ [Eq. (16)] and static $\Delta\omega^s$ [Eqs. (18a) and (26)] shifts, it is not difficult to conclude that in the limiting case of strong magnetic fields, $\omega_H > \omega_q^{\max}$, the static shift is noticeably greater than the dynamic. In fact,

$$\delta = \frac{\Delta\omega^d}{\Delta\omega^s} = \frac{6.77 (m\omega_H/\hbar)^{3/4} \alpha}{(\alpha\rho)^{1/2} \omega_H}. \tag{27}$$

Taking into account the corresponding parameters and setting $\omega_H \approx 10^{11} - 10^{12}$ sec^{-1}, we obtain from (27) the value $\delta \approx 10^{-1} - 10^{-2}$. However, it must be kept in mind that the temperature effect (16b) on the value of the dynamic shift $\Delta\omega$ becomes significant in the region $T > 10^{-3}$ K with increase in the temperature. As a result, the contribution of the dynamic shift to the total value of $\Delta\omega$ in the temperature range $T \leq 0.5$ K begins to compete with the static $\Delta\omega^s$.

In the region of quasiclassical behavior of the electron, when it becomes possible to speak of its trajectory, the determination of the local pressure (21) exerted by the electron on the helium surface loses meaning and the static shift $\Delta\omega^s$ of the cyclotron frequency becomes insignificant. So far as the dynamic shift is concerned, it takes the form $\Delta\omega$ from (10) and continues to be accessible to experimental study under the conditions $\hbar\omega_H < T_e$. A similar condition holds either in a weak magnetic field when the electron subsystem is heated in an external electric field $E_\parallel$ parallel to the helium surface. However, it is seen from the definition of $\Delta\omega$ (10) that the electron temperature should not be very high, because the change in the cyclotron mass $(m^* - m)/m$ vanishes as $T/\varepsilon \to 0$, $\varepsilon \propto T_e$.

CYCLOTRON FREQUENCY SHIFT AT FINITE DENSITY OF SURFACE ELECTRONS

With increase in the density of the surface electrons, free motion along the helium surface becomes difficult and the calculations of the cyclotron frequency shift,

based on the use of the concept of free motion of the electrons along the surface, become unsuitable. In this case, the solution of the problem should be sought in terms that are more adequate for the situation being discussed.

1. The situation becomes definite in the case of sufficiently large concentrations of surface electrons, when the possibility appears of speaking of a crystallization of a two-dimensional electron system on the helium surface. Under similar conditions, the electrons are localized at the sites of a definite lattice (for simplicity, a quadratic one) and oscillate about the position of equilibrium with mean square amplitude $\langle u^2\rangle$. The scale of this oscillation amplitude is determined in the general case by the Coulomb interaction, by the effect of deformation phenomena, and by the magnetic field. As a tentative relation, which gives a qualitative idea of the role of the various interactions in the formation of $\langle u^2\rangle$, we present here the result for $\langle u^2\rangle$ obtained from Ref. 12 under the assumption that all the electrons of a Wigner crystal, except the given one, are fixed in their equilibrium positions:

$$\langle u^2\rangle^{-1}=1/2\langle u^2\rangle_t^{-1}+(1/4\langle u^2\rangle_t^{-2}+\langle u^2\rangle_c^{-2}+\langle u^2\rangle_H^{-2})^{1/2},$$
$$\langle u^2\rangle_c=\hbar^2a^3/5.4me^2, \quad a^2=n_s^{-1}. \tag{28}$$

The expression (28) generalizes the definition (3) of $\langle u^2\rangle$ to include the case of a finite density of the surface electrons. Using (28) and the definition of the static shift of the cyclotron frequency from (18)

$$\Delta\omega=\frac{\omega_t^2}{\omega_H}, \quad \omega_t^2=\frac{e^2E_\perp^2}{2\pi\alpha m\langle u^2\rangle}, \tag{28a}$$

which is valid in the harmonic approximation, we can trace the influence of the finite density of the surface electrons on the shift of the cyclotron frequency.

