Boundary conditions and critical current of SNS junctions

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The boundary condition for the Ginzburg-Landau equation at the interface between a superconductor and a normal metal is obtained for a superconductor with a short mean free path. The expression for the critical current of the SNS junction is investigated at arbitrary ratios of the parameters characterizing the superconductor and the normal metal.

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1. INTRODUCTION

The boundary conditions for the order parameter Δ on the interface between a superconductor and a normal metal were considered by a number of workers.[1-3] If the normal metal is a superconductor with a transition temperature T_{cN} close to T_{cS}, then in the temperature region $|T_{cS}-T|\ll T$ the Ginzburg–Landau equation for the order parameter Δ is valid in both the superconductor and in the normal metal. For dirty superconductors, the boundary conditions for Δ in this case were obtained by de Gennes. If the normal metal is not a superconductor or its transition temperature is not close to T, then to calculate the critical current of the junction and to find the exact boundary conditions for Δ in the superconductor it is necessary to find the Green's function in the normal metal. We confine ourselves below to an examination of the most interesting case—a superconductor with small electron mean free path.

At a low transparency of the interface, the critical current of the SNS junction was obtained in Ref. 4. For superconductors with small electron mean free paths, the critical current of the SNS junction and the boundary condition for the order parameter Δ in the superconductor do not depend on the transparency of the boundary, if the transparency is not too small.[3] In this case it is possible to express all the physical quantities in terms of a solution of a system of algebraic equations whose coefficients depend only on two dimensionless parameters, one of which is the ratio T_{cN}/T. If $T_{cN}=0$, then the system of equations becomes much simpler.

2. GREEN'S FUNCTION FOR SNS JUNCTION

To describe the SNS system we use the equations for the Green's functions, integrated with respect to the energy variable ξ.[5,6] For superconductors with small electron mean free paths, this system reduces to a single differential equation.[7,8] To derive the boundary conditions, the approximation linear in Δ is sufficient. In the linear approximation, the equation for the Green's function β is of the form

$$\left\{-(2\nu)^{-1}\frac{\partial}{\partial \mathbf{r}}\left(\nu D\frac{\partial}{\partial \mathbf{r}}\right)+|\omega|\right\}\beta=\Delta, \tag{1}$$

where $\nu=mp/2\pi^2$ and $D=vl_{tr}/3$ are the state density on the Fermi surface and the coefficient of diffusion, both of which depend on the coordinates. Equation (1) is valid also in the case of jumplike changes of the parameters ν and D. The Green's function β on both sides of the discontinuity satisfies the continuity relations

$$\beta_+=\beta_-,\quad \left(\nu D\frac{\partial\beta}{\partial \mathbf{r}}\right)_+=\left(\nu D\frac{\partial\beta}{\partial \mathbf{r}}\right)_-. \tag{2}$$

Relations (2) were obtained in Refs 2 and 9.

The order parameter Δ and the current density $\mathbf{j}$ are expressed in terms of the Green's function β by means of the formulas

$$\Delta=-\nu\lambda 2\pi T\sum_{\omega>0}\beta,\qquad \mathbf{j}=ie\nu D2\pi T\sum_{\omega>0}(\beta\partial_+\beta^*-\beta^*\partial_-\beta), \tag{3}$$

where $\partial_\pm=\partial/\partial\mathbf{r}\pm 2ie\mathbf{A}$, and λ is the electron–phonon interaction constant. From the system (1), (3) we get

$$\Delta=-\nu\lambda 2\pi T\sum_{\omega>0}\left[-\frac{1}{2}\frac{\partial}{\partial r}\left(\nu D\frac{\partial}{\partial r}\right)+\nu|\omega|\right]^{-1}(\nu\Delta)$$
$$=-\nu\lambda 2\pi T\sum_{\omega>0}\int_{-\infty}^{\infty}G(x,x_1)\Delta(x_1)\,dx_1. \tag{4}$$

