TABLE III

	N					N			
	1	2	3	4		1	2	3	4
$\eta_N^{pt}(1)$	–	0.0185	0.0372	0.0289	$\nu_N^{pt}(1)$	0.6	0.645	0.597	0.731
$\eta_N(1)$	–	0.0320	0.0332	0.0333	$\nu_N(1)$	0.620	0.626	0.625	0.628

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Oscillation of the conductivity of carriers with anistropic energy spectrum in a quantizing magnetic field

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A system of carriers is considered, having an anisotropic energy spectrum and scattered by randomly disposed attraction centers of finite radius $a \lesssim l (l = (c\hbar)/|e|H)^{1/2}$ is the magnetic length) in a quantizing magnetic field H. A new type of oscillations of the kinetic coefficients as a function of the magnetic field is observed. The oscillations are due to the dependence of the one-dimensional (on account of the magnetic field) scattering potential of each individual center on H and to the anisotropy of the effective mass of the carriers. The longitudinal and transverse conductivities of a gas of interacting electrons with soroidal equal surfaces ($m_1 = m_2 = m_\perp$, $m_3 = m_\parallel > m_\perp$) in a weak electric field $\mathbf{E}_0$ and in a quantizing magnetic field parallel to the spheroid axis $\mathbf{H} \| m_\parallel \| z$ are calculated. It is shown that both the longitudinal and transverse conductivities oscillate with changing magnetic fields, and the period is mainly $\propto H^{1/2}$. For definite values of the magnetic field intensity, this effect leads to a negative longitudinal magnetoresistance [L. S. Dubinskaya, Sov. Phys. JETP 29, 436 (1969); M. M. Aksel'rod et al., Phys. Stat. Sol. 9, k91, 1965)]. The possibility of experimentally observing the oscillations is discussed.

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1. We consider the spectrum of the states of an electron whose mass is highly anisotropic, $m_1 = m_2 = m_\perp \ll m_\parallel = m_3$, in a quantizing magnetic field $\mathbf{H} \| m_\parallel \| z$ and in a spherically symmetrical attraction field $U(r) < 0$ of finite radius $a \lesssim l$. In the absence of a center, the motion of the electron in the (x, y) plane perpendicular to H is quantized, and the energy difference between any two neighboring levels is $\hbar\omega_\perp$ ($\omega_\perp = eH/m_\perp c$). By virtue of the axial symmetry of the problem, at $U \neq 0$, the projection m of the orbital angular momentum of the electron on the direction of H is conserved. If the mixing of the levels $N = n + \frac{1}{2}(|m| + m)$ and $K = k + \frac{1}{2}(|m| + m)$ (Ref. 3) by the center is small[4]:

$$U_m^{kn} = \int_0^\infty \rho\, d\rho R_{km}(\rho)\, |U(\rho, 0)|\, R_{nm}(\rho) \ll \hbar\omega_\perp, \qquad (1)$$

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where R are the radial wave functions, n and k are the radial quantum numbers,[3] and $\rho=(x,y)$, then the potential U can be regarded as a perturbation for the motion in the (x,y) plane.

The electron motion (in the Landau band N with definite angular-momentum projection m) along H is determined by the potential averaged over the radial functions R_{nm} (Refs. 3, 4):

$$U_m^N(z)=\int_0^\infty \rho\, d\rho\, R_{nm}^2(\rho)\,U(\rho,z), \tag{2}$$

which can be represented at $a<l$ in the form

$$U_m^N(z)=(a^2/l^2)^{|m|+1}V_m(z)\sim H^{|m|+1},$$
$$V_m(z)=\frac{1}{m!}\int_0^\infty d\left(\frac{\rho^2}{2a^2}\right)\left(\frac{\rho^2}{2a^2}\right)^{|m|}U(\rho,z). \tag{3}$$

The potential $U_m(z)$ is an even function of z and decreases along z at characteristic distances on the order of the effective radius a of the field. If the well (3) is deep enough,

$$\hbar^2/m_\parallel a^2\ll U_m(0)\ll\hbar^2/m_\perp a^2, \tag{4}$$

then the one-dimensional potential (3) contains many levels. The right-hand side of the inequality (4) follows directly from (1) and is a more stringent condition than (1). With decreasing magnetic field intensity, the depth of the well (3) decreases in proportion to $\sim H^{|m|+1}$, i.e., the levels leave the well. When a level with zero energy appears in the well, the one-dimensional well becomes transparent.[3] It follows therefore that all the kinetic coefficients determined by the scattering should oscillate when H is varied. The effect is more pronounced the better the inequalities (1) and (4) are satisfied.

For an isotropic carrier mass, $m_\perp=m_\parallel=m^*$, the condition (4) is violated: if $U\lesssim\hbar^2/m^*a^2$, then the mixing of the levels is weak, but the one-dimensional well (3) is shallow and contains only one state,[4] and the corresponding level does not leave the well when H is changed, but if the center is strong, i.e., $U\gg\hbar^2/m^*a^2$, then the well (3) is deep, but the radial-motion levels are strongly intermixed in this case. In either case there are no oscillations. One should therefore expect these oscillations to be realized for a strongly anisotropic energy spectrum of the carriers, for example in semiconductors of the fourth group (see Sec. 5).