It is obvious that the role of the Coulomb electron–electron interaction in the investigated problem becomes significant under conditions in which the Coulomb length $\langle u^2\rangle_c$ becomes comparable with the magnetic length $\langle u^2\rangle_H$. The inequality $\langle u^2\rangle_c \lesssim \langle u^2\rangle_H$ is equivalent to the requirement

$$n_s>n_s^*, \quad n_s^*\approx(H/4.64m^{1/2}c)^{4/3}. \tag{29}$$

In the case of a magnetic field $H \gtrsim 5\times10^3$ Oe, the inequality (29) gives a numerical estimate of the threshold density, $n_s^* \gtrsim 10^7$ cm^{-2}. Such a density corresponds to an applied field $E_\perp^* \gtrsim 200$ V/cm. Along with this, the critical density n_s^c at which instability of charged surface develops, has the scale $n_s^c \lesssim 10^{10}$ cm^{-2}. Consequently, there exists a wide range of densities $n_s^* \lesssim n_s \lesssim n_s^c$, in which the Coulomb localization of the electrons near the position of equilibrium at the sites of the Wigner lattice is fundamental. Neglecting the quantity $\langle u^2\rangle$ in the definition (28) of $\langle u^2\rangle_t$ (the smallness of this contribution to $\langle u^2\rangle$ is easily estimated), we represent the expression for $\Delta\omega$ in the following form:

$$\Delta\omega=\frac{e^2E_\perp^2}{4\pi\alpha\hbar}\left[1+\frac{\langle u^2\rangle_H^2}{\langle u^2\rangle_c^2}\right]^{1/2}. \tag{30}$$

In the limiting case $n_s \ll n_s^*$, this expression for the shift transforms into the expression for $\Delta\omega$ from (18a). In the case $n_s \gg n_s^*$ the structure of $\Delta\omega$ changes:

$$\Delta\omega=\begin{cases} e^2E_\perp^2/4\pi\alpha\hbar, & n_s\ll n_s^* \quad (30a)\\ \dfrac{e^2E_\perp^2}{4\pi\alpha m\omega_H}\left(\dfrac{5.4me^2}{\hbar^2a^3}\right)^{1/2}, & n_s\gg n_s^*. \quad (30b)\end{cases}$$

If the parameters n_s and $E_\perp$ are connected by the expression $E_\perp=2\pi en_s$, then the field dependence of $\Delta\omega$ (30b) takes the form $\Delta\omega \propto E_\perp^{2.75}$. Moreover, in this limiting case, a dependence of $\Delta\omega$ on H sets in: $\Delta\omega \propto H^{-1}$.

2. The preliminary analysis of the situation with finite density n_s of the surface electrons enables us to draw one interesting conclusion. It turns out that under conditions when the external field above the charged helium surface is cancelled out, i.e., under the conditions $E_\perp=2\pi en_s$ and $\omega_H \gg \omega_t$ we can carry out the calculation of the quantity $\langle u^2\rangle$ which enters into the determination of $\Delta\omega$ from (28a), without taking into account of the self-consistent deformation of the helium surface under the action of the electron pressure. In other words, to determine $\langle u^2\rangle$ it suffices to know the value of $\langle u^2\rangle$ for the electron lattice over the plane surface. Such a problem admits of a consistent solution, free of the model assumptions on which the calculation of $\langle u^2\rangle$ from (28) is based. We have in mind the calculation of the spectrum of collective excitations of a Wigner crystal, and the determination of $\langle u^2\rangle$ in terms of this spectrum. Using the results of Fukuyama,[13] we write down the quantity $\langle u^2\rangle$ of interest to us at $T=0$ in the following form:

$$\langle u^2\rangle=\frac{1}{2mN}\sum_q\frac{(\omega_l+\omega_t)^2}{\omega_l\omega_t(\omega_++\omega_-)}, \tag{31}$$

$$\omega_\pm^2=1/2\{\omega_l^2+\omega_t^2+\omega_H^2\pm[(\omega_l^2+\omega_t^2+\omega_H^2)^2-4\omega_l^2\omega_t^2]^{1/2}\},$$
$$\omega_l^2(q)\approx\frac{2\pi e^2n_s}{m}q, \quad \omega_t\approx sq, \quad s=0.82em^{-1/2}n_s^{1/4}. \tag{31a}$$

