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Translated by W. H. Furry

Instability of cholesteric liquid crystals in an electric field

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(Submitted 25 May 1979)
Zh. Eksp. Teor. Fiz. 77, 2081–2092 (November 1979)

The field instability threshold (U_c) and the deformation period (T_c) have been calculated for four Grandjean bands (the layer thickness L of the cholesteric liquid crystal (CLC) was comparable with the helix pitch p_0) by numerical integration, starting out from the general equations of continual theory of CLC. The calculations were made in the case of initial planar and twist orientations of the director of the CLC at the substrate for domains with different directions. For Grandjean bands with large numbers ($L \gg p_0$) analytic formulas are proposed for the calculation of T_c and U_c with account of the nonequilibrium pitch of the cholesteric helix. Detailed experimental investigation of the instability threshold and the deformation period for the cases mentioned above have shown excellent agreement of experimental results and theoretical calculations.

PACS numbers: 61.30.Gd

1. INTRODUCTION

A field (zero-current) instability is observed in the planar texture of a cholesteric liquid crystal (CLC) with positive dielectric anisotropy ($\Delta\varepsilon = \varepsilon_\parallel - \varepsilon_\perp > 0$) at some threshold voltage U_c upon application of an electric field parallel to the helix axis. This instability appears in the form of a spatially periodic deformation of the initial orientation of the director of the liquid crystal and is due to the destabilizing moment, which is proportional to $E^2\Delta\varepsilon^1$ (E is the intensity of the electric field). In nematic liquid crystals (NLC) the threshold voltage of the analogous instability (Freedericsz transition[1]) is determined by the formula

$$U_F = 2\pi(\pi K_{11}/\Delta\varepsilon)^{1/2}, \tag{1}$$

where K_{11} is the elastic modulus for a transverse flexure deformation; the wave vector of the deformation is equal to zero in this case. For CLC the theoretical value of the threshold U_c and the period of the deformation $T_0 = 2\pi/k$ (k is the wave vector of the deformation) were obtained by Helfrich[2] and refined by Hurault:[3]

$$U_c^2 = \frac{8\pi^3}{\Delta\varepsilon}(6K_{22}K_{33})^{1/2}\frac{L}{p_0}, \tag{2}$$

$$T_c^2 = \left(\frac{3K_{33}}{2K_{22}}\right)^{1/2} p_0 L; \tag{3}$$

here L is the thickness of the CLC, p_0 is the equilibrium helical pitch, K_{33} and K_{22} are the elastic moduli for deformations of longitudinal bending and torsional deformations, respectively.

The dependences $U_c \propto (L/p_0)^{1/2}$ and $T_c \propto (p_0 L)^{1/2}$ that follow from (2) and (3) have been verified experimentally.[4,5] Formulas (2) and (3) were obtained, however, under the assumption that $L \gg p_0$ and without account of the difference of the real (induced) helical pitch p, which arises as a result of the orienting influence of the walls of the vessel,[1,6] from the equilibrium value p_0, and therefore cannot be used directly for the estimation of the instability threshold in the case of a thickness of the CLC layer that is comparable with the helical pitch ($L \sim p_0$). In the case $L \sim p_0$, only the electrohydrodynamic instability has been investigated experimentally in detail.[6]

The purpose of the present work is a systematic theoretical and experimental study of the field instability of planar texture of CLC in the case of arbitrary relations between the layer thickness L and the helical pitch p_0, with account of the real pitch and the boundary conditions.

2. THEORETICAL CALCULATION

Theoretical consideration of the field instability in planar texture of CLC has been carried out under the assumption of a rigid connection of the CLC molecules with the surface of the cell at the boundaries of the layer. Two cases are considered: the directions of orientations of the molecules on the boundaries of the surface are parallel (planar orientation) or perpendic-

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ular (twist orientation). The distribution of the director over the thickness of the layer and the dependence of the real (induced) pitch of the cholesteric helix p on the layer thickness L are shown in Figs. 1 and 2. The functions $p(L)$ are calculated by analogy with the Refs. 1,6 from the conditions $2L/p = m$ (planar orientation) and $2L/p = m + \frac{1}{2}$ (twist orientation), where $m = 0, 1, 2, \ldots$ is the number of the Grandjean band or the number of half-turns of the helix spanned by the thickness of the layer. If the CLC is placed in a wedge-shaped vessel, then the addition of the next half-turn of the cholesteric helix upon increase in the thickness will be preceded by a Kan-Grandjean disclination [1,6] that separates two neighboring bands. In the center of each band the helix pitch p is equal to the equilibrium p_0, while to the left and right halves of the band (see Figs. 1,2) the cholesteric helix corresponds to "compression" ($p < p_0$) and "extension" ($p > p_0$), respectively.

