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Phasing of atomic velocities in the field of a traveling electromagnetic wave

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Resonant collisionless gas is considered in the field of a traveling monochromatic wave. The presence of a small parameter $\mu = \hbar k^2/m\gamma_\perp$ in the problem makes possible an asymptotic analysis of the equations that describe the action of the radiation pressure on the gas. It is shown that the selective character of the radiation force leads to a decrease of the random scatter of the atom velocities. The lower limit of the characteristic width $\delta v \sim (2\hbar\gamma/3m)^{1/2}$, of the nonequilibrium structure on the distribution function determined by the competition between the processes of phasing of the velocities and the diffusion spreading, is obtained. The possibility of eliminating the translational motion of the resonant ions in crossed optical and magnetic fields is noted.

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1. INTRODUCTION

Cooling of a gas in the field of a standing electromagnetic wave was predicted and analyzed in detail from various points of view in Refs. 1–9. In the present paper we show that in the field of a traveling electromagnetic wave it is also possible to eliminate in part the random motion of the atoms of a resonant gas and that a number of distinguishing features appear which are not encountered in the case of opposing waves. Observation of cooling in the field of a traveling electromagnetic wave is apparently simpler from the point of view of experimental realization.

When the action of resonant radiation pressure (RP) in the field of the plane traveling wave is considered, one must bear in mind the following aspects: the translational motion of the gas as a whole, the phasing of the atoms in velocity space, which leads to cooling, and diffusion in velocity space. The first of these factors is quite obvious. Induced absorption and emission of photons are characterized by a preferred direction, whereas scattered photons are on the average isotropically distributed.

Phasing in velocity space is due to the inhomogeneous character of the force of the spontaneous radiation pressure in the sense of its sharp dependence on the velocity, which manifests itself most strongly in weak fields ($|d\mathbf{E}_0|/\hbar<\gamma$). The expression for the force in the case of exact tuning to resonance takes in this case the form

$$F\approx\hbar k\gamma\frac{1}{x^2+1}\frac{|d\mathbf{E}_0|^2}{\hbar^2\gamma_\perp^2},\tag{1}$$

where $x=kv_x/\gamma_\perp$, v_x is the velocity of the atom in the direction of propagation of the radiation, k is the wave vector, $\gamma_\perp$ is the transverse relaxation rate, d is the dipole-moment matrix element, and $\mathbf{E}_0$ is the amplitude of the field. It prevails for the resonant particles ($k|v_x|\lesssim\gamma_\perp$) and decreases rapidly with increasing Doppler shift.

We consider, at the instant of time t_0, two particles with mass m and somewhat different velocities $x_1, x_2 (x_1-x_2=\delta x(t_0))$ in the region $x>0$. Then, obviously, at the instant of time t we have

$$\delta x(t)\approx\delta x(t_0)\exp\left(F'(x)m^{-1}(t-t_0)\right).\tag{2}$$

Since $F'(x)<0 (x>0)$, the distance between the chosen particles in velocity space decreases with time, and monochromatization in velocity (phasing of the velocities) takes place as a result. This can be easily understood with the aid of the following simple arguments. The atoms having a smaller velocity projection on the radiation-propagation direction and accordingly a smaller Doppler shift, move more rapidly in velocity space under the influence of the RP than the atoms having a larger velocity projection. Therefore the "slow" atoms ($x<1$) gradually overtake the "fast" ones ($x>1$), forming a narrow structure in the velocity distribution. This was first pointed out in Ref. 10, where the translational disequilibrium under the action of RP was in-

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vestigated. The possibility of phasing the velocities by an RP force of mixed type in the field of a standing wave was noted in Ref. 1, and the influence of optical nutation without allowance for relaxation on the translational motion of the atoms in a traveling wave is the subject of Ref. 11.

It should be noted that the competing process that hinders infinite phasing is diffusion in velocity space due to fluctuations of the RP force.[1] Allowance for this circumstance is decisive for the determination of the maximum narrowing of the atom distribution in the velocity.

