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# Effect of surface waves on the reflection of sound by a rough surface

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The reflection of sound incident from a liquid on the rough surface of an isotropic solid is considered. For angles of incidence corresponding to total internal reflection, the reflection coefficient has a sharp minimum due to the excitation of surface waves. The minimum is deep for small (compared with the sound wavelength) and weakly sloping roughnesses. This can be due to the competition of a roughness with a small ratio of the acoustic impedances of the liquid and the solid. The location and shape of the minimum have been studied as a function of the sound frequency and the parameters of the roughness. The considered phenomenon can serve as an experimental method for study of the structure of the surface of a solid.

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## 1. INTRODUCTION

The problem of the scattering of various waves by a rough surface has been considered in a large number of researches (see, for example, Ref. 1). In one of the latest papers on this subject,[2] the reflection of sound from the rough surface of a solid body is described. Since the solid in this research was assumed to be absolutely rigid, the effect of any waves propagating in it was not taken into account. However, the problem discussed represents great practical interest,[3] in particular as one of the methods of nondestructive testing. Figure 1 (taken from the work of Rollins[3]) shows the experimental dependence of the reflection coefficient of ultrasound incident from a liquid on the surface of a solid as a function of the angle of incidence (the angle of reflection was chosen equal to the angle of incidence). In the region of total reflection, when volume sound waves cannot propagate in the solid, there exists a minimum, the location and shape of which depend on the properties of the surface.

It is natural to connect the origin of a minimum with

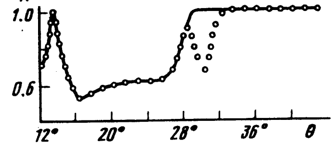


FIG. 1. Dependence of the reflection coefficient $R$ on the angle of incidence $\theta$. The curve is constructed according to Eq. (1), the points are the experimental data from Ref. 3 for a water-aluminum interface and a frequency of 5 MHz.

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the excitation of surface waves in the solid. These waves are similar to Rayleigh waves, which propagate along the surface of a solid bounded by a vacuum. In the present case, when the solid borders on a liquid, the surface waves are damped because of the radiation into the liquid. In spite of this radiation, the reflection coefficient for an ideally plane surface should go to unity even in the region where excitation of surface waves is possible.

Actually, the amplitude of the wave reflected from an ideally rough surface is given by the well-known expression[4]

$$\frac{A_1}{A_0}=\frac{\Delta(k)-i\gamma(k)\omega^4/c_t^4}{\Delta(k)+i\gamma(k)\omega^4/c_t^4}, \qquad (1)$$

where

$$\Delta(k)=(k_t^2-k^2)^2+4k^2k_tk_l, \quad \gamma(k)=\rho k_l/i\rho_s k_1,$$
$$k_{t,l}=(\omega^2/c_{t,l}^2-k^2)^{1/2}, \quad k_1=(\omega^2/c^2-k^2)^{1/2},$$

$\omega$ is the frequency of the sound, $k$ is the component of the wave vector parallel to the surface, $c_t$, $c_l$ and $c$ are the velocities of transverse and longitudinal sound waves in the solid and sound waves in the liquid, respectively, $\rho$ and $\rho_s$ are the densities of the liquid and the solid.

It is seen from Eq. (1) that the reflection coefficient $R=|A_1/A_0|^2$ goes to unity in the region of total reflection, when $k_t$ and $k_l$ are purely imaginary. Along with this, the quantity $\Delta$ goes to zero at some value of the angle. This condition determines the spectrum of the surface Rayleigh waves. Here the quantity

$$\gamma=\frac{\rho c}{\rho_s c_R}\left(\frac{1-c_R^2/c_l^2}{1-c^2/c_R^2}\right)^{1/2} \qquad (2)$$

actually represents in order of magnitude the ratio of the sound impedances of the liquid and the solid; $c_R$ is the velocity of Rayleigh waves. The quantity $\gamma$ is usually small, thanks to which the damping of the excited surface waves because of the radiation turns out to be small.

If the reflecting surface is not ideally rough, then the radiation into the liquid takes place at angles different from the angle of incidence, which should lead to the appearance of a minimum in the reflection coefficient under the condition of excitation of surface waves.

In the present work, we consider the reflection of sound incident from a liquid on a random rough solid surface. In the mean, the surface is assumed to be planar, and the mean amplitude of the roughnesses is assumed to be small both in comparison with the normal (relative to the surface) component of the sound wavelength and in comparison with the distance along the surface over which the roughness changes appreciably. These conditions allow us to expand the boundary conditions in powers of the deviation of the surface from an ideally plane one, as is usually done in the well known method of small perturbations (see Ref. 1).