Let us consider the formation of an instability in the form of spatially periodic deformations (domains). We direct the z axis along the axis of the cholesteric helix, the x axis along the direction of preferred orientation of the molecules on the lower base of the cell ($z = 0$) and the y axis perpendicular to x and z (Figs. 1 and 2). The equilibrium orientation of the molecules in the domains is determined by the unit vector (director)

$$\mathbf{n}_0 = \{\cos\psi, \sin\psi, 0\}, \quad \psi = tz, \quad t = 2\pi/p.$$

If the domains are directed along some (arbitrary) axis $\mathbf{l} = \{\cos\alpha, \sin\alpha, 0\}$, lying in the xy plane, then the small angular deviations of φ and θ ($|\varphi|, |\theta| \ll 1$) of the orientations of the molecules in the domains from the equilibrium values are functions of the three coordinates x, y, z and the director of the perturbed molecular orientation has the form

$$\mathbf{n} = \{\cos(\psi+\varphi)\cos\theta, \sin(\psi+\varphi)\cos\theta, \sin\theta\}.$$

For the case of periodic deformations, the dependence of φ and θ on x and y can be represented in the form[3] $\varphi, \theta \propto \exp[i(k_x x + k_y y)]$, for which $k_x \cos\alpha - k_y \sin\alpha = 0$. The dependence of φ and θ on x is more complicated and can be found from the condition that the functional of the free energy of the CLC in an electric field E be a minimum, and written down with accuracy to within terms that are linear in the perturbations φ and θ:

$$\begin{aligned} K_{11}\frac{d^2\theta}{dz^2} &= \Big[(K_{22}\sin^2\psi + K_{33}\cos^2\psi)k_x^2 + (K_{22}\cos^2\psi + K_{33}\sin^2\psi)k_y^2 \\ &\quad + k_x k_y (K_{33} - K_{22})\sin 2\psi + K_{33}t^2 - 2K_{22}t\,\Delta t - \frac{\Delta\varepsilon E^2}{4\pi}\Big]\theta \\ &\quad + (k_x\cos\psi + k_y\sin\psi)[2K_{22}\Delta t - t(K_{11}+K_{33})]\varphi \\ &\quad + \frac{d\varphi}{dz}(K_{22}-K_{11})(k_x\sin\psi - k_y\cos\psi), \\ K_{22}\frac{d^2\varphi}{dz^2} &= [(K_{11}\cos^2\psi + K_{33}\sin^2\psi)k_y^2 + (K_{11}\sin^2\psi + K_{33}\cos^2\psi)k_x^2 \\ &\quad + k_x k_y (K_{33} - K_{11})\sin 2\psi]\varphi + [2K_{22}\Delta t - t(K_{22}+K_{33})] \\ &\quad \times (k_x\cos\psi + k_y\sin\psi)\theta + \frac{d\theta}{dz}(K_{11}-K_{22})(k_x\sin\psi - k_y\cos\psi), \end{aligned} \tag{4}$$

where $\Delta t - t_0 = 2\pi(1/p - 1/p_0)$. Here we can set

$$\theta \sim \sin(k_x x + k_y y), \quad \varphi \sim \cos(k_x x + k_y y).$$

without limitation on the generality. The boundary conditions in the case of a rigid connection of the molecules with the surface of the cell at $z = 0$, L have the form

$$\theta(0) = \theta(L) = \varphi(0) = \varphi(L) = 0. \tag{5}$$

In the case of a voltage $U = EL$ less than the threshold value U_c, Eqs. (4) and (5) have only the trivial solution $\varphi \equiv \theta \equiv 0$. Nonzero perturbations φ and θ appear in the case of voltages $U \geq U_c$, leading to deformation with wave vector $k = 2\pi/T$, where $k_x = k\sin\alpha$, $k_y = k\cos\alpha$, T is the period of the domains. The threshold U_c and the deformation vector k_c corresponding to it are found from the minimum of the curve $U(k)$ (see Fig. 3). The dependences of $U(k)$ can be calculated from Eqs. (4) and (5) by a numerical method with a computer for a

