Effect of the interaction between nondegenerate electrons localized in a thin surface layer on the cyclotron resonance and on the magnetoconductance

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(Submitted 10 April 1979)
Zh. Eksp. Teor. Fiz. 77, 1488–1502 (October 1979)

It is shown that the interaction between two-dimensional electrons alters significantly the character of their scattering by impurities and phonons in quantizing magnetic fields. The reason is the transverse drift of the electron cyclotron-resonance orbit centers in the electric field produced by the carrier-density fluctuations. In sufficiently strong magnetic fields and at not too low temperatures, scattering by a phonon constitutes an act of Cerenkov scattering by the drifting electron, and elastic scattering by an impurity corresponds to a displacement of the cyclotron orbit by a distance of the order of the quantum magnetic length in a direction perpendicular to the fluctuatuion electric field. It is shown that when account is taken of the interaction between the electrons the peak of the cyclotron resonance can have a Lorentz shape. The half-width and the shift of the peak are calculated. The profile of the N-shaped current-voltage characteristic in the absence of heating is analyzed.

PACS numbers: 76.40. + b, 72.20.My, 72.20.Dp

A distinguishing feature of several presently known systems is the presence of electrons moving in a one-dimensional potential well. These are semiconductor or semimetal films, inversion layers near semiconductor surfaces, thin charged layers over a surface of liquid helium, and others. If the distance between the quantum levels in the well is $\Delta\varepsilon \gg T$ and $\Delta\varepsilon/\hbar$ exceeds appreciably all the characteristic frequencies and the reciprocal relaxation times, then the electrons, if on the lowest level, behave as two-dimensional ones in a number of effects. One such effect is the cyclotron resonance observed experimentally for electrons over liquid helium[1] and in the p-Si inversion layer[2,3] when a magnetic field is applied perpendicular to the surface.

A transverse magnetic field affects strongly the kinetic phenomena in two-dimensional systems, since the electron energy spectrum becomes discrete. Obviously, in this case one should speak not of, say, scattering by defects, but of lifting of the Landau-level degeneracy by the electron-impurity interaction, and of localization of the electrons. Correspondingly, if the impurity concentration n_s is low enough $n_s \ll (2\pi l^2)^{-1}$ ($l = (c\hbar/eH)^{1/2}$ is the magnetic length), and the impurities are short-range, then, neglecting electron-electron and electron-phonon interactions, the static conductivity $\sigma(0)$ of the system vanishes.[4] Allowance for the interaction leads to a finite value of $\sigma(0)$ at $T \neq 0$ and the CR line width.[5,6] The CR peak can then have a complicated non-Lorentzian profile, and the dependence of the conductivity on the temperature and on the coupling constant turns out to be quite unusual.

The interaction between the electrons can alter qualitatively the character of the scattering by impurities and phonons even at carrier densities N such that $R \gg l$ ($R = N^{-1/2}$) and the electron gas is far from degeneracy. This change is due to concentration fluctuations, which cause a fluctuation field $E(r)$ (which is effectively two-dimensional) to act on each individual electron. If the temperatures are not too low, $T \gg e^2 l^2/R^3$, then $E(r)$ is practically homogeneous over the length l at $l \ll R$. In such a field, the center of the cyclotron orbit drifts with a velocity cE/H. Since the electron is "smeared" over a region with dimension $\sim l$, the characteristic time of the interaction of the electron with the short-range defect (the "time of flight") is equal to $\tau_e = \langle E^{-1}\rangle lH/c$ (the estimate of τ_e does not really require that the impurity potential be δ-like). If the impurity level is shallow, $\tau_e \Delta \ll \hbar$ (Δ is the level depth), there is obviously no localization of the electron on the impurity, and elastic scattering takes place instead. Similarly, if the electron characteristic velocity l/τ_e exceeds the phase velocity of the surface phonons, then we get electron-phonon scattering of the Cerenkov-radiation type (in the opposite case, when the coupling is weak, one should more readily speak of quasielastic scattering of the phonons by electrons with a discrete spectrum[6]).

Thus, although the interaction between the electrons does not lead directly to either a shift or a broadening of the CR level, nor to a change of the static conductivity, it does interfere, via the fluctuation field, with the elementary act of scattering by impurities and phonons, and it must therefore be kept in mind when kinetic coefficients are calculated. We note that this pertains also to three-dimensional systems, where the interaction between the electrons eliminates the logarithmic divergence that appears in the Born approximation in the expression for $\sigma(0)$. Since $\tau_e \propto H^{1/2}$, the role of the considered mechanism is smaller in sufficiently strong fields. However, as shown by the estimates that follow (which are, to be sure, quite rough), this mechanism remains important up to fields $\lesssim 10^5$ Oe; In addition, it is of fundamental importance in the investigation of the static conductivity in the case of scattering by impurities if $N \gg n_s$.

The fact that the electron-electron interaction broadens the Landau levels and consequently influences the conductivity of the two-dimensional system was noted in an interesting paper by Ryzhyĭ.[7] He did not explain, however, the mechanism of this interaction, and introduced, instead of the parameter τ_e which is essential for

 0038-5646/79/100747-07$02.40

our problem (and is determined by mean inverse fluctuation-field intensity), the phenomenological lifetime γ^{-1} of the single-electron states. The parameters γ^{-1} and τ_e have different physical meanings and are not directly connected. The expression obtained in Ref. 7 for the current differs greatly from our present result, obtained with heating neglected, see Sec. 3 (cyclotron resonance was not considered in Ref. 7, and heating was not taken into account).

The most thoroughly investigated system of nondegenerate two-dimensional electrons is that localized over the surface of liquid helium.[8] In Sec. 4 we analyze the shift and broadening of the CR peak as applied to the conditions realized in experiment (see, e.g., Ref. 9).

1. GENERAL EXPRESSION FOR THE CONDUCTIVITY NEAR THE CR PEAK

The Hamiltonian of nondegenerate two-dimensional electrons in a quantizing magnetic field takes in the case of interaction with surface phonons (see Ref. 6) the form

$$\mathcal{H}=H_0+H_{ph}+H_i,$$

$$H_0=\omega_c\sum_n p_{1n}p_{-1n}+\frac{1}{2}\sum_{n\neq m}\frac{e_0^2}{|\mathbf{r}_n-\mathbf{r}_m|},\quad \hbar=1;$$

$$H_{ph}=\sum_{\mathbf{q}}\omega_q b_{\mathbf{q}}^+ b_{\mathbf{q}},\quad H_i=\sum_n\sum_{\mathbf{q}}V_{\mathbf{q}}\exp(i\mathbf{q}\mathbf{r}_n)(b_{\mathbf{q}}+b_{-\mathbf{q}}^+); \tag{1}$$

$$p_{\alpha n}=\frac{l}{2^{1/2}}\left[-i\frac{\partial}{\partial x_n}-\frac{e_0}{c}A_x(\mathbf{r}_n)-i\alpha\left(-i\frac{\partial}{\partial y_n}-\frac{e_0}{c}A_y(\mathbf{r}_n)\right)\right],\quad \alpha=\pm1;$$

$$[p_{-1n},p_{1m}]=\delta_{nm},\quad \omega_c=|e_0H|/mc,\quad l^2=(m\omega_c)^{-1}.$$

Here e_0 is the electron charge, x_n and y_n are its two-dimensional coordinates, $\mathbf{q}$ is the two-dimensional phonon momentum, and ω_q is its frequency (the system is assumed to be two-dimensionally isotropic). The gauge of the vector potential $\mathbf{A}(\mathbf{r})$ of the transverse magnetic field H is of no importance in what follows, and $H=(\mathrm{curl}\,\mathbf{A})_z$.