The difference from other similar problems considered earlier lies in the fact that, owing to the presence of the other small parameter $\gamma$ the effect of the surface is large here and one must carry out summation of the principal terms of the perturbation theory series. The method that we have used is analogous to that applied by us earlier[5] for the study of the propagation of Rayleigh waves on a rough surface separating a solid from a vacuum.

## 2. THEORY

We now consider a sound wave of frequency $\omega$ incident from a liquid on an isotropic solid, the equation of the surface of which we write in the form

$$x=\xi(\mathbf{s}), \qquad (3)$$

where $\mathbf{s}$ is a two-dimensional vector lying in the tangent plane. We assume the surface to be plane on the average, i.e., we assume that $\langle\xi(\mathbf{s})\rangle=0$. The averaging is carried out here over some distribution of random functions. Since we limit ourselves to the case in which the quantity $\langle\xi^2\rangle$ is small in comparison with the square of the normal component of the wavelength, the result of the calculations will be expressed in terms of the correlation function

$$W(\mathbf{s}-\mathbf{s}')=\langle\xi(\mathbf{s})\xi(\mathbf{s}')\rangle. \qquad (4)$$

This correlator is the only characteristic of the surface in the considered approximation and we shall assume it to be known. For estimates we shall specify $W(\mathbf{s})$ as usual by two parameters: the value $W(0)=\langle\xi^2\rangle=a^2$ at $s=0$, and also the radius $d$ of the region in which $W(\mathbf{s})$ is basically different from zero. For a surface having a cross section in the form of a saw with teeth of variable height and different distances between them, $d$ is the mean distance between the teeth.

We represent the displacement of the medium $\mathbf{u}(\mathbf{r})$, expanded in a Fourier integral in the coordinate $\mathbf{s}$, in the form of a sum over vibrations with definite polarization:

$$u_\alpha(x\mathbf{k})=\sum_\gamma e_\alpha(\gamma\mathbf{k})A_\gamma(\mathbf{k})\exp(ik_\gamma x), \qquad (5)$$

where $e_\alpha(\gamma\mathbf{k})$ are unit vectors, and the definition of $\mathbf{k}$ is given following Eq. (1). For the solid, towards which is directed the $x$ axis, the summation in (5) is carried out over three values of $\gamma$, corresponding to two transverse $(A_{t1}, A_{t2})$ and one longitudinal $(A_l)$ polarizations; for the liquid, there are two waves in (5): incident $A_0$ and reflected $A_1$. In contrast with the vector $\mathbf{k}$, the frequency $\omega$ is preserved in the scattering from static roughnesses and we shall not write it as an argument of $A$.

For the determination of the four unknown amplitudes, we use the boundary conditions, which consists of the equality of the surface forces and (upon neglect of the viscosity of the liquid) of the normal components of the displacement. The first three conditions can be written as the condition of continuity of matter:

$$\rho c_t^2 n_\beta\left(\frac{\partial u_\alpha}{\partial r_\beta}+\frac{\partial u_\beta}{\partial r_\alpha}\right)+\rho(c_l^2-2c_t^2)n_\alpha\frac{\partial u_\beta}{\partial r_\beta}, \qquad (6)$$

where $\mathbf{n} \sim (1, -\partial\xi/\partial y, -\partial\xi/\partial z)$ is a vector normal to the surface (3).

The boundary conditions are satisfied on the rough surface (3). In the approximation that we have used, we expand them in a series in $\xi$. The conditions obtained in such fashion should be satisfied at $x=0$.

Expanding the quantity (6) in a Fourier integral in $\mathbf{s}$, and using (5), we write down the boundary conditions with accuracy to within terms of first order in $\xi$:

$$
\begin{aligned}
&H_{ij}(k)A_j(k)+\int\frac{d^2q}{(2\pi)^2}\xi(\mathbf{k}-\mathbf{q})V_{ij}(kq)A_j(q)\\
&=H_{i0}(k)A_0(k)+\int\frac{d^2q}{(2\pi)^2}\xi(\mathbf{k}-\mathbf{q})V_{i0}(kq)A_0(q),
\end{aligned} \tag{7}
$$

where $\xi(\mathbf{k})$ is the Fourier component of the function $\xi(\mathbf{s})$. On the left side of (7), we have retained the terms with the reflected and transmitted waves, and summation is carried out over the index $j$, which runs over the corresponding four values.