We calculate below the static conductivity of a gas of noninteracting electrons with ellipsoidal equal-energy surfaces in a quantizing magnetic field parallel to the spheroid axis, $\mathbf{H}\|m_\parallel$, in a weak electric field $\mathbf{E}_0$ and in longitudinal ($\mathbf{E}_0\|\mathbf{H}$) and transverse ($\mathbf{E}_0\perp\mathbf{H}$) fields. It is assumed that the electrons are scattered by randomly disposed attraction centers $U(r)$ of radius $a\lesssim l$. We shall show that both the longitudinal and the transverse components of the conductivity tensor oscillate with changing H, and that the oscillations are in the main periodic in $H^{1/2}$, in contrast to the Gurevich and Firsov magnetophonon resonance,[5] where the period of the oscillations is linear in H^{-1}. For different values of the magnetic field intensity, the effect indicated in the present paper leads to the negative longitudinal magnetoresistance investigated by Dubinskaya[1] for scattering by a long-range Coulomb potential, and by Aksel'rod *et al.*[2] for scattering by acoustic phonons, but it is of an entirely different character.

2. We assume that the scatterer concentration n_0 is small enough:

$$n_0 l^2\lambda_\parallel\ll 1 \tag{5}$$

($\lambda_\parallel=\hbar/(m_\parallel T)^{1/2}$, T the electron temperature), so that the neighboring centers do not influence the interaction of the electron with any scatterer. Inasmuch as in a quantizing magnetic field $\hbar\omega\gg T$ it is the lower Landau band $n=0$, $m\leq 0$ which is mainly populated, the analysis that follows pertains only to this band. A Boltzmann distribution function is assumed hereafter for the electrons with respect to the longitudinal energy in this band.[6]

The Schrödinger integral equation that describes the scattering of the electron of the ground Landau band $N=0$ in homogeneous magnetic field $A_x=-Hy$, $A_y=A_z=0$ by the potential $U(\mathbf{r})$ is given by (in the isotropic case $m_\parallel=m_\perp=m^*$, see the paper by Skobov[7])

$$\psi_{y_0p_z}(\mathbf{r})=\varphi_{0y_0p_z}(\mathbf{r})-\iint dV'G_{y_0'p}(\mathbf{r},\mathbf{r}')\psi_{y_0p_z}(\mathbf{r}')\,dy_0'\,dp\,U(\mathbf{r}'), \tag{6}$$

$$G_{y_0p}(\mathbf{r},\mathbf{r}')=\sum_{k=0}^{\infty}\frac{\varphi_{kpy_0}(\mathbf{r})\varphi^*_{kpy_0}(\mathbf{r}')}{(p^2-p_z^2)/2m_\parallel+\hbar\omega_\perp k-i\delta}, \tag{7}$$

where

$$\varphi_{0py_0}(\mathbf{r})=(2\pi\hbar)^{-1/2}e^{ipz/\hbar}\chi_{0y_0}(\rho)$$

is the wave function of the electron in a magnetic field with momentum $p\|\mathbf{H}$,[3] A is the vector potential, k, p, and y_0 are the Landau numbers,

$$\langle kpy_0|np'y_0'\rangle=\delta_{kn}\delta(y_0-y_0')\delta(p-p').$$

In the case of weak mixing of the Landau levels by the center it suffices to retain in (7) only the term pertaining to the zeroth band with $k=0$.

For the probability per unit time of the transition from the state y_0p_0 into the state $y_0'p$ we have from (6) and (7)

$$dW(y_0p_z\to y_0'p)=\frac{8\pi^3l^2}{V^{2/3}}\left|\int dV\psi_{y_0p_z}(\mathbf{r})U(\mathbf{r})\varphi^*_{0py_0'}(\mathbf{r})\right|^2\delta\left\{\frac{p^2-p_z^2}{2m_\parallel}\right\}dy_0'\,dp. \tag{8}$$

In the derivation of (8) we must change over from a delta-function normalization of φ to a normalization with one electron in the volume.

For a spherically symmetrical potential $U(r)$, Eq. (6) can be reduced to a system of uncoupled integral (or differential) one-dimensional equations, which describe the scattering of an electron with a definite momentum $p_z\|\mathbf{H}$ and with different m by a one-dimensional potential $U_m(z)$ (3):. We change over to a new function $g_{p_z}(z;\rho;\rho_1)$:

$$g_{p_z}(z,\rho,\rho_1)=\int_{-\infty}^{+\infty}dy_0\psi_{y_0p_z}(z,\rho)\chi^*_{0y_0}(\rho_1), \tag{9}$$

meaning physically a transition to a wave packet that describes an electron concentrated in a region of order l and with center at the point ρ_1. Such a packet constitutes a superposition of states with different projections m of the orbital angular momentum of the electron on the direction of H.

