# Low-frequency excitations in systems such as a doped excitonic dielectric

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The effect of doping on the excitation spectrum in Fermi-liquid systems of the excitonic dielectric type is considered. Doping leads to suppression of new oscillation modes in the low-frequency region $\omega < \Delta$ and $kv < \Delta$, to hybridization of the existing modes, and to a change in the region of stability relative to excitation of zero-sound waves. Analogous effects in nonequilibrium systems are also analyzed.

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1. The spectrum of longitudinal excitations in an excitonic dielectric with one gap was already considered earlier in the high-density limit by Kozlov and Maximov,[1] and in a two-gap Fermi liquid in our earlier paper.[2] We have assumed there that the doping, which is needed in principle for the coexistence of the singlet and triplet pairing, has little effect on the collective-excitation spectrum, and in Ref. 1 they did not consider doping at all. Yet it will be shown below that doping leads, even in a single-gap system, to the appearance of new oscillation modes, and strong doping leads to hybridization of the excitations that exist in the normal (without pairing) and gap systems. Kopaev[3] has shown that no gap state is possible at all at doping impurity concentrations $\tilde{n}$ exceeding $\tilde{n}_k = mp_0\Delta_0/4\pi^2$ or, in energy units, $n \geq \Delta_0/2$ ($\Delta_0$ is the gap in the undoped dielectric). By strong doping we mean hereafter the limit $|2n - \Delta_0| \ll \Delta_0$.

We note that all the results apply also with practically no change to the dielectric phase of an electron-hole liquid in a semiconductor, the so-called "drops,"[4] in the presence of an additional metal-dielectric transition. Similar effects should take place also in equilibrium systems, of the excitonic dielectric type, which are produced by laser background illumination.[5] In this case, however, the role of the doping is assumed by disequilibrium effects that lead in the general case to inequality of the effective Fermi quasilevels of the electrons and holes, $\mu_e \neq \mu_h$. We are interested in the present paper in the indicated group of questions connected with the influence of doping or inverted population on the spectrum of the collective excitations in the electron-hole Fermi liquid of a single-gap excitonic dielectric.[2] We confine ourselves to homogeneous (i.e., commensurate) systems with constant order parameter.

2. Consider a system with pairing in the region of low frequencies and small wave numbers, $\omega \ll \Delta$ and $kv \ll \Delta$. In this case the excitation spectrum is determined in fact by the poles of the corresponding polarization operator, which coincide with the poles of the vertex functions.[2,6]

The system of equations for the vertex functions and for the gaps, which must be solved simultaneously, was derived earlier [see Eqs. (12) and (19) of Ref. 2]. In the general case the corresponding equations are quite unwieldy. But in the case of isotropic interaction, which is assumed hereafter for simplicity, and in the absence of conditions for the coexistence of two types of pairing, the equations for the amplitudes of the interaction of the electron with the external field without ($R$) and with spin flip ($\Phi$) are uncoupled.

Let, for example, $\Delta_s \neq 0$ and $\Delta_t = 0$, where $\Delta_s$ and $\Delta_t$ are the singlet and triplet gaps in the single-particle spectrum. This situation is possible in principle when $\Delta_{s0} > \Delta_{t0}$, or if there is no triplet pairing at all ($\Delta_{t0} = 0$) because the coherent interaction constant in the triplet state is negative. In this case the amplitude $\Phi$ is a vector quantity proportional to the vector $\boldsymbol{\sigma}$. As a result the system (19) of Ref. 2 takes a somewhat different form, since such amplitudes have other properties with respect to space inversion:

$$R = R^\omega + f_0^\omega \int_{-1}^{1} [LR + M\tilde{R}]\,dz, \qquad \tilde{R} = f_0^\xi \int_{-1}^{1} [N\tilde{R} + OR]\,dz,$$
$$\Phi = \Phi_s^\omega + g_0^\omega \int_{-1}^{1} [L\Phi + M\tilde{\Phi}]\,dz, \qquad \tilde{\Phi} = g_0^\xi \int_{-1}^{1} [N\tilde{\Phi} + O\Phi]\,dz. \tag{1}$$