Here q is the wave number of the excitations and N is the total number of electrons. Taking into account the explicit form of the spectrum (31a) we must note that no finite shift of the cyclotron frequency, independent of q, arises in the solution of the equation of motion of a Wigner crystal above the plane of the surface because

$$\omega_+ \underset{q\to0}{\longrightarrow} \omega_H, \quad \omega_- \underset{q\to0}{\longrightarrow} 0.$$

In this sense, the assertion that it is impossible to obtain the cyclotron frequency shift that is dependent on n_s, made in Refs. 5 and 6, remains in force. However, the indirect effect of the Coulomb interaction on the quantity $\langle u^2\rangle$, which follows from the definition (31) of $\langle u^2\rangle$, is sufficient onset of a dependence of $\Delta\omega$ on n_s.

In the limiting case $n_s > n_s^*$, where n_s^* is taken from (29), the dependence of $\langle u^2\rangle$ (31) reduces to the expression (see Ref. 14):

$$\langle u^2\rangle \approx \langle u^2\rangle_c=\frac{\hbar}{em^{1/2}n_s^{3/4}}\frac{\text{const}}{4\pi}, \quad \text{const}\approx1,$$

which is identical, apart from a numerical factor (≈1), with $\langle u_c^2\rangle$ from (28). Thus, the approximate scheme of calculation (28)–(30) has an accuracy no worse than the scheme (31) if in the latter we use the long-wave approximation for the functions $\omega_l(q)$ and $\omega_t(q)$. More accurate results for $\langle u^2\rangle$ and $\Delta\omega$ can be obtained with account of the correct behavior of the dependences $\omega_l(q)$

and $\omega_t(q)$ in the region of large values of the wave numbers. The weakest point in this and the other approximations is the use of the harmonic approximation for the connection between $\Delta\omega$ and $\langle u^2\rangle$. There is still no consistent determination of $\Delta\omega$ as a function of n_s, similar to $\Delta\omega$ from (26) in the single-particle approximation. A discussion of this question will be given in a separate paper.

3. In concluding the discussion of the problem of the static shift of $\Delta\omega$ under conditions of finite density of the surface electrons, it is necessary to estimate the role of the temperature in the formation of $\langle u^2\rangle_c$. Assuming, for simplicity, a system of electrons in the liquid state, i.e., neglecting the contribution to $\langle u^2\rangle_c$ by the transverse mode of oscillations, we can write down the following expression for the mean square shift $\langle u^2\rangle_c$[14]:

$$\langle u^2\rangle_c \approx \langle u^2\rangle_c|_{T=0} + \frac{\zeta(3)}{2\pi^3 b n_s^{1/2}}\left(\frac{T}{V_c}\right)^3, \qquad (32)$$

$$b=\hbar^2/me^2, \quad V_c=e^2n_s^{1/2}, \quad \zeta(3)\approx 1.2.$$

Estimates based on (32) show that the thermal part of the mean square shift $\langle u^2\rangle_c$ under the conditions $n_s \approx 10^9$ cm^{-2} and $T \lesssim 0.5$ K amounts to 1–10% of the value of $\langle u^2\rangle_c$ at $T=0$. This estimate enables us to conclude that the role of the temperature in the formulation of $\langle u^2\rangle_c$ is not large in the region of parameters $T \ll V_c$ of interest to us.

4. In the case of finite dimensions L^2 of the area occupied by the electrons on the helium surface, the observation of a plasma shift of the cyclotron frequency of the form

$$\Delta\omega = \pi e^2 n_s L^{-1}/m\omega_H$$

is possible. Estimates of this cause of the shift $\Delta\omega$ are contained in the work of Édel'man.[2] For the concrete of the electron-system dimensions used in the experiments of Ref. 1, the plasma shift $\Delta\omega$ turns out to be unimportant.

DISCUSSION OF THE RESULTS AND COMPARISON WITH THE EXPERIMENTAL DATA

The presented analysis shows that, of the two possible types of renormalization of the cyclotron frequency of the surface electrons (dynamic and static) under conditions of weak heating of the electron system and sufficiently low temperatures, the principal role is played by the static renormalization. This circumstance qualitatively distinguishes the electron system on the surface of the liquid helium from the other electron systems with electron-phonon interaction (semiconductors, metals), for which there exists only the dynamic shift of the location of the cyclotron frequency.