The inverse transformation is of the form

$$\psi_{y_0 p_z}(\mathbf{r}) = \int d^2\rho_1 \chi_{0y_0}(\rho_1) g_{p_z}(z, \rho, \rho_1). \tag{10}$$

Multiplying (6) by $\chi^*_{0y_0}(\rho_1)$ and integrating the resultant equation over all y_0, we arrive at the equation for the function g_{p_z}(9), whose solution, as can be easily verified by direct substitution, can be written in series form[1)]

$$g_{p_z}(z, \rho, \rho_1) = \frac{1}{2\pi l^2} \exp\left[-\frac{\rho^2+\rho_1^2}{4l^2} - \frac{i}{2l^2}(xy - x_1y_1)\right] \times \sum_{m=0}^{+\infty} \frac{1}{m!}\left(\frac{\rho\rho_1}{2l^2}\right)^m \exp[-im(\varphi-\varphi_1)] C_m(z), \tag{11}$$

where the coefficients C_m satisfy the equations

$$C_m(z) = \frac{\exp(ip_z z/\hbar)}{(2\pi\hbar)^{1/2}} + \frac{im_\parallel}{\hbar|p_z|} \int_{-\infty}^{+\infty} dz' C_m(z') U_m(z') \exp\left(\frac{i|p_z||z-z'|}{\hbar}\right), \tag{12}$$

$$U_m(z) \equiv U_m^{N=0}(z).$$

The integral equation (12) describes the one-dimensional problem of the scattering of a particle with momentum p_z and mass $m_\parallel$ by a potential $U_m(z) < 0$. To solve this equation we must specify the concrete form of the scattering potential $U(\mathbf{r})$. We solve below this equation for a model potential $U(\mathbf{r})$ specified in the form of a screened Coulomb potential.

3. We calculate now the longitudinal component σ_{zz} of the electric conductivity tensor in a magnetic field, following Ref. 1. The density of the longitudinal current in the magnetic field is

$$j_z = -eg \int_{-\infty}^{+\infty} dp_z \frac{p_z}{m} f_1(p_z) = \sigma_{zz} E_0, \tag{13}$$

where f_1 is an increment, linear in the field $\mathbf{E}_0$, to the equilibrium distribution function, while g is defined by the equation

$$n_e = g \int_{-\infty}^{+\infty} dp_z f_0(\varepsilon),$$

and n_e is the concentration of the conduction electrons; $f_0(\varepsilon)$ is the equilibrium distribution function with respect to the longitudinal energy $\varepsilon = p_z^2/2m_\parallel$. The function f_1 is a solution of the kinetic equation

$$eE_0 \frac{\partial f_0}{\partial p_z} = N_0 \int_{-\infty}^{+\infty} \frac{dy_0}{V^{1/2}} dW(y_0 p_z \to y_0 p)[f_1(p) - f_1(p_z)], \tag{14}$$

where $N_0 = n_0 V$ is the total number of scatterers in the volume V.

In the right-hand side of (14), after substituting (10) and (11) in Eq. (8), we can carry out the integration, in general form, with respect to the coordinates of the center of the electron orbit in the initial and final states y_0 and y_0'. The integration with respect to p can also be carried out, owing to the presence of the δ function, after which Eq. (14) reduces to an algebraic one whose solution enables us to write the function f_1 in the form

$$f_1(p_z) = \frac{eE_0\hbar|p_z|p_z}{8\pi^2 n_0 l^2 m_\parallel^2}\left(-\frac{\partial f_0}{\partial \varepsilon}\right) \Big/ \sum_{m=0}^{\infty} |g_m(-p_z)|^2. \tag{15}$$

Substituting (15) in (11) we obtain for the longitudinal conductivity

$$\sigma_{zz} = \frac{e^2\hbar}{2\pi^2 l^2 n_0 m_\parallel} \int_0^\infty d\varepsilon\, \varepsilon\left(-\frac{\partial f_0}{\partial \varepsilon}\right) \Big/ \sum_{m=0}^{\infty} |g_m(-p_z)|^2, \tag{16}$$

$$g_m(p) = \int_{-\infty}^{+\infty} dz\, e^{-ipz/\hbar} C_m(z) U_m(z), \tag{17}$$

$|g_m(-p_z)|^2$ coincides, apart from a factor, with the reflection coefficient of an electron with a definite projection of the angular momentum m on the direction of H and with a momentum $p_z \| \mathbf{H}$ from the potential U. On the other hand, the sum

$$\sum_m |g_m(-p_z)|^2$$

contained in (16) is, accurate to the same factor, the coefficient of reflection, from the potential $U(r)$, of a broad beam of electrons situated at the ground Landau level.

We now calculate the conductivity for the case $\mathbf{E}_0 \perp \mathbf{H}$, $\mathbf{E}_0 \| y$. The transverse-conductivity tensor component connected with the current $j_y \| \mathbf{E}_0$ is due to jump-over of the center of the electron orbit y_0 as a result of collision of the latter with the scattering centers.

The probability of the transition $y_0 p_z \to y_0' p$ due to collisions with individual centers depends on the electric field intensity E_0 and can be obtained, in analogy with (8), from the integral equation for the wave function of the electrons in the field of the individual center and in mutually perpendicular electric and magnetic fields:

$$\frac{dW(y_0 p_z \to y_0' p)}{dy_0'\, dp} = \frac{8\pi^3 l^2}{V^{2/3}} \left| \int dV \psi^{E_0}_{y_0 p_z}(r) U(r) \varphi^{E_0}_{0 y_0'}(r) \right|^2 \times \delta\left[\frac{p^2 - p_z^2}{2m_\parallel} + eE_0(y_0 - y_0')\right], \tag{18}$$

where φ^{E_0} is the eigenfunction of the electron in mutually perpendicular electric and magnetic fields,[8] and ψ^{E_0} is the exact wave function of the electron in the field of the center $U(r)$ and $\mathbf{E}_0 \perp \mathbf{H}$.