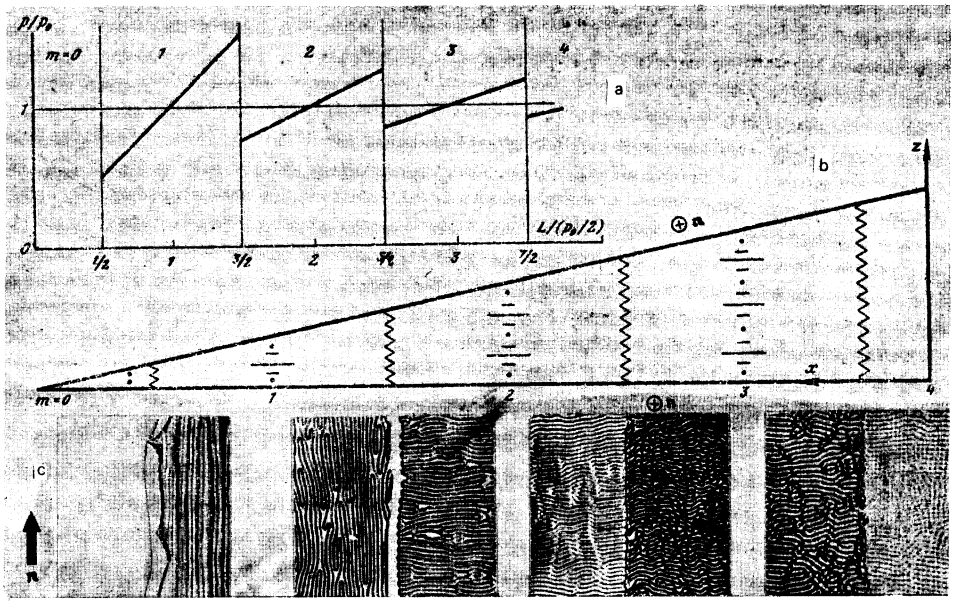

FIG. 1. Initial planar orientation. a) Dependence of the induced pitch of the cholesteric helix p on the thickness of the cell L. b) Distribution scheme of the director $\mathbf{n}$ of CLC (projection on the xz plane) according to the Grandjean bands in a wedge-shaped cell. Form of the threshold deformation (domains) in various Grandjean bands, $p_0 = 30\,\mu$, potential $U \gtrsim U_c$.

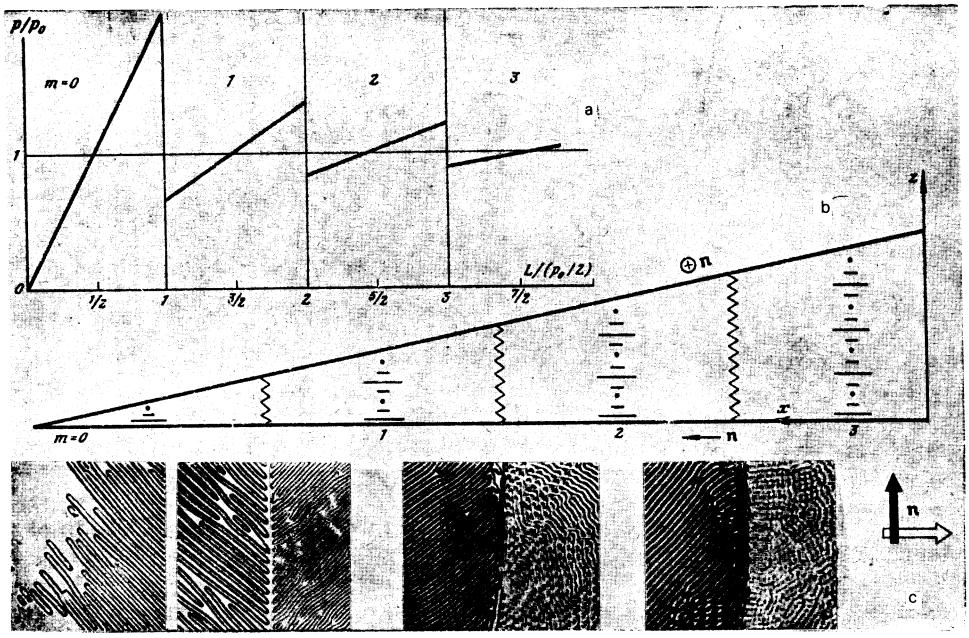

FIG. 2. Initial twist orientation. Notation the same as in Fig. 1.

specified direction of the domains 1.

