Radiative corrections to the vertex form factors in the two-dimensional approximation of quantum electrodynamics

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An investigation is made into the asymptotic behavior of the radiative corrections to the vertex form factors of an electron in a superstrong magnetic field $B > m^2/e = 4.41 \times 10^{13}$G. The calculations are made in the formalism of a method called the two-dimensional approximation of quantum electrodynamics. The technique for calculating the field asymptotic behavior of complicated diagrams in the two-dimensional formalism is demonstrated. The estimates made for the contributions of the fifth-order diagrams to the vertex form factors give, when compared with the contribution of the third-order diagrams, grounds for assuming that the perturbation theory for the given type of compact blocks is invalid at fields higher than 10^{17}–10^{18}G.

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1. INTRODUCTION

In recent years, there has been increasing interest shown in two-dimensional field-theory models, since these models, which fairly adequately reflect some characteristic features of interactions in four-dimensional space-time, involve much simpler analysis and calculation of the necessary diagrams. In Refs. 1–11 we have shown that under certain conditions the quantum electrodynamics (QED) of a charged fermion in a strong magnetic field also degenerates into a two-dimensional form. The two-dimensional approximation of QED developed in Refs. 1–11 made it possible to simplify significantly the problem of calculating various matrix elements of the associated Feynman diagrams. The method proved very effective in the study of a number of astrophysical problems[2-4,8] and is convenient and appropriate for investigating one of the key problems of QED—the problem of the applicability of perturbation theory and the finding of an expansion parameter in the presence of a strong external electromagnetic field (and the problem of the existence of such a parameter). The point is that at large values of the magnetic field induction B, exceeding the characteristic value $B_0 = m^2/e = 4.41 \times 10^{13}$ G, the actual expansion parameter begins to depend on $\xi = B/B_0$, and the form of this dependence may change in each step of the expansion. Sufficiently reliable information about the limits of applicability of perturbation theory and the very nature of the expansion can be obtained by comparing the successive radiative corrections to the various forms of compact diagrams. Corresponding analysis of the radiative corrections to the polarization operator $P(q^2, q_1^2)$ (Refs. 1 and 6) and the mass operator $M(p)$ (Refs. 8 and 10), and also to diagrams with weak vertices,[2,11] shows that for $B \sim 10^{17}$–10^{18} G the contribution of diagrams of order α^{n+1} is comparable with the contribution of diagrams of order α^n.

Investigation of the convergence of the radiative corrections to the vertex operator $\bar{\Gamma}_\mu$ is also one of the important aspects of the problem.

In Ref. 5 we showed that for on-shell electron ends the vertex function of third order (in e) is determined

in the two-dimensional approximation of QED by two form factors, $f(t)$ and $g(t)$, which depend on the square of the two-dimensional momentum $t \equiv k^2 = k_0^2 - k_3^2$ of the virtual photon (the axis 3 is taken parallel to the field B):

$$\tilde{\Lambda}_\mu = \bar{\Gamma}_\mu - \tilde{\gamma}_\mu = f(t)\tilde{\gamma}_\mu - \frac{1}{2m} g(t)\tilde{\sigma}_{\mu\nu}k^\nu, \tag{1}$$

where $\tilde{\sigma}_{\mu\nu} = 1/2(\tilde{\gamma}_\mu\tilde{\gamma}_\nu - \tilde{\gamma}_\nu\tilde{\gamma}_\mu)$, and the tilde on a letter means that the corresponding matrices have two rows.[1] Note that in the two-dimensional matrix notation, the electron Green's function will be expressed by the matrix $(\check{p} - m)^{-1}, \check{p} = \tilde{\gamma}^0 p_0 + \tilde{\gamma}^3 p_3$, and the two-row spinor $v(p)$ satisfies the equation

$$(\check{p}-m)v(p)=0, \quad \bar{v}v=2m, \quad \bar{v}=v^+\tilde{\gamma}^0, \tag{2}$$

and the density matrix is[2]

$$\rho=(\check{p}+m). \tag{3}$$

In the following, fifth order in e, contributions to the vertex function $\tilde{\Lambda}_\mu$ are made by four types of reducible diagram (diagrams a–d in Fig. 1) and one irreducible diagram (e). The contribution of the polarization diagram a to the form factor $f(t)$ was investigated in Ref. 9. In the nonrelativistic limit ($t \ll m^2$), it was found to be proportional to $\alpha^2 \xi$, which confirms the results mentioned above concerning the limits of applicability of perturbation theory, since in the third order (in e) we
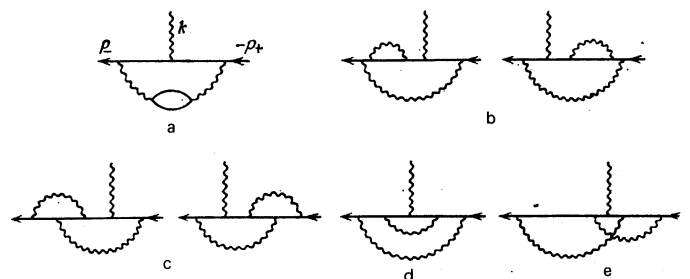