The matrices $H$ and $V$ have the following form:

$$
H(k)=\left|\begin{matrix} -e_x(1k) & e_x(\gamma k) \\ -n_\alpha^0 \nu e(1k) & n_\alpha^0[k_\gamma e_x(\gamma k)+\delta_{\gamma l}e(lk)(c_l^2/c_t^2-2)\omega/c_l]+ \\ & +k_\gamma e_\alpha(\gamma k)+k_\alpha e_x(\gamma k)\end{matrix}\right|,
$$

$$
V(kq)=i\left|\begin{matrix} -\frac{\omega}{c}e(1q)+\mathbf{k}e(1q) & \frac{\omega}{c_l}e(lq)\delta_{\gamma l}-\mathbf{k}e(\gamma q) \\ (k_\alpha-q_\alpha+q_1 n_\alpha^0)\nu e(1q) & e_\alpha(\gamma q)(\omega^2/c_\gamma^2-\mathbf{kq})-\mathbf{k}e(\gamma q)(n_\alpha^0 q_\gamma+q_\alpha)+ \\ & +\delta_{\gamma l}e(lq)[(c_l^2/c_t^2-1)(q_\alpha+n_\alpha^0 q_l-k_\alpha)+k_\alpha]\omega/c_l\end{matrix}\right|. \tag{8}
$$

where $n_\alpha^0=(1,0,0)$, $\nu=\rho c\omega/\rho_s c_t^2$, have reduced the columns 2–4 (the index $\gamma = l, t_1, t_2$) and rows (2–4) ($\alpha = x, y, z$) to a single representation. The columns $H_{i0}$ and $V_{0i}$ are obtained from the first columns of the matrices $H_{1j}$ and $V_{1j}$ by change of sign and by the substitutions $e(1q)\to e(0q)$, $q_1\to -q_1$.

We are interested in the mean square of the amplitude of the reflected wave. It is easy to show (this will be done below) that in the case in which the angular width of the incident sound wave is small in comparison with the width of the minimum shown in Fig. 1, the mean square can be replaced by the square of the mean amplitude of the reflected wave. Here we assume that the reflected wave is observed at an angle equal to the angle of incidence.

We shall solve Eq. (7) by iteration in $\xi$, carrying out the averaging at each stage. In zeroth order we get the expression (1). The first-order term vanishes by virtue of the condition $\langle\xi\rangle=0$. In second order, we have the mean $\langle\xi(\mathbf{q})\xi(\mathbf{q}')\rangle$, which, because of the uniformity of the surface in the mean [this has already been used in the definition (3)] can be reduced to the form

$$
\langle\xi(\mathbf{q})\xi(\mathbf{q}')\rangle=w(q)2\pi\delta(\mathbf{q}+\mathbf{q}'),
$$

where $w(q)$ is the Fourier component of the function $W(\mathbf{s})$ [Eq. (4)].

It is impossible, however, to restrict oneself to the term of second order, since the small factor $w$ has a denominator which is small only in comparison with the term of zero order and which is proportional to the determinant of the matrix $H_{ij}$ and equal to the denominator of the fraction (1). The summation of the dangerous components is accomplished in a way similar to what is done in the theory of alloys[6] and is described in Ref. 5 as applied to the similar problem of the damping of Rayleigh waves.

The condition that is necessary for separating the principal terms is smallness of the damping of the excited surface wave, i.e., of the quantity $\gamma$. A certain additional condition is due to the fact that near the Rayleigh spectrum $\Delta(k)=0$ the numerator of the fraction (1) is also small. This leads to the necessity of taking into account the terms with $\xi$ in the right side of Eq. (7).

As a result of the described calculations we get

$$
\begin{aligned}
\frac{\langle A_i(k)\rangle}{A_0(k)}=&\left[H(k)-\int\frac{d^2q}{(2\pi)^2}w(\mathbf{k}-\mathbf{q})V(kq)H^{-1}(q)V(qk)\right]^{-1}_{ii}\\
&\times\left[H_{i0}(k)-\int\frac{d^2q}{(2\pi)^2}w(\mathbf{k}-\mathbf{q})V_{ij}(kq)H^{-1}_{jk}(q)V_{k0}(qk)\right].
\end{aligned} \tag{9}
$$