Here $R^\omega$ and $\Phi_s^\omega$ are the renormalized bare amplitudes without and with spin flip, respectively, $f_0$ and $g_0$ are the "zeroth" coefficients of the expansion of the Landau function in Legendre polynomials, and $f_0^\omega$ and $g_0^\omega$ are the singlet and triplet coherent-interaction constants. $\tilde{R}$ and $\tilde{\Phi}$ are the amplitudes of the interaction with the field with production of electron-hole pairs from the vacuum, $kvz = \mathbf{k}\cdot\mathbf{v}$, and

$$L = \frac{z}{s-z}(1-\varphi) - \varphi\frac{1+\hat{P}}{2}, \qquad M = \frac{kv}{2\Delta}(s+z)\left[\varphi\frac{1+\hat{P}}{2} + \lambda\frac{1-\hat{P}}{2}\right],$$
$$N = \ln\frac{2\xi}{\Delta_{s0}} - x^2\varphi - \frac{1-\hat{P}}{2}\varphi, \qquad O = -\frac{kv}{4\Delta}[(s+z) + (s-z)\hat{P}](\varphi+\lambda), \tag{2}$$

where $\Delta_{s0} \ll \xi \ll \varepsilon_F$, $\hat{P}$ is the space-inversion operator,

$$2x(1-x^2)^{1/2}\varphi = 2\arcsin x - \operatorname{arctg}\frac{nx(1-x^2)^{1/2}}{\omega/2+\mu x^2} + \operatorname{arctg}\frac{nx(1-x^2)^{1/2}}{\omega/2-\mu x^2},$$
$$4x^2\lambda = \ln\frac{(\mu kv)^2(s^2-\varepsilon^2 z^2) - 4(x\Delta)^4 + 4\mu kv\varepsilon(x\Delta)^2}{(\mu kv)^2(s^2-\varepsilon^2 z^2) - 4(x\Delta)^4 - 4\mu kv\varepsilon(x\Delta)^2}, \tag{3}$$
$$x^2 = [\omega^2 - (kv)^2](2\Delta)^{-2}, \quad skv = \omega, \quad \mu\varepsilon = (\mu^2 - \Delta^2)^{1/2},$$

and $\mu$ is the shift of the electron Fermi energy because of the doping. In addition, the condition for electroneutrality of the system yields a connection between $n$

  

and $\mu$ (Ref. 7):

$$n=(\mu^2-\Delta^2)^{1/2}. \tag{4}$$

Since the vector amplitudes $\Phi$ and $\tilde{\Phi}$ are proportional to to the axial vector $\sigma$, while the operator $\hat{P}$ reverses their sign. In contrast, in Ref. 2 we introduced for a two-gap system the scalar amplitudes $\Phi$ and $\tilde{\Phi}$ proportional to $\sigma\cdot\Delta_t$, invariant to the $\hat{P}$-transformation, and having poles that described magnetoelastic waves in a ferromagnet.

3. In the absence of doping, when $x \ll 1$ and $\varphi \approx 1$ at $\mu=0$ and $\varepsilon=0$, an analysis of Eqs. (1) shows that the only type of low-frequency excitation consists of ordinary acoustic oscillations corresponding to a pole of the amplitude $R$:

$$R=R^{\omega}\frac{\omega^2-{}^1/_3(kv)^2}{\omega^2-{}^1/_3(1+f_0^{\omega})(kv)^2}. \tag{5}$$

It must be noted that the interband transitions disregarded by us produce a threshold in the sound spectrum of an equilibrium excitonic dielectric.[1,8] No such threshold appears in the dielectric phase of a nonequilibrium electron-hole liquid.