In the approximation (5), the scattering of the electron by different centers is independent. Following Davydov and Pomeranchuk[9] and Titeica,[10] we calculate the current along the field E_0 as the algebraic sum of currents connected with the transitions $y_0 p_z \rightleftarrows y_0' p$.

In the approximation linear in the field $\mathbf{E}_0$, the only terms contributing to the current are those obtained by expanding the δ function (18) in powers of the electric field.[8] In this approximation, the functions ψ^{E_0} and φ^{E_0} in (18) can be replaced by their expressions at $\mathbf{E}_0 = 0$ and ψ can be found from Eq. (6). The transition to the function g_{p_z} (9) enables us to integrate the expression for the current density with respect to the initial and final coordinates y_0 and y_0' of the center of the orbit. Next, multiplying the current from one scatterer by the number of scatterers N_0, we obtain for the diagonal component of the transverse-conductivity tensor

$$\sigma_{yy} = \frac{4\pi^2 l^4 e^2 n_0 m_\parallel}{\hbar} \int_{-\infty}^{+\infty} \frac{dp_z}{|p_z|}\left(-\frac{\partial f_0}{\partial \varepsilon}\right) \sum_{m=0}^{\infty} [A_m^+ + A_m^-],$$

$$A_m^\pm = |g_m^\pm|^2 + 2(m+1)[|g_{m+1}^\pm|^2 - \mathrm{Re}(g_{m+1}^\pm g_m^{\pm *})], \quad g_m^\pm = g_m(\pm p_z). \tag{19}$$

4. We now analyze the derived expressions. The transverse conductivity of the electron gas with isotropic spectrum in a quantizing magnetic field, due to scat-

tering by centers of small radius $a \ll l$, was calculated by Skobov.[11] Replacing in (12) and (17) the potential $U(r)$ by a delta function in z, it is easy to calculate the functions g_m.[4] Next, retaining in (19) only the term with $m=0$, putting $m_\perp = m_\parallel = m^*$, we obtain for the transverse conductivity an expression that coincides with the result of Skobov[11] for shallow ($U \ll \hbar^2/m^*a^2$) centers. For sufficiently deep centers, $U \gtrsim \hbar^2/m^*a^2$, agreement with the result of (11) can also be easily obtained by taking into account the higher Landau bands; this leads to renormalization of the constant of the interaction between the electron and the center.

To investigate the oscillations, we specify the concrete form of the scattering potential $U(r)$ in the form of a screened Coulomb potential:

$$U(r) = -\frac{\alpha}{r} e^{-r/a}, \quad \alpha > 0. \tag{20}$$

The oscillations investigated here are due to the finite effective radius of the one-dimensional potential $U_m(z)$ (3). They are produced whenever a zero-energy level appears in one of the $U_m(z)$ wells. Since the physical picture of the oscillations is the same for all m, we confine ourselves to a calculation of the longitudinal and transverse conductivity, retaining only the term with $m=0$. In addition, the potential (3) decreases with increasing m like $\exp(-m^{1/2})$, and this leads to a decrease of the depth of the one-dimensional well U_m with increasing m and to violation of the condition (4). Finally, at $a < l$ the potential $U_m \propto (a^2/l^2)^{m+1}$ becomes shallower with increasing m, and this likewise violates the condition (4). Substituting (20) in (3) we obtain at $m=0$ ($al^{-1} \lesssim 1$)

$$U_0(z) = -\frac{\alpha}{a}\left(\frac{a}{l}\right)^2 e^{-|z|/a} = -U_0 e^{-|z|/a}\left(\frac{a}{l}\right)^2. \tag{21}$$

The solution of Eq. (12) with $m=0$ is expressed for the one-dimensional potential (21) in terms of Bessel functions. The integrals (17) are evaluated in this case in general form and, omitting the intermediate results, we get for the longitudinal (16) and transverse (19) components of the conductivity tensor

$$\sigma_{zz} = \frac{e^2\hbar}{2\pi^2 l^2 n_0 m_\parallel}\int_0^\infty \varepsilon\left(-\frac{\partial f_0}{\partial\varepsilon}\right) \nu \operatorname{sh}\pi\nu \Big/ \left[a^2\left(\frac{\hbar^2}{2m_\parallel a^2}\right)^2 |D|^2 + \sum_{m=+1}^{\infty} |g_m^{-1}|^2\right] d\varepsilon, \tag{22}$$

$$\sigma_{yy} = \frac{\pi\hbar^2}{m_\parallel a^2} a^4 e^2 \int_{-\infty}^{+\infty} \frac{dp_z}{|p_z|}\left(-\frac{\partial f_0}{\partial\varepsilon}\right) \nu^2\{1+F(\nu,\beta)\}, \tag{23}$$