The observed value of $\Delta\omega$ is closely connected with the behavior of the mean square displacement of the surface electrons in a magnetic field and, consequently, can be used for the study of this important characteristic of a multiparticle electron system on a helium surface.

Rigorous calculation of the quantity $\Delta\omega$ requires a departure from the framework of the harmonic approximation in the description of the self-consistent deformation of the helium surface and its effect on the motion of the electron system, and also complete information on the electron spectrum in a magnetic field over the plane helium surface for all wave numbers.

The qualitative behavior of $\Delta\omega$ as a function of the applied field $E_\perp$, which is connected with n_s by the relation $E_\perp - 2\pi e n_s$, is the following. If $T \ll \hbar\omega_H$ (more accurately, if the inequality (16b) is satisfied), then, throughout the entire range of clamping fields, the shift $\Delta\omega$ is of static origin. In the range $n_s \ll n_s^*$ it is described by the expression (26), and on going to $n_s > n_s^*$ it grows rapidly in correspondence with (30), owing to the additional localization of the electron wave functions, which is governed by the Coulomb interaction. In the case $T \lesssim \hbar\omega_H$, in the region $n_s < n_s^*$, the principal role is played by the dynamic shift $\Delta\omega$. Its behavior is described approximately by identical formulas, (10) with $\varepsilon \approx \hbar\omega_H/2$ and (16b). On going to the region $n_s > n_s^*$, the expression (30) for $\Delta\omega$ with $\langle u^2\rangle_c$ from (28) is again valid, because, as already noted in the comments on the derivation of $\langle u^2\rangle_c$ (32), the temperature $T \ll e^2 n_s^{1/2}$ has little effect on the $\langle u^2\rangle_c$ of Coulomb origin, while no dynamic shift $\Delta\omega$ occurs in the region $n_s > n_s^*$ in accord with the considerations given above. The third possibility $T \gg \hbar\omega_H$ is of little interest from the viewpoint of the study of $\Delta\omega$.

Turning to the discussion of the experimental data[2] for $\Delta\omega$, it is not difficult to make it clear[3)] that they refer to the intermediate case $T \lesssim \hbar\omega_H$. In fact, for the two frequencies $f_1 = \omega_H^{(1)}/2\pi \doteq 18.5\times 10^9$ sec^{-1} and $f_2 = 37.75\times 10^9$ sec^{-1} and for the minimum temperature $T \lesssim 0.4$ K, used in Ref. 2, the ratio $T/\hbar\omega_H$ has a scale 0.3–0.5. Consequently, in the region $n_s < n_s^*$, along with the static shift $\Delta\omega^s$ (26), we need to take into account the dynamic shift (16b). The resulting expression for $\Delta\omega$ takes the following form:

$$\Delta\omega = \frac{e^2E_\perp^2}{8\pi\alpha\hbar}\left(1+4\frac{T}{\hbar\omega_H}\right). \qquad (33)$$

In comments on this limiting case it is stated in Ref. 2 that the expression $\Delta\omega^s = e^2E_\perp^2/8\pi\alpha\hbar$ describes correctly the parametric dependence of the shift $\Delta\omega$ for electrons above the surface of He^4, but is numerically somewhat below the observed shift $\Delta\omega$. It is obvious that the presence of a temperature term in $\Delta\omega$ (33) solves the problem of the increase in the theoretical value of $\Delta\omega$ by a factor of 2–3 (at $T/\hbar\omega_H \approx 0.3$–0.5) in comparison with $\Delta\omega^s$. However, an additional dependence of $\Delta\omega$ on the temperature and magnetic field arises here, the existence of which in the experiment has as yet not been clearly determined.