If the cyclotron frequency $\omega_c\gg\Gamma$ (Γ is the width of the CR peak; it determines the characteristic relaxation time $\tau_r=\Gamma^{-1}$ of the total momentum of the electrons), then the conductivity of the electron gas near the CR peak is

$$\sigma_{xx}(\omega)\approx\frac{e_0^2}{2m}Q(\omega),\quad \omega\sim\omega_c;$$

$$Q(\omega)=[\bar{n}(\omega_c)+1]^{-1}\,\mathrm{Re}\int_0^\infty dt\,e^{i\omega t}Q(t); \tag{2}$$

$$Q(t)=S^{-1}\langle\!\langle\mathcal{P}_{-1}(t)\mathcal{P}_1(0)\rangle\!\rangle,\quad \mathcal{P}_\alpha=\sum_n p_{\alpha n};$$

$$\bar{n}(\omega)=(e^{\lambda\omega}-1)^{-1},\quad \lambda=1/T,$$

where S is the area of the system and $\langle\!\langle\cdots\rangle\!\rangle$ denotes quantum-mechanical averaging.

In the zeroth approximation in H_i, as seen from (1) and (2)

$$\mathcal{P}_\alpha(t)=\mathcal{P}_\alpha(0)\exp(i\alpha\omega_c t),$$

whence $Q(\omega)\,N\delta(\omega-\omega_c)$. Scattering broadens and shifts the CR peak. The form of $Q(\omega)$ near the maximum is determined, according to (2), by the behavior of the time correlation function $Q(t)$ in the region of long times $t\gg\omega_c^{-1}$, $t\sim\tau_r$. In this region it is impossible to calculate $Q(t)$ by the usual perturbation theory in H_i, since the terms proportional to $|V_q|^2t$ turn out to be of the order of unity. At the same time since the interaction between the electrons is not weak, it is impossible to calculate $Q(t)$ by using the single-electron kinetic equation obtained in Refs. 5 and 6. The problem simplifies, however, if $T\gg e_0^2l^2/R^3$ and the average distance between the electrons R is much larger than l. Then the interelectron interaction can be treated in a certain sense quasiclassically.

To calculate $Q(t)$ it is convenient to change over to the interaction representation by introducing the operators

$$U(t)=T_t\exp\left[-i\int_0^t H_i(\tau)\,d\tau\right],$$
$$H_i(\tau)=\exp[i(H_0+H_{ph})\tau]H_i\exp[-i(H_0+H_{ph})\tau], \tag{3}$$

where T_t is the chronological-ordering operator. Equation (2) for $Q(t)$ can then be written in the form

$$Q(t)=\exp(-i\omega_c t)\bar{Q}(t),$$
$$\bar{Q}(t)=S^{-1}[\mathrm{Sp}\,e^{-\lambda\mathcal{H}}]^{-1}\,\mathrm{Sp}\{\mathcal{P}_{-1}U(t)\mathcal{P}_1U^{-1}(t+i\lambda)\exp[-\lambda(H_0+H_{ph})]\}. \tag{4}$$

It is convenient to calculate the trace in (4) over a system of wave functions that represent the products of the wave functions of the non-interacting electron and phonon subsystems, while the trace over the phonons can be easily calculated directly (see, e.g., Ref. 10) by using Wick's theorem. As a result we have at $T\gg\tau_r^{-1}$

$$\bar{Q}(t)\approx S^{-1}Z^{-1}\,\mathrm{Sp}_0[\mathcal{P}_{-1}F(t;\,\mathcal{P}_1)\exp(-\lambda H_0)],\quad Z=\mathrm{Sp}_0\,e^{-\lambda H_0},$$

$$F(t;\mathcal{P}_1)=T_t\bar{T}_t\{\exp[G(t)]\mathcal{P}_1\},\quad G(t)=\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2 g(\tau_1,\tau_2),$$

$$g(\tau_1,\tau_2)=-\sum_{\mathbf{q}}|V_q|^2\sum_{m,n}[a_n(\mathbf{q},\tau_1)\varphi_q(\tau_1-\tau_2)$$
$$-\bar{a}_n(\mathbf{q},\tau_1)\varphi_q(\tau_2-\tau_1)][a_m(-\mathbf{q},\tau_2)-\bar{a}_m(-\mathbf{q},\tau_2)],\quad a_n(\mathbf{q},\tau)=\exp[i\mathbf{q}\mathbf{r}_n(\tau)],$$
$$\mathbf{r}_n(\tau)=\exp(iH_0\tau)\mathbf{r}_n\exp(-iH_0\tau), \tag{5}$$
$$\varphi_q(\tau)=\bar{n}(\omega_q)\exp(i\omega_q\tau)+[\bar{n}(\omega_q)+1]\exp(-i\omega_q\tau).$$

We have introduced here a symbolic notation according to the operators $\bar{a}_n$ (with the bar) are placed on the right of $\mathcal{P}_1$ and are ordered by the operator $\bar{T}_t$ from right to left in decreasing order of time (they "stem" from the operator $U^{-1}(t+i\lambda)$, while in the approximation $T\gg\tau_r^{-1}$ the argument $t+i\lambda$ is replaced by t). The operators a_n (without the bar) are placed to the left of $\mathcal{P}_1$ and are ordered by the operator T_t from left to right in decreasing order of time (they stem from the operator $U(t)$ in (4)); Sp_0 means the trace on the wave functions of the isolated electron subsystem.

In the time region $t\sim\tau_r$, the matrix elements of $G(t)$ are of the order of unity, and to calculate $F(t)$ it is necessary to gather together the entire series obtained from the expansion of the exponential in (5). This can be done in the zeroth approximation in τ_e/τ_r, $(\omega_c\tau_r)^{-1}\ll1$, by retaining in the n-th term of the expansion only the terms $\sim(t/\tau_r)^n$.