$$F(\nu,\beta) = \frac{\nu \operatorname{sh}\pi\nu}{\pi} \operatorname{Re}\left[\left(\frac{2}{\beta}\right)^{2i\nu} \Gamma^2(i\nu) \frac{[J_{i\nu}(\beta)/J_{-i\nu}(\beta)]' J_{-i\nu}^2(\beta)}{[J_{-i\nu}^2(\beta)]'}\right], \tag{24}$$

$$\nu = \frac{2p_z a}{\hbar}, \quad \beta = \frac{1}{\hbar}(8a^2 m_\parallel U_0)^{1/2}\frac{a}{l}, \tag{25}$$

$$D = -C\frac{[J_{i\nu}(\beta)J_{-i\nu}(\beta)]'}{[J_{-i\nu}^2(\beta)]'}, \quad C = \left(\frac{\pi\nu}{2\pi\hbar \operatorname{sh}\pi\nu}\right)^{1/2}$$

$J_{\pm i\nu}$ are Bessel functions. The spectrum of the bound states on the same potential (21) is determined by the equation

$$[J_\nu^2(\beta) + J_{-\nu}^2(\beta)]' = 0. \tag{26}$$

Equations (22)–(26) become much simpler in the limiting case of sufficiently small $\nu \ll 1$, at temperatures $T \ll \hbar^2/m_\parallel a^2$, and at large $\beta \gg 1$ (we use the asymptotic forms of the Bessel functions[2)]:

$$\sigma_{zz}(H) = \frac{e^2\hbar m_\parallel^{-1}}{2\pi^2 l^2 n_0}\int_0^\infty \pi\nu^2\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\varepsilon\, d\varepsilon \Big/ \left[\frac{a^2}{2\pi\hbar}\left(\frac{\hbar^2}{2m_\parallel a^2}\right)^2 \times \frac{\cos^2 2\beta}{\cos^2 2\beta + \pi^2\nu^2} + \sum_{m=1}^{\infty} |g_m^{-1}|^2\right], \tag{27}$$

$$\sigma_{yy}(H) = 8\pi e^2 n_0 l^4 \int_0^\infty d\varepsilon\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\frac{(1-\sin 2\beta)\pi^2\nu^2}{\cos^2 2\beta + \pi^2\nu^2}. \tag{28}$$

It follows from (27) that for magnetic field intensities satisfying the relation

$$2\beta = \pi\left(n + \frac{1}{2}\right),$$

i.e.,

$$H = \frac{c\hbar}{|e|a^2}\frac{\hbar^2}{m_\parallel a^2 U_0}\frac{\pi^2}{32}\left(n+\frac{1}{2}\right)^2, \tag{29}$$

the longitudinal conductivity increases. The condition (29) corresponds to the appearance of a level with zero energy in a one-dimensional well (21), as can be easily seen from (26). The amplitude of the oscillations is limited because of the contribution made to the scattering by states with $m \geq 1$. The functions g_m^-, as indicated above, decrease rapidly with increasing m. Confining ourselves in (27) to the term with $m=1$, we obtain for the maximum amplitude of the longitudinal conductivity

$$\sigma_{zz}^{\text{макс}} = \frac{e^2\hbar}{2\pi l^2 n_0 m_\parallel}\int_0^\infty \frac{\nu^2\varepsilon}{|g_1^-|^2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right) d\varepsilon. \tag{30}$$

The transverse conductivity (28) also oscillates; σ_{yy} in (28) vanishes at values of H (29) corresponding to even $n=2l$, and reaches for odd values $n=2l+1$ a maximum

$$\sigma^{max} = 16\pi e^2 n_0 l^4 f_0(0).$$

This behavior is due to the different parities of the wave functions of the bound states of the one-dimensional well for successive n. In contrast to the magnetophonon oscillations of Gurevich and Firsov[5] and to the Shubnikov–de Haas oscillations,[13] which are periodic in H^{-1}, the period of the investigated oscillations at $a < l$ is proportional to

$$\propto \left(U_0 \Big/ \frac{\hbar^2}{m_\parallel a^2}\right)^{1/2}\left(\frac{a}{l}\right) \propto H^{1/2}.$$

At $a \sim l$ this relation does not hold and the period of the oscillation is a complicated function of H.

Next, for values of H satisfying (29), the longitudinal magnetoresistance is minimal: $\rho_\parallel \sim (\sigma_{zz}^{\text{ампл}})^{-1}$. This can lead to a negative longitudinal magnetoresistance:

$$\frac{\rho_\parallel(H) - \rho_\parallel(0)}{\rho_\parallel(0)} < 0, \quad \rho_\parallel = \sigma_{zz}^{-1},$$

which was previously observed[1,2] in scattering of electrons by ions and acoustic phonons. We emphasize that the negative longitudinal magnetoresistance,[1,2] not being due to oscillations, is of different physical origin than in our case.

The scattering of carriers with spheroidal equal-energy surfaces by neutral atoms was investigated by Brooks.[14] According to Brooks, the momentum relaxation time due to the scattering of the electrons by such

atoms is of the form

$$\tau^{-1}=\frac{20\hbar^3\varkappa n_0}{3e^2(m_\parallel m_\perp^2)^{1/3}}\left(\frac{1}{m_\parallel}+\frac{2}{m_\perp}\right), \qquad (31)$$

where $\varkappa$ is the dielectric constant.