2. LIMITING CHARACTERISTICS OF THE PHASING

The evolution of the distribution function (DF) of the atoms in velocity in the field of a traveling electromagnetic wave, over times exceeding the phase relaxation time ($t>\gamma_\perp^{-1}$), and at exact resonance, is described by equations that follow from the equations for the density matrix after eliminating the off-diagonal elements,[4,7] namely

$$df_2(\mathbf{v},t)/dt=-\gamma f_2+IL(\mathbf{v}-\hbar\mathbf{k}/2m)\,[f_1(\mathbf{v}-\hbar\mathbf{k}/m)-f_2(\mathbf{v})\,], \tag{3}$$

$$df_1(\mathbf{v},t)/dt=\gamma\int d\mathbf{n}F(\mathbf{n})f_2(\mathbf{v}+\hbar k\mathbf{n}/m)$$
$$-IL(\mathbf{v}+\hbar\mathbf{k}/2m)\,[f_1(\mathbf{v})-f_2(\mathbf{v}+\hbar\mathbf{k}/m)\,],$$
$$L(v)=L(kv/\gamma_\perp)=1/(x^2+1), \tag{4}$$

$f_1(\mathbf{v},t)$ and $f_2(\mathbf{v},t)$ are the distribution functions of the atoms in the lower and upper states (the diagonal elements of the density matrix in the Wigner representation), L is the form factor of the spectral line, $F(\mathbf{n})$ is the probability of emission of a quantum in the direction $\mathbf{n}$, and I is the rate of the induced transitions. In (3) and (4) we take into account both the transfer of momentum from the radiation and the shift of the spectral line as a result of the recoil.

Let $\mathbf{k}=\mathbf{e}_x k$, and let the distribution of the scattered photons be isotropic:

$$F(\mathbf{n})\,d^2\mathbf{n}=d\Omega/4\pi.$$

We consider the distribution of the atomic velocities along the x axis, and then the corresponding DF are equal to

$$f_i(v_x,t)=\int f_i(\mathbf{v})\,dv_z\,dv_y.$$

In weak saturation ($I\ll\gamma,\ f_2\ll f_1$), when the selectivity of the radiation force is maximal, we obtain in the quasistationary situation[4] from (3) and (4)

$$\frac{df_1(v_x,t)}{dt}=IL\left(v_x-\frac{\hbar k}{2m}\right)f_1\left(v_x-\frac{\hbar k}{m}\right)-IL\left(v_x+\frac{\hbar k}{m}\right)f_1(v_x)$$
$$+(\hat{\gamma}-\gamma)\gamma^{-1}\left[f_1\left(v_x-\frac{\hbar k}{m}\right)IL\left(v_x-\frac{\hbar k}{2m}\right)\right], \tag{5}$$

$$\gamma f_1\left(v_x-\frac{\hbar k}{m}\right)L\left(v_x-\frac{\hbar k}{2m}\right)$$
$$=\frac{\gamma}{4\pi}\int\sin\theta\,d\theta\,d\varphi\,f_1\left(v_x-\frac{\hbar k}{m}+\frac{\hbar k}{m}\cos\theta\right)L\left(v_x-\frac{\hbar k}{2m}+\frac{\hbar k}{m}\cos\theta\right). \tag{6}$$

We change over to the dimensionless variables $x=kv_x/\gamma_\perp$, $\tau=I\mu t$, $\mu=\hbar k^2/m\gamma_\perp$ and introduce the auxiliary function

$$\varphi(x,\tau)=L(x+\mu/2)f_1(x,\tau). \tag{7}$$

Then, taking into account the smallness of the parameter μ, we get from (5) the following equation [$\mu\sim(10^{-3}-10^{-4})$ for optical transitions]:

$$\frac{\partial\varphi}{\partial\tau}+L(x)\frac{\partial\varphi}{\partial x}-\frac{2}{3}\mu L(x)\frac{\partial^2\varphi}{\partial x^2}+\mu\frac{L'(x)}{2}\frac{\partial\varphi}{\partial x}+R\left(\frac{\partial^i\varphi}{\partial x^i},x,\mu\right)=0,\quad i\geqslant 1,$$
$$\varphi(x,0)=f_0(x)\left[L(x)+\frac{\mu}{2}L'(x)+O(\mu^2)\right], \tag{8}$$

where $R=O(\mu^2)$ [by $O(\mu^\alpha)$ we denote the terms $\sim\mu^\alpha$]. We assume henceforth an equilibrium initial velocity distribution, therefore

$$f_0(x)=\frac{\exp[-(x/a)^2]}{\pi^{1/2}a}.$$

We note that if we discard the terms $\sim\mu^2$ in (9), then this equation becomes the Fokker–Planck equation.[1,12]