We now give the explicit form of the matrices $H$ and $V$ (8). Inasmuch as the boundary conditions involve polarization of the waves even in the zeroth order in $\xi$, and in the case of scattering from roughnesses, the plane of propagation changes, there is no necessity of being concerned with the orthogonality and normalization of the vectors $\mathbf{e}(\gamma k)$. The only condition which these vectors must satisfy is that the corresponding wave be longitudinal or transverse. Therefore, we choose them for the solid state in the form

$$
e_\alpha(\gamma k)=\left|\begin{matrix} k_l & k_y & k_z \\ k_y & -k_t & 0 \\ k_z & 0 & -k_t\end{matrix}\right|, \tag{10}
$$

where the index $\gamma$ enumerates the columns, and the first column corresponds to longitudinal polarization. We recall that the $yz$ plane is the mean boundary of the solid, while $k_{l,t}$ are the normal projections of the wave vector corresponding to the given tangential projection of $\mathbf{k}$, polarization $\gamma$, and frequency $\omega$. The vectors $\mathbf{e}$ corresponding to the liquid are obtained from the first column of the matrix (10) by substituting $c$ for $c_l$ in $k_l$ and reversing the sign of $k_l$ for the reflected wave.

In the basis (10), the matrix $H_{ij}$ is equal to

$$
H_{ij}(k)=\left|\begin{matrix} k_1 & k_l & k_y & k_z \\ -\delta & k_t^2-k^2 & 2k_yk_t & 2k_zk_t \\ 0 & 2k_yk_l & k_y^2-k_t^2 & k_yk_z \\ 0 & 2k_zk_l & k_yk_z & k_z^2-k_t^2\end{matrix}\right|, \tag{11}
$$

where $\delta=\rho\omega^2/\rho_s c_t^2$. The first row in (11) corresponds to the condition of continuity of the normal components of the displacements in Eq. (7)

The matrix $V_{ij}$ is determined by the expression

$$
V_{ij}(kq)=i\left|\begin{matrix} \mathbf{kq}-\omega^2/c^2 & \omega^2/c_l^2-\mathbf{kq} & k_yq_t & k_zq_t \\ q_1\delta & q_l\alpha & q_y(\alpha+\mathbf{kq})+k_yq_t^2 & q_z(\alpha+\mathbf{kq})+k_zq_t^2 \\ (k_y-q_y)\delta & q_y\alpha-k_y\beta & -q_t(\alpha+k_zq_z) & k_zq_yq_t \\ (k_z-q_z)\delta & q_z\alpha-k_z\beta & k_yq_zq_t & -q_t(\alpha+k_yq_y)\end{matrix}\right|, \tag{12}
$$

where

$$
\alpha=\omega^2/c_t^2-2\mathbf{kq},\qquad \beta=\omega^2/c_t^2-2\omega^2/c_l^2.
$$

The columns of $H_{i0}$ and $V_{i0}$ have the form

$$
H_{i0}(k)=\left|\begin{matrix} k_1 \\ \delta \\ 0 \\ 0\end{matrix}\right|,\quad V_{i0}(kq)=i\left|\begin{matrix} \omega^2/c^2-\mathbf{kq} \\ q_1\delta \\ (q_y-k_y)\delta \\ (q_z-k_z)\delta\end{matrix}\right|. \tag{13}
$$

Equation (9) acquires integrals of the type

$$I(k)=\int\frac{d^2q}{(2\pi)^2}\frac{w(\mathbf{k}-\mathbf{q})f(kq)}{\Delta(q)+i\gamma(q)\omega^4/c_t^4}; \quad (14)$$

the definitions of $\Delta$ and $\gamma$ are given in (1). We are interested in the vicinity of the minimum (see Fig. 1), where $\Delta(k)=0$. The roots of this equation—the spectrum of the Rayleigh waves—we write in the form $\omega=c_Rk$. As is known,[7] $c_R=\zeta c_t$, where the number $\zeta$, which depends on the modulus of the elasticity, lies in the range 0.87–0.96.

The real part of the integral (14) determines the small displacement (proportional to the roughness) of the minimum from the position corresponding to a planar surface: $\omega/c_R=k=(\omega/c)\sin\theta$, i.e., $c/c_R=\sin\theta$, where $\theta$ is the angle of incidence.