The longitudinal conductivity (at $H=0$) is given by

$$\sigma_{zz}(0)=\frac{n_e e^2\tau}{m_\parallel},$$

and the effect of negative magnetoresistance can be observed at a ratio

$$\rho_\parallel(H)/\rho_\parallel(0)=\sigma_{zz}^{(0)}/\sigma_{zz}^{ampl}<1.$$

Taking (30) and (31) into account, the last relation takes the form

$$\frac{3\pi^2}{40}n_e e^2 m_\perp(m_\parallel m_\perp^2)^{1/3}l^3/\hbar^4\varkappa\int_0^\infty\frac{\pi v^2\varepsilon}{|g_1^-|^2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)d\varepsilon<1, \quad m_\parallel\gg m_\perp. \qquad (32)$$

5. The effects considered here are realized in the group-IV semiconductors Ge and Si in which electrons interact with hydrogenlike neutral impurities. The mass parameter $m_\parallel/m_\perp=19.5$ is maximal for the electrons in n-Ge. The constant-energy surfaces of the electrons in n-Ge are ellipsoids of revolution with major axes in the $\langle 111\rangle$ direction. The shallow-donor radius is $a=4\times10^{-7}$ cm. The magnetic length is of the order $l\approx a$ for the magnetic-field intensity values $H=4\times10^5$ G. The ionization potential of the shallow donors in n-Ge is $U_{ion}=0.01$ eV $=120$ K.

If the magnetic field is directed along the principal axis of one of the ellipsoids, then for the electrons of this valley the degree of mixing of the Landau levels by the center (1) is

$$U^{00}_{m+s0}/\hbar\omega_\perp<U_0/\hbar\omega_\perp=0,2<1,$$

and the depth of the one-dimensional well that determines the number of bound states [see (4)] is

$$U_0(0)\Big/\frac{\hbar^2}{m_\parallel a^2}\approx 5.$$

Because of numerical factors, the last estimate is too low by at least a factor of two, i.e., a one-dimensional well contains according to (29) four bound states. When the magnetic field intensity is decreased to $H=4\times10^4$ G we have $a^2l^{-2}=0.1$, $U_0(0)\sim\hbar^2/m_\parallel a^2$, and one bound state remains in the well, so that three oscillations should be observed in experiment.

The amplitude of the oscillations of the longitudinal conductivity turns out to be finite for the following reasons:

1) the temperature is finite, leading to a thermal broadening of the one-dimensional well characterized by a parameter

$$\xi_1=T\Big/\frac{\hbar^2}{m_\parallel a^2};$$

2) a contribution is made to scattering by states with higher $m\geq 1$ (27), as determined by the parameter

$$\xi_2^m\simeq\left(\frac{a}{l}\right)^{4m}, \quad m\geq 1;$$

3) other scatterers are present, such as ionized impurities, acoustic phonons, etc..

For temperatures $T\approx 1$ K and at magnetic field intensities $H=6\times10^4$ G and $a^2l^{-2}=0.16$, the chopping of the amplitude of the longitudinal-conductivity oscillation, due to mechanisms (1) and (2), is of the same order: $\xi_1=0.03\sim0.027=\xi_2^1\gg\xi_2^2$.

In addition to scattering by neutral impurities, a contribution to the resistance is made by scattering by other impurities, particularly by ionized donors, which ensure the presence of electrons in the conduction band. An estimate of this scattering mechanism shows that at ionized impurity concentrations n_+ satisfying the inequalities

$$n_0\frac{m_\parallel Tl^2}{\hbar^2}\gg n_+\gg n_0\frac{m_\parallel Tl^2}{\hbar^2}\left(\frac{a}{l}\right)^4, \qquad (33)$$

the oscillations should be distinctly observed, but the amplitude of the oscillations is determined by the scattering from ionized impurities and is of the order of

$$\sigma\approx\frac{2}{\pi^{3/2}}\frac{n_e}{n_+}\frac{e^2(m_\parallel T)^{1/2}}{\hbar^2}\left(\frac{T^2}{U_0^2}\right)\left(\frac{l}{a}\right)^2.$$

In the derivation of the estimate (33), the scattering of the electrons by the ionized impurities was considered in the Born approximation, which overestimates somewhat the contribution made to the resistance by the ionized impurities.[15] The parameter $(m_\parallel T)^{1/2}l/\hbar$, which enters in (33), is of the order of unity for n-Ge at $T=1$ K, $l=10^{-6}$ cm, and $a^2l^{-2}=0.16$.

We estimate also the contribution made to the resistance by the electron scattering from the acoustic phonons. Using the corresponding expression for σ_{zz}^{phon} (Ref. 16), we find that scattering by neutral impurities prevails at concentrations corresponding to the inequality

$$n_0\gg\frac{a^{-3}}{15}\left(\frac{a^2m_\parallel T}{\hbar^2}\right)^2\frac{MC_0a^2}{\hbar^2}\frac{m_\parallel}{M}\left(\frac{a}{l}\right)^2\left(\frac{C_0}{Ms}\right)^2\frac{\hbar\omega_\perp}{T}\frac{m_\perp}{m_\parallel},$$

where s is the speed of sound, c_0 is the constant of the deformation potential, and M is the mass of the unit-cell atom.