We consider first the solution of the degenerate equation ($\mu=0$)

$$\frac{\partial\varphi_0}{\partial\tau}+\frac{1}{1+x^2}\frac{\partial\varphi_0}{\partial x}=0,\qquad \varphi_0(x,0)=\frac{f_0(x)}{1+x^2}. \tag{9}$$

It takes the form

$$\varphi_0(x,t)=\Phi(1/3x^3+x-\tau), \tag{10}$$

where Φ is determined by the initial condition $\Phi(\frac{1}{3}x^3+x)=\varphi_0(x,0)$. For short irradiation times $\tau\ll 1$ and for $a\gg\gamma$ we obtain from (10)

$$f_1(x,\tau)\approx\frac{\exp[-(x/a)^2]}{\pi^{1/2}a}\left[1+\frac{2x\tau}{(1+x^2)^2}\right]. \tag{11}$$

Thus, even at short irradiation times a nonequilibrium structure is produced on the DF and consists of a dip with a center at $x\approx -1\sqrt{3}$ and with a peak centered at $x\approx 1/\sqrt{3}$ and of width ~ 1. In the course of the subsequent evolution, the dip broadens and the peak narrows down and moves in the radiation-propagation direction. This is clearly seen in Figs. 1–3 which show the numerical solution of the degenerate equation (9) for different detunings $\Delta'=(\omega-\omega_{21})/\gamma_\perp$. Analyzing (10), we easily note that $\varphi_0(x,\tau)$ has a sharp maximum at $x=x_0(\tau)$ where $x_0(\tau)$ satisfies the equation

$$1/3x_0^3+x_0=\tau. \tag{12}$$

$\tau\to\infty$ we have $x_0(\tau)\sim(3\tau)^{1/3}$, therefore in the region of the nonequilibrium peak we have

$$\varphi_0(x,\tau)\approx\Phi((1+x_0^2)y),\quad y=x-x_0(\tau).$$

Thus, the width of the sharp nonequilibrium peak on the DF decreases with time like $\sim(x_0^2+1)^{-1}$, while for the wings of the peak ($1/x_0^2\lesssim y<1/x_0$) the following expres-

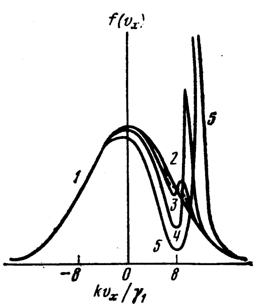

FIG. 1. $t=0$ (1), $0.2\omega_R^{-1}$ (2), ω_R^{-1} (3), $5\omega_R^{-1}$ (4), $25\omega_R^{-1}$ (5); $|dE_0|/\hbar=\gamma_\perp/2$; $a=10$, $\gamma_\perp=\gamma/2$, $\Delta=8$

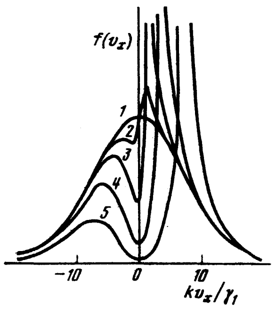

FIG. 2. $t=0$ (1), ω_R^{-1} (2), $5\omega_R^{-1}$ (3), $25\omega_R^{-1}$ (4), $125\omega_R^{-1}$ (5); $|dE_0|/\hbar=\gamma_\perp/2$, $a=10$, $\gamma_\perp=\gamma/2$, $\Delta=0$

sion is valid:

$$\varphi_0(x_0, y)\approx\exp\left[-(x_0^2 3y)^{2/3}/a^2\right]\pi^{-1/2}a^{-1}((3yx_0)^{2/3}+1)^{-1}. \tag{13}$$

Thus, the degenerate solution of Eq. (8) leads to an infinitesimally narrow DF, but in the course of time the action of the diffusion becomes noticeable. Mathematically this manifests itself in the increase of the term $\sim\mu\partial^2\varphi/\partial x^2$ in (8), which can become comparable with the first two terms because of the rapid change of the DF in the region of the maximum. This takes place only in the region of the nonequilibrium structure, so that to take the diffusion into account it is convenient to change over to a coordinate system that moves together with the peak, by making the change of variables

$$x-x_0(\tau)=y, \quad \tau=\tau'.$$

In addition, we introduce the dilatation transformation $y_1=y/\mu^{1/2}$, which takes into account the rapid change of the DF near the maximum. Then the evolution of the nonequilibrium structure, with account taken of the diffusion spreading, is described by the following equation obtained from (8) by taking the limit ($\mu^{1/2}\to 0$, y_1 is fixed):

$$\frac{\partial\varphi}{\partial\tau}-\frac{2x_0y_1}{(1+x_0^2)^2}\frac{\partial\varphi}{\partial y_1}-\frac{2}{3}\frac{1}{(1+x_0^2)}\frac{\partial^2\varphi}{\partial y_1^2}=0. \tag{14}$$