2. CALCULATION OF THE CORRELATION FUNCTION OF THE MOMENTUM OPERATORS $Q(t)$

We consider first the term linear in $G(t)$ in (5):

$$\tilde{Q}^{(1)}(t)=S^{-1}Z^{-1}\,\mathrm{Sp}_0\,[\mathscr{P}_{-1}T_t\bar{T}_t\{G(t)\mathscr{P}_1\}\exp(-\lambda H_0)], \tag{6}$$

which constitutes actually a sum of skeleton diagrams (cf. Ref. 6. It is important, however, that in the problem under consideration it is not convenient to use single-particle Green's functions). Using the momentum amplitudes $P_{\alpha n}$ and the positions $\mathbf{R}_n$ of the cyclotron-orbit centers, which vary slowly over the time ω_c^{-1} and are given in the Appendix, the operators a_n in G can be represented, taking (A.3)–(A.5) into account, in the form

$$\begin{aligned} a_n(\mathbf{q},\tau)&\equiv\exp[i\mathbf{q}\mathbf{r}_n(\tau)]=\exp(-{}^1\!/_4 l^2q^2)\\ &\times\exp[2^{-1/2}lq_{-1}P_{1n}(\tau)\exp(i\omega_c\tau)]\\ \times\exp[-2^{-1/2}lq_1P_{-1n}(\tau)\exp(-i\omega_c\tau)]&\exp[i\mathbf{q}\mathbf{R}_n(\tau)],\quad q_\alpha=q_x-iq_y. \end{aligned} \tag{7}$$

The double integrals with respect to time in the expression for $G(t)$ have, according to (5), the following structure:

$$\int_0^t d\tau_1\int_0^{\tau_1} d\tau_2\, a_n(\mathbf{q},\tau_1)a_n(-\mathbf{q},\tau_2),$$

where $a_n(\mathbf{q},\tau)$ are determined by Eq. (7).[1)] In the series expansions of the four exponentials of the rapidly oscillating expressions

$$2^{-1/2}lq_\alpha^{*}P_{\alpha n}(\tau_{1,2})\exp(i\alpha\omega_c\tau_{1,2})$$

it is necessary to retain in the zeroth approximation in $(\omega_c\tau_r)^{-1}$ only the terms $\sim\exp[\mathrm{im}\omega_c(\tau_1-\tau_2)]$. If $m\neq0$, then the main contribution to the integral is made by the region $\tau_1-\tau_2\approx\omega_c^{-1}$. Therefore τ_2 in $P_{\alpha n}(\tau_2)$ and $\mathbf{R}_n(\tau_2)$ can be replaced by τ_1, so that the integral with respect to τ_2 can be immediately calculated. On the other hand if $m=0$, then we must take into account the dependence of $\mathbf{R}_n$ on the time (it is through this dependence that the interaction between electrons affects the scattering), and it turns out that it suffices to substitute

$$\mathbf{R}_n(\tau_2)\approx\mathbf{R}_n(\tau_1)-(\tau_1-\tau_2)\dot{\mathbf{R}}_n(\tau_1).$$

At $m=0$ the main contribution to the integral is made by the time interval $\tau_1-\tau_2\sim l/|\dot{\mathbf{R}}_n|$.

Calculating in this manner the integrals in $G(t)$, we can reduce expression (6) to the form

$$\begin{aligned} &\tilde{Q}^{(1)}(t)=-N[iPt+\Gamma t],\quad t\gg\tau_e\gg\omega_c^{-1},\quad \tau_e=l\langle|\dot{\mathbf{R}}_n|^{-1}\rangle,\\ &\Gamma=2\pi T\sum_{\mathbf{q}}\frac{|V_{\mathbf{q}}|^2}{\omega_{\mathbf{q}}}\exp\left(-\frac{l^2q^2}{2}\right)\left(\frac{l^2q^2}{2}\right)^2\langle\delta(\mathbf{q}\dot{\mathbf{R}}_n-\omega_{\mathbf{q}})\rangle,\\ &P=P_v+p,\quad P_v=\omega_c^{-1}\sum_{\mathbf{q}}|V_{\mathbf{q}}|^2[2\bar{n}(\omega_{\mathbf{q}})+1]\\ &\times\exp(-{}^1\!/_2l^2q^2)\left\{{}^1\!/_2l^2q^2-\sum_{m=1}^{\infty}({}^1\!/_2l^2q^2)^m\right.\\ &\left.\times[m(m+1)-(m+1)l^2q^2+({}^1\!/_2l^2q^2)^2][m(m+1)!]^{-1}\right\},\\ &p=\frac{S}{(2\pi)^2}\int d\mathbf{q}|V_{\mathbf{q}}|^2\exp(-{}^1\!/_2l^2q^2)l^2q^2(1-{}^1\!/_4l^2q^2)\langle(\omega_q-\mathbf{q}\dot{\mathbf{R}}_n)^{-1}\rangle, \end{aligned} \tag{8}$$

where N is the electron concentration, $2\pi Nl^2\ll1$; it is assumed that $\exp(\omega_c/T)\gg1$, i.e., the electrons are on the lowest Landau level and that $\omega_c\gg\omega_q$. The angle brackets in (8) denote the following averaging:

$$\begin{aligned} &\langle f(\dot{\mathbf{R}}_n)\rangle=\int f(\dot{\mathbf{R}}_n)\exp(-\lambda H_{ee})\prod_m d\mathbf{R}_m\Big[\int\exp(-\lambda H_{ee})\prod_m d\mathbf{R}_m\Big]^{-1},\\ &H_{ee}=\frac{1}{2}e_0^2\sum_{n\neq m}|\mathbf{R}_n-\mathbf{R}_m|^{-1},\\ &\dot{X}_n=-l^2\frac{\partial H_{ee}}{\partial Y_n},\quad \dot{Y}_n=l^2\frac{\partial H_{ee}}{\partial X_n},\quad \mathbf{R}_n=(X_n,Y_n). \end{aligned} \tag{9}$$

The parameters Γ and P in (8) have the meaning of the broadening and the shift of the CR peak. The damping is due to transitions between states of the electron subsystem with emission or absorption of a phonon, and is possible only when the drift velocity in the fluctuation field exceeds the phase velocity of the phonon. The shift P_v is due to virtual processes, wherein the electron hops through m Landau levels, and this in fact is the reason for the sum over m in the expression for P_v. These processes are fast, and at $\omega_c\tau_e\gg1$ the electron-electron interaction does not affect the value of P_v, so that the expression for P_v coincides with that obtained in Refs. 5 and 6. The p term describes the shift due to virtual processes not connected with transitions between Landau levels.