With increase in the intensity of the applied field and consequently, in the density of the surface electrons, a kink is observed on the experimental plot of $\Delta\omega(E_\perp)$, in the direction of an increased dependence of $\Delta\omega$ on $E_\perp$. From there on the value of $\Delta\omega$ continues to be quadratic in $E_\perp$ and independent of H. Only the slope of the plot of $\Delta\omega$ against $E_\perp^2$ changes (see Fig. 4 in Ref. 2). The onset of a similar kink in the $\Delta\omega(E_\perp^2)$ curve can be put into correspondence with a transition of the system from the region $n_s < n_s^*$ to the region $n_s > n_s^*$. Here, in accord with (30), the mean square displacement of the

electrons becomes a function of the field $E_\perp$; this leads to an additional dependence of $\Delta\omega$ on $E_\perp$ and can be interpreted as a change in the slope of the $\Delta\omega(E_\perp^2)$ curve. However, in this case, the theoretical value of $\Delta\omega$ ceases to be a quadratic function of $E_\perp$ and begins to depend on the magnetic field, in qualitative disagreement with the observations. In this situation, it is meaningful to carry out a numerical comparison of the theory and experiment in the region $n_s > n_s^*$ with the aim of clarification of the possible scales of the discrepancy.

The behavior of the combination $(1+\langle u^2\rangle_H^2/\langle u^2\rangle_c^2)^{1/2}$, which enters into the expression (30), as a function of $E_\perp$ for the magnetic fields H_1 and H_2, corresponding to the frequencies $f_1 = 18.5\times10^9$ sec^{-1} and $f_2 = 37.75\times10^9$ sec^{-1} used in Ref. 2, is shown in Fig. 1 (curves 1, 2). The departure of this expression from unity begins in the range of fields $E_\perp \approx 100$–200 V/cm, which agrees qualitatively with the location of the transition region on the experimental plot of $\Delta\omega$ against $E_\perp$ in the case of electrons on an He^4 surface (see Fig. 4 in Ref. 2). The corresponding $\Delta\omega(E_\perp^2)$ curve, constructed on the basis of (30)[4)] and the numerical data of Fig. 1, is shown in Fig. 2. For convenience in the comparison with the experimental graph of Fig. 4 from Ref. 2 the quantity $\delta H = -(mc/e)\Delta\omega$ is used in Fig. 2 in place of $\Delta\omega$. The solid curves 1 and 2 are the results of calculation of δH for the magnetic fields H_1 and H_2. The dashed line e between the curves 1 and 2 corresponds to the position of the experimental data from Ref. 2 for $\delta H(E_\perp)$ in the case of electrons on an He^4 surface and for various directions of the magnetic field. The additional curve o illustrates the $\Delta\omega(E_\perp^2)$ dependence (26) in the case $n_s < n_s^*$. The experimental data at low intensities $E_\perp$, lie somewhat lower than the curve o, and then make the transition to the curve e in the region of the kink. Moreover, the open circles in Fig. 2 indicate another series of experimental values, obtained in the following fashion. Beginning with some density $n_s^0 > n_s^*$, the connection between $E_\perp$ and n_s was discontinued, after which only the intensity of the clamping field changed, without an increase of the number of electrons on the surface of the helium.

On the basis of the data of Fig. 2, we can conclude that the static shift $\Delta\omega$ (30), with account of the influence of electron-electron interaction on $\langle u^2\rangle$, gives the correct scale of the value of the observed effect over a wide range of clamping fields and densities of the surface electrons. A reasonable explanation is found for the appearance of a kink (in actuality, we are dealing with a smooth transition from the asymptote form at $n_s < n_s^*$ to the opposite limiting case $n_s > n_s^*$), and also for

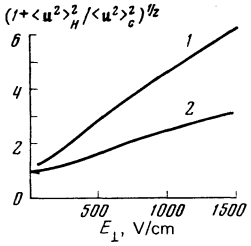

FIG. 1.

