values corresponding to $T_{cN}/T=0$. Therefore in the vicinity of the point $T_{cN}/T=0$ there is observed an abrupt change of the parameters γ and F when T_{cN} deviates little from zero.

In conclusion, the authors thank A. I. Larkin for useful remarks.

[1]N. R. Werthamer, Phys. Rev. **132**, 2440 (1963).
[2]P. G. de Gennes, Rev. Mod. Phys. **36**, 225 (1964).
[3]R. O. Zaĭtsev, Zh. Eksp. Teor. Fiz. **50**, 1055 (1966) [Sov. Phys. JETP **23**, 702 (1966)].
[4]L. G. Aslamazov, A. I. Larkin, and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 323 (1968) [Sov. Phys. JETP **28**, 171 (1969)].
[5]G. Eilenberger, Z. Phys. **214**, 195 (1968).
[6]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys. JETP **28**, 1200 (1969)].
[7]K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[8]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **61**, 2147 (1971) [Sov. Phys. JETP **34**, 144 (1972)].
[9]Z. G. Ivanov, M. Yu. Kupriyanov, A. K. Liharev, and O. V. Snigirev, J. Phys. (Paris) **39**, C6-556 (1978).
[10]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **61**, 1221 (1971) [Sov. Phys. JETP **34**, 651 (1972)].

Translated by J. G. Adashko

Quantum defects in superconductors

A. I. Morozov

Moscow Institute of Radio Engineering, Electronics, and Automation

(Submitted 10 April 1979)
Zh. Eksp. Teor. Fiz. 77, 1471–1478 (October 1979)

The possibility of the existence of quantum defects in metals is studied. Because of collisions with electrons, the quantum defects in a normal metal are localized at temperatures which exceed their band width. The ranges of concentration and temperature at which the quantum defects in a superconductor are localized have been found. It is shown that the electron-defecton interaction leads to an increase in the gap in the electron excitation spectrum.

PACS numbers: 74.90. + n, 71.50. + t

It has been suggested by A. F. Andreev and I. M. Lifshitz that hydrogen can be a quantum impurity in certain metals. A large number of papers has been devoted to quantum defects in He^4 (see the review of Ref. 2 and elsewhere), but quantum defects in metals have been little studied.

In the first part of the present work, the interaction of quantum defects with electrons and with one another is considered, and the region of their existence is found. In the second part, the effect of quantum defects on the superconducting characteristics is studied, and it is shown that the interaction with quantum defects leads to an increase in the gap in the spectrum of electronic excitations of the superconductor.

QUANTUM DEFECTS IN METALS

The atoms of hydrogen occupy voids in the metal matrix. As a consequence of quantum tunneling, the impurity level diffuses into the energy band. Similarly to electrons in a metal, the defection is characterized by a quasimomentum p and a dispersion law $\varphi(\mathbf{p})$. In the case of low hydrogen concentration, the elementary excitation is the hydrogen atom-impuriton, but the results are applicable with some reservations to vacancies in the hydrogen sublattice, when their number is small, and the metal + hydrogen combination is nearly stoichiometric. Both the vacancion and the impuriton are quantum defects, to which the analysis is in fact devoted.

The Hamiltonian of a system of defectons has the form

$$H=H_0+H_{\text{int}} \tag{1}$$

$$H_0=\sum_{\mathbf{p}}\varphi(\mathbf{p})d^+(\mathbf{p})d(\mathbf{p}), \tag{2}$$

H_0 describes the system of noninteracting defectons, $d^+(\mathbf{p})$ and $d(\mathbf{p})$ are the second-quantization operators of the defectons, H_{int} includes the interaction of the defectons with phonons, with electrons, and with one another.

It has been shown in Refs. 1–3 that at temperatures T much lower than the Debye temperature Θ_D but far exceeding the bandwidth of the defections ε_0, the mean free path of the defectons between collisions with phonons l_g behaves as $a(\Theta_D/8T)^9$, where a is the interatomic distance. We shall be interested in temperatures $T\leqslant T_c$, where T_c is the temperature of the superconducting transition. For these temperatures, $l_g\gg a$ and the defecton-phonon interaction can be neglected.