In the derivation of (8) we used a quasiclassical approximation: the fact that the operators $R_{\alpha n}=X_n-i\alpha Y_n$ and $\dot{R}_{\alpha n}$ do not commute and the higher-order derivatives were discarded from the expansion of $\mathbf{R}_n(\tau_2)$ in terms of $\tau_2-\tau_1$. It is seen from (A.3) and (A.4) that this is equivalent to neglecting the inhomogeneity of the fluctuation field over the length l:

$$\langle|\mathbf{E}(\mathbf{R}_n)|\rangle\gg l\langle|\partial\mathbf{E}(\mathbf{R}_n)/\partial R_{\alpha n}|\rangle\sim e_0l/R^3.$$

The corrections $\sim e_0^2l^2/(R^3T)\ll1$ in (8) and (9) were also discarded. In this approximation, the calculation of the trace in (5) and (6) breaks up into summation over the numbers of the Landau levels and integration over the positions of the centers of the cyclotron orbits:

$$\exp(-\lambda H_0)\approx\exp\Big[-\lambda\omega_c\sum_n P_{1n}P_{-1n}\Big]\exp(-\lambda H_{ee}),$$

wherein the operators $P_{\alpha n}$ and $R_{\alpha n}$ commute. The operators (P_{1n}) (P_{-1n}), when acting on the wave function, increase (decrease) the number of the Landau level of the n-th electron by unity (they are analogous to the creation and annihilation operators for the harmonic oscillator). In the derivation of (8) we took account of the fact that in the temperature region $\exp(\omega_c/T)\gg1$ the contribution made to $\tilde{Q}^{(1)}$ by the normal product of these operators is equal to zero.

The condition for the applicability of (8) and (9) is, however, not only the homogeneity but also the smallness of the fluctuation field:

$$\langle|e_0\mathbf{E}(\mathbf{R}_n)|\rangle l=\langle|\partial H_{ee}/\partial\mathbf{R}_n|\rangle l\ll T, \tag{10}$$

whence $\tau_e^{-1}\ll T$ (if the criterion (10) were not satisfied, then the electrons would emit continuously Cerenkov phonons of energy $\omega_q>T$ and thermodynamic equilibrium would be impossible). Strictly speaking, the inequality (10) and the condition that the field be homogeneous should take place simultaneously in a self-consistent treatment of the fluctuations of the electron concentra-

tion. This can be done roughly by assuming that short-range order in the positions of the particles exists at $e_0^2 N^{1/2} \gg T$. Then $E \sim e_0 N^{3/2}\delta$, where δ is the displacement of the electron from the equilibrium position in the "molecule." Associated with this displacement is an energy $\sim e_0^2\delta^2 N^{3/2}$, which can be naturally set equal to the temperature. Hence $E \sim N^{3/4}T^{1/2}$. The field can be regarded as homogeneous, and the quasiclassical approximation can be regarded as valid if $\delta \gg l$, from which follows (10). The criterion (10) is the condition for the quasi-elasticity of the scattering and establishes the relation between the concentration of the electrons, the magnetic field, and the temperature, at which the expressions (8) and (9) are valid.

It is seen from the derivation of (8) that the largest contribution to $G(t)$ is made by the time region $\tau_1 - \tau_2 \sim \tau_e$, $\omega_c^{-1} \ll t$. Therefore in the expansion of $F(t; \mathscr{P}_1)$ of (5) in terms of $G(t)$ we can confine ourselves in the zeroth approximation in $\Gamma\tau_e \ll 1$ and $\Gamma\omega_c^{-1} \ll 1$ to only those terms in which the double integrals do not break up when ordered in time, i.e., the product $g(\tau_1, \tau_2)\ldots g(\tau_n, \tau_{n+1})$ is integrated over the region $t \geqslant \tau_1 \geqslant \tau_2 \geqslant \ldots \geqslant \tau_n \geqslant \tau_{n+1}$. This is equivalent to taking into account only non-intersecting diagrams. The obtained series can be easily summed:

$$F(t;\mathscr{P}_1) = \mathscr{P}_1 + \int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2\, T_t\bar{T}_t\{g(\tau_1,\tau_2)F(\tau_2;\mathscr{P}_1)\}. \tag{11}$$

Substituting (11) in (5), differentiating with respect to time, and recognizing that $g(\tau_1, \tau_2)$ is a rapidly oscillating function of $\tau_1 - \tau_2$ with a period $\sim \tau_c, \omega_c^{-1}$, and $F(\tau_2; \mathscr{P}_1)$ is a smooth function of τ_2, we get

$$d\tilde{Q}(t)/dt = -(\Gamma + iP)\tilde{Q}(t), \quad \tilde{Q}(0) = N, \tag{12}$$

whence

$$\tilde{Q}(t) = N\exp[-(\Gamma+iP)t], \quad t \gg \tau_e \gg \omega^{-1},$$
$$Q(\omega) = N\frac{\Gamma}{\Gamma^2 + (\omega-\omega_c-P)^2}, \quad |\omega-\omega_c| \ll \tau_e^{-1}. \tag{13}$$

In the derivation of (12) we have performed calculations similar to those that have led to Eq. (8). In addition, it was assumed that over times $\sim\Gamma^{-1}$ there is no correlation between the electron velocities, and assumption quite natural if $\Gamma < e_0^2 l^2/R^3 \ll T$.

Thus, according to (13), the form of the CR peak of nondegenerate two-dimensional electrons turns out to be Lorentzian, when account is taken of their interaction with one another. The half-width Γ of the peak is determined both by the interaction with the phonons (or impurities, see below), and by the intensity E of the fluctuation field. It is easy to analyze the dependence of Γ on $\langle E\rangle$ in the important case of relatively strong fields, when $\tau_e^{-1} \gg \omega_q$ at $q \lesssim l^{-1}$. It follows then from (8) that

$$\Gamma = \frac{S}{\pi}\tau_e \int_0^\infty dq \frac{T|V_q|^2}{l\omega_q}\left(\frac{1}{2}l^2q^2\right)^2 \exp\left(-\frac{1}{2}l^2q^2\right), \quad \tau_e^{-1} \gg \omega_q. \tag{14}$$

According to (14), (8), and (9), the half-width Γ is proportional to the average reciprocal intensity of the fluctuation field, and if we estimate the field at $\langle E\rangle \sim N^3/\sim N^{3/4}T^{1/2}$, then $\Gamma \propto N^{-3/4}T^{1/2}$, even though Eq. (13) is of the single-electron form $Q(\omega) \propto N$, the relaxation parameter $\Gamma = \tau_r^{-1}$ itself depends quite strongly on the electron density. The shift of the CR peak P is likewise, generally speaking, dependent on N.