The spin excitations have a threshold even if no account is taken of the interband transitions, and appear in the region of frequencies on the order of $2\Delta$ (Ref. 2; cf. Ref. 9):

$$\omega^2(0)=4\Delta^2[1-{}^1/_4\pi^2(f_0^t-f_0^{\omega})^2]. \tag{6}$$

In the presence of doping, the situation changes radically. We consider first the spin waves. Inasmuch as in the case of isotropic interaction $\Phi$ and $\tilde{\Phi}$ do not depend on the angles, the system (1) for the spin vertices turns out to be algebraic, so that the dispersion equation of the spin excitations is obtained by setting the determinant of this system equal to zero:

$$\left[1-\frac{g_0^{\omega}}{2}\int_{-1}^{1}\frac{dz\,z}{s-z}(1-\varphi)\right]\left[\ln\frac{\Delta_{s0}}{\Delta_{t0}}+\int_0^1 dz\,\varphi\right]+g_0^{\omega}\left(\frac{kv}{2\Delta}\right)^2\left(\int_0^1 dz\,z\lambda\right)^2=0. \tag{7}$$

In the derivation of (7) we made use of the fact that $\varphi$ is even and $\lambda$ is odd in $z$, and neglected terms small in the parameter $x \ll 1$.

In the region of very weak doping $1 \gg |x| \gg \varepsilon \gg |x|^2$ at frequencies $\omega \sim x^2\Delta \ll kv$, Eq. (7) takes the form

$$1-\frac{h\Delta^{1/2}}{akv}\left\{\ln\frac{a+1}{a-1}-2\,\mathrm{arctg}\frac{1}{a}\right\}=0, \tag{8}$$

where

$$a^2=2\omega\Delta(kv)^{-2},\quad h=\left[1-g_0^{\omega}\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1\right)\right]\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1\right)^{-1}.$$

It is easily seen that in the considered limit, if the constant $h$ is positive, zero-sound excitations of the magnon type exist in the system, with a frequency proportional to the square of the wave vector:

$$\omega=\frac{(kv)^2}{2\Delta}\left\{1+4\exp\left[-\frac{(kv)^2}{\varepsilon h\Delta^2}\right]-\left(\frac{kv}{2\Delta}\right)^2\right\}. \tag{9}$$

It follows from (9), that the condition for the existence of the obtained mode, besides the possibility of neglecting the collision dissipation $\omega \gg \nu$, is that $\varepsilon h$ be small enough. Recognizing that the parameter $\varepsilon$ is in fact bounded from below by the requirement that the impurity and ground bands overlap,[3] this condition imposes an upper bound on the constant $h$, $\varepsilon h \ll |x|^2$. In addition, if the condition

$$4\exp[-(kv)^2/\varepsilon h\Delta^2] \gg \varepsilon\Delta/kv$$

is violated, there is no singularity at all in the formula for $\varphi$ at $\omega \sim (kv)^2/2\Delta$, and this imposes a lower bound on the constant $h$.

With increasing density of the doping impurity, when $1 \gg \varepsilon \gg |x|$, the dispersion equation takes the form

$$\varepsilon\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1-\varepsilon\right)=\left[g_0^{\omega}\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1-\varepsilon\right)-1+\varepsilon^2\right]\eta\left(\frac{s}{\varepsilon}\right), \tag{10}$$

where

$$\eta(y)=\frac{y}{2}\ln\frac{y+1}{y-1}-1.$$

At relatively low densities or for strong interaction, $h \gg \varepsilon$, the spin excitations acquire an acoustic character

$$\omega=(\varepsilon h/3)^{1/2}kv,\quad h>0. \tag{11}$$

We have $\omega \ll kv$ as before, but now in the region $1 \gg \varepsilon h \gg |x|$.

Finally, in the case of strong doping, or more accurately at low values of the ratio $h/\varepsilon$, Eq. (10) has a solution

$$\omega=\varepsilon kv\left\{1+2\exp\left[-\frac{2\varepsilon(\ln(\Delta_{s0}/\Delta_{t0})+1-\varepsilon)}{g_0^{\omega}(\ln(\Delta_{s0}/\Delta_{t0})+1-\varepsilon)-1+\varepsilon^2}\right]\right\}, \tag{12}$$

that differs from the known dispersion relation for spin zero sound in the normal phase (i.e., at $\varepsilon \to 1$, $\Delta \to 0$) of effective renormalization of the velocity of the quasiparticle excitations $v \to \varepsilon v$ and of the Fermi-liquid interaction constant.