At $T=1$ K and $H=6\times10^4$ G we have $n_0\gtrsim10^{11}$–10^{12} cm^3. The contribution made to the electron oscillations by the three remaining valleys, the axes of which do not coincide with the direction of the magnetic field at the same intensities H, is small. The magnetic length l_1 for the electrons of these valleys is connected with the magnetic length l of the electrons of the principal $m_\parallel\|H$ valley by the relation

$$l_1^2=l^2\Big/\left(\cos^2\vartheta+\frac{m_\perp}{m_\parallel}\sin^2\vartheta\right)^{1/2},$$

ϑ is the angle between the direction of the principal axes of the valleys. In Ge we have $\vartheta=120°$ and $l_1^2=2l^2$, by virtue of which the parameter (a^2/l_1^2) is small, and the corresponding potential turns out to be shallow and contains only one or two bound states. The electrons of these three valleys, just as the acoustic phonons, give rise to a conduction background. Measurements on deformed samples amke it possible to "exclude" the inessential valleys[17] and by the same token decrease the

background.

We now examine the possibility of observing in n-Ge a negative longitudinal magnetoresistance due to the oscillations. As indicated above, at $a<l$ the one dimensional potential $U_m(z)\sim(a^2/l^2)^{m+1}$ is shallow for all $m\geq 1$ and can be replaced by a delta-function potential in z. The corresponding one-dimensional equation is then easily solved[4] and the function g_1 can be calculated. The estimate (32) (for a Boltzmann distribution function f_0) then takes the form

$$\frac{\rho_{\parallel}(H)}{\rho_{\parallel}(0)}=\left(\frac{U_0}{T}\right)^2\left(\frac{a}{l}\right)^8\frac{1}{\varkappa}\frac{e^2}{\hbar^2}m_{\perp}l\left(\frac{m_{\perp}}{m_{\parallel}}\right)^{1/2}, \tag{34}$$

from which we get $\rho_{\parallel}(H)/\rho_{\parallel}(0)\sim 5\cdot 10^{-2}$ for $T=1$ K and $H=6\times 10^4$ G.

The possibility of observing oscillations in Si is much worse than in Ge. The mass anisotropy parameter in Si is $m_{\parallel}/m_{\perp}=5.15$. For shallow donors in Si we have $a\approx 1.5\times 10^{-7}$ cm. The magnetic length is $l\approx a$ at $H=27\times 10^5$ G. At these values of H we have $U_0/(\hbar^2/m_{\parallel}a^2)\approx 3$ even for the deepest impurities Bi, but the mixing of the Landau levels by the center is much more substantial than for Ge, where $U_0/\hbar\omega_{\perp}=0.64$.

It is also of interest to observe the oscillations when electrons are scattered by deep impurities in semiconductors. In germanium such impurities are, in particular, Se and Te.[18] For these impurities, $a\approx 1.7\times 10^{-7}$ cm, the magnetic length is $l\approx a$ at $H=22\times 10^5$ G, and at these magnetic-field intensities the parameters characterizing the mixing of the Landau levels by the center and the number of levels in the one-dimensional well are respectively $U_0/\hbar\omega_{\perp}=0.44$, $U_0/(\hbar^2/m_{\parallel}a^2)=11.3$. From the last estimate it is seen that the number of bound states produced in the one-dimensional well by the deep impurity is approximately double the number of states in the one-dimensional well of a shallow donor.

We must now point out the following circumstances. First, the inequality (1), which ensures smallness of the mixing of the Landau levels by the center, reduces at $a<l$, as already indicated, to $U_0\ll\hbar^2/m_{\perp}a^2$ and is independent of the magnetic field intensity. There is, however, a lower bound on the permissible values of H at which the Landau level mixing is small. Shallow neutral H-impurities in the absence of a magnetic field can be attached to an electron, forming H^- centers with binding energy $E_f\sim\hbar^2/m^*f^2$ ($f\gg a$ is the amplitude of scattering of an electron with energy $E\to 0$ by a neutral center). So long as the binding energy E_f is small

$$\hbar^2/m^*f^2\ll\hbar^2/m^*l^2=\hbar\omega_H,$$

the electron of the Landau ground band with $m=0$ is localized in the region $\sim l\ll f$ determined by the magnetic field, and the mixing of the Landau levels is small. At small H, when $l\gg f\gg a$, the region of the localization of the electron is determined by the center, and the Landau-level mixing is substantial. Thus, inclusion of only one Landau band is correct for fields

$$l^2\ll f^2, \text{ i.e. } H\gg c\hbar/|e|f^2. \tag{35}$$

In the case of a strongly anisotropic carrier mass at $H=0$, the energy of the bound state of the electron on the center is close to the binding energy of the electron with mass $m_{\perp}$ on a shallow two-dimensional potential

$$U_0(\rho,0)\ll\hbar^2/m_{\perp}a^2.$$

The corresponding bound state exists for an arbitrarily shallow potential, is exponentially small,[3] and the inequality (35) must be replaced by