It is easy to generalize the theory of the shape of the CR peak to include the case of electron scattering by impurities in the linear approximation in the impurity concentration n_s. In this approximation, the Hamiltonian of the interaction of the electrons with the scatterers is conveniently written in the form

$$H_i = \sum_n \sum_{\mathbf{q}} v_{\mathbf{q}} \exp(i\mathbf{q}\mathbf{r}_n), \quad \langle\langle v_{\mathbf{q}}\rangle\rangle = 0, \tag{15}$$
$$\langle\langle v_{\mathbf{q}} v_{\mathbf{q}'}\rangle\rangle = n_s|u_{\mathbf{q}}|^2\delta_{\mathbf{q},-\mathbf{q}'},$$

where u_q is the Fourier component of the impurity potential. Calculations for the model of the interaction (15) are similar to those made above for the case of the interaction with the phonons. As a result we obtain for $Q(\omega)$ again expression (13), with Γ and P described respectively by Eq. (14) and by expression (8) in which it is necessary to replace $2T|V_q|^2/\omega_q$ and $[2\bar{n}(\omega_q)+1]|V_q|^2$ and $n_s|u_q|^2$. The factors l^2q^2 in (8) and (14) lead to cut-off of the impurity potential at distances on the order of l.

3. CURRENT-VOLTAGE CHARACTERISTIC OF NONDEGENERATE ELECTRONS IN THE ABSENCE OF HEATING

An interesting feature of two-dimensional electrons is the possibility of obtaining an N-shaped current-voltage characteristic in the case when the Hall field is equal to zero. It is known[1,2] that the conductivity in quantizing magnetic fields is proportional to the cross section for the scattering by impurities and phonons. This cross section, according to (14), is in turn proportional to the time of flight τ_e of the electron near the defect. If the external field E_D is strong and exceeds the fluctuation field E, then τ_e decreases, $\tau_e \sim lH/cE_0$. At the same time, the possibility of the electron of "hopping away" as a result of scattering along the field decreases: in elastic scattering, by virtue of the energy conservation law, the electron can hop away only transversely to the total field $\mathbf{E}_0 + \mathbf{E}$; if $E_0 \gg E$, then the projection of the displacement of the electron on the direction of E_0 is small. Therefore with increasing field the current decreases at $E_0 > E$, and in the case of elastic scattering we have $j \sim E_0^{-2}$ at $E_0 \gg E$.

The Hamiltonian of two dimensional electrons in an electric field takes in the case of scattering by phonons the form

$$\tilde{\mathscr{H}}(t) = H_{E_0}(t) + H_i;$$
$$H_{E_0}(t) = H_0 - \frac{e_0 l\omega_c}{2^{1/2}c}A_0(t)(\mathscr{P}_1 + \mathscr{P}_{-1}) + \frac{e_0^2}{2mc^2}NSA_0^2(t); \tag{16}$$
$$E_0 = -\frac{1}{c}\frac{\partial A_0}{\partial t}, \qquad E_0 \equiv E_{0x},$$

where H_0 and H_i are defined in (1). The vector potential $A_0(t)$ of the electric field is assumed to be homogeneous in space.

The expression for the current density j_x along the field

$$j_x=e_0S^{-1}\left\langle\!\left\langle \frac{l\omega_c}{2^{1/2}}(\mathscr{P}_1+\mathscr{P}_{-1})\right\rangle\!\right\rangle -N\frac{e_0^2}{mc}A_0 \tag{17}$$

is best transformed, using the solution of the equation of motion

$$\dot{\mathscr{P}}_\alpha=i\alpha\omega_c\mathscr{P}_\alpha-i\alpha\frac{e_0l\omega_c}{2^{1/2}c}A_0NS-\frac{il}{2^{1/2}}\sum_n\sum_q V_q q_\alpha(b_q+b_{-q}^+)\exp(i\mathbf{q}\mathbf{r}_n),$$
$$q_\alpha=q_x-i\alpha q_y, \tag{18}$$

into

$$j_x=-ie_0l^2S^{-1}\left\langle\!\left\langle \sum_n\sum_q V_q q_y(b_q+b_{-q}^+)\exp(i\mathbf{q}\mathbf{r}_n)\right\rangle\!\right\rangle, \tag{19}$$

[we have taken into account here the fact that $A_0(t)$ and $\langle\langle b_q\exp(i\mathbf{q}\mathbf{r}_n)\rangle\rangle$ vary slowly over the time ω_c^{-1} and that the increment $\sim\mathscr{P}_\alpha/\omega_c$ to $\mathscr{P}_\alpha$ was discarded when $\mathscr{P}_\alpha$ was substituted in (17)].

If the current is calculated by Eq. (19) in the case of a weak interaction and the heating is neglected, it suffices to find the density matrix of the system in first order in H_t, i.e., to find the linear response to H_t. It is convenient to do this by the standard method, "turning on" H_t adiabatically at the instant $t\to-\infty$. We assume also that there is no electric field at $t<t_0$, i.e., $A_0(t<t_0)=0$, where t_0 is a certain remote ($-t_0\gg\tau_e$), but finite instant of time. Then, obviously

$$\rho(t)=\rho_0,\quad t\to-\infty;$$
$$\rho_0=Z^{-1}\prod_q[\bar{n}(\omega_q)+1]^{-1}\exp[-\lambda(H_0+H_{ph})], \tag{20}$$

where ρ is the density matrix of the system, and Z is the partition function defined in (5) [it actually immaterial what is turned on first, the field or H_t; it is important only that at the turning-on instant the density matrix have the equilibrium form (20)].

Calculating in the usual manner the increment $\propto H_t$ to ρ, substituting in (19), and retaining only the terms diagonal in q (all others drop out when the decoupling of $\langle\langle b_q b_{q'}^+\rangle\rangle$ presented below is carried out), we get

$$j_x(t)=e_0l^2S^{-1}\sum_q|V_q|^2q_y\int_{-\infty}^{t}d\tau\,e^{\varepsilon\tau}$$
$$\times\mathrm{Sp}\left\{\rho_0\sum_{n,m}[a_m'(-\mathbf{q},\tau)(b_{-q}(\tau)+b_q^+(\tau)),\ a_n'(\mathbf{q},t)(b_q(t)+b_{-q}^\pm(t))]\right\};$$
$$\varepsilon\to+0,\quad a_n'(\mathbf{q},\tau)=K^+(\tau,t_0)\exp(i\mathbf{q}\mathbf{r}_n)K(\tau,t_0), \tag{21}$$
$$K(\tau,t_0)=T_t\exp\left[-i\int_{t_0}^{\tau}d\tau_1H_{e_0}(\tau_1)\right].$$

The propagators $K(t,t_0)$ are conveniently written in the form

$$K(t,t_0)=\exp[-iH_0(t-t_c)]\exp\left\{\frac{ie_0l\omega_c}{2^{1/2}c}\mathscr{P}_1\int_{t_0}^{t}A_0(\tau)\exp[i\omega_c(\tau-t_0)]d\tau\right\}$$
$$\times\exp\left\{\frac{ie_0l\omega_c}{2^{1/2}c}\mathscr{P}_{-1}\int_{t_0}^{t}A_0(\tau)\exp[-i\omega_c(\tau-t_0)]d\tau\right\}\exp[\Phi(t,t_0)], \tag{22}$$

where $\Phi(t,t_0)$ is a c-number which plays no role hereafter.

