Bound states of a heavy hole and a phonon in the case of a valence band with a degeneracy point

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It is shown that in the case of an energy spectrum with a degeneracy point, which is typical of the valence band of a semiconductor such as Ge, there exist bound states of a heavy hole and a phonon near the optical-phonon energy. The spectrum of a polaron with the bare mass of a light hole contains additional spectrum branches that are due entirely to the electron-phonon interaction. They condense as the optical-phonon energy is approached.

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1. INTRODUCTION

Bound states of a polaron, in the case of a simple parabolic band, exist in the tight-binding limit.[1] Analysis by Matulis[2] has shown that in the limit of weak binding the polaron produced by interaction with polar phonons has no bound states. The problem of polaron in the case of an energy spectrum with a degeneracy point which is characteristic of the valence band of semiconductors such as Ge or InSb, was considered previously.[3-5] The renormalization of the spectrum of light and heavy holes was determined in the limit of weak[3,5] and intermediate couplings.[4]

It is shown in the present article that near the threshold energy $\hbar\omega_0$, where ω_0 is the frequency of the optical phonon, a radical restructuring of the energy spectrum takes place of the energy spectrum with bare light-hole mass m_l as a result virtual transitions from a state with the light-hole mass to the state with the heavmole mass m_h. In addition, in the near-threshold region there appear additional spectrum branches whose existence is due entirely to the electron-phonon interaction. These additional branches, in analogy with the interpretation offered by Levinson and Rashba,[1] can be regarded as bound states of a heavy hole and a phonon. The difference between the threshold energy $\hbar\omega_0$ and the binding energy is of the order of the Born energy of the heavy hole

$$E_h=m_h e^4/2(\hbar\varkappa^*)^2,$$

where the effective dielectric constant is $\varkappa^*=(1/\varkappa_\infty-1/\varkappa_0)^{-1}$, $\varkappa_\infty$ and $\varkappa_0$ are the high- and low-energy dielectric constants. The energy levels of the bound states condense in proportion to $1/n^6$ (n is the number of the level) as the threshold energy $\hbar\omega_0$ is approached.

2. EQUATIONS FOR THE POLARON SPECTRUM NEAR THRESHOLD

We solve the polaron problem in the case of a valence band with degeneracy point in the spherical approximation. The Hamiltonian of the system is the sum of the Hamiltonians H_0 of the free holes, H_{ph} of the optical phonons, and H_{int} of the interaction of the holes with the phonons

$$H=H_0+H_{ph}+H_{int}. \tag{1}$$

The Hamiltonian that describes the spectrum of the holes in the valence band has according to Ref. 4 the form

$$H_0=\frac{k^2}{2m_l}\Lambda^{(l)}(\mathbf{k})+\frac{k^2}{2m_h}\Lambda^{(h)}(\mathbf{k}), \tag{2}$$

where k is the wave vector, $\Lambda^{(l)}(\mathbf{k})$ and $\Lambda^{(h)}(\mathbf{k})$ are the operators of projection on the states of the light and heavy hole band. They are expressed in terms of the 4×4 matrices J_x, J_y, and J_z of the angular momentum with eigen values $J=3/2$ in the following manner:

$$\Lambda^{(h)}(\mathbf{k})={}^1/_2[k^{-2}(\mathbf{kJ})^2-{}^1/_4],\quad \Lambda^{(l)}(\mathbf{k})=1-\Lambda^{(h)}(\mathbf{k}). \tag{3}$$

(We put hereafter $\hbar=1$). In the Hamiltonian of the system of optical phonons

$$H_{ph}=\sum_q \omega_0 b_\mathbf{q}^+ b_\mathbf{q}, \tag{4}$$

where b_q^+ and b_q are the optical-phonon creation and annihilation operators, we disregard effects connected with phonon dispersion. The interaction of the holes with the optical phonons is described by the Fröhlich Hamiltonian

$$H_{int}=(2\pi\omega_0 e^2/V\varkappa^*)^{1/2}\sum_q \frac{1}{q}(b_\mathbf{q}e^{i\mathbf{qr}}+b_\mathbf{q}^+e^{-i\mathbf{qr}}), \tag{5}$$

where V is the volume of the crystal.