The interaction of the defectons with electrons was considered in Ref. 3, but the electrons were assumed to be nondegenerate in that case. In metals, the electrons are strongly degenerate and these results are not applicable. The Hamiltonian of the electron-defecton interaction is equal to

$$H_{\text{int}}=\sum_{\mathbf{pp'q}}V(\mathbf{q})b^+(\mathbf{p})d^+(\mathbf{p'})d(\mathbf{p'}-\mathbf{q})b(\mathbf{p}+\mathbf{q}), \tag{3}$$

$b^+(\mathbf{p})$ and $b(\mathbf{p})$ are the second-quantization operators of the electrons, while $V(\mathbf{q})$ describes the defecton-electron interaction. Since the bandwidth of the defectons $\varepsilon_0 \ll \varepsilon_F$, where ε_F is the Fermi energy of the electrons. the velocity of the defectons is much less than the velocity of the electrons. Therefore, with accuracy to within $\varepsilon_0/\varepsilon_F$ we can assume that $V(\mathbf{q})$ corresponds to the interaction of the electron with a static defect,[3]

We consider the electron-defecton interaction on the basis of the temperature diagram technique.[4] We shall assume here that $T \gg \varepsilon_0$, and that the defectons obey Fermi statistics. At $T \gg \varepsilon_0$, this is not important, since the Boltzmann limit for both statistics is identical.

Collisions with electrons limit the path length of the defecton l_g. We are interested in the case in which the interaction with electrons is weak and $l_g \gg a$. In the opposite case, the defectons are localized, and we get the case of static defects.

First, we shall study the electron-defecton interaction in the normal metal. Simple analysis of the graphs shows that the digram shown in Fig. 1a leads only to renormalization of the chemical potential of the electron and the basic contribution to the electron self-energy part $\Sigma_{ig}(\mathbf{p}, \varepsilon_k)$ is made in Born approximation by the diagram shown in Fig. 1b.

The thin and heavy solid lines correspond to the zeroth, $G_0(\mathbf{p}, \varepsilon_k)$, and total, $G(\mathbf{p}, \varepsilon)$, Green's functions of the electrons, the wavy lines to $V(\mathbf{q})$, and the dashed line to the Green's function of the defecton $\psi(\mathbf{p}, \varepsilon_k)$:

$$G_0(\mathbf{p}, \varepsilon_k) = [i\varepsilon_k - \varepsilon(\mathbf{p}) + \mu]^{-1}, \tag{4}$$

$$G^{-1}(\mathbf{p}, \varepsilon_k) = G_0^{-1}(\mathbf{p}, \varepsilon_k) - \Sigma_{1g}(\mathbf{p}, \varepsilon_k), \tag{5}$$

$$\psi(\mathbf{p}, \varepsilon_k) = [i\varepsilon_k - \varphi(\mathbf{p}) - \chi(\mathbf{p}, \varepsilon_k) + \zeta]^{-1}, \tag{6}$$

where $\varepsilon_k = T(2k+1)$, $k = 0, \pm 1, \pm 2, \ldots$, $\varepsilon(\mathbf{p})$ is the dispersion law of the electrons, and μ and ζ are the chemical potentials of the electrons and defectons. Since $l_g \gg a$, the self-energy part of the defectons $\chi(\mathbf{p}, \varepsilon_k)$ satisfies the condition

$$|\operatorname{Im} \chi(\mathbf{p}, \varepsilon_k)| \ll \varepsilon_0, \tag{7}$$

and we can neglect it. The real part of χ, as will be shown below, leads to the renormalization of ζ.

The next terms of perturbation theory, which contain new defecton loops, are equivalent to crossing graphs in the case of static impurities, and for the same reason contain an extra power of the defecton concentration x. Graphs containing a single defecton loop but a larger number of wavy lines, are equivalent to account of more than two crosses from a single atom of the impurity in the case of static defects and are small in the Born approximation.