For this purpose, we consider four fundamental solutions of the linear system (4), which differ in their initial values. Setting

$$\theta=y_1,\quad \frac{d\theta}{dz}=y_2,\quad \varphi=y_3,\quad \frac{d\varphi}{dz}=y_4,\quad \mathbf{Y}=\{y_1,y_2,y_3,y_4\},$$

we define

$$\mathbf{Y}_1(z=0)=(\varepsilon,0,0,0),\quad \mathbf{Y}_2(z=0)=(0,\varepsilon,0,0),$$
$$\mathbf{Y}_3(z=0)=(0,0,\varepsilon,0),\quad \mathbf{Y}_4(z=0)=(0,0,0,\varepsilon),$$

where ε const $\neq 0$. Then the general solution of the system (4) $\sum_{i=1}^{4} c_i\mathbf{Y}_i$ does not depend on the choice of ε. In order to satisfy (5), we set $c_1=c_3=0$ and obtain a homogeneous set of linear equations for c_2 and c_4:

$$c_2\tilde{Y}_{21}+c_4\tilde{Y}_{41}=0,\quad c_2\tilde{Y}_{23}+c_4\tilde{Y}_{43}=0, \tag{6}$$

where $\tilde{Y}_{ij}$ is the j-th component of the vector solution $\tilde{\mathbf{Y}}_i$ at $z=L$. The nonzero values of the coefficients c_2 and c_4 are obtained from the condition

$$\tilde{Y}_{21}\tilde{Y}_{43}-\tilde{Y}_{23}\tilde{Y}_{41}=0. \tag{7}$$

Equation (7) has an infinite set of solutions in the form of the $U(k)$ dependences. The instability threshold U_c and the corresponding deformation wave vector k_c are determined from the lower branch of the solutions $U(k)$

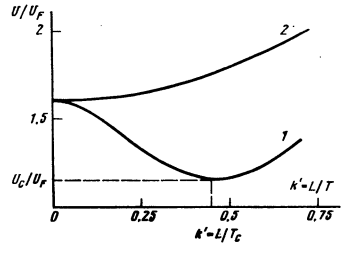

FIG. 3. Lower branch of the solution $U(k)$ of Eq. (7): the results of a numerical calculation: planar orientation, a $2L/p_0=1.25$ (first Grandjean band): curve 1—$\alpha=0$ (longitudinal domains), curve 2—$\alpha=\pi/2$ (transverse domains).

(see Fig. 3). The perturbations corresponding to the upper branches of the solutions $U(k)$ can arise only in the case in which the distortions of the orientation as a consequence of the domain structure remain small upon increase in the voltage above the threshold, which is determined by the lower branch of the solutions. The values of the orientation perturbations φ and θ at each point $U(k)$ are determined to within a constant factor. Calculation shows that the very lowest branch of the solutions (7) corresponds to the functions $\varphi(z)$ and $\theta(z)$ with a single maximum at $z=L/2$, while $\varphi>\theta$ (Fig. 4). For the next higher branch of the solutions $U(k)$, the $\varphi(z)$ and $\theta(z)$ dependences have a more complicated form (Fig. 4).

The values of the threshold voltage U_c and the deformation vector of the field instability of planar texture

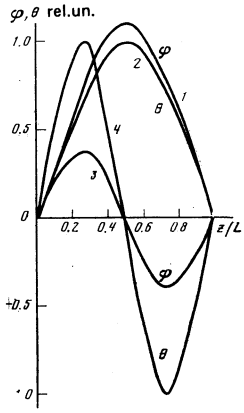

FIG. 4. Form of the perturbations $\varphi(z)$ (curves 1 and 4) and $\theta(z)$ (curves 2 and 3) for the solutions U(k) of Eq. (7), the results of numerical calculation; planar orientation, $2L/p_0=1.25$, $\alpha=0$. Curves 1 and 2 are the lower branch of the solutions $U(k)$, $U_c=1.16\ U_F$, $k_c'=L/T_c=0.45$; curves 3 and 4 are the upper (second) branch of the solutions $U(k)$, $U_c'=2.4\ U_F$.