Equation (14) can be obtained also without resorting to the procedures employed in modern asymptotic analysis,[13] by assuming that after a sufficiently long time peak (of width $\delta y\ll 1$) becomes so narrow that the photon absorption cross section $\sim L(x)$ for all the atoms grouped in the peak will differ insignificantly, and the linear terms of the expansion of $L(x+y)$ in y are sufficient. Determining the mean squared scatter of the velocities in the region Ω of the nonequilibrium struc-

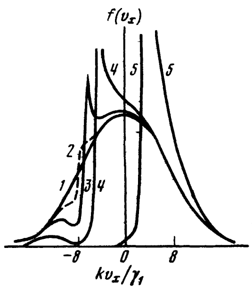

FIG. 3. $t=0$ (1), ω_R^{-1} (2), $5\omega_R^{-1}$ (3), $25\omega_R^{-1}$ (4), $500\omega_R^{-1}$ (5); $|dE_0|/\hbar=\gamma_\perp/2$, $a=10$, $\gamma_\perp=\gamma/2$, $\Delta=-8$

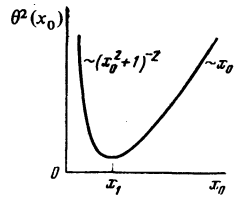

FIG. 4.

ture by means of

$$\int_\Omega y_1^2 f_1(y_1,\tau)\,dy_1\Big/\int_\Omega f_1(y_1,\tau)\,dy_1=\langle y_1^2\rangle=\theta^2(x_0) \tag{15}$$

and assuming that the limit of the integration region does not make a substantial contribution to the integrals (very sharp and high peak), we obtain from (14), taking (7) into account, the differential equation

$$d\theta^2(x_0)/dx_0=-4\theta^2(x_0)x_0/(1+x_0^2)^2+4/3, \tag{16}$$

which has the following solution:

$$\theta^2(x_0)=\frac{C+{}^4/_3(x_0+{}^2/_3x_0^2+{}^1/_5x_0^4)x_0}{(1+x_0^2)^2}, \tag{17}$$

C is the integration constant. Figure 4 shows the dependence of the width of the peak on the position of its center x_0. Thus, the diffusion leads ultimately to a smearing of the nonequilibrium structure on the DF (demonochromatization of the atomic velocities). There exists an instant of time $\tau=t_1$, $x_1=x_0(t_1)$ when maximum phasing is reached. Recognizing that in this case

$$\left.\frac{d\theta^2(x_0)}{dx_0}\right|_{x_0=x_1}=0,$$

we obtain from (16)

$$\theta^2(x_1)={}^1/_3(1/x_1+x_1)\geqslant\theta_m^2={}^2/_3. \tag{18}$$

Changing over to physical variables, we can write down the following inequality for the characteristic width of the nonequilibrium peak on the DF:

$$\delta v\geqslant(2\hbar\gamma/3m)^{1/2}. \tag{19}$$

We note that in essence an analogous expression for the minimum scatter of the velocities was obtained for the case of a standing wave.[5,9] This seems to be some general law for the limiting characteristics of gas cooling by radiation pressure due to the spontaneous emission. We note that (19) agrees in form with the fundamental quantum-mechanical uncertainty relation for the time and energy $\Delta E\Delta t\geqslant\hbar$, if $\Delta E=\frac{1}{2}m(\delta v)^2$ and $\Delta t\sim\gamma^{-1}$.

The constant C in (17) is determined from the obvious joining principle: at sufficiently short times (when the diffusion spreading has not yet manifested itself) the width of the nonequilibrium peak should coincide with the values given by the solution of the degenerate equation, therefore $C=C_0/\mu$, where $C_0\sim 1$. It follows then from (17) that the limiting narrowing of the peak will be observed as soon as its center leaves the resonance region $x_0\sim\mu^{-1/5}$. The irradiation time needed for this purpose is determined from the relation (12).