In a superconductor where, along with $G(\mathbf{p}, \varepsilon_k)$, there exist the anomalous Green's functions $F^+(\mathbf{p}, \varepsilon_k)$ and $F(\mathbf{p}, \varepsilon_k)$ of the electrons, the electron-defecton interaction leads to the appearance of the self-energy parts $\Sigma_{2g}^+(\mathbf{p}, \varepsilon_k)$ and $\Sigma_{2g}(\mathbf{p}, \varepsilon_k)$. With the same accuracy as $\Sigma_{ig}(\mathbf{p}, \varepsilon_k)$, they are shown by the diagrams of Fig. 2.

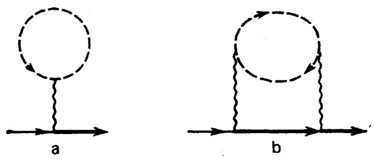

FIG. 1.

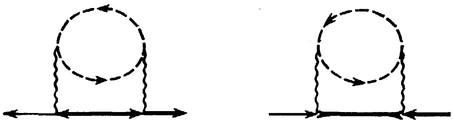

FIG. 2.

In the zeroth approximation in ε_k/T, the quantity $\Sigma_{1g}(\mathbf{p}, \varepsilon_k)$ is identical with the results for the static impurities[4]:

$$\Sigma_{1g}(\varepsilon_k) = -\frac{i \operatorname{sign} \varepsilon_k}{2\tau_{el}}, \tag{8}$$

where

$$\tau_{el}^{-1} = \frac{xmp_F}{4\pi^2 W_{cell}} \int \left| V\left(2p_F \sin\frac{\theta}{2}\right)\right|^2 d\Omega, \tag{9}$$

W_{cell} is the volume of the elementary cell, x is the concentration of the defects, m and p_F are the mass and the Fermi momentum of the electrons. Integration is carried out over the Fermi surface. Departure from the limits of the Born approximation leads to replacement of the zero amplitude scattering $V(\mathbf{q})$ by the total amplitude $\tilde{V}(\mathbf{q})$. Therefore, in the following, we shall not limit ourselves to the Born approximation. The next orders of ε_0/T in $\Sigma_{1g}(\varepsilon_k)$ are considered in the second part of this paper.

We now consider the defecton self-energy part $\chi(\mathbf{p}, \varepsilon_k)$. The first terms of perturbation theory correspond to diagrams similar to those shown in Fig. 1, but now the electron and defecton Green's functions must change places. Then the diagram of Fig. 1a gives the renormalization of the chemical potential of the defectons ζ, and the diagram of Fig. 1b, gives also the damping of the defectons $\operatorname{Im}\chi(\mathbf{p}, \varepsilon_k)$. Corrections containing new electron loops and wavy lines are small in the Born approximation.

The corrections to the electron-defecton vertex, which contain new defecton loops (Fig. 3), compensate, in the zeroth approximation in ε_0/T, for the appearance of $\Sigma_{1g}(\varepsilon_k)$ in the electron Green's function (5). Therefore, in the calculation of χ we can use the Green's function of the free electrons and disregard these corrections. Then

$$\chi(\mathbf{p}, \varepsilon_k) = -2T^2 \sum_{\omega_n \varepsilon_m} \int \frac{d\mathbf{q}d\mathbf{p}'}{(2\pi)^6} |V(\mathbf{q})|^2$$
$$\times [i(\varepsilon_m - \omega_n) - \varepsilon(\mathbf{p}' - \mathbf{q}) + \mu]^{-1} [i\varepsilon_m - \varepsilon(\mathbf{p}') + \mu]^{-1}$$
$$\times [i(\varepsilon_k - \omega_n) - \varphi(\mathbf{p} - \mathbf{q}) + \zeta - \chi(\mathbf{p} - \mathbf{q}, \varepsilon_k - \omega_n)]^{-1};$$
$$\omega_n = 2\pi T n, \quad n = 0, \pm 1, \pm 2, \ldots \tag{10}$$

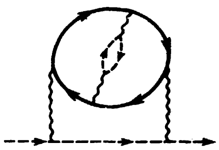

FIG. 3.