CLC corresponding to it, were calculated by the above method for the first four Grandjean bands ($L \sim p_0$) and the two initial orientations of the liquid crystal in the wedge-shaped cell: planar (Fig. 1) and twist (Fig. 2). In the first case here, the calculation was carried out for longitudinal ($\alpha = 0$) and transverse ($\alpha = \pi/2$) domains, while in the second case, it was carried out also for domains located at an angle of 45° to the initial orientation of the director of the substrates ($\alpha = \pi/2$ and $\alpha = 3\pi/4$).[1)] The results of the calculations are given in Figs. 5 and 6 and discussed below.

Equations (4) and (5) allow an approximate analytic solution under the condition $L/p_0 \gg 1$. Following Hurault,[3] we make the following changes in (4):

$$\theta \to \theta' \cos\psi, \quad k_x \to k, \quad k_y \to 0,$$

and we seek the values of θ' and φ to within terms proportional to $\cos 4\psi$ and $\sin 4\psi$ in the form

$$\begin{aligned} \theta' &= (\theta_0 + \theta_1 \cos 2\psi + \theta_2 \sin 2\psi) e^{iqz}, \\ \varphi &= (\varphi_0 + \varphi_1 \cos 2\psi + \varphi_2 \sin 2\psi) e^{iqz}, \end{aligned} \tag{8}$$

where $q = \pi/L$ is the z-coordinate deformation wave vector, such that $q \ll k \ll t$.

Substituting the expressions (8) for θ' and φ in the equations obtained from (4), and equating the coefficients in zeroth and first order of the harmonic expansion, we obtain six relations connecting the amplitudes θ_0, θ_1, θ_2, φ_0, φ_1 and φ_2. The asymptotic expressions

$$\begin{aligned} &\varphi_0 \approx t\theta_0/k, \quad \varphi_1 \approx k\theta_0/4t, \quad \varphi_2 \sim kq\theta_0/t^2 \ll \varphi_1, \\ &\theta_1 \sim k^2\theta_0/t^2, \quad \theta_2 \sim q\theta_0/t \sim \theta_1, \end{aligned} \tag{9}$$

follow from these relations and the condition $q \ll k \ll t$. The last three of these dependences are accurate to within a numerical coefficient of the order of unity.

Expressing the amplitudes φ_0, φ_1, φ_2, θ_1 and θ_2 in terms of θ_0 with the help of (9), and substituting the results in the initial relations, we obtain the condition

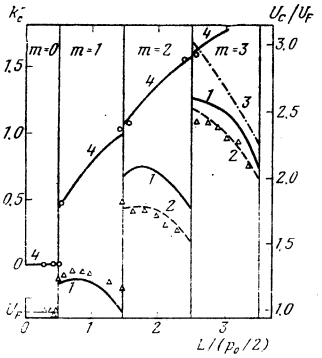

FIG. 5. Dependence of the instability threshold U_c (in relative units U_c/U_F) and the deformation wave vector $k_c' = L/T_c$ on the ratio $L/(p_0/2)$ for Grandjean bands with small number ($m < 4$); planar orientation, $p_0 = 30\ \mu$. Curve 1—numerical calculation of U_c for longitudinal domains ($\alpha = 0$); curve 2—numerical calculation of U_c for transverse domains ($\alpha = \pi/2$); curve 3—calculation of U_c by Eq. (12); curve 4—calculation of k_c'. Experimental values: △—U_c, ○—k_c'.