To determine the structure of the peak with allowance for the initial conditions and diffusion in the far field ($\mu x_0^5\geqslant 1$), we use the method of joining together asymptotic expansions[13] (see the Appendix). The principal term of the asymptotic expansion is

$$\varphi(y,x_0)=\int_{-\infty}^{\infty}\Phi(\xi)G\left(\xi-\eta(x_0,y),\frac{2}{3}\mu g(x_0)\right)d\xi, \qquad (20)$$

$$G(\xi,\tau)=\frac{\exp[-\xi^2/4\tau]}{2\pi^{1/2}\tau^{1/2}},\quad \eta(x_0,y)=(1+x_0^2)y+x_0y^2+\frac{1}{3}y^3,$$

$$g(x_0)=\mu\int_0^{\infty}(1+x_0^2)^2\,dx_0.$$

It follows from (20) that at short irradiation times [near field, $\mu g(x_0)\ll 1$] we have

$$\varphi(y,x_0)\approx\Phi(\eta)$$

[this corresponds to the degenerate solution (10)], and at long times (far field), when $\mu g(x_0)\gtrsim a$, the DF in the region of the peak does not depend on the initial distribution and is given by

$$\varphi\approx\exp\left[-15y^2/8x_0\mu\right]/({}^8/_3 g(x_0)\mu\pi)^{1/2}. \qquad (21)$$

Consequently, in the far field the width of the peak is $\sim(\langle y^2\rangle)^{1/2}\sim(x_0\mu)^{1/2}$ in full agreement with (17).

We call attention to the fact that the spreading of the inhomogeneous peak as the result of the diffusion is a very slow process, since $\theta(x_0)\sim x_0^{1/2}\sim\tau^{1/6}$. At the same time in the near zone, where the velocity phasing predominates, we have $\theta(x_0)\sim x_0^{-2}\lesssim\tau^{-2/3}$.

3. MOTION OF RESONANT IONS IN CROSSED OPTICAL AND MAGNETIC FIELDS

As shown in the preceding section, when a field of a traveling electromagnetic wave acts on a collisionless gas, the random velocity decreases and the directional velocity increases. It is known that the translational motion of the particle can be limited by ionizing it and placing it in a magnetic field. It is of interest to consider the joint action of optical and magnetic fields on a resonant ion, since this uncovers additional possibilities of controlling the translational degrees of freedom of an ionized gas.

The action of the magnetic field in this case is twofold. First, it influences directly the acceleration of the ion and enters in the form of the Lorentz force into the equation for the DF. Second, it changes the contour of the spectral line and therefore enters in the equation for the polarization and the population difference. We now estimate the influence of each of these factors. If the characteristic dimensions of the region of the sharp change of the DF is $\delta v\ll\gamma/k$, then we can show, by using the procedure of Ref. 14, that the magnetic field in the first case must be taken into account if $\omega_L\sim\omega_1=\hbar k\gamma_\perp/mv$ (ω_L is the Larmor frequency), and in the second case, when $\omega_L\sim\omega_2=\gamma_\perp/kv$, the ratio is

$$\omega_1/\omega_2=\omega_R/\gamma_\perp=2\mu\ll 1,\quad \omega_R=\hbar k^2/2m,$$

and therefore for a magnetic field $H<mc_2/e$ its influence on the contour of the spectral line can be neglected. (For the limiting characteristics of the cooling by the optical field $\omega_1/\omega_2\sim\mu^{1/2}\ll 1$. We confine ourselves to consideration of an individual particle, ignoring the fluctuations of the radiation force. The equations of motion take the form (the magnetic field is directed along the z axis, $\gamma/\gamma_\perp=2$)

$$\frac{dv_x}{dt}=\frac{F(v_x)}{m}+\omega_L v_y,\quad F(v_x)=\hbar k\gamma_\perp\frac{|dE_0|^2}{\hbar^2[(\Delta-kv_x)^2+\gamma_\perp^2]+2|dE_0|^2}, \qquad (22)$$

$$dv_y/dt=-\omega_L v_x. \qquad (23)$$

Differentiating (22) with respect to t and using (23), we obtain the equation of an oscillator with friction. For $|\Delta|\gg|kv|$, $\gamma_\perp$, we can easily find its solution