Using the known relations that connect the amplitude $\Phi$ with the magnetic susceptibility of the system $\chi$ (Ref. 6), we can obtain an expression for $\chi$ as a function of the frequency, of the wave vector, and of the parameter $\varepsilon$. In particular, in the region where Eq. (10) is valid, this expression takes the form ($\Delta_{t0} \neq 0$)

$$\chi(\omega,k)=4m^2\frac{\partial G^{-1}}{\partial\varepsilon_p}N(\varepsilon_F)\times\frac{-[\ln(\Delta_{s0}/\Delta_{t0})+1-\varepsilon]\eta(s/\varepsilon)}{[\varepsilon-g_0^{\omega}\eta(s/\varepsilon)][\ln(\Delta_{s0}/\Delta_{t0})+1-\varepsilon]+(1-\varepsilon^2)\eta(s/\varepsilon)}. \tag{13}$$

Equation (13) is a generalization of the known results to include the case of alternating fields and Fermi-liquid interaction. In fact, in the static limit $s=0$ this equation leads at $\varepsilon \to 1$, in particular, to an expression for the susceptibility of the normal system $\chi=\chi_0(1+g_0^{\omega})^{-1}$, and if liquid effects are neglected Eq. (13) goes over at $s=0$ into the corresponding equation derived by Volkov, Kopaev, and Rusinov[7]:

$$\chi(\omega=k=0)=4m^2N(\varepsilon_F)\left\{\frac{1}{\varepsilon}+\frac{\Delta^2}{n(n\ln(\Delta_{s0}/\Delta_{t0})-\Delta_{s0}+2n)}\right\}. \tag{14}$$

We note also that in the case of weak interaction at $s=0$ the denominator $\chi$ coincides with the expression used in Ref. 10 as the criterion for ferromagnetic instability.

A simple analysis shows that in the region

$$n(\varepsilon+g_0^\omega)\left(n\ln\frac{\Delta_{s0}}{\Delta_{t0}}-\Delta_{s0}+2n\right)+\varepsilon g_0^\omega\Delta^2<0\ . \qquad (15)$$

Eq. (10) has a pure imaginary solution. Thus, in this region the dielectric phase becomes unstable to spin zero-sound excitations—a transition takes place to a two-gap ferromagnetic state. In other words, the condition (15) is a generalized criterion for the ferromagnetic transition in a doped Fermi-liquid system with electron-hole pairing (we recall that we are considering a commensurate phase).

The situation is different in systems with $\Delta_{t0}=0$. A transition to a two-gap system is impossible in principle, so that the amplitude $\tilde{\Phi}$ responsible for pairing in the triplet state vanishes identically, as follows directly from the corresponding equation of the system (1). A detailed analysis of such systems is analogous to a considerable degree to the one given above for the case $\Delta_{t0}\neq 0$. In particular, it is easy to show that in such a dielectric there propagate modes of the type (9), (11), and (12) with the parameter change $h\to g_0^\omega$. The expression for $\chi$ now takes the form

$$\chi(\omega,k)=4m^2N(\varepsilon_F)\frac{\partial G^{-1}}{\partial\varepsilon_F}\left\{\frac{\eta(s/\varepsilon)}{g_0^\omega\eta(s/\varepsilon)-\varepsilon}\right\} \qquad (16)$$

and leads to oscillations with a dispersion equation

$$1=\frac{g_0^\omega}{\varepsilon}\eta\left(\frac{s}{\varepsilon}\right). \qquad (17)$$

At positive values of $g_0^\omega$ Eq. (17) describes spin zero-sound oscillations of the type (12). In the region $g_0^\omega<-\varepsilon$ the solution of (17) is pure imaginary—the system becomes unstable. However, since there is no two-gap phase in the considered case, a transition to a Stoner ferromagnet takes place, and the condition for such a transition is much less stringent than the Stoner ferromagnetism criterion $g_0^\omega<-1$ for a normal system. Since $\varepsilon$ is in principle bounded from below only by the requirement that the ground and impurity bands overlap,[3] such a system can be doped with causing restructuring, only in a very narrow doping region, if at all.