$$a^2\exp\left[-\hbar^2U_0^{-1}/m_{\perp}a^2\right]\gg l^2.$$

Second, the oscillations can become enhanced because of the increase of the concentration of the carriers in the conduction band with decreasing depth of the one-dimensional well and with appearance of a zero energy level. The corresponding calculation was carried out for a model in which the neutral centers, which are responsible for the oscillations, are deep impurities with concentration n_0, and the electrons are supplied to the conduction band via thermal ionization of the shallow donors whose concentration is n_+. The neutral centers in a magnetic field play the role of acceptors and allowance for the change of the carrier density is essential in the case of strong "compensation" $n_+-n_0\gg n_0$. In this case, however, the electron density in the conduction band is small, making it difficult to observe the oscillations in experiment. In the case of weak compensation $n_+\gg n_0$, the change of the carrier density in the conduction band following the appearance of a zero energy level can be neglected. The corresponding parameter is $\exp(-\hbar^2/m_{\parallel}a^2T)$.

Third, we have considered the scattering of slow electrons by neutral atoms in a magnetic field. It is known[19] that $H=0$ the polarization of the atom by the incident electron is significant. The polarization potential, at distances larger than atomic, decreases in proportion to r^{-4}. The presence of such a tail in the three-dimensional case (at $H=0$) does not influence the very existence of the bound states, but leads only to their shift.[19] In a magnetic field, for slow particles, the situation is similar, does not affect the very existence of the oscillations considered in this paper, and leads only to different values of H at which the oscillations take place. At $a<l$, however, and at sufficiently high temperatures, the contribution of the polarization potential to the scattering turns out to be small. The point is that the corresponding one-dimensional polarization potential $U_p(z)$ is determined by the matrix element of the polarization interaction, averaged over the radial motion, i.e.,

$$U_p(z)\approx\frac{e^4}{3}\sum_{\alpha}\frac{|r_{0\alpha}|^2}{(E_{\alpha}-E_0)}\left[\int_0^{\infty}\rho\,d\rho R_{00}^2(\rho)\frac{z}{(\rho^2+z^2)^{1/2}}\right]^2$$
$$\approx\frac{e^2}{3a\varkappa}\left(\frac{a}{l}\right)^4\left(\frac{1}{(2l^2+z^2)^{1/2}}-\frac{1}{|z|}\right)^2,$$

α is the set of quantum numbers of the excited state of the atom, E_0 is the energy of the ground state of the atom, and $\mathbf{r}$ is the radius vector of the atomic electron.

We can approximately express $U_p(z)$ in the form

$$U_p(z)=\frac{e^2}{3a\varkappa}\begin{cases}(a/l)^4, & |z|\ll l\\ (a/z)^4, & |z|\gg l\end{cases}\quad\frac{e^2}{a\varkappa}\approx U_0.$$

If the electron temperature is such that

$$\frac{e^2}{3a\varkappa}\left(\frac{a}{l}\right)^4\ll U_0\left(\frac{a}{l}\right)^4\ll T\ll U_0\left(\frac{a}{l}\right)^2,$$

then in the solution of the one-dimensional scattering problem the polarization potential $U_p(z)$ can be omitted.

Finally, the conditions for the applicability of the single-center approximation (5) is usually well satisfied in semiconductors. Thus, at $H=6\times10^4$ G and $T=1$ K the estimate (5) yields $n_0 \ll 10^{18}$ cm^{-3} in Ge.

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[1)]The substitution $m \to -m$ has been made below everywhere.

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High-frequency properties of $ErFeO_3$ in the ordering region of rare-earth systems

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Magnetic resonance at frequencies 10–37 GHz is used to investigate the behavior of the spin-wave frequency in the vicinity of the low-temperature spin flip in a zero magnetic field. A decrease in the frequency of the "soft mode" down to 15 GHz at a temperature 4 K is observed, in correspondence with the ordering of the spin system of Er.

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One of the distinguishing features of a definite class of ordered magnets is the presence of an energy gap in the spin-wave spectrum even in a zero external magnetic field H (see, e.g., Ref. 1). This circumstance is connected with the presence of a nonzero anisotropy field H_A. Among these magnets is included, in particular, the rare-earth (RE) orthoferrite $ErFeO_3$, which is a weak ferromagnet with a residual moment m. When no account is taken of the magnetoelastic interaction and of anisotropy of order higher than the fourth, the field H_A vanishes in various kinds of phase transitions (PT), including orientational PT which take place in this crystal. This means that the frequency ν_0 of the "soft mode" of the spin-wave spectrum should also vanish in the PT ($\nu_0 \sim H_A$ at $H=0$).[2] A direct method of determining the spin-wave frequency is the method of homogeneous antiferromagnetic resonance (AFMR).

It is known that in $ErFeO_3$ there take place a number of orientational PT with change of only the temperature ($H=0$), and these were investigated in sufficient detail by various methods.[3–6] Two such PT occur in the temperature region 90–100 K and manifest themselves, with decreasing temperature, in a smooth rotation of the magnetic moment of the iron sublattice by an angle $\pi/2$–from the a axis to the c axis. The change of orien-

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