Taking into account the smoothness of the field over the times ω_c^{-1} and discarding the derivatives of $A_0(t)$, we can simplify (22) still further:

$$K(t,t_0)=\exp\left[\frac{e_0l}{2^{1/2}c}\mathscr{P}_1A_0(t)\right]$$
$$\times\exp\left[-\frac{e_0l}{2^{1/2}c}\mathscr{P}_{-1}A_0(t)\right]\exp[-iH_0(t-t_0)+\Phi(t,t_0)]. \tag{23}$$

It follows therefore that

$$K^+(t,t_0)\exp(i\mathbf{q}\mathbf{r}_n)K(t,t_0)=\exp[i\mathbf{q}\mathbf{r}_n(t-t_0)]\exp\left[\frac{ie_0l^2}{c}q_yA_0(t)\right],$$
$$\mathbf{r}_n(t)=\exp(iH_0t)\mathbf{r}_n\exp(-iH_0t), \tag{24}$$

i.e., the contribution of the field to the dependence of the electron coordinate on the time can be exactly separated, and this contribution corresponds simply to a drift of all the electrons across the field with equal velocity. Substituting (24) in (21), replacing $A_0(t)-A_0(\tau)$ by $-(t-\tau)cE_0$ in the argument of the exponential, and putting $t=0$, we obtain after simple transformations, taking (20) into account,

$$j_x=\frac{e_0^2E_0l^4}{T}S^{-1}Z^{-1}\sum_q|V_q|^2q_y^2\,\mathrm{Re}\int_0^\infty d\tau\,\varphi_q(\tau)\exp(-\varepsilon\tau-ie_0l^2q_yE_0\tau)$$
$$\times\mathrm{Sp}\sum_{n,m}\{\exp(-\lambda H_0)a_n(\mathbf{q},\tau)a_m(-\mathbf{q},0)\},\qquad |e_0lE_0|\ll T, \tag{25}$$

where $a_n(\mathbf{q},\tau)$ and $\varphi_q(\tau)$ are defined in (5). The factor $\exp(-ie_0E_0l^2q_y\tau)$ in (25) describes the influence of the external field on the scattering act and leads to a deviation from Ohm's law. If we discard this factor, then the expression that follows from (25) for the static conductivity $\sigma(0)$ coincides, in the lowest order in H_t, with that obtained in Ref. 6 by an entirely different method.

Transforming $a_n(\mathbf{q},\tau)$ with the aid of (7) and using the quasiclassical approximation developed above, we can calculate the integral in (25):

$$j_x=2\pi e_0^2E_0Nl^4\sum_q\frac{|V_q|^2}{\omega}q_y^2\exp\left(-\frac{1}{2}l^2q^2\right)\langle\delta(q\dot{\mathbf{R}}_n-e_0E_0l^2q_y-\omega_q)\rangle. \tag{26}$$

Thus, the influence of the field E_0 on the scattering act reduces simply to an increment to the drift velocity $\dot{\mathbf{R}}_n$ in the fluctuation field $\mathbf{E}$ of the drift velocity $e_0E_0l^2$ in the external field.

It is of interest to analyze expression (26) for relatively strong fluctuation fields, when $\tau_e^{-1}\gg\omega_q$ at $q\lesssim l^{-1}$. If at the same time $E_0\ll\langle E\rangle$, then

$$j_x=\sigma_{xx}(0)E_0,\quad \sigma_{xx}(0)=e_0^2Nl^3\tau_e\frac{S}{2\pi}\int_0^\infty dq\,\frac{|V_q|^2}{\omega_q}q^2\exp\left(-\frac{1}{2}l^2q^2\right). \tag{27}$$

It follows from (27) that $\sigma(0)\propto\tau_e$. If we use the estimate $\langle E\rangle\sim N^{3/4}T^{1/2}$, then $\sigma(0)\propto N^{1/4}T^{-1/2}$ (in the case of scattering by impurities it is necessary to replace $|V_q|^2/\omega_q$ in (27) by $n_s|u_q|^2/2T$, and then $\sigma(0)\propto T^{-3/2}$). Thus, although the scattering is quasi-single-electron in character, the conductivity varies very slowly with the electron concentration.

In the case of a strong drawing field, when $cE_0/H\gg\langle|\dot{\mathbf{R}}_n|\rangle$ (but $\tau_e^{-1}\gg\omega_q$), expression (26) is transformed into

$$j_x=\frac{NE_0}{|e_0E_0|}\langle\dot{\mathbf{R}}_n^2\rangle\frac{S}{2\pi l^2}\int_0^\infty q^2\,dq\,\frac{|V_q|^2}{\omega_q}\exp\left(-\frac{1}{2}l^2q^2\right), \tag{28}$$

i.e., j_x decreases like E_0^{-2}. The maximum value of j_x in the field region $|e_0E_0|l\ll\omega_c$ is reached at $|e_0E_0|l^2\sim\langle|\dot{\mathbf{R}}_n|\rangle$

and amounts to $\sim\sigma(0)/e_0 l\tau_e$. It is seen from (26) that the decrease of the field $j_x \propto E_0^{-2}$ in strong fields E_0 takes place at arbitrary τ_e^{-1}/ω_q. We note that in the field region $e_0 E_0 l \sim \omega_c$ the current increases sharply.[11]

An N-shaped current-voltage characteristic of two-dimensional electrons in quantizing magnetic fields in the absence of heating was first obtained in Ref. 7. A calculation based on a phenomenological introduction of the electron lifetime γ^{-1} yielded in Ref. 7 an expression for the conductivity $\sigma(0)$ an expression similar to (27), in which τ_e must be replaced by γ^{-1}. If collective oscillations take place in the electron system, then γ can be estimated as being the limiting oscillation frequency. For the two-dimensional Wigner crystal model it amounts to $\sim e_0^2 l^2/R^3$ (the high frequency oscillations in the crystals with frequency $\gtrsim\omega_c$ are not excited in the crystal in the case of slow motion). From the condition that the electric field be homogeneous over the magnetic length at $T \gg e_0^2 l^2/R^3$ it follows that $\tau_e^{-1} \gg e_0^2 l^2/R^3$ and consequently $\tau_e \ll \gamma^{-1}$, i.e., although the expressions for $\sigma(0)$ are outwardly similar, Eq. (27) differs greatly from the result of Ref. 7. The general and quite clear expression (26) for the current, when applied to the case considered in Ref. 7, that of scattering by impurities, differs from the results of Ref. 7. In particular, in Ref. 7 we have $j_x \propto E_0^{-1}$ in strong fields.