It is easy to verify that a dielectric with $\Delta_{t0}>\Delta_{s0}$ has collective modes that differ by the substitution $\Delta_t\rightleftarrows\Delta_s$ from the excitations considered before. In fact, in this case the system of equations for the vertex spin function takes the form

$$\Phi=\Phi^\omega+g_0^\omega\int_{-1}^{1}dz[L\Phi+MR],\quad R=f_0^\omega\int_{-1}^{1}[NR+O\Phi]dz. \qquad (18)$$

The substitution $\Delta_t\rightleftarrows\Delta_s$ changes the system (18) into the corresponding equations of the system (1). The subsequent analysis and its results agree, apart from the indicated substitution, with those given above.

Thus, the violation, due to doping, of the symmetry of the electrons and holes leads to the appearance of low-frequency spin waves $\omega\ll\Delta$ in a system with pairing. As indicated above, there are no such waves in the absence of doping. The physical meaning of this phenomenon is that the ground state of the carriers in such a system is a hybrid of the states of the gap and normal phases. The amount of doping determines the degree of presence of "normal" properties—with increasing $\varepsilon$ the properties of an excitonic dielectric approach those of an ordinary semiconductor or semimetal, reaching the normal phase in the limit $\varepsilon=1$ $(2n\geq\Delta_0)$.

4. The spectrum of the zero-spin excitations is determined by the poles of the amplitude $R$. The corresponding dispersion equation that follows from the first two equations of the system (1) takes the form:

$$1-\frac{f_0^\omega}{2}\left\{\int_{-1}^{1}dz\left[\frac{z}{s-z}(1-\varphi)-\varphi\right]+2s^2\left[\int_0^1 dz\,\varphi\right]^2\left[\int_0^1 dz(s^2-z^2)\varphi\right]^{-1}\right\}=0. \qquad (19)$$

In the region of very low concentrations of the doping impurity $\varepsilon\ll|x|\ll1$, in contrast to the case of spin waves, this equation has no low-frequency zero-sound solutions—only ordinary sound propagates in the system [see, incidentally, the remark concerning Eq. (5)].

With increasing degree of doping, $\varepsilon$ increases and in the concentration region $\varepsilon\gg|x|$ the equation for zero-spin excitations can be written in the form

$$3s^2(\varepsilon^2-1)(\varepsilon+1)\eta(s/\varepsilon)+\varepsilon^2(3\varepsilon s^2+1)=f_0^\omega\{3s^2+\varepsilon^2)\eta(s/\varepsilon)+\varepsilon^2(\varepsilon-1)\}. \qquad (20)$$

Equation (20) describes bound oscillations of zero-sound and ordinary acoustic types. In fact, in contrast to the limit of strong doping $(\Delta/n)\to 0$ (i.e., $\varepsilon\to 1$), when Eq. (20) goes over into the known equation for zero-sound oscillations in a normal system, we arrive in the absence of doping, $\varepsilon=0$, at the acoustic solution described by the pole in Eq. (5).

Thus, by varying the concentration of the doping impurity (and by the same token the parameter $\varepsilon$) we can observe a continuous transition from the acoustic to the zero-sound oscillation regime. As already noted above in connection with the analysis of spin excitations, this behavior is quite natural, since an increase of $\varepsilon$ leads to an increase of the "normal" properties of the system.

It should be noted that the effects indicated take place in the low-frequency region $\omega\ll\Delta$ and $kv\ll\Delta$, while the difference between the high-frequency collective modes and the corresponding oscillations in the undoped system is of little effect.

For frequencies $\omega\sim\varepsilon kv$ Eq. (20) admits of a solution of the type

$$\omega=\varepsilon kv\left\{1+2\exp\left[\frac{f_0^\omega(\varepsilon-1)-1-3\varepsilon^2}{f_0^\omega(\varepsilon+3)-3(\varepsilon^2-1)(\varepsilon+1)}\right]\right\}, \qquad (21)$$

which is similar to the corresponding spin excitation (12). Equation (21) is valid only in the case of a large negative argument of the exponential, which reduces at $|f_0^\omega|<1$ to the inequality

$$f_0^\omega(3+\varepsilon)+3(1-\varepsilon^2)(1+\varepsilon)\ll1. \qquad (22)$$

The condition (22) is satisfied only in the case of negative parameters $f_0^\omega$ or small $f_0^\omega$ and $1-\varepsilon\equiv\delta\ll1$. In the latter case Eq. (21) can be reduced to the form

$$\omega=\varepsilon kv\left\{1+2\exp\left[-\frac{2}{f_0^\omega+3\delta}\right]\right\}. \qquad (23)$$