Since we are interested in the region in which $|\mathrm{Im}\chi| \ll \varepsilon_0$ and $l_g \gg a$, then we neglect $\chi(\mathbf{p}-\mathbf{q},\ \varepsilon_k-\omega_k)$ in the right side of (10). After summation over the frequencies, we obtain

$$\chi(\mathbf{p},\varepsilon_k)=-\int\frac{d\mathbf{p}'d\mathbf{q}}{(2\pi)^6}|V(\mathbf{q})|^2\frac{n(\mathbf{p}'-\mathbf{q})(1-n(\mathbf{p}'))}{i\varepsilon_k-\varepsilon(\mathbf{p}')+\varepsilon(\mathbf{p}'-\mathbf{q})-\varphi(\mathbf{p}-\mathbf{q})+\zeta}, \quad (11)$$

where

$$n(\mathbf{p}')=\left[1+\exp\frac{\varepsilon(\mathbf{p}')-\mu}{T}\right]^{-1}. \quad (12)$$

We continue $\chi(\mathbf{p},\varepsilon_k)$ from the points $\varepsilon=i\varepsilon_k$ to the real axis. Integration over $\mathbf{p}'$ and $\mathbf{q}'$ over regions far from the pole of (11) gives a constant renormalization of ζ. Integration near the pole gives

$$\mathrm{Im}\,\chi=-\pi\int\frac{d\mathbf{p}'d\mathbf{q}}{(2\pi)^6}|V(\mathbf{q})|^2 n(\mathbf{p}'-\mathbf{q})(1-n(\mathbf{p}'))\delta(\varepsilon(\mathbf{p}'-\mathbf{q})-\varepsilon(\mathbf{p}'))=\mathrm{const}, \quad (13)$$

where $\delta(y)$ is the Dirac delta function, and we can neglect the value of $\varphi(\mathbf{p})$ in its argument[2] in comparison with the characteristic values

$$|\varepsilon(\mathbf{p}'-\mathbf{q})-\varepsilon(\mathbf{p}')|\sim T.$$

In (13) only $|\mathbf{p}'|$ and $|\mathbf{p}'-\mathbf{q}|$ are important close to p_F. Then the integration over $\xi=\varepsilon(\mathbf{p}')-\mu$ can be continued to infinity. We obtain

$$\mathrm{Im}\,\chi=-T\frac{(mp_F)^2}{(2\pi)^4}\int\left|V\left(2p_F\sin\frac{\theta}{2}\right)\right|^2 d\Omega. \quad (14)$$

The condition for the existence of the free defectons takes the form

$$T\frac{(mp_F)^2}{(2\pi)^4}\int\left|V\left(2p_F\sin\frac{\theta}{2}\right)\right|^2 d\Omega\ll\varepsilon_0\ll T. \quad (15)$$

In real metals, the quantity

$$\frac{(mp_F)^2}{(2\pi)^4}\int\left|V\left(2p_F\sin\frac{\theta}{2}\right)\right|^2 d\Omega\sim 1,$$

and there is no region of existence of free defectons at $T\gg\varepsilon_0$.

In superconductors at temperatures below the temperature of the superconducting transition T_c, the number of electron excitations falls off rapidly with decrease in the temperature. At $T\ll T_c$, it is exponentially small. The damping of the defectons is found similar to (14), and we must take into account, along with the ordinary polarization operator, a polarization operator consisting of two anomalous Green's functions of the electrons. We have

$$\mathrm{Im}\,\chi=-T\frac{(mp_F)^2}{(2\pi)^4}\int\left|V\left(2p_F\sin\frac{\theta}{2}\right)\right|^2 d\Omega\frac{2}{1+\exp(\Delta(T)/T)}, \quad (16)$$

where $\Delta(T)$ is the gap in the spectrum of electronic excitations. Here we continue to assume that $T\gg\varepsilon_0$. Since $|\mathrm{Im}\gamma|$ falls off rapidly with decrease in the temperature, the condition $|\mathrm{Im}\chi|\ll\varepsilon_0$ begins to be satisfied at $\varepsilon_0\ll T\ll T_c$.