Of most interest is the width of the minimum, which depends on the imaginary part of the integral (14). If the function $w(\mathbf{k}-\mathbf{q})$ is sufficiently steep, the imaginary part is due only to the bypass of the pole in (14). The pole contribution takes into account the excitation of the surface waves. Upon increase in the radius $d^{-1}$ of the circle in which $w(\mathbf{k}-\mathbf{q})$ is different from zero, an imaginary part arises also from that region where the $q_l$ or $q_t$ become real (see, for example, $\Delta(q)$ in (10)]. This term represents the contribution of the volume waves excited in the solid.

In the linear approximation in $(ka)^2$, $\gamma$, and $\Delta(k)$, the contribution of the roughness to the numerator and denominator of the fraction (9) is the same. Thus, we find the mean amplitude of the reflected wave:

$$\langle A_1\rangle/A_0=[\Delta(k)+i(\tau-\gamma)\omega^4c_t^{-4}]/[\Delta(k)+i(\tau+\gamma)\omega^4c_t^{-4}], \quad (15)$$

where $\tau=-\operatorname{Im}I(k)$. It can be seen that the omitted terms lead to the appearance in $\Delta$ and $\gamma$ of (15) of factors that differ from unity by an amount of the order of $(ak)^2$.

The function $f(kq)$, which determines the integral $I(k)$ of (14) has a very cumbersome form in the general case. On the Rayleigh wave spectrum, i.e., at $q=k=\omega/\zeta c_t$, we have

$$f(kq)=-k_t^2\Big[k^2-k_t^2+\frac{\mathbf{kq}}{k^2}\Big(\frac{\omega^2}{c_t^2}-2\mathbf{kq}\Big)\Big]^2. \quad (16)$$

At $|\mathbf{q}-\mathbf{k}|\ll k$ this expression is still further simplified:

$$f(kq)=(\mathbf{kq}-k^2)^2k^2(1-\zeta^2)(4-\zeta^2)^2. \quad (17)$$

We estimate $\tau$, using (14), (17) and the parameters $a$ and $d$ introduced earlier. The estimate depends on the relation between the wavelength $2\pi/k$ of the excited waves, the length $(\gamma k)^{-1}$ of their damping due to the radiation into the liquid, and the scale of roughness $d$, which determines the change of $|\mathbf{q}-\mathbf{k}|\sim d^{-1}$ of the wave vector of the waves scattered by the roughnesses.

For extremely gently sloping roughnesses ($kd\gg1$), only the surface waves make a contribution:

$$\tau\sim(ak)^2\gamma^{-1}(kd)^{-2} \quad \text{at} \quad (kd)^{-1}\ll\gamma/2\ll1, \quad (18)$$

$$\tau\sim(ak)^2\max\{\gamma,\ (kd)^{-3}\} \quad \text{at} \quad \gamma/2\ll(kd)^{-1}\ll1, \quad (19)$$

the quantity $\gamma$ is given in (2). The proportionality constant $C$ in formula (19) for the case $\gamma\ll(kd)^{-3}$ can be calculated with the help of (17) if we assume that the correlator $W(s)$ has Gaussian shape:

$$W(s)=a^2\exp(-s^2/d^2).$$

We find

$$C=\frac{3\pi^{1/2}}{4}\frac{(1-\zeta^2)(4-\zeta^2)^2(2-\zeta^2)^2}{\zeta^2[\zeta^6-4\zeta^4+8(1-c_t^2/c_l^2)]}.$$

Finally, for a sufficiently ribbed surface ($kd\ll1$), when the contributions of the surface and volume waves have the same order, we get

$$\tau\sim(adk^2)^2. \quad (20)$$

The graph of $\tau$ (18)–(20) is shown in Fig. 2. We note again that, along with the conditions determining the regions of applicability of formulas (18)–(20), we have assumed that the inequalities

$$ak_1=a\omega\cos\theta/c\ll1, \quad a\ll d,$$

are satisfied. These are necessary for expansion of the boundary conditions in a series.

## 3. DISCUSSION OF RESULTS

It is seen frmo Eq. (15) that the reflection coefficient

$$R=\frac{\Delta^2(k)+(\tau-\gamma)^2\omega^8c_t^{-8}}{\Delta^2(k)+(\tau+\gamma)^2\omega^8c_l^{-8}} \quad (21)$$

can be significantly different from unity:

$$R_{min}=(\tau-\gamma)^2/(\tau+\gamma)^2$$

at small roughnesses $\tau\sim\gamma$. Figure 1 corresponds to $\tau=0.1\gamma$, and, according to the data of Rollins,[3] a very deep minimum is often observed. It is curious that under the condition $\gamma kd/2\ll1$ the minimum is deepest at some definite value of the frequency, as follows from the relations (19)–(20). This conclusion agrees with experiment.[3] In the region $\gamma kd/2\gg1$ the width of the minimum does not depend on the frequency (18). The fact that the effect of the roughness turns out to be greatest at a certain relation among its parameters $a, d$ and the relative impedance $\gamma$ has a simple physical meaning. It is understood that the reflection coefficient (21) should, with decrease in the roughness, approach unity in the region of internal reflection. On the other hand, as the liquid density decreases, $R$ approaches unity independently of the quality of the surface, simply because of the large difference in the acoustical impedances of the liquid and the solid.