In the absence of interaction, the single-particle Green's function is given by the 4×4 matrix

$$G(\varepsilon,\mathbf{k})=G^{(l)}(\varepsilon,\mathbf{k})+G^{(h)}(\varepsilon,\mathbf{k}),\quad G^{(l)}(\varepsilon,\mathbf{k})=(\varepsilon-k^2/2m_l+i0)^{-1}\Lambda^{(l)}(\mathbf{k}),$$
$$G^{(h)}(\varepsilon,\mathbf{k})=(\varepsilon-k^2/2m_h+i0)^{-1}\Lambda^{(h)}(\mathbf{k}). \tag{6}$$

We are interested in the spectrum in the near-threshold region, when the polaron energy is $\varepsilon\sim\omega_0$. Because of the small mass ratio $m_l/m_h\ll1$ the light hole with energy $\omega_0-\varepsilon\ll\omega_0$ has a small energy deficit for the production of an optical phonon with transition into a heavy hole. Therefore the dangerous cross sections in the mass operator $M(\varepsilon,\mathbf{k})$ are connected precisely with this virtual transition

$$M(\varepsilon,\mathbf{k})=\frac{e^2\omega_0}{(2\pi)^2\varkappa^*}\int\frac{d^3q}{q^2}\left[\varepsilon-\omega_0-\frac{(\mathbf{k}-\mathbf{q})^2}{2m_h}\right]^{-1}\Lambda^{(h)}(\mathbf{k}-\mathbf{q})\Gamma(\mathbf{q};\varepsilon,\mathbf{k}). \tag{7}$$

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Here $\Gamma(\mathbf{q};\varepsilon,\mathbf{k})$ is the vertex.

If we confine ourselves to the zeroth approximation for the vertex, i.e., we put $\Gamma=1$, then

$$M(\varepsilon,\mathbf{k})=\frac{e^2\omega_0}{(2\pi)^2\varkappa^*}\int\frac{d^3q}{q^2}\left[\varepsilon-\omega_0-\frac{(\mathbf{k}-\mathbf{q})^2}{2m_h}\right]^{-1}\Lambda^{(h)}(\mathbf{k}-\mathbf{q}). \tag{8}$$

For the light-hole spectrum in the near-threshold region, when $k\approx(2m_l\omega_0)^{1/2}$, we can neglect k compared with q if $\omega_0-\varepsilon\gg m_l\omega_0/m_h$. In this case the renormalized energy of the light hole is determined from the equation

$$\varepsilon-k^2/2m_l+e^2\omega_0(2m_h)^{1/2}/4\varkappa^*(\omega_0-\varepsilon)^{1/2}=0. \tag{9}$$

Equation (9) has one real root in the region $\varepsilon<\omega_0$. Figure 1 shows a plot of $x=(\omega_0-\varepsilon)(\omega_0-k^2/2m_l)^{-1}$ against $\beta=1/4\,\omega_0^2E_h(\omega_0-k^2/2m_l)^{-3}$. As follows from (9), the small parameter of the expansion in our problem is the ratio $E_h/4\omega_0$, and therefore the renormalization becomes significant at $\omega_0-\varepsilon\lesssim E_h^{1/3}\omega_0^{2/3}$. Equation (9) for the polaron dispersion law is valid up to energies $\omega_0-\varepsilon>E_h$. At energies $\omega_0-\varepsilon\lesssim E_h$ the renormalization of Γ becomes essential, and it is just in this region that bound states of a heavy hole and a phonon exist.