We now consider the interaction of the defectons with one another. According to Ref. 2, the condition of applicability of the gas approximation $\langle r\rangle\gg R_{\mathrm{int}}$, where $\langle r\rangle$ is the mean distance between the defectons and R_{int} is the characteristic radius of their interaction, yields a limit on the concentration of defects:

$$x\ll(a/R_{\mathrm{int}})^3, \quad (17)$$

R_{int} is found from the equation

$$U(R_{\mathrm{int}})=\varepsilon_0, \quad (18)$$

where $U(r)$ is the potential energy of interaction of the defectons. At high concentrations, the defecton-defecton interaction leads to localization of the defectons.

It is seen from (18) that the region of concentration of free defectons depends on the state of the hydrogen in the metal lattice. Since the hydrogen atom is completely or partially ionized in the metal,[5] we shall assume that the defects interact as point charges screened by s electrons. Their deformation interaction is significantly weaker than the Coulomb interaction.[5] The principal role is played by the interaction of the defect with the Friedel s-electron density oscillations produced by another defect. The potential energy of this interaction is equal to[6]

$$U(r)\approx\frac{W_{\mathrm{eff}}me^2V(2p_F)}{4\pi^2\hbar p_F\bar{\varepsilon}^2(2p_F)}\frac{\cos(2p_Fr)}{r^3}, \quad (19)$$

where $\bar{\varepsilon}(2p_F)$ is the permittivity. From (17), with account of (18) and (19), we obtain

$$x\ll 4\pi^2\hbar p_F\bar{\varepsilon}^2(2p_F)\varepsilon_0/me^2V(2p_F). \quad (20)$$

At these concentrations, the quantum defects in the superconductor are free.

If the quantum defect is a vacancy in the hydrogen sublattice, then it undergoes additional scattering due to the presence of nuclear spin in the hydrogen atoms. At $T\gg\varepsilon_0$ these spins are disordered, which leads to destruction of coherence upon tunneling and to the appearance of an additional damping. Its estimate was given in Ref. 7. We have $|\mathrm{Im}\chi|=0$ at $|\varepsilon-\zeta|>\varepsilon_0$.

In what follows, we shall be interested in the characteristic energies of the order of $\Delta\gg\varepsilon_0$. For these values of the energy, we can neglect the scattering and assume the vacancies to be free.

EFFECT OF QUANTUM DEFECTS ON SUPERCONDUCTIVITY

We consider a superconductor which contains free defectons at temperatures $\varepsilon_0\ll T\ll T_c$. If the scattering of the electron by the static defect was not accompanied by a change in its energy and in the isotropic case had no affect on the superconductivity, then the scattering by the defecton is accompanied by a change in the energy by an amount of the order of ε_0. As a result the gap in the spectrum of the electronic excitations begins to depend on the concentration of the defects.

Thus, we now take the electron-defect on interaction into account together with the electron-phonon interaction. As has been shown above, the latter leads to the appearance of $\Sigma_{1g}(\mathbf{p}, \varepsilon_k)$ and $\Sigma_{2g}(\mathbf{p}, \Sigma_k)$, which are shown in Figs. 1b and 2. The electron-phonon corrections to the electron-defecton vertex are small as $(m/M)^{1/2}$, where M is the mass of the ion.[8] Therefore, the electron self-energy parts of $\Sigma_1(\mathbf{p}, \varepsilon_k)$ and $\Sigma_2(\mathbf{p}, \varepsilon_k)$ are simply the sum of the phonon and defecton parts. The electron Green's functions are expressed by Σ_1 and Σ_2 in the usual way.[9]