$$v_x=A_0\exp({}^1/_2\alpha\Delta t)\sin(\Omega t-\varphi), \qquad (24)$$

$$\alpha=2\frac{\hbar k^2\gamma}{m}\frac{|\mathbf{d}E_0|^2\hbar^2}{2|dE_0|^2+\hbar^2\Delta^2},\quad \Omega=(\omega_L^2-\alpha^2\Delta^2/4)^{1/2},$$

$$A_0=\left(v_x^2(0)+\frac{1}{\Omega^2}\left[\frac{\hbar k\gamma}{m}\frac{|\mathbf{d}E_0|^2}{\hbar^2\Delta^2+2|\mathbf{d}E_0|^2}-\frac{\alpha\Delta}{2}v_x(0)\right]^2\right)^{1/2},$$

$$\operatorname{tg}\varphi=v_x(0)\Omega\left[\frac{\hbar k\gamma}{m}\frac{|dE_0|^2}{2|dE_0|^2+\hbar^2\Delta^2}-\frac{\alpha\Delta}{2}v_x(0)\right]^{-1}.$$

It follows from (24) that in an optical field crossed with a magnetic field the projections of the velocities of all the ions on the x axis oscillate at the same frequency and with an amplitude that decreases ($\Delta<0$) or increases ($\Delta>0$), depending on the sign of the detuning. In this respect the situation is similar to the case of a standing wave, when the resonant particle is decelerated at $\Delta<0$ regardless of the velocity direction.

The Larmor precession in a plane perpendicular to the direction of the magnetic field causes the particle to interact with the optical field, in the course of one half-period of the motion on its circular orbit, more strongly than during the second half-period, when it goes off-resonance because of the Doppler shift. Therefore along the direction of the electromagnetic wave the ions are decelerated at $\Delta<0$ regardless of the sign of the initial projection of the velocity and are bunched in velocity space near $v_x=0$.

Since the magnetic field constantly rotates the component of the velocity vector $\mathbf{v}$ in the xy plane, the radiation pressure acts also on the projection of the velocity v_y in a direction perpendicular to the propagation of the radiation and to the magnetic field:

$$v_y=v_y(0)-\frac{\omega_\pi}{\Omega}\frac{A_0}{(\alpha\Delta/2\Omega)^2+1}\left\{\exp\left(\frac{\alpha\Delta}{2}t\right)\left[\frac{\alpha\Delta}{2\Omega}\sin(\Omega t-\varphi)-\cos(\Omega t-\varphi)+\frac{\alpha\Delta}{2\Omega}\sin\varphi+\cos\varphi\right]\right\}. \qquad (25)$$

The ion deceleration in crossed fields can be used to eliminate the Doppler broadening of the spectral line. When there is no optical field, this is possible only for strong fields ($\omega_L\gg kv$) on account of the Dicke effect.[15]

CONCLUSION

The phasing of the atomic velocities by the field of a traveling electromagnetic wave, considered in the present article, extends the possibility of controlling the translational degrees of freedom of a gas with the aid of light and can be used, in particular, for monochromatization of longitudinal velocities in a molecular beam without substantially changing its intensity. The mean squared scatter δv of the projections of the velocities can in this case not be less than a value determined by the lifetime of the upper state $\delta v_m\sim(\hbar\gamma/m)^{1/2}$. For example, for the Sr^{88} atom ($\lambda = 4607$ Å, transition $5\,{}^1S_0 - 5\,{}^1P_1$):

$$2\omega_R = \hbar k^2/m \sim 22\text{ kHz},\quad \gamma_\perp \sim 10^7\text{ Hz},\quad I/\gamma_\perp \sim 10^{-1},\quad \mu = 10^{-3},$$

the characteristic time of peak formation on the DF is $t \sim 5\times10^{-4}$ sec, and the limiting width $\delta v_m \sim 3$ cm/sec is reached after $t \sim 5\times10^{-2}$ sec. After the lapse of a time $t \sim 1$ sec, when diffusion in velocity space becomes noticable, the peak is still quite narrow: $\delta v \sim 7$ cm/sec $\ll \gamma_\perp/k = 100$ cm/sec. We note that the increase of the radiation power leads to a shortening of the time of formation of the peak and increases the number of cooled particles, but because of the saturation of the radiation-pressure force ($|\mathbf{dE}_0|/\hbar \gtrsim \gamma_\perp$) the minimal velocity scatter increases:

$$\delta v_m \sim [\hbar\gamma(1+G_1)/m]^{1/2}$$

(G_1 is the saturation parameter).