The kernel $G(x,x_1)$ of the integral equation (4) coincides with the corresponding expression of the de Gennes' paper[2]

$$G(x>0,x_1>0)=\frac{\mu_1}{2|\omega|}[\exp\{-\mu_1|x-x_1|\}+\alpha\exp\{-\mu_1(x+x_1)\}],$$
$$G(x<0,x_1<0)=\frac{\mu_2}{2|\omega|}[\exp\{-\mu_2|x-x_1|\}-\alpha\exp\{\mu_2(x+x_1)\}],$$
$$G(x>0,x_1<0)=\frac{\mu_2}{2|\omega|}(1-\alpha)\exp\{-\mu_1x+\mu_2x_1\},$$
$$G(x<0,x_1>0)=\frac{\mu_1}{2|\omega|}(1+\alpha)\exp\{-\mu_1x_1+\mu_2x\}, \tag{5}$$

where

$$\mu_{1,2}=\left(\frac{2|\omega|}{D_{1,2}}\right)^{1/2},\qquad \alpha=\frac{\nu_1(D_1/D_2)^{1/2}-\nu_2}{\nu_1(D_1/D_2)^{1/2}+\nu_2}. \tag{6}$$

The subscripts 1 and 2 pertain respectively to the superconductor and the normal metal.

It is easy to verify that if both T_{cN} and T_{cS} are close to T, then the exact solution of the integral equation (4) near the boundary is of the form

$$\Delta(x>0)=\Delta_++x(\partial\Delta/\partial x)_+,\quad \Delta(x<0)=\Delta_-+x(\partial\Delta/\partial x)_-. \tag{7}$$

The coefficients $\Delta_\pm$ and $(\partial\Delta/\partial x)_\pm$ satisfy in this case the relations

$$\Delta_+=\Delta_-,\quad (\nu D\partial\Delta/\partial x)_+=(\nu D\partial\Delta/\partial x)_-. \tag{8}$$

By virtue of the conditions (7), the Ginzburg–Landau equation, which is valid for all x, can be written in the

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form[10]

$$\left(1-\frac{T}{T_c(x)}\right)\Delta+\frac{\pi}{8T\nu(x)}\partial_-(\nu D\partial_-\Delta)-\frac{7\zeta(3)}{8\pi^2T^2}|\Delta|^2\Delta=0, \tag{9}$$

where ζ is the Riemann zeta function.

We proceed now to consider the case when the normal metal is either not a superconductor, or else its transition temperature is not close to T. We assume also that the thickness d of the normal-metal layer is large compared with the correlation length k_N^{-1}, the value of which will be determined below.

From formulas (4) and (5) we obtain

$$\begin{aligned}\Delta(x>0)&=\nu_1|\lambda_1|2\pi T\sum_{\omega>0}\Bigg(\frac{\mu_1}{2\omega}\int_0^\infty dx_1\Delta(x_1)\exp\{-\mu_1|x-x_1|\}\\&\quad+\frac{e^{-\mu_1x}}{2\omega}[\alpha\mu_1C(\omega)+(1-\alpha)\mu_2B(\omega)]\Bigg);\\ \Delta(x<0)&=-\nu_2\lambda_2 2\pi T\sum_{\omega>0}\Bigg(\frac{\mu_2}{2\omega}\int_{-\infty}^0 dx_1\Delta(x_1)\exp\{-\mu_2|x-x_1|\}\\&\quad+\frac{e^{\mu_2x}}{2\omega}[-\alpha\mu_2B(\omega)+(1+\alpha)\mu_1C(\omega)]\Bigg),\end{aligned} \tag{10}$$

where

$$C(\omega)=\int_0^\infty dx\Delta(x)e^{-\mu_1x},\quad B(\omega)=\int_{-\infty}^0 dx\Delta(x)e^{\mu_2x}. \tag{11}$$