FIG. 1. Contribution of fifth-order diagrams to the electron vertex function.

have $f(t) \sim \alpha$ and $f(t)$ does not depend on the field. In the present paper, we investigate the contribution of the remaining diagrams (b–e) and, in particular, establish the expansion parameter of the form factor $g(t)$.

2. GENERAL RELATIONS

According to Ref. 5, the exact expression in α (but in the leading order in ξ) for the contribution to the vertex operator of the reducible diagrams has the form

$$\tilde{\Lambda}_\mu^{(n)} = \Gamma_\mu^{(n)} - \tilde{\gamma}_\mu = -\frac{i\alpha}{2(2\pi)^3} \int dq_\perp^2 \exp\left(-\frac{q_\perp^2}{2\gamma}\right) \int d^2p$$

$$\times \Gamma^\nu(p_-,p) G(p) \Gamma_\mu(p,p-k) G(p-k) \Gamma^\epsilon(p-k, p_--k) D_{\nu\epsilon}(p-p_-, q_\perp^2), \quad (4)$$

where $\gamma \equiv |eB|$, and all contractions are made in the two-dimensional subspace (0, 3). The outer integral in (4) is over the square of the transverse momentum of the heavy photon line, and the inner integral is over the two-dimensional momentum of the loop.

When allowance is made for only the first radiative corrections (in the posed problem, this is sufficient) the functions in the integrands are as follows:

a) the vertex operator[5]

$$\Gamma_\mu(p',p'-k) = \tilde{\gamma}_\mu - \frac{\alpha}{(2\pi)^2} \int_0^\infty dq_\perp^2 \exp\left(-\frac{q_\perp^2}{2\gamma}\right) \int d^2p$$

$$\times \tilde{\gamma}^\epsilon \frac{\tilde{p}+m}{p^2-m^2} \tilde{\gamma}_\mu \frac{\tilde{p}-\tilde{k}+m}{(p-k)^2-m^2} \tilde{\gamma}_\epsilon \frac{i}{(p'-p)^2-q_\perp^2}; \quad (5)$$

b) the electron propagator[1,8]

$$G(p) = \frac{\tilde{p}+m}{p^2-m^2} + \frac{\tilde{p}+m}{p^2-m^2} M_0(p) \frac{\tilde{p}+m}{p^2-m^2}, \quad (6a)$$

where we have the second-order mass operator

$$M_0(p) = \frac{\alpha}{(2\pi)^2} \int_0^\infty dq_\perp^2 \exp\left(-\frac{q_\perp^2}{2\gamma}\right) \int d^2k \tilde{\gamma}^\epsilon \frac{\tilde{k}+m}{k^2-m^2} \tilde{\gamma}_\epsilon \frac{-i}{(k-p)^2-q_\perp^2}; \quad (6b)$$

c) the photon propagator (in a special gauge)[9]

$$D_{\nu\epsilon}(q^2, q_\perp^2) = g_{\nu\epsilon}\left[\frac{4\pi}{q^2-q_\perp^2} + \left(\frac{4\pi}{q^2-q_\perp^2}\right)^2 \frac{P_R(q^2, q_\perp^2)}{4\pi}\right], \quad (7a)$$

with the regularized second-order polarization operator (see Ref. 1)

$$P_R(q^2, q_\perp^2) = P(q^2, q_\perp^2) - P(0, q_\perp^2), \quad (7b)$$

$$P(q^2, q_\perp^2) = \frac{2i\alpha\gamma}{(2\pi)^2} \exp\left(-\frac{q_\perp^2}{2\gamma}\right) \int d^2k \, Sp\left\{\tilde{\gamma}^\epsilon \frac{\tilde{k}+m}{k^2-m^2} \tilde{\gamma}_\epsilon \frac{\tilde{k}-\tilde{q}+m}{(k-q)^2-m^2}\right\}.$$

Substituting (5)–(7) in (4) and retaining the terms of fifth order (in e), we can obtain the contribution of the reducible diagrams to the required vertex function (to calculate the form factor $f(t)$, we also require a subtraction operation to make $f(0) = 0$).