The current voltage characteristic can be of interesting form if the interaction between the electrons is weak and Cerenkov scattering in the absence of a drawing field E_0 is forbidden, $c\langle E\rangle/H < \omega_q/q$. Then in weak fields E_0 the conductivity appears only when account is taken of fourth order terms in V_q (in the case of weak coupling with the phonons), and is small.[6] At $E_0 \sim H\omega_q/qc$ the current increases strongly because of the appearance of the new scattering mechanism (26), and then decreases with increasing E_0 (like E_0^{-2} in strong fields).

The form of the current-voltage characteristic can change noticeably when allowance is made for heating, but in strong fields the heating is obviously slow. The calculation of j_x with heating taken into account will be carried out separately.

4. DISCUSSION OF RESULTS

The foregoing analysis of the influence of the interaction between the electrons on the electron-phonon scattering is not connected with the model of the electron system, and is apparently valid for a Wigner single crystal or polycrystal, as well as for an electron liquid. It is merely required that the distance between carriers be large, $R \gg l$, that the temperature not be too low, so as to satisfy the inequality (10), and that the fluctuation field be homogeneous over the length l. The interaction with the phonons and with the impurities is assumed to be weak, so that $\Gamma\tau_e \ll 1$, i.e., the impact collision time τ_e is short compared with the reciprocal collision frequency Γ^{-1}, and that there be no formation of strongly bound polarons or capture by impurities (if the interaction with the phonons, for example, turns out to be stronger than the electron-electron interaction, i.e., the half-width of the CR peak calculated neglecting the electron-electron interaction is much larger than τ_e^{-1}, then the single-electron approximation is valid.[5,6]).

In the model with ordering we obtain from the condition (10)

$$(e_0^2 N^{1/2}/T)^{1/2} \ll (m\omega_c/\hbar)^{1/2}, \quad \langle E\rangle \sim N^{1/4} T^{1/2}. \tag{29}$$

This inequality imposes a lower bound on the magnetic field. The upper bound is imposed by the Cerenkov radiation condition

$$cN^{1/4}T^{1/2} > H\omega_q/q, \quad q \sim l^{-1}. \tag{30}$$

It is of greatest interest to estimate the concentrations and the magnetic fields, using Eqs. (29) and (30), for two-dimensional electrons localized over the surface of liquid helium and interacting with the surface oscillations (ripplons).[6] The phase velocity of the ripplons is small: $\omega_q/q \lesssim 1.6\cdot 10^3$ cm/sec in the interval of q from 20 to 10^6 cm^{-1}. At $T = 1$ K and $N = 10^8$ cm^{-2} we get from (30) $H < 2\cdot 10^5$ Oe. The lower bound (29) of the magnetic field is quite lax: it yields $H \gg 10$ Oe. Much more restrictive are the conditions $\omega_c\tau_e \gg 1$ and $\omega_c \gg \Gamma$. In addition, a more stringent upper bound than (30) may be the inequality $\Gamma\tau_e \ll 1$. The condition $\exp(\omega_c/T) \gg 1$ used above in the calculation is of no significance for the assessment of the role of the considered relaxation mechanism.

All the results can be directly generalized to include the case of interaction with volume phonons. The corresponding contribution to Γ and P for electrons localized over liquid helium turns out to be small, since the phase velocity of the volume phonons is large. The interaction with these phonons can, however, be very substantial for the energy relaxation of the electrons under CR conditions in heating fields: it induces transitions between the Landau levels,[5,6] on which the electron-electron interaction has little effect at $\omega_c\tau_e \gg 1$.

Explicit expressions for the width and shift of the CR peak and for the magnetic conductivity of the electrons over liquid helium take the simplest form in the important case $\tau_e^{-1} \gg \omega_q$, when the inelasticity of the scattering by the ripplons can be neglected. This case is of interest from the experimental point of view. In Ref. 9, for example, $T = 0.4$ K, $\omega_c = 1.13\cdot 10^{11}$ sec^{-1} and $N \sim 10^8$ cm^{-2}. In this case $\omega_q \lesssim 10^8$ sec^{-1}, and if the fluctuation field is estimated from (29), then $\tau_e \sim 10^{-10}$ sec. If we use the electron-ripplon interaction model[8] and assume the helium film to be thick compared with l ($l \approx 3\cdot 10^{-6}$ cm in Ref. 9), then

$$V_q = S^{-1/2} e_0 E_\perp (q/2\rho\omega_q)^{1/2}, \quad \omega_q = (\alpha q^3/\rho)^{1/2}, \tag{31}$$

where α is the surface tension of the helium, ρ is its density, and $E_\perp$ is the field that clamps the electrons to the surface. Substituting (31) in (27), (14), and (8) we get

$$\sigma_{xx}(0) = \frac{Ne_0^2 l^2 \Gamma}{2T}, \qquad \Gamma = \frac{e_0^2 E_\perp^2 T}{8\alpha\hbar^2\sqrt{2\pi}}\tau_e,$$
$$P \approx P_v = e_0^2 E_\perp^2 T/4\pi\alpha\hbar^2\omega_c. \tag{32}$$

If no account is taken of the electron-electron interaction, the shift calculated by the method of moments is larger than in (32) (by an approximate factor of 3 under the conditions of Ref. 9). The characteristic half-width

of the peak is determined in this case by the expression $\tilde{\Gamma}\sim(\Gamma\tau_e^{-1})^{1/2}\propto E_\perp$. Since $\tilde{\Gamma}\gg\Gamma$ and they have different dependences on the clamping field $E_\perp$ as well as on the magnetic field ($\Gamma\propto H^{1/2}$), it becomes possible to verify in experiment the extent to which the interelectron interaction influences the relaxation (it is necessary, however, to take into account the possible "softening" of the ripplon spectrum by the electron-ripplon interaction[8,12]). Another experimental check on the considered mechanism of the CR peak broadening might be the measurement of the dependence of the width of the peak on the value of the static (or quasistatic field E_0). Such a field produces an additional transverse drift, the total drift velocity increases, and in the expression for τ_e it is necessary to replace the average fluctuation field $\langle E^{-1}\rangle$ by $\langle|\mathbf{E}+\mathbf{E}_0|^{-1}\rangle$, i.e., τ_e decreases and the half-width of the CR peak should decrease.

The authors are deeply grateful to M. A. Krivoglax for a discussion of the work.