The system (10) can be solved by the Wiener–Hopf method. The solution is then expressed in terms of the still unknown functions $C(\omega)$ and $B(\omega)$:

$$\begin{aligned}Z(n)&=C+\sum_{n_1=0}^\infty K(n,n_1)I_0(n_1)\left\{\frac{\alpha}{2}I_0(n_1)Z(n_1)+\frac{1-\alpha}{2}I_1(n_1)Y(n_1)\right\},\\ Y(n)&=-\operatorname{sign}\lambda_2\cdot\sum_{n_1=0}^\infty K(n,n_1)I_1(n_1)\left\{-\frac{\alpha}{2}I_1(n_1)Y(n_1)+\frac{1+\alpha}{2}I_0(n_1)Z(n_1)\right\};\end{aligned} \tag{12}$$

here

$$\begin{aligned}K(n,n_1)&=(n+{}^1/_2)^{1/2}\{(n_1+{}^1/_2)[(n_1+{}^1/_2)^{1/2}+(n+{}^1/_2)^{1/2}]\}^{-1},\\ I_0(n)&=\exp\left\{-\frac{1}{\pi}\int_0^\infty\frac{dt}{1+t^2}\ln[\psi({}^1/_2+(n+{}^1/_2)t^2)-\psi({}^1/_2)]\right\},\\ I_1(n)&=\exp\Bigg\{-\frac{1}{\pi}\int_0^\infty\frac{dt}{1+t^2}\ln\Bigg[\frac{1}{\nu_2|\lambda_2|}\\&\quad+\operatorname{sign}\lambda_2\cdot\left(\ln\left(\frac{2\gamma\omega_D}{\pi T}\right)-\psi({}^1/_2+(n+{}^1/_2)t^2)+\psi({}^1/_2)\right)\Bigg]\Bigg\};\end{aligned} \tag{13}$$

$\psi(x)$ is the psi function and $\ln\gamma=0.577$ is the Euler constant.

We have introduced in (12) new quantities Z and Y, which are connected with $C(\omega)$ and $B(\omega)$ by the relations

$$B(\omega)=Y(n)I_1(n)/\mu_2(\omega),\quad C(\omega)=Z(n)I_0(n)/\mu_1(\omega),\qquad \omega=2\pi T(n+{}^1/_2). \tag{14}$$

We note that the linear system of algebraic equations (12) depends only on two parameters: α and the ratio T_{cN}/T. The quantity C is the integration constant.

At distances large compared with $(D_1/2\pi T)^{1/2}$, the order parameter $\Delta(x)$ in the superconductor is equal to

$$\Delta(x)=C[x(8T/\pi D_1)^{1/2}+\gamma_0]+\frac{1}{\pi 2^{1/2}}\sum_{n=0}\frac{I_0(n)}{(n+{}^1/_2)^{3/2}}[\alpha Z(n)I_0(n)+(1-\alpha)Y(n)I_1(n)]. \tag{15}$$

Here

$$\gamma_0=\frac{2^{1/2}}{\pi^2}\int_0^\infty dx\{x^{-2}-\psi'({}^1/_2+x^2)[\psi({}^1/_2+x^2)-\psi({}^1/_2)]^{-1}\}=0.525. \tag{16}$$

Inside the normal metal the Green's function β is the sum of damped exponential $\exp(k_Nx)$, where the quantities k_N are obtained from the equation

$$1+\nu_2\lambda_2\left[\ln\left(\frac{2\gamma\omega_D}{\pi T}\right)-\psi\left({}^1/_2-\frac{D_2k_N^2}{4\pi T}\right)+\psi({}^1/_2)\right]=0. \tag{17}$$

At large distances there is left the exponential with the smallest value of k_N:

$$\begin{aligned}\beta(x<0)&=\frac{\mu_2^2(\omega)}{\mu_2^2(\omega)-k_N^2}\beta_0(k_N)\frac{\exp(k_Nx)}{4\omega},\\ \beta_0(k_N)&=\frac{I_2(k_N)}{\chi\psi'({}^1/_2-\chi^2)}\sum_{n=0}^\infty\frac{I_1(n)}{(n+{}^1/_2)((n+{}^1/_2)^{1/2}-\chi)}\\&\quad\times\{-\alpha Y(n)I_1(n)+(1+\alpha)Z(n)I_0(n)\},\end{aligned} \tag{18}$$

where $\chi=(k_N^2D_2/4\pi T)^{1/2}$, $\psi(x)$ is the psi function, and

$$\begin{aligned}I_2(k_N)&=\exp\Bigg\{\frac{1}{\pi}\int_0^\infty\frac{dt}{1+t^2}\ln\Bigg[\frac{1}{\nu_2|\lambda_2|}\\&\quad+\operatorname{sign}\lambda_2\cdot\left(\ln\left(\frac{2\gamma\omega_D}{\pi T}\right)-\psi({}^1/_2+t^2\chi^2)+\psi({}^1/_2)\right)\Bigg]\Bigg\}.\end{aligned} \tag{19}$$