The explicit form of the solution depends substantially on the relation between $\delta$ and $f_0^\omega$. If the doping is "weak," when $3\delta \gg |f_0^\omega|$, the solution seems at first glance not to depend at all on the interaction. At small $f_0^\omega$, however, it must be borne in mind that[9] $f_0^\omega \approx -f_0^\xi$, and the usual connection holds between the gap and the interaction, $\Delta \sim \exp(-1/f_0^\xi)$, in which case $\delta \sim \Delta^2/2\mu^2$ and is a function of $f_0^\omega < 0$. For strong doping, when $3\delta \ll |f_0^\omega|$, the oscillations actually correspond at zero sound in the normal system, and the Landau parameter $f_0^\omega$ should naturally be positive.

Thus, the argument of the exponential of Eqs. (21) and (23) contains in fact the liquid interaction renormalized by the effective interaction of the paired and unpaired electrons. The modes (21) and (23) describe zero-sound oscillations in a system which such an interaction. An experimental investigation of the excitation spectrum could yield information on the sign and magnitude of the liquid interaction constant $f_0^\omega$.

The situation is different in systems with $\Delta_{t0} \neq 0$, $\Delta_{s0} = 0$. As already indicated above in connection with the system (18), the dispersion equation for the collective modes is obtained in this case from (20) by making the substitution $\Delta_s \to \Delta_t$. In particular, Eq. (23) is valid, where $\delta$ should now be taken to mean $\Delta_t^2/2n^2$. Since the gap $\Delta_t$ is connected with the triplet excitation, the mode (23) propagates also at $f_0^\omega \approx 0$.

In concluding this section it should be noted that besides the spinless excitations considered above, plasma oscillations can propagate in the system. In the absence of doping these oscillations, naturally, have high frequencies.[1,2] Doping, however, makes possible oscillations of the density of the unpaired electrons with a frequency

$$\omega^2 = \varepsilon \omega_{pl}^2.$$

Obviously in the case of weaker doping, $\varepsilon \ll 1$, these modes are of low frequency, $\varepsilon \ll \Delta$.

5. We consider now nonequilibrium excitonic dielectrics. As already noted above, a detailed analysis of such system was carried out by Kapaev and Kopaev.[5] They have shown that in nonequilibrium systems the role of doping is assumed by the presence of excess electrons above the dielectric gap, as a result of which no additional doping is necessary for the coexistence of the singlet and triplet pairings. On the other hand, the resultant state is not, generally speaking, ferromagnetic. In addition, unlike in superconductors, the gap is determined by the Coulomb interaction of the electrons and holes, as a result of which the nonequilibrium electrons have a Fermi distribution.[5]

The equations for the vertex functions $R$, $\tilde{R}$, $\Phi$ and $\tilde{\Phi}$ are of the same form as in the equilibrium case, but since the Green's functions of the nonequilibrium electrons is different, the functions $\tilde{\varphi}$ and $\tilde{\lambda}$ will also be different. The connection of these operators $\tilde{\varphi}$ and $\tilde{\lambda}$ in the nonequilibrium case with the corresponding operators in an equilibrium excitonic dielectric $\varphi$ and $\lambda$ is of the form

$$1-\tilde{\Phi}=2(1-\varphi), \quad \tilde{\lambda}=2\lambda. \tag{24}$$

To determine the collective excitations in such a system it is necessary to solve the system (1) jointly with the equation for the gap. This equation, say for the case $\Delta_t = 0$, can be written in the form (see Ref. 5)

$$1=f_0^\xi \ln\left[\frac{2\xi}{\Delta}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{1/2}\right], \tag{25}$$

where, as before, $\varepsilon = (\mu^2 - \Delta^2)^{1/2}\mu$. It must be noted that the role of the increment to the chemical potential $\mu$ is played by the Fermi quasi-level of the nonequilibrium electrons. Since the solution of this system of equations is perfectly analogous to the one given above for the equilibrium case, we shall dwell only on the results.