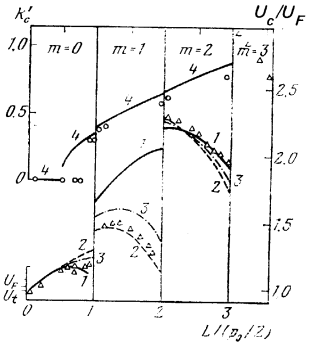

FIG. 6. Dependence of the instability threshold U_c (in relative units U_c/U_F) and the deformation wave vector $k_c' = L/T_c$ on the ratio $L/(p_0/2)$ for Grandjean bands with small number ($m < 4$); twist orientation, $p_0 = 30\mu$. Curve 1—numerical calculation of U_c for domains with $\alpha = \pi/4$; curve 2—numerical calculation of U_c for domains with $\alpha = 3\pi/4$; curve 3—numerical calculation of U_c for domains with $\alpha = 0, \pi/2$; curve 4—calculation of k_c'. Experimental values: △—U_c, ○—k_c'.

for the existence of a nontrivial solution for θ_0, written down with accuracy to within terms of higher order of the expansion in powers of the ratios k/t and q/t:

$$\frac{3}{8} K_{33} k^2 + \frac{K_{22} q^2 t^2}{k^2} + K_{22} t \Delta t = \frac{\Delta\varepsilon U^2}{8\pi L^2}. \tag{10}$$

The left side of (10) represents the elastic energy of the CLC, which is a function of the deformation wave vector k and has a minimum value at $k_c^2 = ((8/3)K_{22}/K_{33})^{1/2} tq$. With account of the relations $q = \pi/L$, $t = 2\pi/p$, $\Delta t = 2\pi(1/p - 1/p_0)$, we obtain the following expressions for the period of the deformation and the threshold voltage of the instability under the condition $L/p_0 \gg 1$:

$$T_c^2 = (3K_{33}/2K_{22})^{1/2} pL, \tag{11}$$

$$U_c^2 = \frac{8\pi^3}{\Delta\varepsilon}\left[(6K_{22}K_{33})^{1/2} + 4K_{22}\left(\frac{L}{p} - \frac{L}{p_0}\right)\right]\frac{L}{p}. \tag{12}$$

Estimates show that for the Grandjean band with large number ($m > 4$) the approximate solution of (11) and (12) gives an error no greater than 5–7% relative to the exact value.

Equations (11) and (12), in contrast with (2) and (3) obtained in Ref. 3, take into account the nonequilibrium pitch of the cholesteric helix p, which is equal to p_0 only at the center of the corresponding Grandjean bands. A work has recently been published[7] in which Eqs. (11) and (12) were obtained by use of a large-scale variant of the continual theory of CLC.[1] The formula for the threshold with account of the induced helical pitch, proposed in Ref. 8, does not, in contrast with (12) reduce to (3) at $p = p_0$ and is in worse agreement with experiment.

3. METHOD OF EXPERIMENT

The experimental investigations were carried out with nematic–cholesteric mixtures on a base of MBBA (4-methoxybenzyldiene-4′-n′-butylaniline). The positive value of the dielectric anisotropy of the nematic omponent ($\Delta\varepsilon = +0.74$) was guaranteed by the addition to the MBBA of 5% n'-cyanophenyl ether n-heptyl benzoic acid. The value of the elastic modulus K_{11} for this mixture, determined from the Frederiks transition thres-

hold U_F, is equal to 5.6×10^{-7} dyn, which is close to the value K_{11} for pure MBBA.[1] Upon addition of 0.25 and 1% of cholesteryl caprinate to the nematic component, a nematic cholesteric mixture with half pitches ($p_0/2$) corresponding to 30 and 6 μ respectively, determined by the wedge method.[6] The first mixture was used in the study of the field instability in Grandjean bands with small number ($m=0$–4), and the second, those with large number ($m=4$–12). Use of the mixtures on the base of MBBA with small additions of other liquid crystals enabled us to compare the experimental results with the results of calculations in which the ratios between elastic constants that are characteristic for pure MBBA ($K_{22}/K_{11}=0.67$, $K_{33}/K_{11}=1.25$, Ref. 1) were used.

Measurements of the instability threshold and the period of the deformation were carried out on wedge-shaped cells, the assembly and control of which are described in Ref. 6. The frequency of the electric field was 1–2 kHz, which assured the cutoff of electrohydrodynamic effects connected with the flow of current.[6] The threshold potential U_c of the field instability for the first four Grandjean bands were determined from the dependence of the phase lag on the potential in the irradiation of the cell by linearly polarized monochromatic light[9] (laser beam $\lambda=633$ nm) and was identical with the U_c determined from the appearance of the diffraction picture on the domains. For Grandjean bands with numbers from 4 to 12, U_c was determined with a microscope according to the appearance of a domain picture. The period of the deformation T_c for all the Grandjean bands is determined with the help of a polarization microscope. The accuracy of the measurement of U_c amounted to 5% for the first four Grandjean bands and 10% for bands with a larger number, the accuracy of determination of the period of deformation T_c amounted to 10–15%.