If the thickness d of the layer of the normal metal is large compared with k_N^{-1}, then we can neglect the mutual influence of the boundaries, and the Green's function β is a sum of two terms, each of which connected only with "its own" boundary:

$$\begin{aligned}\beta&=\frac{\mu_2^2(\omega)}{\mu_2^2(\omega)-k_N^2}\frac{1}{4\omega}\left[\beta_0^l(k_N)\exp\left\{k_Nx+i\frac{\varphi}{2}\right\}\right.\\&\quad\left.+\beta_0^r(k_N)\exp\left\{-k_N(d+x)-i\frac{\varphi}{2}\right\}\right],\end{aligned} \tag{20}$$

where $\pm\varphi/2$ is the phase of the order parameter of the superconductors, $\beta_0^L(k_N)$ and $\beta_0^r(k_N)$ are amplitudes defined by formula (18), in which it is necessary to substitute the parameters of the "right" or "left" superconductor, respectively.

From (3) and (20) we obtain an expression for the current flowing through the normal-metal layer:

$$j=e\nu_2\chi^2\sin\varphi\,\beta_0^r(k_N)\beta_0^l(k_N)\psi'({}^1/_2-\chi^2)\exp\{-k_Nd\}/2k_N. \tag{21}$$

In the particular case when the electron–phonon interaction $\lambda_2\to0$, we obtain from (12), (18), and (21)

$$j=\frac{4e\nu_2}{k_N}\sin\varphi\exp\left\{-d\left(\frac{2\pi T}{D_2}\right)^{1/2}\right\}I_0^2(0)Z^r(0)Z^l(0)(1+\alpha^r)(1+\alpha^l). \tag{22}$$

The order parameter $\Delta(x)$ is determined by Eq. (15).

The system (12) can be quite easily solved with a computer. The results of the numerical calculation for the quantities $B^{\pm1}$, $A^{\pm1}$, F/C, γ, and χ are listed in Tables I and II. The quantities F and γ are defined as follows:

$$F=\chi\beta_0(k_N)\left[\psi'\left(\frac{1}{2}-\chi^2\right)\right]^{1/2},\quad \Delta(x>0)=C\left[x\left(\frac{8T}{\pi D_1}\right)^{1/2}+\gamma\right], \tag{23}$$

where C is an integration constant obtained by solving the Ginzburg–Landau equation in the superconductor. The coefficients $A^{\pm1}$ and $B^{\pm1}$ determine the behavior of the functions F/C and γ at values of the parameter α

TABLE I.

T_{cN}/T	$A^{(-1)}$	$B^{(-1)}$	$A^{(1)}$	$B^{(1)}$	χ	T_{cN}/T	$A^{(-1)}$	$B^{(-1)}$	$A^{(1)}$	$B^{(1)}$	χ
0	1.801	0.285	5.262	0.93	0.707	0.4	1.991	0.598	7.731	2.337	0.374
0.05	1.953	0.408	7.124	1.534	0.535	0.45	1.993	0.631	7.771	2.472	0.355
0.1	1.965	0.438	7.299	1.665	0.502	0.5	1.994	0.668	7.803	2.623	0.336
0.15	1.973	0.464	7.418	1.777	0.476	0.55	1.996	0.709	7.832	2.794	0.316
0.2	1.978	0.489	7.506	1.883	0.453	0.6	1.997	0.758	7.859	2.991	0.296
0.25	1.983	0.514	7.579	1.989	0.433	0.65	1.998	0.816	7.877	3.225	0.275
0.3	1.986	0.54	7.637	2.098	0.413	0.7	1.998	0.887	7.898	3.511	0.253
0.35	1.989	0.568	7.69	2.213	0.393						

close to ± 1 [Eqs. (28) and (31)].