3. EQUATION FOR THE VERTEX

The integral equation for Γ is shown in Fig. 2. It is of the form

$$\Gamma(\mathbf{q};\varepsilon,\mathbf{k})=1+\frac{e^2\omega_0}{(2\pi)^2\varkappa^*}\int\frac{d^3q_1}{q_1^2}\left[\varepsilon-\omega_0-\frac{(\mathbf{k}-\mathbf{q}_1)^2}{2m_h}\right]^{-1}$$
$$\times\Delta^{(h)}(\varepsilon,\mathbf{k};\mathbf{q},\mathbf{q}_1)\Lambda^{(h)}(\mathbf{k}-\mathbf{q}_1)\Gamma(\mathbf{q}_1;\varepsilon,\mathbf{k}). \tag{10}$$

Here $\Delta^{(h)}$ is a sum of a diagram with two external phonon lines and two internal heavy-hole Green's functions (incoming and outgoing), which contain in the cross section no less than two phonon lines. In (10) we have again retained only the graphs that contain dangerous cross sections. The quantity $\Delta^{(h)}$ does not contain them and in the approximation of lowest order in the interaction we have at $m_l/m_h\ll1$

$$\Delta^{(h)}=-\omega_0^{-1}\Lambda^{(h)}(\mathbf{k}-\mathbf{q}-\mathbf{q}_1).$$

Neglecting in (10) the value of k compared with q, a procedure that will be shown to be valid at $k\ll m_he^2/$

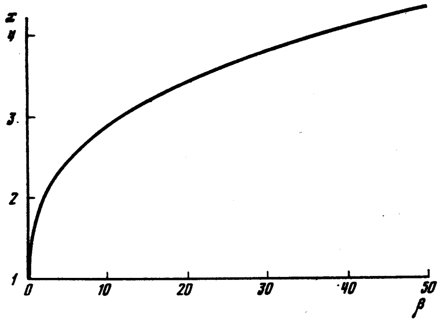

FIG. 1. Plot of $x=(\omega_0-\varepsilon)(\omega_0-k^2/2m_l)^{-1}$ against $\beta=\frac{1}{4}E_h(\omega_0-k^2/2m_l)^{-3}\omega_0^2$.

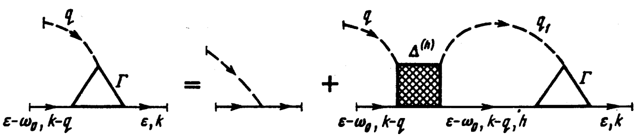

FIG. 2

$\varkappa^*$, we simplify (10) to

$$\Gamma(-\mathbf{q};\varepsilon,\mathbf{k})=1+\frac{e^2}{(2\pi)^2\varkappa^*}\int\frac{d^3q_1}{q_1^2}\left(\omega_0-\varepsilon+\frac{q_1^2}{2m_h}\right)^{-1}$$
$$\times\Lambda^{(h)}(\mathbf{q}+\mathbf{q}_1)\Lambda^{(h)}(\mathbf{q}_1)\Gamma(-\mathbf{q}_1;\varepsilon,\mathbf{k}). \tag{11}$$

The wave vector $\mathbf{k}$ enters in the simplified equation only as a parameter, and Γ is independent of $\mathbf{k}$ in this approximation.

We now obtain the eigenvalues of the integral homogeneous equation corresponding to the inhomogeneous equation (11)

$$\psi(\mathbf{q})=\frac{e^2}{(2\pi)^2\varkappa^*}\int\frac{d^3q_1}{q_1^2}\left(\omega_0-\varepsilon+\frac{q_1^2}{2m_h}\right)^{-1}\Lambda^{(h)}(\mathbf{q}+\mathbf{q}_1)\Lambda^{(h)}(\mathbf{q}_1)\psi(\mathbf{q}_1). \tag{12}$$

we represent ψ as a sum of functions, each of which is a wave packet made up of the wave functions of the light and heavy holes

$$\psi(\mathbf{q})=q^2(\omega_0-\varepsilon+q^2/2m_h)[\varphi^{(h)}(\mathbf{q})+\varphi^{(l)}(\mathbf{q})],$$
$$\varphi^{(h)}(\mathbf{q})=q^{-2}(\omega_0-\varepsilon+q^2/2m_h)^{-1}\Lambda^{(h)}(\mathbf{q})\psi(\mathbf{q}),$$
$$\varphi^{(l)}(\mathbf{q})=q^{-2}(\omega_0-\varepsilon+q^2/2m_h)^{-1}\Lambda^{(l)}(\mathbf{q})\psi(\mathbf{q}). \tag{13}$$