The dispersion equation of the spin excitation is of the same form as in a doped dielectric, apart from the substitution (24) [cf. (7)]. In particular, in the region $\varepsilon \ll |x|$ there exists a magnon-type mode analogous to (9):

$$\omega = \frac{(kv)^2}{2\Delta}\left\{1+4\exp\left[-\frac{(kv)^2}{2\varepsilon h\Delta^2}\right]\right\}. \tag{26}$$

At the parameter values $1 \gg \varepsilon \gg |x|$ the solution takes the acoustic form

$$\omega = (2\varepsilon h/3)^{1/2} kv. \tag{27}$$

The two indicated solutions differ from the corresponding solution in the equilibrium system by the substitution $h \to 2h$, where $h$ coincides with equilibrium value [see (8)]. The appearance of the factor 2 is natural, since the effective state density in the nonequilibrium case is twice as large because of the simultaneous change of the chemical potentials of both the electrons and holes.

Finally, at $\varepsilon \sim 1$ there appears a solution analogous to the zero-sound mode (12):

$$\omega = \varepsilon kv\{1+2\exp(-\varepsilon/\tilde{h})\}, \tag{28}$$

where

$$\tilde{h} = \left[g_0^\omega\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1-2\varepsilon\right)-1+\varepsilon^2\right]\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1-2\varepsilon\right)^{-1}.$$

In this case $\tilde{h}$ must satisfy all the conditions that are imposed on $h$.

In the region $\varepsilon \gg kv/2\Delta$ in the case of attraction $f_0^\xi > 0$ the magnetic susceptibility of the system is given by

$$\chi(\omega, k) = 4m^2 N(\varepsilon_F)\frac{\partial G^{-1}}{\partial \varepsilon_p}\frac{2\eta(s/\varepsilon)}{2\tilde{h}\eta(s/\varepsilon)-\varepsilon}. \tag{29}$$

This expression differs from the corresponding equilibrium function (13) because of the already indicated substitution (24). We note that in the limit as $\Delta \to 0$ the susceptibility $\xi$ does not have the Stoner form, since the system is in disequilibrium.

In the range of parameters were the inequality

$$(\varepsilon+2g_0^\omega)\left(\ln\frac{\Delta_{s0}}{\Delta_{t0}}+1-2\varepsilon\right)-2(1-\varepsilon^2)<0, \tag{30}$$

holds there appears an instability to formation of a triplet gap, and a transition to the magnetic phase takes place. The condition, (30), neglecting the Fermi-liquid interaction $g_0^\omega$, goes over into the previously obtained magnetic-ordering criterion.[5]

The dispersion equation for zero-spin excitation is similar to (20) and of the form

$$6s^2(1-\varepsilon^2)^2\eta(s/\varepsilon)+\varepsilon^2(\varepsilon-2)-3\varepsilon^2s^2(1-2\varepsilon)$$
$$=-f_0^\omega\{2[3s^2(1-2\varepsilon)+\varepsilon^2(2-\varepsilon)]\eta(s/\varepsilon)+\varepsilon^2(1-2\varepsilon)(2-\varepsilon)\}. \quad (31)$$

Proceeding to an analysis of (31), we must recall that in system with inverted population the pairing at low pump levels takes place in the case of attraction $f_0^\xi > 0$, and at high pumps [when $\varepsilon \approx 1$] it takes place for repulsion $f_0^\xi < 0$ [see (25)].