The current density j is expressed in terms of the function F in accordance with the formula

$$j=ev_2\sin\varphi\exp(-k_Nd)F^rF^l/2k_N. \tag{24}$$

The quantity γ determines the boundary condition for the Ginzburg–Landau equation:

$$\mathbf{n}\frac{\partial\Delta}{\partial r}=\Delta\left(\frac{8T}{\pi D_1}\right)^{1/2}\gamma^{-1}, \tag{25}$$

where **n** is the direction of the inward normal to the superconductor.

The system (12) admits of an exact solution at parameter values $\alpha=\pm1$. At $\alpha=-1$ the solution of the system (12) is of the form

$$\Delta(x<0)=0,\quad \Delta(x>0)=Cx(8T/\pi D_1)^{1/2}. \tag{26}$$

The coefficients Z and Y are then

$$Z(n)=C\cdot2^{1/2}[\pi I_0(n)(n+{}^1/_2)^{1/2}]^{-1},\quad Y(n)=0. \tag{27}$$

It follows from (12) and (27) that at small values of $(1+\alpha)\ll1$ we have

$$\gamma=B^{(-1)}(1+\alpha),\quad F/C=A^{(-1)}(1+\alpha). \tag{28}$$

In the other limiting case at $\alpha=1$

$$\Delta(x>0)=\text{const}.$$

$\alpha=1$ is an eigenvalue of the system (12). The solution of the homogeneous system is of the form

$$Z(n)=1/I_0(n),\quad Z^{\text{conj}}(n)=[I_0(n)(n+{}^1/_2)^{3/2}]^{-1}, \tag{29}$$

where $Z_{\text{conj}}(n)$ is the solution of the system conjugate to (12). It follows therefore that at small values of the parameter $(1-\alpha)\ll1$ we have

$$F/C=A^{(1)}/(1-\alpha),\quad \gamma=B^{(1)}/(1-\alpha),\quad Z(n)=\gamma/I_0(n). \tag{30}$$

From the system (10) in the vicinity of the points $\alpha=\pm1$ we obtain after simple calculations

$$A^{(-1)}=2^{3/2}[\psi({}^1/_2)-\psi({}^1/_2-\chi^2)]/\pi\chi^2[\psi'({}^1/_2-\chi^2)]^{1/2},$$

$$A^{(1)}=4B^{(1)}[\psi({}^1/_2)-\psi({}^1/_2-\chi^2)]/\chi[\psi'({}^1/_2-\chi^2)]^{1/2},$$

$$B^{(-1)}=\frac{2^{1/2}}{\pi^3}\sum_{n=0}^{\infty}(n+{}^1/_2)^{-5/2}$$

$$+\frac{2^{3/2}}{\pi^4}\int_0^\infty\frac{dt}{t^4}[\psi({}^1/_2+t^2)-\psi({}^1/_2)]^2\Big/\Big[-\frac{1}{v_2\lambda_2}-\sum_0^{\omega_D/2\pi T}\frac{1}{n+{}^1/_2+t^2}\Big];$$

$$B^{(1)}=2^{1/2}\pi\Big\{\sum_0^\infty(n+{}^1/_2)^{-3/2} \tag{31}$$

$$-\frac{2}{\pi}\int_0^\infty\frac{dt}{t^2}[\psi({}^1/_2+t^2)-\psi({}^1/_2)]^2\Big/\Big[-\frac{1}{v_2\lambda_2}-\sum_0^{\omega_D/2\pi T}\frac{1}{n+{}^1/_2+t^2}\Big]\Big\}^{-1}.$$

At values of the parameter $\lambda_2<0$ we have

$$-\frac{1}{v_2\lambda_2}-\sum_0^{\omega_D/2\pi T}\frac{1}{n+{}^1/_2+t^2}=\ln\left(\frac{T}{T_{cN}}\right)+\psi({}^1/_2+t^2)-\varphi({}^1/_2), \tag{32}$$

where T_{cN} is the temperature of the transition of the normal metal into the superconducting state.