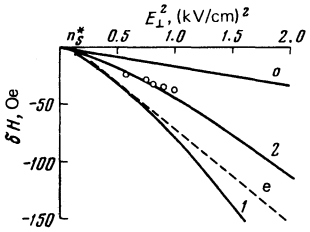

FIG. 2.

the appearance of experimental points in the intermediate region between the line o and the line 1 in the case when there is linear connection between $E_\perp$ and n_s and n_s increases more slowly than $E_\perp$. It is meaningful also to note that the non-quadratic theoretical behavior of $\Delta\omega$ as a function of $E_\perp$ is almost not noticeable in the coordinates $\delta H(E_\perp^2)$ and at limits of $E_\perp$ used in the experiment. A more serious problem is the absence in the experiment of a dependence of δH on the magnetic field H in the region $n_s > n_s^*$. The accuracy of the experimental data in the wide range of densities should enable us to observe the difference between curves 1 and 2, but this is actually not the case. Still another difficulty within the framework of the proposed interpretation arises when the experimental data for δH are considered in the case of electrons over an He^3 surface. No initial section of $\delta H(E_\perp)$ dependence along the line o, with slope corrected to another value of the surface tension, appears in these data. The experimental points δH for electrons above He^3 therefore immediately undergo transition to a regime which could describe the limiting case $n_s > n_s^*$ (see Fig. 4 in Ref. 2). A possible explanation of the similar behavior of $\delta H(E_\perp)$ in the case of electrons above He^3 can be connected with the loss of the homogeneity of the mean density of electrons on the helium surface and the appearance of electron drops having a local density greater than the average. The onset of such formations is facilitated for the case of liquid He^3 and can prevent the creation of the situation $n_s < n_s^*$.

An interesting possibility of explaining the different behavior of the localized electrons above the He^3 or He^4 surfaces appears if we turn to an earlier[15] calculation of the profile of the vapor-liquid boundary of liquid helium. According to this calculation, the profile of the He^3 boundary is much more friable (stretched out by about 20 Å) than in He^4 (the thickness of the transition layer for He^4 is about 2–3 Å). This circumstance can lead to a significant difference in the behavior of the localized electrons over the He^3 and He^4 surfaces.

The author is sincerely grateful to Yu. P. Monarkha and V. S. Édel'man for numerous discussions, useful remarks and for furnishing the data of researches found in press.

[1)] For the thermal part of the problem, the series (11) and (12) converge much better than in the case $T = 0$. Here the characteristic $\tilde{q}$, at which the integration over q is cut off, can be determined from the condition $x = \hbar\tilde{q}^2/m\omega_H \approx 1$. For $\omega_H \sim 10^{10}$ sec^{-1}, the quantity $\tilde{q} \approx 10^5$ cm^{-1}. As a result, in the thermal part of the problem, the neglect of energies $\omega(q)$ in comparison with ω_H in the denominators of (11) and (12) is valid for much weaker magnetic fields than if the inequality

(13) is used.

[2)]As one of the possible states that must be taken into account in the dipole excitation of electrons in a magnetic field, it is also necessary to consider the state

$$\psi_{11}=\frac{2r}{(2\pi)^{1/2}r_H^2}\left(2-\frac{r^2}{r_H^2}\right)\exp\left(-\frac{r^2}{2r_H^2}\right)e^{-i\varphi}.$$

However, the matrix element for the exciting alternating field $eE_{\parallel}r\cos\varphi$ between the states ψ_{00} and ψ_{11} vanishes:

$$\int\psi_{00}eE_{\parallel}r\cos\varphi\psi_{11}r\,dr\,d\varphi=0$$

as a result of integration over r.

[3)]We are dealing with information obtained under the conditions $\tau_{\parallel}\gg\tau$ (see (19b)).

[4)]Actually, a definition of $\Delta\omega$ lower by a factor of two than $\Delta\omega$ in (30) is used in the calculation of $\Delta\omega$, in order that the value of $\Delta\omega$ in the region $n_s<n_s^*$ coincide with the value of $\Delta\omega$ from (26) and not with the classical result (18a).

[1]V. S. Édel'man, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 510 (1976); **26**, 647 (1977) [JETP Lett. **24**, 468 (1976); **26**, 493 (1977)].

[2]V. S. Édel'man, Zh. Eksp. Teor. Fiz. **77**, No. 2 (1979).

[3]V. B. Shikin and Yu. P. Monarkha, Zh. Eksp. Teor. Fiz. **65**, 751 (1973) [Sov. Phys. JETP **38**, 373 (1974)].

[4]V. B. Shikin and Yu. P. Monarkha, Fiz. Nizk. Temp. **1**, 957 (1975) [Sov. J. Low Temp. Phys. **1**, 459 (1975)].