The experimental dependences of the threshold of the field instability U_c and the deformation period T_c on the ratio $L/(p_0/2)$ (upon increase in L, find ourselves in various portions of the Grandjean bands, beginning with the zeroth) are shown in Figs. 5–7.

4. DISCUSSION OF THE RESULTS

Grandjean bands with small number (m = 0 - 3). Planar orientation

Numerical calculation of the field instability in the initial planar orientation of the liquid crystal (Fig. 1)

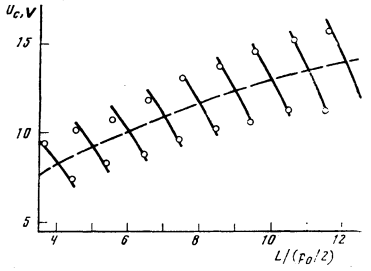

FIG. 7. Values of U_c for Grandjean bands with large numbers ($m>4$), calculated from Eq. (12) (solid lines) and experimentally obtained (○) planar orientation, $p_0=6$ μ. The dashed curve corresponds to calculation from the formula of Helfrich-Hurault (2).

was carried out by the above method for two possible directions of the domains: along the direction of the initial orientation of the director ($\alpha=0$, longitudinal domains) and perpendicular to it ($\alpha=\pi/2$, transverse domains). Just such domains are observed experimentally in the first Grandjean bands: longitudinal and transverse and, beginning with the fourth band, a network is observed which can be regarded as the superpostion of both systems of domains (Fig. 1). We shall consider systematically each Grandjean band, beginning with the zeroth.

In the zeroth band ($0<2L/p_0<\frac{1}{2}$) the CLC is completely "untwisted" by the walls of the cell. Theory predicts the absence of a domain structure ($k=0$) and the independence of the threshold on the thickness, while the quantity U is equal to the quantity U_F, i.e., the threshold of the Freedericsz transition in CLC. The experimental results completely correspond with the theoretical Figs. 1 and 5). When a voltage higher than threshold in the right side of the zeroth band is applied, a nonstationary, periodic structure is formed (longitudinal domains), which disappears rapidly (within 1–2 sec), testifying to the more complicated character of the CLC deformation in comparison with NLC, in spite of the equality of thresholds ($U_c=U_F$).

In the first Grandjean band, theory predicts the appearance only of longitudinal domains (see the dependences $U(k)$ in Fig. 3), and the dependence of U_c on L has a sharply expressed maximum to the left of center of the band, which is also confirmed experimentally (Fig. 5). We note that in a narrow region of the first band, which borders on the zeroth), deformation without formation of domains is observed experimentally ($k=0$) (Fig. 1).

In the second and third Grandjean bands, according to the theory, the appearance of both longitudinal and transverse domains is possible but the threshold for the latter is much lower. Transverse domains are observed experimentally in these bands (Fig. 1 and 5). Curve 4 in Fig. 5 in the third Grandjean band corresponds to calculation of U_c from Eq. (12) and lies considerably above the experimental values of U_c, and the threshold, calculated from the exact theory, and this divergence is still more significant for the second and first zones.

The calculated and experimentally obtained values of the deformation period T_c for the first four Grandjean bands are in excellent agreement (Fig. 5),

Grandjean band with small number (m = 0 - 3). Twist orientation

In the case of an initial twist orientation, numerical calculation of the instability threshold has been carried out for the following possible directions of the domains: $\alpha=\pi/4$, $\alpha=3\pi/4$, and also $\alpha=0$ and $\alpha=\pi/2$. Domains have been observed experimentally in the first three Grandjean bands with $\alpha=\pi/4$ and $\alpha=3\pi/4$ (Fig. 2). As in the previous case, we consider each Grandjean band systematically.