APPENDIX

The Heisenberg equations of motion for the coordinates and momenta of the electrons in the absence of interaction with the phonons are of the form

$$\dot{r}_{\alpha n}=\frac{2^{1/2}}{ml}p_{\alpha n},\quad \dot{p}_{\alpha n}=i\alpha\omega_c p_{\alpha n}+\frac{le_0^2}{2^{1/2}}{\sum_m}'\frac{r_{\alpha nm}}{(r_{1nm}r_{-1nm})^{3/2}},\quad \alpha=\pm1,$$
$$r_{\alpha n}=x_n-i\alpha y_n,\quad r_{\alpha nm}=r_{\alpha n}-r_{\alpha m},\qquad (A.1)$$
$$[r_{\alpha n},\,p_{\beta m}]=i\delta_{nm}(1-\delta_{\alpha\beta})l\cdot2^{1/2}.$$

The prime on the summation sign denotes exclusion of terms with $m=n$.

In the case of large distances between particles, $R\gg1$ and strong magnetic fields $\omega_c\gg\gamma_e$, $\gamma_e=e_0^2l^2/R^3$, the system of nonlinear equations (A.1) is best solved in the region of long times $t\sim\gamma_e^{-1}\gg\omega_c^{-1}$ with the aid of the known averaging method of nonlinear mechanics.[19] We seek the solution in the form

$$p_{\alpha n}(t)=\sum_\nu p_{\alpha n}^{(\nu)}(t)\exp(i\nu\omega_c t),$$
$$r_{\alpha n}(t)=\sum_\nu r_{\alpha n}^{(\nu)}(t)\exp(i\nu\omega_c t),\qquad (A.2)$$
$$|\dot{p}_{\alpha n}^{(\nu)}|\ll\omega_c|p_{\alpha n}|\sim\omega_c,\quad |\dot{r}_{\alpha n}^{(\nu)}|\ll\omega_c l,\quad \nu=0,\pm1,\ldots.$$

The system of equations for $p_{\alpha n}^{(\nu)}$ and $r_{\alpha n}^{(\nu)}$ can be obtained by substituting the expansion (A.2) in (A.1) and equating term by term the terms of like powers of $\exp(i\omega_c t)$. It is clear from qualitative considerations (which are confirmed by Eq. (A.3)) that $r_{\alpha n}^{(\nu)}\sim l$ at $\nu\neq0$. We can therefore expand $r_{\alpha nm}$ in (A.1) in powers of $r_{\alpha nm}^{(\nu)}/r_{\alpha nm}^{(0)}$ and confine ourselves in the right-hand side of the second equation in (A.1) to the first nonvanishing terms of the expansion. As a result, the system (A.1) takes the form

$$r_{\alpha n}^{(\alpha)}=-i\alpha l\cdot2^{1/2}P_{\alpha n},\quad \dot{P}_{\alpha n}=\frac{i\alpha e_0^2l^2}{2}{\sum_m}'\frac{P_{\alpha nm}}{|R_{\alpha nm}|^3},$$
$$P_{\alpha n}\equiv p_{\alpha n}^{(\alpha)},\quad P_{\alpha nm}=P_{\alpha n}-P_{\alpha m},$$
$$p_{\alpha n}^{(0)}=\frac{ml}{2^{1/2}}\dot{R}_{\alpha n},\quad \dot{R}_{\alpha n}=i\alpha e_0^2l^2{\sum_m}'\frac{R_{\alpha nm}}{|R_{\alpha nm}|^3},\qquad (A.3)$$
$$R_{\alpha n}\equiv r_{\alpha n}^{(0)},\quad R_{\alpha nm}=R_{\alpha n}-R_{\alpha n}.$$
$$r_{\alpha n}(t)\approx R_{\alpha n}(t)-i\alpha l\cdot2^{1/2}P_{\alpha n}(t)\exp\{i\alpha\omega_c t\},$$
$$p_{\alpha n}(t)\approx P_{\alpha n}(t)\exp(i\alpha\omega_c t)+p_{\alpha n}^{(0)}(t),\quad \gamma_e\ll\omega_c.$$

The momentum components (and the corresponding coordinates) not written out in (A.3) are $p_{\alpha n}^{(-\alpha)}\sim(\gamma_e/\omega_c)P_{\alpha n}$, while the components $p_{\alpha n}^{(\nu)}$ contain at $|\nu|\geq2$ even higher powers of the small parameter l/R. Neglecting them, we obtain from the commutation condition of the operators $p_{\alpha n}$ and $r_{\alpha n}$, taking (A.3) into account, the commutation conditions for the operators $p_{\alpha n}^{(\nu)}$ and $r_{\alpha n}^{(\nu)}$:

$$[P_{-1n}(t),P_{1m}(t)]=\delta_{nm},\quad [P_{\alpha n}(t),R_{\alpha' m}(t)]=0,$$
$$[r_{-1n}^{(-1)},r_{1m}^{(1)}]=2l^2\delta_{nm},\quad [r_{\alpha n}^{(\alpha)},R_{\alpha' m}]=0,\quad [R_{-1n}(t),R_{1m}(t)]=-2l^2\delta_{nm}.\qquad (A.4)$$

The smooth motion described by the operators $R_{\alpha_n}(+)$ corresponds to the transverse drift of the center of the cyclotron orbit in the fluctuation electric field

$$\mathbf{E}(\mathbf{R}_n)=-\frac{e_0}{2}\frac{\partial}{\partial\mathbf{R}_n}{\sum_{n\neq m}}|\mathbf{R}_n-\mathbf{R}_m|^{-1},$$
$$\mathbf{R}_n=(X_n,Y_n),\quad R_{\alpha n}=X_n-i\alpha Y_n,\qquad (A.5)$$

where X_n and Y_n are the coordinates of the center of the orbit.

We note that the drift of the center can be described as one-dimensional motion by introducing the canonically conjugate momenta π_n and coordinates ξ_n of certain one-dimensional particles with the aid of the relations

$$\pi_n=l^{-1}X_n,\quad \xi_n=l^{-1}Y_n,\quad [\pi_n,\xi_n]=-i,$$
$$\tilde{\mathscr{H}}=\frac{e_0^2}{2l}{\sum_{n\neq m}}[(\pi_n-\pi_m)^2+(\xi_n-\xi_m)^2]^{-1/2}.\qquad (A.6)$$

Here $\tilde{\mathscr{H}}$ is the effective Hamiltonian of these particles, and the Hamilton equations for π_n and ξ_n, which are obtained from (A.6), coincide with Eqs. (A.3).

It is obvious that all the foregoing calculations can be directly applied to the case of a more general interaction of two-dimensional electrons, with image forces taken into account (see, e.g., Ref. 14).

[1)]The terms off-diagonal in the electron number are small at $l\ll R$ and are not taken into account, i.e., the scattering has, as it were, a single-particle character. However, the possibility of scattering is itself due to the interelectron interaction, which leads to a drift of the centers of the cyclotron orbits.

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Translated by J. G. Adashko