From the point of view of the applicability of the method used here, it must be borne in mind that it is the more effective, the smaller is the region of integration with respect to the two-dimensional momenta of the internal electron lines making the main contribution to the integral. If the effective contribution derives from the region $|p^2 - m^2| \ll \gamma$, then the condition

$\xi \gg 1$ must at least be satisfied. However, as has frequently been emphasized earlier as well, these restrictions are sometimes satisfied only for the calculation of some of the first terms, the principal terms in ξ, which usually have a logarithmic dependence on ξ. In such cases, the condition of applicability of the method is more stringent: $\ln\xi \gg 1$.

We mention in passing one further circumstance associated with the calculation of the integrals over the internal photon lines. In their calculation, as in four-dimensional electrodynamics, an infrared divergence can arise. In the framework of the two-dimensional approximation, it is eliminated by the generally adopted cutoff procedure $q_\perp^2 \to q_\perp^2 + \delta^2$, which does not affect the final results (see, for example, Ref. 5).

3. CALCULATION OF THE RADIATIVE CORRECTIONS

A. Correction of mass type

Using (4)–(7) and the algebra of the $\tilde{\gamma}$ matrices, we can readily obtain the following expression for the contribution to $\tilde{\Lambda}^\mu$ of the first of the mass-type diagrams (diagram b):

$$\tilde{\Lambda}_{m_1}^\mu = -\frac{m\alpha^2}{4\pi^4} \int_0^\infty dx\, dy \exp\left(-\frac{x+y}{2\gamma}\right) \int d^2p\, d^2q$$

$$\times \frac{(p^2+m^2)(\tilde{p}-\tilde{k})\tilde{\gamma}^\mu + 2m^2\tilde{\gamma}^\mu\tilde{p}}{(p^2-m^2)^2(q^2-m^2)[(p-k)^2-m^2][(p-q)^2-y][(p-p_-)^2-x]}. \quad (8)$$

The corresponding expression for the contribution of the second mass diagram will differ from the given expression by reversal of the common sign, permutation of the matrices $\tilde{\gamma}^\epsilon$, and the replacement of p_- by p_+. As is readily seen, both expressions can lead here to a finite result, which is valid only with doubly logarithmic accuracy in ξ, since, on the one hand, the integral over q contains a logarithmic contribution of values $q^2 \sim y$, and, on the other, the integral over y contains a logarithmic contribution of values $y \sim \gamma$. Restricting ourselves in this connection to determining the leading contribution in ξ and noting that to this accuracy we can make the substitution

$$\int_0^\infty dx \frac{\exp(-x/2\gamma)}{(p-p_-)^2-x} \to -\ln\xi, \quad (9)$$

since the effective contribution to the integral over p is made by the region $p^2 \lesssim \max|k^2, m^2|$, for the total contribution of the two mass diagrams we obtain the expression

$$\tilde{\Lambda}_m^\mu \approx -\frac{m\alpha^2}{2\pi^4} \ln\xi \int_0^\infty dy \exp(-y/2\gamma) \int d^2p\, d^2q$$

$$\times \frac{(p^2+m^2)(p-k)_\nu - 2m^2 p_\nu}{(p^2-m^2)^2(q^2-m^2)[(q-p)^2-y][(p-k)^2-m^2]} \delta^{\mu\nu}, \quad (10)$$

which, as can be seen, leads in its turn to a contribution only to the form factor $g(t)$. Hence, in accordance with the definition (1) in the nonrelativistic limit $t \ll m^2$ we find the following expression for the contribution of diagrams of this type to the form factor $g(t)$:

$$g_m(0) \approx -(\alpha/2\pi)^2 \ln^3\xi. \quad (11)$$

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B. Correction from vertex diagrams of type c

The contribution of these diagrams to $\bar{\Lambda}_\mu$ can be readily obtained by using the general formulas of Sec. 2. For one of them, the result has the form

$$\bar{\Lambda}_{vi}{}^\mu = -\frac{\alpha^2}{(2\pi)^4}\int_0^\infty dx\, dy \exp\left(-\frac{x+y}{2\gamma}\right)\int d^2p\, d^2q\, \frac{\bar{\gamma}^v(\check{p}+m)}{p^2-m^2}$$

(12)

$$\times \frac{\bar{\gamma}^\mu(\check{p}-\check{k}+m)\bar{\gamma}^v(\check{q}+m)\bar{\gamma}_v(\check{q}-\check{p}+\check{p}_-+m)\bar{\gamma}_\varepsilon}{[(p-k)^2-m^2](q^2-m^2)[(q-p+p_-)^2-m^2][(p-p_-)^2-x][(p-k-q)^2-y]}.$$

The contribution of the second diagram is obtained from (12) by the well-known replacements and permutations.