At $T_{cN}/T=0$, the expression for the coefficient γ, determined by formulas (30) and (31), goes over into the expression obtained in Ref. 9.

It follows from (28) and (30) that when the diffusion coefficient D_1 in the superconductor increases, the boundary condition (25) changes and goes over from $\Delta=0$ to $\Delta'=0$.

At intermediate values of the parameter α, the quantities γ and F, which determine the boundary condition of the critical current of the SNS junction, are given in Table II.

We note that in the region of small values of the parameter T_{cN}/T the transition to the limiting state corresponding to $T_{cN}=0$ is effected in accord with the parameter $\Gamma=\ln(T/T_{cN})$. Only at $\Gamma\gg1$ do the expressions for the functions γ and F assume their limiting

TABLE II.

T_{cN}/T	α									
		−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1
0		0.187 0.032	0.391 0.065	0.613 0.101	0.858 0.14	1.13 0.185	1.43 0.235	1.78 0.292	2.17 0.358	2.63 0.434
0.05		0.205 0.045	0.431 0.093	0.681 0.145	0.961 0.204	1.28 0.271	1.63 0.346	2.04 0.433	2.52 0.534	3.07 0.653
0.1		0.206 0.048	0.434 0.099	0.687 0.156	0.97 0.22	1.29 0.291	1.65 0.373	2.07 0.467	2.55 0.576	3.12 0.704
0.15		0.207 0.051	0.437 0.105	0.692 0.165	0.977 0.233	1.3 0.309	1.67 0.396	2.09 0.496	2.57 0.612	3.15 0.749
0.2		0.208 0.054	0.439 0.111	0.695 0.174	0.981 0.246	1.31 0.326	1.67 0.418	2.1 0.524	2.59 0.646	3.17 0.791
0.3		0.209 0.059	0.441 0.122	0.699 0.193	0.988 0.272	1.32 0.361	1.69 0.463	2.12 0.58	2.61 0.716	3.2 0.877
0.4		0.21 0.065	0.443 0.135	0.702 0.214	0.994 0.301	1.32 0.4	1.7 0.513	2.13 0.643	2.63 0.795	3.23 0.974
0.5		0.211 0.073	0.445 0.151	0.704 0.239	0.997 0.337	1.33 0.448	1.71 0.574	2.14 0.72	2.65 0.89	3.25 1.09
0.6		0.211 0.083	0.445 0.172	0.707 0.271	1 0.383	1.33 0.509	1.71 0.653	2.15 0.819	2.66 1.01	3.26 1.24
0.7		0.212 0.097	0.448 0.201	0.711 0.318	1.01 0.449	1.34 0.598	1.72 0.767	2.16 0.963	2.68 1.19	3.28 1.46
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	3.16 0.525	3.8 0.635	4.59 0.77	5.59 0.942	6.9 1.17	8.71 1.48	11.38 1.95	15.78 2.73	24.48 4.26	50.18 8.8
0.05	3.74 0.795	4.55 0.968	5.55 1.18	6.84 1.46	8.54 1.82	10.92 2.34	14.47 3.1	20.36 4.36	32.03 6.88	66.43 14.28
0.1	3.8 0.858	4.62 1.05	5.65 1.28	6.96 1.58	8.71 1.97	11.14 2.53	14.78 3.36	20.8 4.73	32.76 7.46	67.96 15.48
0.15	3.83 0.912	4.67 1.11	5.71 1.36	7.04 1.68	8.82 2.1	11.29 2.69	14.98 3.58	21.1 5.04	33.24 7.95	68.93 16.5
0.2	3.86 0.964	4.71 1.18	5.76 1.44	7.11 1.78	8.9 2.23	11.4 2.85	15.13 3.79	21.32 5.34	33.6 8.42	69.65 17.46
0.3	3.9 1.07	4.76 1.31	5.83 1.6	7.2 1.97	9.02 2.47	11.56 3.17	15.36 4.22	21.65 5.94	34.13 9.37	70.68 19.42
0.4	3.94 1.19	4.8 1.45	5.88 1.78	7.27 2.2	9.11 2.75	11.68 3.53	15.53 4.69	21.89 6.62	34.5 10.45	71.35 21.57
0.5	3.96 1.33	4.84 1.63	5.93 1.99	7.33 2.46	9.18 3.09	11.78 3.96	15.66 5.26	22.08 7.42	34.77 11.69	71.76 24.14
0.6	3.98 1.52	4.86 1.85	5.96 2.27	7.37 2.81	9.25 3.52	11.86 4.51	15.76 6	22.22 8.46	34.97 13.32	71.95 27.43
0.7	4.01 1.78	4.89 2.18	6 2.67	7.41 3.3	9.3 4.13	11.92 5.3	15.84 7.05	22.32 9.93	35 13 15.61	71.82 32.03