In the zeroth band, four turns of the cholesteric helix fit between the electrodes of the cell, and the initial

structure is analogous to the twist structure of NLC. According to the exact theory, as $L/p_0 \to 0$,

$$U_c(L\to 0)=\pi\left\{\frac{4\pi}{\Delta\varepsilon}\left[K_{11}+\frac{1}{4}(K_{33}-2K_{22})\right]\right\}^{1/2},$$

i.e., U_c corresponds to the threshold of deformation of the twist structure in NLC[1]

$$U_t=U_F\left(1+\frac{K_{33}-2K_{22}}{4K_{11}}\right)^{1/2}.$$

The experimentally observed threshold of deformation in the twist cell with NLC (without the cholesteric component) does not depend on L and corresponds to the calculated value of U_t. At $0<2L/p_0<\frac{1}{2}$ (the left half of the zeroth zone) the instability threshold U_c increases according to theory (Fig. 6); here $k=0$ (there are no domains). In the right half of the zeroth ($2L/p_0>\frac{1}{2}$) the instability appears in the form of domains (a remarkable fact! —dielectric domains in a nonperiodic structure[10]); here the threshold of domains with different directions at $2L/p_0=\frac{1}{2}$ coincide and upon increase in L, the smaller values of U_c correspond to domains with $\alpha=\pi/4$. The experimentally observed situation (Figs. 2 and 6) corresponds to theory with only this difference that the domains ($\alpha=\pi/4$) in the zeroth band are observed upon increase of L not from the middle of the band but close to the boundary with the first band. Upon increase in the potential ($U\approx(2-3)U_c$), the domains are transformed into a region with a different angle of twist of the director, separated by disclinations, although the initial structure of the liquid crystal in the absence of the field is uniform.

In the first Grandjean band, according to the theory, the smaller values of the threshold correspond to domains with $\alpha=3\pi/4$, which are also observed experimentally (Figs. 2 and 6). The threshold of domains with $\alpha=\pi/4$ lies significantly above and the threshold of domains with $\alpha=0$ ($\pi/2$) has an intermediate value (Fig. 6) – these domains are not observed experimentally.

In the second Grandjean band, the theoretical values of the threshold for domains with different directions are very close and at a thickness $L=2.25$ ($p_0/2$) are identical, and only on the edge of the band near the boundary with the third band is the threshold of domains with $\alpha=3\pi/4$ significantly smaller than the U_c for the other domains (Fig. 6). The experimental values of the threshold of the field instability are close to the theoretical, while on the right edge of the band domains are observed with $\alpha=3\pi/4$, and there is a network in the remaining portion of the band (Fig. 2). On the boundary with the second zone (in the left side of the third band) domains are observed with $\alpha=\pi/4$, distorted by the superposition of a network, as expected from theory (Figs. 2 and 6).

The experimental values of the deformation period are close to those calculated (Fig. 6) with the exception of the already discussed singularities in the zeroth Grandjean band.

Grandjean bands with large numbers (m = 4 - 12)

Beginning with the fourth band and up to the twelfth, the experimental values of the threshold of the field instability were comparable with those calculated from Eq. (12), since it can be assumed that $L \gg p_0$ for these bands. The initial orientation (planar of twist) does not play an important role here. Excellent agreement is observed between the experimental and theoretical values of U_c (Fig. 7). We turn our attention to the characteristic oscillating form of the dependence of U_c on L/p_0; the swing of the oscillations does not decrease upon increase in L. For example, in the limits of the eleventh Grandjean band, in the case of a thickness of 66 μ in the middle of the band, a change in thickness by 6 μ leads to a decrease in the threshold potential to 4 V. Upon transition from band to band, U_c changes jumpwise. Qualitatively, this is explained by the fact that although the difference of the nonequilibrium pitch of the cholesteric helix p from the equilibrium p_0 decreases with increase in the number of the band (Figs. 1 and 2), the total change in the elastic energy associated with the inequality of p and p_0 remains significant, since small changes in the elastic energy for a single half-turn of the helix in Grandjean bands with large number add up over all the half turns that fit into a given thickness of the cell L.

The experimental values of the deformation period, which has the form of a grid, exceeds the values of T_c calculated from Eq. (11) by about 10–20%. However, we note that the accuracy of determination of T_c decreases with increase in L.

In conclustion, the authors thank L. M. Blinov for useful discussions and G. G. Slashchev for help in the research.

[1)]For domains with another direction ($\alpha \neq 0$, $\pi/4 \cdot \pi/2$, $3\pi/4$), the calculation gives intermediate values of U_c; such domains are not observed experimentally.

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Translated by R. T. Beyer