The Éliashberg equations[9] with account of electron-defecton interaction have the form

$$f_1(\mathbf{p}, \varepsilon_k)=\pi T\sum_{\varepsilon_m}\int\frac{|\Gamma(\mathbf{p}-\mathbf{p}_1)|^2 mp_F d\Omega_1}{(2\pi)^3\tilde{\varepsilon}^2(\mathbf{p}-\mathbf{p}_1)}\frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_k-\varepsilon_m)^2}$$
$$\times\frac{i\varepsilon_m}{[\varepsilon_m^2+\Delta^2(\mathbf{p}_1, \varepsilon_m)]^{1/2}}-xT\sum_{\varepsilon_m}\int\frac{mp_F d\Omega_1 d\mathbf{p}'|V(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6}$$
$$\times\frac{\exp(-\varphi(\mathbf{p}')/T)-\exp(-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)/T)}{i(\varepsilon_k-\varepsilon_m)-\varphi(\mathbf{p}')+\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)}\int_{-\infty}^{\infty}\frac{i\varepsilon_m d\xi}{\varepsilon_m^2+\xi^2+\Delta^2(\mathbf{p}_1, \varepsilon_m)}, \quad (21)$$

$$\Delta(\mathbf{p}, \varepsilon_k)=\frac{\pi T}{1+if_1(\mathbf{p}, \varepsilon_k)/\varepsilon_k}\sum_{\varepsilon_m}\int\frac{|\Gamma(\mathbf{p}-\mathbf{p}_1)|^2 mp_F d\Omega_1}{(2\pi)^2\tilde{\varepsilon}^2(\mathbf{p}-\mathbf{p}_1)}$$
$$\times\frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_k-\varepsilon_m)^2}\frac{\Delta(\mathbf{p}_1, \varepsilon_m)}{[\varepsilon_m^2+\Delta^2(\mathbf{p}_1, \varepsilon_m)]^{1/2}}$$
$$+\frac{xT}{1+if_1(\mathbf{p}, \varepsilon_k)/\varepsilon_k}\sum_{\varepsilon_m}\int\frac{mp_F d\Omega_1 d\mathbf{p}'|V(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6}$$
$$\times\frac{\exp(-\varphi(\mathbf{p}')/T)-\exp(-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)/T)}{i(\varepsilon_k-\varepsilon_m)-\varphi(\mathbf{p}')+\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)}\int_{-\infty}^{\infty}\frac{\Delta(\mathbf{p}_1, \varepsilon_m)d\xi}{\varepsilon_m^2+\xi^2+\Delta^2(\mathbf{p}_1, \varepsilon_m)}, \quad (22)$$

$f_1(\mathbf{p}, \varepsilon_k)$ is the part of $\Sigma_1(\mathbf{p}, \varepsilon_k)$ that is odd in ε_k,

$$\Delta(\mathbf{p}, \varepsilon_k)=\Sigma_2(\mathbf{p}, \varepsilon_k)[1+if_1(\mathbf{p}, \varepsilon_k)/\varepsilon_k]^{-1}, \quad (23)$$

$\Gamma(\mathbf{p}-\mathbf{p}_1)$ is the electron-phonon matrix element, and $\omega_0(\mathbf{p}-\mathbf{p}_1)$ is the phonon frequency. Integration with respect to $\mathbf{p}_1$ is carried out over the Fermi surface, and with respect to $\mathbf{p}'$, over the first Brillouin zone. Moreover, we have used the fact that, with accuracy to term of order x^2,

$$\text{th}\frac{\varphi(\mathbf{p}')-\zeta}{2T}=1-bx\exp(-\varphi(\mathbf{p}')/T), \quad (24)$$

where

$$b=1+O(\varepsilon_0/T)\approx 1.$$

We shall solve the set of equations (21), (22) by perturbation theory assuming the correction from the electron defecton interaction to Δ to be small. In what follows, we limit ourselves to the case of small coupling, when the constant of electron-phonon interaction $\lambda \ll 1$ and the phonon contribution to f_1 can be neglected. Then