[5]V. B. Shikin, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 328 (1975) [JETP Lett. **22**, 154 (1975)].

[6]A. Cheng and P. M. Platzman, Solid Stat. Comm. **25**, 813 (1978).

[7]I. M. Lifshitz, M. Ya. Azvel' and M. I. Kaganov, Élektronnaya teoriya metallov (Electron Theory of Metals) Nauka, 1971.

[8]Yu. P. Monarkha and S. S. Sokolov, Fiz. Nizk. Temp. **5**, (1979) [Sov. J. Low Temp. Phys. **5**, (1979)].

[9]D. M. Larsen, Phys. Rev. **A135**, 419 (1964).

[10]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics) Fizmatgiz, 1963 [Pergamon, 1968].

[11]I. S. Gradshtein and I. M. Ryzhik, Tablitsy integralov summ, ryadov i proizvedenii (Tables of Integrals, Sums, Series, and Products) Fizmatgiz, 1962.

[12]Yu. P. Monarkha and V. B. Shikin, Zh. Eksp. Teor. Fiz. **68**, 1423 (1975) [Sov. Phys. JETP **41**, 710 (1975)].

[13]H. Fukuyama, Solid Stat. Comm. **19**, 551 (1976).

[14]V. B. Shikin, Zh. Eksp. Teor. Fiz. **72**, 1619 (1977) [Sov. Phys. JETP **45**, 850 (1977)].

[15]F. D. Mackie and Chia-Wei Woo, Phys. Rev. **B18**, 529 (1978); Legesse Senbetu and Chia-Wei Woo, Phys. Rev. **B18**, 3251 (1978).

Translated by R. T. Beyer

Thermal conductivity of pure lead crystals at low temperatures

L. P. Mezahov-Deglin

Institute of Solid State Physics, USSR Academy of Sciences

(Submitted 29 March 1979)
Zh. Eksp. Teor. Fiz. **77**, 733–751 (August 1979)

Results are presented of measurements of the thermal conductivity of perfect and of plastically deformed crystals of pure lead (certified 99.9999% Pb) in the normal and superconducting states in the temperature interval 0.6–7.5 K. Judging from the thermal conductivity in perfect crystals of 2–4 mm diameter in a zero magnetic field, the maximum phonon mean free paths are limited by scattering from the surface, and the maximum electron mean free paths are limited by scattering from impurity atoms. In a strongly deformed sample at temperatures $T\sim1$ K the quasiparticle free paths are limited by scattering from dislocations, and the principal role in the phonon thermal conductivity is played by the flutter effect, i.e., scattering of phonons by vibrating dislocations. At temperatures $T>2$ K the phonon and electron mean free paths are limited by the mutual scattering of the quasiparticles. A comparison of the experimental data with the predictions of the theory of thermal conductivity of pure superconductors shows that the behavior of the phonon component of the thermal conductivity agrees in practice with the theory, while the temperature dependences of the thermal conductivity of the electronic component differ substantially from the theoretical ones.

PACS numbers: 74.30.Ek, 74.70.Gj, 72.15.Eb, 72.15.Qm

1. INTRODUCTION

This work is a continuation of a cycle of studies, initiated by us earlier,[1-3] of kinetic phenomena in perfect bulky crystals at low temperatures. Such investigations are essential for a better understanding of the role of various relaxation mechanisms of excitation and of energy-transport mechanisms in perfect crystalline structures at low temperature; they are also of interest from the point of view of utilitarian material study, and can serve as a basis for the production of crystals with specified physical properties, for the development of physical methods for the analysis of highly purified materials, etc. The objects of the preceding measurements were solid helium and bismuth, a nearly perfect dielectric and semimetal, respectively, with low carrier density ($\sim10^{-5}$ per atom), whose thermal conductivity at helium temperatures is determined by the relaxation processes in the phonon system. It was natural to choose as the next object a superconductor, since the thermal conductivity of a superconductor in the normal state, just as that of a normal metal, is determined mainly by the electronic component $\varkappa_e^n$, while in the superconducting state at $T\ll T_c$ it is determined