Multiplying (12) from the right by $\Lambda^{(h)}(\mathbf{q})$ and $\Lambda^{(l)}(\mathbf{q})$, we obtain equations for $\varphi^{(h)}$ and $\varphi^{(l)}$:

$$q^2\left(\omega_0-\varepsilon+\frac{q^2}{2m_h}\right)\varphi^{(h)}(\mathbf{q})=\frac{e^2}{(2\pi)^2\varkappa^*}\Lambda^{(h)}(\mathbf{q})\int d^3q_1\Lambda^{(h)}(\mathbf{q}+\mathbf{q}_1)\varphi^{(h)}(\mathbf{q}_1), \tag{14a}$$

$$q^2\left(\omega_0-\varepsilon+\frac{q^2}{2m_h}\right)\varphi^{(l)}(\mathbf{q})=\frac{e^2}{(2\pi)^2\varkappa^*}\Lambda^{(l)}(\mathbf{q})\int d^3q_1\Lambda^{(h)}(\mathbf{q}+\mathbf{q}_1)\varphi^{(h)}(\mathbf{q}_1). \tag{14b}$$

From (14a) and (14b) it follows that the eigenvalues of the problem are determined from the solution of Eq. (14a). We can then obtain $\varphi^{(l)}(\mathbf{q})$ from (14b). Equation (14a) can be transformed into a differential equation, and then expansion in the wave functions of a particle with spin 3/2 (Ref. 7) can be used to separate the angular dependence of $\varphi^{(h)}(\mathbf{q})$ and obtain a system of ordinary differential equations for the radial functions. In the present article, however, we obtain with the aid of a variational principle the smallest eigenvalue, and then show how the spectrum of the highly excited states behaves. We are interested in wave functions of the form[8]

$$\varphi_\alpha^{(h)}(q)=f(q)\Lambda_{\alpha\beta}^{(h)}(\mathbf{q})\chi_\beta, \tag{15}$$

where $f(q)$ is a scalar function that depends only on the modulus of q, χ_α is a third-rank spinor, and it is convenient to choose the quantization axis z along $\mathbf{k}$. The states with the wave function (15) are fourfold degenerate.

With the aid of the relation

$$\mathrm{Sp}\,\Lambda^{(h)}(\mathbf{q}_1)\Lambda^{(h)}(\mathbf{q}_2)\Lambda^{(h)}(\mathbf{q}_3)={}^3/_4[(\mathbf{q}_1\mathbf{q}_2)^2/q_1^2q_2^2+(\mathbf{q}_2\mathbf{q}_3)^2/q_2^2q_3^2+(\mathbf{q}_3\mathbf{q}_1)^2/q_3^2q_1^2]-{}^1/_4 \tag{16}$$

we obtain an equation for $f(q)$

$$q^2\left(\omega_0-\varepsilon+\frac{q^2}{2m_h}\right)f(q)=\frac{3e^2}{8(2\pi)^2\varkappa^*}\int d^3q_1$$
$$\times\left[\frac{(\mathbf{q}\mathbf{q}_1)^2}{q^2q_1^2}+\frac{(q^2+\mathbf{q}\mathbf{q}_1)^2}{q^2(\mathbf{q}+\mathbf{q}_1)^2}+\frac{(q_1^2+\mathbf{q}\mathbf{q}_1)^2}{q_1^2(\mathbf{q}+\mathbf{q}_1)^2}-\frac{1}{3}\right]f(q_1). \tag{17}$$

The smallest eigenvalue ε_0 of Eq. (17) will be obtained by a variational principle, choosing the trial function in the form

$$f(q_1)=\exp(-q^2/q_0^2), \tag{18}$$

where q_0 is the variational parameter. We obtain for q_0 the equation

$$\omega_0-\varepsilon=-\frac{5}{8m_h}q_0^2+\frac{e^2}{2\varkappa^*}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{\pi}{2}-1\right)q_0. \tag{19}$$

Minimizing with respect to q_0 we get

$$\omega_0-\varepsilon_0=\frac{2}{5\pi}\left(\frac{\pi}{2}-1\right)^2E_h\approx0.04E_h. \tag{20}$$