New possibilities are uncovered also for the control of the translational motion of ions in the combined action of resonant light and a magnetic field. Here, for example, to eliminate the Doppler broadening the fields required are $\sim 10^2$ Oe ($\omega_L \geqslant \omega_R > \alpha\Delta$), whereas the use of the Dicke effect for these purposes[15] calls for fields $\sim 10^5$ Oe ($\omega_L > |kv|$).

The variety of effects of resonant radiation pressure makes it possible to change the state of the translational degrees of freedom of the gas in a very wide range of directions and points with optimism to prospects for using these effects for both scientific and applied purposes. Further progress in this direction will be brought about by the organization of new experiments.

APPENDIX

We now derive the asymptotic expression (20). We introduce to this end the new variables

$$\eta = (1+x_0^2)y + x_0y^2 + y^3/3,\quad x_0 = x_0(\tau). \tag{A.1}$$

Then, taking into account the following obvious relations

$$\frac{\partial}{\partial y} = [1+(x_0+y)^2]\frac{\partial}{\partial\eta},$$
$$\frac{\partial^2}{\partial y^2} = [1+(x_0+y)^2]^2\frac{\partial^2}{\partial\eta^2} + 2(x_0+y)\frac{\partial}{\partial y},$$
$$\frac{dx_0}{d\tau} = \frac{1}{x_0^2+1}, \tag{A.2}$$

we rewrite (8) in terms of the variables η and x_0:

$$\frac{\partial\varphi_0}{\partial x_0} = \frac{2}{3}\mu(1+x_0^2)[1+(x_0+y)^2]\frac{\partial^2\varphi}{\partial\eta^2} + \frac{7}{3}\mu\frac{(1+x_0^2)(x_0+y)}{1+(x_0+y)^2}\frac{\partial\varphi}{\partial\eta} + O(\mu^2) = \mu\hat{M}\varphi + O(\mu^2). \tag{A.3}$$

In the near field (region of small x_0) the direct asymptotic expansion (we are considering the limiting process $\mu\to0$, with x_0 and η fixed) of the solution of (3) can be sought in the form

$$\varphi = \varphi_0 + \mu\varphi_1 + \ldots\,. \tag{A.4}$$

The expansion (A.4) generates a sequence of linear problems

$$\partial\varphi_0/\partial x_0 = 0,\quad \partial\varphi_1/\partial x_0 = \hat{M}(\varphi_0), \ldots, \tag{A.5}$$

which must be solved under boundary and initial conditions corresponding to (8). The expansion (A.4) becomes non-uniform in the far field, when $\mu x_0^5 \sim O(1)$. Thus, the direct asymptotic expansion is not valid after a sufficiently large time interval, when the nonequilibrium peak goes out of the resonance region ($x_0 > 1$). This is due to the cumulative character of the manfestation of diffusion.

To construct a solution suitable in the far field up to $\mu x_0^5 \geqslant 1$, we introduce the slow variable $\mu g(x_0) = s$ and consider another limiting process: η and s are fixed and $\mu\to0$, where

$$s = \mu g(x_0) = \int_0^{x_0} \mu(1+x_0^2)^2\,dx_0. \tag{A.6}$$

Equation (A.3) is transformed into the following (with account taken of the terms $\sim\mu^2$ in (A.3) and of the relation $y = u(x_0, \eta, \mu) = \tilde{u}(s, \eta, \mu) \sim \eta\mu^{2/5}/s^2$):

$$\partial\tilde{\varphi}(s,\eta)/\partial s = {}^2/_3\,\partial^2\tilde{\varphi}(s,\eta)/\partial\eta^2 + O(\mu^{1/5}). \tag{A.7}$$

Consequently, the asymptotic expansion of the solution in the far field takes the form

$$\tilde{\varphi} = \tilde{\varphi}_0(s,\eta) + \mu^{1/5}\tilde{\varphi}_1(s,\eta) + \ldots\,. \tag{A.8}$$

The principal term of the asymptotic expansion (A.8) satisfies the thermal-conductivity equation

$$\partial\tilde{\varphi}_0/\partial s = {}^2/_3\,\partial^2\tilde{\varphi}_0/\partial\eta^2. \tag{A.9}$$

The initial conditions for $\tilde{\varphi}_i$ are determined by joining together the asymptotic expansions (A.8) and (A.4). We consider the intermediate limit [η, $\alpha(\mu)g(x_0) = s_\alpha$ is fixed, $\alpha(\mu)\to0$], where $\mu \ll \alpha(\mu) \ll 1$.