$$\Delta(\mathbf{p}, \varepsilon_k)=\Delta_0+\Delta_1(\mathbf{p}, \varepsilon_k), \quad (25)$$

where Δ_0 is the gap in the absence of electron-defecton interaction. In the first approximation, we get from (21)

$$f_1(\mathbf{p}, \varepsilon_k)=-\frac{ix}{W_{\text{cell}}}\int\frac{mp_F d\Omega_1|V(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3}\frac{\varepsilon_k}{(\varepsilon_k^2+\Delta_0^2)^{1/2}}$$
$$+ix\int\frac{mp_F d\Omega_1 d\mathbf{p}'|V(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6}[\varphi(\mathbf{p}')-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)]^2$$
$$\times\left\{\frac{\varepsilon_k}{T(\varepsilon_k^2+\Delta_0^2)}\left[1+\frac{\Delta_0^2}{2\varepsilon_k(\varepsilon_k^2+\Delta_0^2)^{1/2}}\ln\frac{(\varepsilon_k^2+\Delta_0^2)^{1/2}+\varepsilon_k}{(\varepsilon_k^2+\Delta_0^2)^{1/2}-\varepsilon_k}\right]\right.$$
$$\left.-\frac{3\pi\varepsilon_k}{2(\varepsilon_k^2+\Delta_0^2)^{3/2}}\right\}. \quad (26)$$

In (26), we have limited ourselves to the first non-vanishing term in $\varphi(\mathbf{p}')/T$. The terms that are linear in $\varphi(\mathbf{p}')$ contain the factor

$$[\varphi(\mathbf{p}')-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)]$$

and vanish upon integration with respect to $\mathbf{p}'$, which is carried out over the first Brillouin zone. Moreover, we have assumed that

$$\text{th}\frac{(\Delta_0^2+\xi^2)^{1/2}}{2T}=1.$$

In the region $T \ll T_c$, this assumption is incorrect.

The first term on the right side of (26), as also in the case of static impurities, does not affect the value of the superconducting gap in the isotropic case, canceling with the similar term in (22). The electron-defecton interaction makes $f_1(\mathbf{p}, \varepsilon_k)$ and $\Delta(\mathbf{p}, \varepsilon_k)$ anisotropic; however, the corrections connected with the anistropy appear in the next approximation in x and we can neglect them. Therefore, we can average f_1 over the Fermi surface:

$$f_1(\varepsilon_k)=\int\frac{d\Omega}{4\pi}f_1(\mathbf{p}, \varepsilon_k). \quad (27)$$

Substituting $f_1(\varepsilon_k)$ in (22), we obtain

$$\Delta(\varepsilon_k)=\pi T\sum_{\varepsilon_m}\int_0^{k_D}\frac{|\Gamma(q)|^2 q\,dq}{(2\pi)^2 v_F\tilde{\varepsilon}^2(q)}\frac{\omega_0^2(q)}{\omega_0^2(q)+(\varepsilon_k-\varepsilon_m)^2}$$
$$\times\frac{\Delta(\varepsilon_m)}{(\varepsilon_m^2+\Delta^2(\varepsilon_m))^{1/2}}+x\int\frac{mp_F d\Omega d\Omega_1 d\mathbf{p}'|V(\mathbf{p}-\mathbf{p}_1)|^2}{2(2\pi)^7}$$
$$\times[\varphi(\mathbf{p}')-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)]^2\left\{\frac{\Delta_0}{2\varepsilon_k T(\varepsilon_k^2+\Delta_0^2)^{1/2}}\right.$$
$$\left.\times\ln\frac{(\varepsilon_k^2+\Delta_0^2)^{1/2}+\varepsilon_k}{(\varepsilon_k^2+\Delta_0^2)^{1/2}-\varepsilon_k}-\frac{\pi\Delta_0}{(\varepsilon_k^2+\Delta_0^2)^{3/2}}\right\}. \quad (28)$$

The second term in the first part of (28) is the low-frequency contribution $\Delta_1'(\varepsilon_k)$ to Δ_0. It is easy to see that it falls off rapidly at frequencies $|\varepsilon_k| \gg \Delta_0$.