4. EXCITED-STATE SPECTRUM

For the function $\varphi_1(q)=q^2f(q)$ the equation (17) can be transformed into a differential equation in coordinate space

$$\left(\omega_0-\varepsilon+\frac{\hat{\mathbf{p}}^2}{2m_h}\right)\hat{\mathbf{p}}^6\varphi_1(r)=\frac{3e^2}{64\varkappa^*}\left[\hat{\mathbf{p}}^6\frac{1}{r}\varphi_1(r)-\hat{\mathbf{p}}^4\frac{1}{r}\hat{\mathbf{p}}^2\varphi_1(r)\right.$$
$$\left.-\hat{\mathbf{p}}^2\frac{1}{r}\hat{\mathbf{p}}^4\varphi_1(r)+\frac{1}{r}\hat{\mathbf{p}}^6\varphi_1(r)\right], \tag{21}$$

where $\hat{\mathbf{p}}$ is the momentum operator. The function $\varphi_1(r)$ depends only on the absolute value of r. Therefore

$$\hat{\mathbf{p}}^2=-\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{d}{dr}=-\left(\frac{d}{dr}+\frac{1}{r}\right)^2=-\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}+1\right). \tag{22}$$

Equation (21) is an ordinary differential equation of seventh order for the function $d\varphi_1(r)/dr$, since it does not contain the non-differentiated function $\varphi_1(r)$. It can be transformed however into a fourth-order equation for the function

$$R_0=r\frac{d}{dr}\hat{\mathbf{p}}^2\varphi_1.$$

To this end we use first the relation

$$\frac{1}{r}\left(\frac{d}{dr}+\frac{1}{r}\right)\varphi_2=\left(\frac{d}{dr}+\frac{2}{r}\right)\frac{1}{r}\varphi_2,$$

where φ_2 is an arbitrary function, to lower the order of Eq. (21)

$$\frac{d}{dr}\left(\omega_0-\varepsilon+\frac{\hat{\mathbf{p}}^2}{2m_h}\right)\hat{\mathbf{p}}^4\varphi_1=\frac{3e^2}{64\varkappa^*}\left\{\frac{d}{dr}\frac{1}{r}\frac{d^2}{dr^2}\left(\frac{d^2}{dr^2}\right.\right.$$
$$\left.\left.+\hat{\mathbf{p}}^2\right)\varphi_1-\frac{d}{dr}\frac{1}{r}\hat{\mathbf{p}}^4\varphi_1+\frac{1}{r}\left(\frac{d}{dr}+\frac{1}{r}\right)\hat{\mathbf{p}}^4\varphi_1\right\}. \tag{23}$$

The order of the equation can be lowered once more by using the equations

$$\frac{d}{dr}\hat{\mathbf{p}}^2=-\left(r\frac{d}{dr}+4\right)\frac{1}{r^3}\left(r\frac{d}{dr}\right)\left(r\frac{d}{dr}-2\right),$$
$$\frac{d}{dr}\frac{1}{r}\frac{d^2}{dr^2}=\left(r\frac{d}{dr}+4\right)\frac{1}{r^4}\left(r\frac{d}{dr}-3\right)\left(r\frac{d}{dr}-1\right),$$
$$\frac{1}{r^2}\hat{\mathbf{p}}^2=-\left(r\frac{d}{dr}+4\right)\frac{1}{r^4}\left(r\frac{d}{dr}+1\right). \tag{24}$$

It follows from these equations that the right- and left-hand sides of (23) are acted upon by the same operator $rd/dr+4$. As a result we get

$$\left(r\frac{d}{dr}-2\right)\left(r\frac{d}{dr}\right)\left(\omega_0-\varepsilon+\frac{\hat{\mathbf{p}}^2}{2m_h}\right)\hat{\mathbf{p}}^2\varphi_1$$
$$=\frac{3e^2}{32\varkappa^*r}\left\{\left(r\frac{d}{dr}-3\right)\left(r\frac{d}{dr}-1\right)\frac{1}{r}\frac{d\varphi_1}{dr}+\left(r\frac{d}{dr}+1\right)\hat{\mathbf{p}}^2\varphi_1\right\}. \tag{25}$$