$$\lim_{\mu\to0}(\mu/\alpha(\mu)) = 0.$$

The joining together of first order leads to the relation

$$\lim_{\mu\to0}\tilde{\varphi}_0(\alpha^{-1}(\mu)\mu S_\alpha, \eta) = \tilde{\varphi}_0(0,\eta) = \varphi_0(\eta) = \Phi(\eta). \tag{A.10}$$

Equation (A.10) yields the initial condition for (A.9). According to Kaplun's continuation theorem[13] the expansion (A.8) is uniformly suitable on the interval $|\eta| \leqslant \sigma(\mu)$, where $\sigma(\mu)\to\infty$. Therefore $\tilde{\varphi}_0(s,\eta)$ is unambiguously given by

$$\tilde{\varphi}(s,\eta) = \int_{-\infty}^{\infty}\Phi(\xi)G\left(\xi-\eta, \frac{2}{3}s\right)d\xi, \tag{A.11}$$

where $G(x, t)$ is the fundamental solution of the heat conduction equation. Changing over in (A.11) to the variables (x_0, y) we obtain (20). We note finally that inasmuch as in the near field ($\mu g(x_0) \ll 1$) the asymptotic expression (20) goes over directly into a degenerate solution φ_0, which is the first term of the direct asymptotic expansion, it is uniformly suitable in the near and far fields in the interval $0 \leqslant \mu g(x_0) \leqslant \sigma_1$ ($\sigma_1 \gg 1$). We point out also that the foregoing calculations show the validity of the Fokker–Planck approximation for the distribution function in the considered time intervals, since neglect of second-order terms in (8) does not change the principal terms of the asymptotic expansions and leads to (20).

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Excitation of ion oscillations by a freely propagating electron beam

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We calculate the mechanism and singularities of the excitation of ion oscillations in a plasma produced by a steady electron beam that propagates in a gas in the absence of an external magnetic field. It is shown that they are governed by the effect of spatial enhancement of system-charge-density perturbations that are almost transverse to the beam velocity. The established experimental facts are explained satisfactorily, in the main, on the basis of a dispersion relation that takes into account, on the one hand, the buildup of the ion oscillations by the cold beam, and on the other, the collisionless damping of these oscillations by the ions and electrons of the plasma.

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Ion oscillations excited in a neutralized electron beam were investigated in a number of studies both in the presence of an external longitudinal magnetic field and without such a field. Whereas for a beam in a magnetic field there exists a fairly complete picture of the excitation of the ion oscillations of different types,[4] the data obtained in the absence of a magnetic field are insufficient to establish on the basis of the results of the theory[6-8] the character of the oscillations and the processes that determine their onset in this case. Yet freely propagating beams are quite frequently encountered physical systems (electron-beam welding, active experiments in the upper atmosphere, their transport is apparently quite strongly subjected to action of oscillations produced in the beams.

We have experimentally investigated ion oscillations in a plasma that is produced by an electron beam passing in the absence of a magnetic field through a gas. The measurements were performed with a setup for electron-beam welding, comprising a metallic cubic chamber measuring $50\times50\times50$ cm, in which it was possible to vary the air pressure in the range from 8×10^{-5} to 1×10^{-3} Torr. In one of the steps of the measuring chambers we placed an electron gun. In almost all the measurements we used a diode focusing system that shaped a practically parallel beam of 2–3 mm diameter with current up to 250 mA and energy 10–20 keV. The only exceptions were experiments with variation of the beam diameter. The latter can be easily effected only by using guns with magnetic focusing lens; to be sure, this resulted in a beam with a clearly pronounced crossover and one could speak of a certain average beam diameter.

A system of three diaphragms with holes of 10 mm diameter, approximately coaxial with the beam was located 18 cm away from the gun anode. The outer diaphragms were grounded, and when necessary it was possible to apply to the inner diaphragm a high-frequency voltage to modulate the beam. The modulator length was 20 mm. The oscillations excited in the beam were received with a high-frequency probe or with a diaphragm-loaded collector. The signal was next applied to the input of an oscilloscope or an S-4-8 spectrum analyzer. Since the investigated beam carried a rather large specific power (up to 10^5 W/cm^2 in a welding beam), the probe was moved in the course of the measurements in a direction transverse to the beam, with linear velocity 1.6×10^3 cm/sec, to prevent

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