We consider the case of attraction $f_0^\xi > 0$. At small $\varepsilon$ Eq. (31) leads to the usual acoustic solution [see (5)]. As $\varepsilon$ approaches 0.5, the character of the solution changes:

$$s^2 \approx \frac{1}{3}(1+f_0^\omega)-\frac{2\varepsilon(1-\varepsilon)}{3(1-2\varepsilon)}. \quad (32)$$

The value $\varepsilon=0.5$ is critical, since near $\varepsilon=0.5$ the solution of (31) turns out to be pure imaginary ($s^2<0$), and the system becomes unstable. To understand the physical meaning of this instability it must be borne in mind that at $n=n_{cr}=\Delta_0/3$ and in the case of attraction the gap with value $\Delta=\Delta_0/3$ vanishes jumpwise.[5] At this point $(\partial n/\partial\Delta)=0$, and with further increase of the pump we find ourselves in the region of the metastable phase $(\partial n/\partial\Delta)=0$. It is easily seen that at $n=n_{cr}$ and at the corresponding value $\varepsilon=0.5$ the instability obtained above describes the jump of the system to the normal state ($\Delta=0$).

In the case of repulsion $f_0^\xi<0$, Eq. (31) has a zero-sound solution in the region $\varepsilon\approx 1$. If the inequality

$$3(\varepsilon^2-1)^2+f_0^\omega(3-4\varepsilon-\varepsilon^2)\ll 1,\quad |f_0^\omega|<1 \quad (33)$$

is satisfied this solution can be written in the form

$$\omega=\varepsilon kv\left\{1+2\exp\left[-\frac{f_0^\omega(1-2\varepsilon)(2-\varepsilon)+3\varepsilon^2(1-2\varepsilon)+2-\varepsilon}{3\varepsilon(\varepsilon^2-1)^2+f_0^\omega\varepsilon(3-4\varepsilon-\varepsilon^2)}\right]\right\}. \quad (34)$$

Recognizing that in the case of pairing with repulsion $f_0^\xi<0$ the parameter $\varepsilon$ should be close to unity [see (25)], the inequality (33) can be satisfied only at small positive values of the Fermi-liquid constant $f_0^\omega$. Thus, zero sound propagates only in the case of repulsion. This is natural, since the already noted connection $f_0^\omega \approx -f_0^\xi$ holds at small $f_0^\omega$.

Generalization to the case $\Delta_s=0$, $\Delta_t\neq 0$ entails no difficulty because it reduces obviously, as in the equilibrium case, to the substitution $\Delta_{s0}\rightleftarrows\Delta_{t0}$.

6. Thus, doping and disequilibrium alter substantially the collective properties of an excitonic dielectric. In particular, the spin-wave spectrum acquires low-frequency magnon and acoustic modes. The stability of the system to these modes, just as in the normal system, is connected with the stability to the formation of the ferromagnetic phase. As seen from (13), there are two possibilities for the formation of this phase: either a transition to the excitonic-ferromagnet phase, or a Stoner transition. Since an excitonic ferromagnet is in fact either a ferromagnet or an incompletely compensated antiferromagnet, it follows that, depending on the degree of doping and on the ratio of the constants $g_0^\xi$ and $g_0^\omega$ of the coherent and incoherent interactions, the system goes in accord with (15) either into the Stoner criterion $1+g_0^\omega<0$, in our case we get the inequality $\varepsilon+g_0^\omega<0$, where $\varepsilon<1$, and as a result the transition takes place at Fermi-liquid interaction constants $g_0^\omega$ of much smaller absolute value. This fact, as already noted, is due to the change of the carrier density on the Fermi surface because of pairing and doping, and agrees in general with the fact that for many metals the transition to the ferromagnetic place occurs at low interaction constants. Analogous results were obtained also for nonequilibrium systems.

In the case of zero-spin excitations, greatest interest attaches to the system with pumping (32). It is precisely the instability to this mode which causes the collapse of the steady-state regime of electron and hole pairing due to their attraction, and the transition to the pairing state in the case of a coherent repulsion interaction.

Finally, notice should be taken of one more feature of doped and nonequilibrium systems—the existence, depending on the degree of doping or inversion, of zero-sound spin and spinless excitations in the case of Fermi-liquid attraction as well as repulsion. By the same token, in contrast to normal or undoped systems, it is possible in principle, by varying the degree of pumping or the degree of doping, to observe acoustic, zero-sound, or magnon mode and assess the sign and magnitude of the corresponding interaction.

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