In addition, the electron-defecton interaction leads to the appearance of a constant correction Δ_1'' to Δ_0, which arises upon the substitution of Δ_1' in the first term of the right side of (28):

$$\Delta_1''=\sum_{\varepsilon_m}\frac{\Delta_1'(\varepsilon_m)\varepsilon_m^2}{(\varepsilon_m^2+\Delta_0^2)^{3/2}}\Big/\sum_{\varepsilon_m}\frac{\Delta_0^2}{(\varepsilon_m^2+\Delta_0^2)^{3/2}}. \quad (29)$$

Since $T \ll \Delta_0$, summation in (28) can be replaced by integration. Then

$$\Delta_1''=\frac{x}{4}\left(\frac{1}{\Delta_0 T}-\frac{\pi^2}{8\Delta_0^2}\right)\int\frac{mp_F d\Omega d\Omega_1 d\mathbf{p}'|V(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^7}$$
$$\times[\varphi(\mathbf{p}')-\varphi(\mathbf{p}'-\mathbf{p}+\mathbf{p}_1)]^2. \quad (30)$$

Both Δ_1' and Δ_1'' are of the order of $\varepsilon_0^2/\Delta_0 T\tau_{\text{el}}$. In the case of a superconductor with strong coupling, the order of Δ_1' and Δ_1'' does not change.

Thus the electron-defecton interaction leads to the appearance of an effective attraction between the electrons at frequencies of the order of Δ_0, which in turn produces an increase in the gap of the spectrum of electronic excitations of the superconductor by an amount of the order of $\varepsilon_0^2/\Delta_0\tau_{\text{el}}T$, and also to the appearance in it of a frequency dependence at frequencies of the order of Δ_0.

An example of a material in which this effect can be observed is palladium hydride PdH_{1-x}, where the quantum defect is a vacancy in the hydrogen sublattice. A rough estimate gives a value of the order of 1 K for ε_0. The region of concentrations for which the vacancions can be regarded as free is $x \ll 10^{-3}-10^{-4}$, while the quantity Δ_1/Δ_0 reaches 10% on the boundary of this region. Since Δ' depends quadratically on ε_0, a preliminary measurement of ε_0 is necessary in the study of this effect.

The author thanks R. O. Zaĭtsev for suggesting the problem and supervision of the work and A. F. Andreev and V. I. Marchenko for discussion of the paper.

[1]A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **56**, 2057 (1969) [Sov. Phys. JETP **29**, 1107 (1969)].

[2]A. F. Andreev, Usp. Fiz. Nauk **118**, 251 (1976) [Sov. Phys. Uspekhi **19**, 137 (1976)].

[3]D. I. Pushkarov, Zh. Eksp. Teor. Fiz. **59**, 1755 (1970) [Sov. Phys. JETP **32**, 954 (1971)].

[4]A. A. Abrikosov, L. P. Gor'kov and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Quantum Field Theoretical Methods in Statistical Physics) Fizmatgiz, 1962 [Pergamon, 1965].

[5]E. G. Maksimov and O. A. Pankratov, Usp. Fiz. Nauk **116**, 385 (1975) [Sov. Phys. Uspekhi **18**, 481 (1975)].

[6]W. A. Harrison, Solid State Theory, McGraw, 1970.

[7]W. F. Brinkman and T. M. Rice, Phys. Rev. **B2**, 1324 (1970).

[8]A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys. JETP **7**, 996 (1958)].

[9]G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960); **39**, 1437 (1960) [Sov. Phys. JETP **11**, 716 (1960); **12**, 1000 (1961)].

Translated by R. T. Beyer