Because of the property of the $\bar{\gamma}$ matrices, namely, $\bar{\gamma}^\varepsilon \check{Q}\bar{\gamma}_\varepsilon = 0$ when there is an odd number of $\bar{\gamma}$ matrices in the operator $\check{Q}$, the numerator in (12) and the analogous second expression contains after the necessary transformations not higher than the third power of the virtual momentum p and the first power of q. Therefore, the effective contribution to the integrals over p and q is made by the regions $q^2, p^2 \lesssim \max|k^2, m^2|$, which agrees with the condition of applicability of the two-dimensional approximation of QED. Thus, in contrast to the preceding case, the expression (12) will correctly give all powers of $\ln\xi$. By virtue of the same circumstance, in the leading order in $\ln\xi$, the integrals over x and y in (12) can be replaced by $\ln^2\xi$ [see (9)], to which the required correction will be proportional.

C. Correction from diagrams of type d and e

The contribution of diagram d, which is also obtained on the basis of the general formulas of Sec. 2, has the form

$$\bar{\Lambda}_{vi}{}^\mu = -\frac{\alpha^2}{(2\pi)^4}\int_0^\infty dx\, dy \exp\left(-\frac{x+y}{2\gamma}\right)\int d^2p\, d^2q$$

$$\times \frac{\bar{\gamma}^v(\check{p}+m)\bar{\gamma}^\varepsilon(\check{q}+m)\bar{\gamma}^\mu(\check{q}-\check{k}+m)\bar{\gamma}_\varepsilon(\check{p}-\check{k}+m)\bar{\gamma}_v}{(p^2-m^2)(q^2-m^2)[(q-k)^2-m^2][(p-k)^2-m^2][(q-p)^2-y][(p-p_-)^2-x]}$$

(13)

and that of diagram e has the form

$$\bar{\Lambda}_{\text{irr}}{}^\mu = -\frac{\alpha^2}{(2\pi)^4}\int_0^\infty dx\, dy \exp\left(-\frac{x+y}{2\gamma}\right)\int d^2p\, d^2q\, \frac{\bar{\gamma}^v(\check{p}+m)}{p^2-m^2}$$

$$\times \frac{\bar{\gamma}^\varepsilon(\check{q}+m)\bar{\gamma}^\mu(\check{q}-\check{k}+m)\bar{\gamma}_v(\check{q}-\check{p}-\check{k}+\check{p}_-+m)\bar{\gamma}_\varepsilon}{(q^2-m^2)[(q-k)^2-m^2][(q-p-k+p_-)^2-m^2][(p-p_-)^2-x][(q-p)^2-y]}.$$

(14)

An analysis of the expressions (13) and (14) similar to the one made in subsections A and B shows that the main terms in (13) and (14) have the order $\ln^3\xi$. However, there is no need to calculate them explicitly, since after summation they are compensated (in the framework of the chosen approximation). Therefore, the total contribution to Λ^μ of these diagrams will not be higher than $\ln^2\xi$.

4. CONCLUSIONS

It follows from the above analysis that at the given step in the expansion the logarithmic contributions to the form factor $f(t)$ do not exceed $\alpha^2 \ln^2\xi$; the logarithmic contribution to the form factor $g(t)$ is determined in the nonrelativistic approximation by the expression (11). Since the contribution of the diagrams of third order to $g(t)$ has the order $\alpha \ln\xi$, the results obtained here do not as a whole change the conclusions drawn earlier about the limits of applicability of perturbation theory for $B \gg B_0$.

It would be interesting to elucidate the nature of the convergence of the compact diagrams of QED in the next step of the expansion, i.e., to investigate the sixth-order diagrams for the mass and polarization operators and the seventh-order diagrams for the vertex function. The method for writing down analytically the corresponding matrix elements by means of the general formulas used in Sec. 2 is fairly obvious, and the idea of the approximate calculations with separation of the leading powers of $\ln\xi$ illustrated above considerably simplifies the calculations. At this stage we can note that factorization of the leading asymptotic behavior in the field occurs in the presence in the complex diagram of only "mass" and "vertex" insertions in the main skeleton diagram,[6,10,11] and in the presence of "polarization" insertions there is no factorization.[9,10]

[1] In Ref. 5, the four-rowed matrices γ_μ were used. However, there is actually no need for this, since the electron Green's function in this approximation is proportional to the matrix $(1-\sigma_3)/2$, and the four-spinor $u(p)$ is an eigenfunction of this matrix.

[2] The normalization of the four-spinor $u(p)$ is the same; the density matrix ρ in four-dimensional notation has the form $\frac{1}{2}(\check{p}+m)(1-\sigma_3)$; in Eq. (2c) in Ref. 5, the factor $(1-\sigma_3)$ was omitted.

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