Note. The upper lines represent F/C, the lower γ.

values corresponding to $T_{cN}/T=0$. Therefore in the vicinity of the point $T_{cN}/T=0$ there is observed an abrupt change of the parameters γ and F when T_{cN} deviates little from zero.

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Quantum defects in superconductors

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The possibility of the existence of quantum defects in metals is studied. Because of collisions with electrons, the quantum defects in a normal metal are localized at temperatures which exceed their band width. The ranges of concentration and temperature at which the quantum defects in a superconductor are localized have been found. It is shown that the electron-defecton interaction leads to an increase in the gap in the electron excitation spectrum.

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It has been suggested by A. F. Andreev and I. M. Lifshitz that hydrogen can be a quantum impurity in certain metals. A large number of papers has been devoted to quantum defects in He^4 (see the review of Ref. 2 and elsewhere), but quantum defects in metals have been little studied.

In the first part of the present work, the interaction of quantum defects with electrons and with one another is considered, and the region of their existence is found. In the second part, the effect of quantum defects on the superconducting characteristics is studied, and it is shown that the interaction with quantum defects leads to an increase in the gap in the spectrum of electronic excitations of the superconductor.

QUANTUM DEFECTS IN METALS

The atoms of hydrogen occupy voids in the metal matrix. As a consequence of quantum tunneling, the impurity level diffuses into the energy band. Similarly to electrons in a metal, the defection is characterized by a quasimomentum p and a dispersion law $\varphi(\mathbf{p})$. In the case of low hydrogen concentration, the elementary excitation is the hydrogen atom-impuriton, but the results are applicable with some reservations to vacancies in the hydrogen sublattice, when their number is small, and the metal + hydrogen combination is nearly stoichiometric. Both the vacancion and the impuriton are quantum defects, to which the analysis is in fact devoted.

The Hamiltonian of a system of defectons has the form

$$H=H_0+H_{\rm int} \tag{1}$$

$$H_0=\sum_{\mathbf{p}}\varphi(\mathbf{p})d^+(\mathbf{p})d(\mathbf{p}), \tag{2}$$

H_0 describes the system of noninteracting defectons, $d^+(\mathbf{p})$ and $d(\mathbf{p})$ are the second-quantization operators of the defectons, $H_{\rm int}$ includes the interaction of the defectons with phonons, with electrons, and with one another.

It has been shown in Refs. 1–3 that at temperatures T much lower than the Debye temperature Θ_D but far exceeding the bandwidth of the defections ε_0, the mean free path of the defectons between collisions with phonons l_g behaves as $a(\Theta_D/8T)^9$, where a is the interatomic distance. We shall be interested in temperatures $T\leqslant T_c$, where T_c is the temperature of the superconducting transition. For these temperatures, $l_g\gg a$ and the defecton-phonon interaction can be neglected.

The interaction of the defectons with electrons was considered in Ref. 3, but the electrons were assumed to be nondegenerate in that case. In metals, the electrons are strongly degenerate and these results are not applicable. The Hamiltonian of the electron-defecton interaction is equal to

$$H_{\rm int}=\sum_{\mathbf{pp'q}}V(\mathbf{q})b^+(\mathbf{p})d^+(\mathbf{p'})d(\mathbf{p'}-\mathbf{q})b(\mathbf{p}+\mathbf{q}), \tag{3}$$

$b^+(\mathbf{p})$ and $b(\mathbf{p})$ are the second-quantization operators of