Applying to the right and left sides of (25) the operator $rd/dr+4$ we obtain a fifth-order equation for the function $\hat{\mathbf{p}}^2\varphi_1$

$$\left(r\frac{d}{dr}+4\right)\left(r\frac{d}{dr}-2\right)\left(r\frac{d}{dr}\right)\left(\omega_0-\varepsilon+\frac{\hat{\mathbf{p}}^2}{2m_h}\right)\hat{\mathbf{p}}^2\varphi_1=\frac{3e^2}{4\varkappa^*}\frac{d}{dr}\hat{\mathbf{p}}^2\varphi_1. \tag{26}$$

Using (22) we can lower once more the degree of (26)

$$\left(r\frac{d}{dr}+4\right)\left(r\frac{d}{dr}-2\right)\left[\omega_0-\varepsilon-\frac{1}{2m_hr^2}\left(r\frac{d}{dr}-2\right)\cdot\right.$$
$$\left.\cdot\left(r\frac{d}{dr}+1\right)\right]R_0-\frac{3e^2}{4\varkappa^*r}R_0=0. \tag{27}$$

The spectrum of the highly excited states can be determined by a quasiclassical method. The wave functions of these states have a large range in space, so that to determine their spectrum we must know the behavior of the potential at large distances. At large r we can simplify (27) to

$$\frac{d^2}{dr^2}\left(\omega_0-\varepsilon-\frac{1}{2m_h}\frac{d^2}{dr^2}\right)R_0-\frac{3e^2}{4\varkappa^*r^3}R_0=0. \tag{28}$$

A quasiclassical solution of this equation is the function

$$R_0=\exp\left[-\int_0^r q_i(r')dr'\right]\sin\left[\int_0^r dr'q_r(r')+\alpha\right], \tag{29a}$$

where

$$q_r=2^{-1/2}[(3/2a_hr^3)^{1/2}-m_h(\omega_0-\varepsilon)]^{1/2}, \tag{29b}$$
$$q_i=2^{-1/2}[(3/2a_hr^3)^{1/2}+m_h(\omega_0-\varepsilon)]^{1/2}, \tag{29c}$$

$a_h=\varkappa^*/m_he^2$. The turning point r_0 for the solution (29a) is the point at which $q_r(r_0)=0$, i.e.,

$$r_0=(3/2m_h^2a_h)^{1/3}(\omega_0-\varepsilon)^{-2/3}. \tag{30}$$

The eigenvalues can be obtained from the Bohr quantization condition

$$\int_0^{r_0}dr\,q_r(r)=\pi n, \tag{31}$$

where n is an integer. Substituting (29b) in (31) we get

$$\omega_0-\varepsilon_n=\frac{9}{n^6}\left(\frac{3}{8\pi^2}\right)^6\left[\Gamma\left(\frac{1}{3}\right)\right]^{18}E_h\approx\frac{1.36}{n^6}E_h, \tag{32}$$

where $\Gamma(x)$ is the Euler function. Thus, in the limit $m_l/m_h \ll 1$ the vertex Γ has an infinite number of poles between ε_0 and ω_0. The poles condense in proportion to $1/n^6$ as ω_0 is approached

5. ADDITIONAL BRANCHES OF THE LIGHT-HOLE SPECTRUM NEAR THRESHOLD

As already mentioned, because of the spherical symmetry of the problem there exists an operator $\mathbf{F}=\mathbf{L}+\mathbf{J}$, where $\mathbf{L}$ is the orbital-momentum operator, which commutes with the Hamiltonian of Eqs. (13) and (14). The position of the pole of the vertex $\varepsilon_{F,n}$ depends on F and on the quantum number n that numbers the levels at the given F. The eigenvalues ε_{Fn} are degenerate in the projection M of the vector $\mathbf{F}$.[7] In Secs. 3 and 4 we have investigated the spectrum at 3/2, when the wave functions have positive parity (with $l=0$ and $l=2$). At $q \gg k$ the expression for the mass operator contains the projection of the vertex on the states of the heavy-hole band

$$\Gamma^{(h)}(-\mathbf{q};\varepsilon,\mathbf{k})=\Lambda^{(h)}(\mathbf{q})\Gamma(-\mathbf{q};\varepsilon,\mathbf{k}).$$