As is seen from the formula (18), the transition to an ideally smooth surface takes place according to the law $d^{-2}a^2$. The factor $a^2$, which is common to all the formulas, is the result of the expansion over the roughness. It is also clear that upon smoothing of the surface, i.e., as $d\to\infty$, the effect of the roughness should

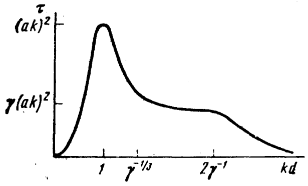


FIG. 2. Dependence of $\tau$ on the scale of roughness $d$.

decrease.

The factor $d^{-2}$ arises in the following manner in other problems of surface roughnesses. Any physical quantity in such cases represents some mean square, and together with the square of the mean there is a second term, which cancels out the first at small transfers of the wave vector, when $d\to\infty$. What has been said is possibly better illustrated by an analogy with the transport time in kinetics, which takes explicitly into account the ineffectiveness of the small-angle scattering. However, in the given problem, this second term, as has already been remarked, is small.

In fact, in second order in the roughness, for example, along with the expression

$$\langle A_1^{(2)}(k)\rangle A_1^0(k)\sim H^{-1}(k)\int d^2q w(k-q) \times V(kq)H^{-1}(q)V(qk)H^{-1}(k)A_0(k)\cdot H^{-1}(k)A_0(k),$$

which represents the second-order correction to the mean square (the superscripts denote the order of perturbation theory, matrix indices are omitted), there is the term

$$\langle (A_1^{(1)}(k))^2\rangle\sim\int d^2q w(k-q)[H^{-1}(k)V(kq)H^{-1}(q)A_0(q)]^2.$$

The ratio of the second to the first is determined by the ratio of the angular width of the incident wave to the width of the minimum of the reflection coefficient. For observation of the minimum, this quantity should be small and on this basis the second term was omitted in the calculations.

The ineffectiveness of the scattering processes with small transfer of the wave factor manifests itself already here in the square of the mean. As is seen from formulas (14)–(17), the probability of scattering tends to zero as $q\to k$. In this case, the function $f(kq)$, being a scalar, vanishes according to the law $(\mathbf{q}-\mathbf{k},\mathbf{k})^2$, and it is this which leads to the factor $d^{-2}$ in $\tau$.

Finally, there is still one question needing explanation. The result of expression (15) is determined by the quantity $w$, which is quadratic in the roughness $\xi$. In this same approximation, terms of second order in $\xi$ would have to be retained in the expansion of the boundary condition (7). However, the direct calculation, which is analogous to that carried out here, shows that these terms lead only to a shift of the minimum, and do not make a contribution to its width and depth. Incidentally, we note that the reflection coefficient from an absolutely rigid surface, obtained by Howe,[2] is given by formula (9) with the same degree of accuracy. In this case, there is only the single condition of continuity of the normal displacements [it is described by the first rows of the matrices (11) and (12)] and there are no transmitted waves lacking. Therefore the quantities (11)–(13) reduce to a single element:

$$H_{ij}(k)=H_{i0}(k)=k_1,\quad V_{ij}(kq)=-V_{i0}(kq)=i(\mathbf{kq}-\omega^2/c^2).$$

With help of (9), it is easily shown that

$$\langle A_1\rangle/A_0=(k_1-\sigma)/(k_1+\sigma),$$

where

$$\sigma=\int\frac{d^2q}{(2\pi)^2q_1}w(\mathbf{k}-\mathbf{q})\left(\mathbf{kq}-\frac{\omega^2}{c^2}\right)^2.$$

We note in conclusion that, similar to what was considered here for the reflection coefficient, we can investigate the effect of roughness on the transmitted wave. This effect is most important near the angle of total internal reflection, where there is no transmitted wave for an ideal surface.


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