The quantity $\Gamma^{(h)}$ can be expressed in terms of the eigenfunctions of Eq. (14)

$$\Gamma_{\alpha\beta}^{(h)}=\sum_{n,F,M}(\varepsilon_{Fn}-\varepsilon)^{-1}q^2\left(\omega_0-\varepsilon+\frac{q^2}{2m_h}\right)\varphi_{FMn\alpha}^{(h)}(\mathbf{q})\int d^3q_1\varphi_{FMn\beta}^{(h)*}(\mathbf{q}_1). \tag{33}$$

Here α and β are the spinor indices.

If we expand the functions $\varphi_{FMn}^{(h)}$ in the eigenvalues of the operator $\mathbf{F}$ (Ref. 7) and substitute (33) in (7), then at $q \gg k$, after integration over the angles of $\mathbf{q}$, there will be left in the sum (33) only even functions corresponding to $F=3/2$. Substituting (33) in (7) and integrating with respect to the angles of $\mathbf{q}$, we obtain the dispersion equation for the polaron spectrum

$$\varepsilon-\frac{k^2}{2m_l}=\frac{(4\pi e)^2\omega_0}{4\varkappa^*(2\pi)^2}\sum_n(\varepsilon-\varepsilon_{3/2,n})^{-1}\int_0^\infty dq\,q^2f_n(q)\int_0^\infty dq_1q_1^2f_n(q_1). \tag{34}$$

A plot of the right-hand side of (34) against ε is shown in Fig. 3. The curve has singularities at $\varepsilon=\varepsilon_{3/2,n}$. These singularities are located in the region $\varepsilon_0 \leqslant \varepsilon < \omega_0$. In the region $\varepsilon<\varepsilon_0$ the curve decreases monotonically. The law governing its decrease is described by the last term of (9) at $\omega_0-\varepsilon_0 \gg E_h$. The left-hand side of (34) as a function of ε is a straight line. Its successive intersections with the sections of the curve between two singularities of the right-hand side of (34) yields the polaron energy.

To determine the polaron energy in the case of additional spectrum branches we can replace ε in the left-hand side by ω_0, since $\omega_0 \gg E_h$, and $\omega_0-\varepsilon \sim E_h$. The polaron energy is determined by Eq. (34) at $k \ll m_he^2/\varkappa^*$. For the branches with large n the region of values

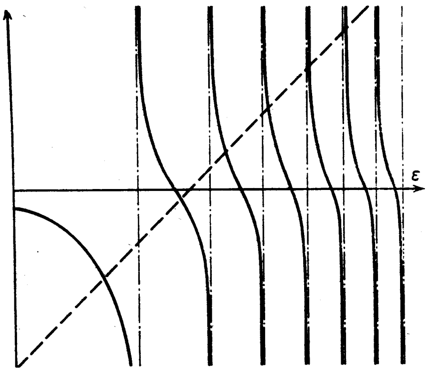

FIG. 3. Plot of the right-hand side of (34) against ε.

of k in which our soltion is valid narrows down to $k \ll m_he^2/\varkappa^*n^3$. Additional spectrum branches exist also for a polaron with bare mass of a light hole, but they exist only in that wave-vector region in which the kinetic energy $k^2/2m_h$ can be neglected.

One of the possible semiconducting compounds in which bound states of a heavy hole and a phonon can be observed is CdTe, in which $\varkappa_0=10.6$, $\varkappa_\infty=7.1$, $\omega_0=21.3$ meV and $m_h=1.5\,m_0$ (Ref. 9), where m_0 is the mass of the free electron. Using (20), we obtain the estimate $\omega_0=\varepsilon_0=1.8$ meV.

The dimensionless parameter used for the expansion needed to obtain our solution is $E_h/4\omega_0=1/2$ in the case of CdTe. Therefore our estimates for CdTe are only approximate. However, the main criterion that points to the possible existence of bound states of a heavy hole and a phonon is small, $m_l/m_h \sim 0.1$.

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