[4]D. L. Lin and M. Y. Chen, Nucl. Phys. A **251**, 457 (1975).
[5]A. N. Moskalev, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 229 (1974) [JETP Lett. **19**, 141 (1974)].
[6]A. N. Moskalev, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 394 (1974) [JETP Lett. **19**, 216 (1974)].
[7]A. N. Moskalev and R. M. Ryndin, Yad. Fiz. **22**, 147 (1975) [Sov. J. Nucl. Phys. **22**, 71 (1975)].
[8]V. G. Gorshkov, A. I. Mikhaĭlov, and A. N. Moskalev, Zh. Eksp. Teor. Fiz. **69**, 1507 (1975) [Sov. Phys. JETP **42**, 769 (1975)].
[9]D. P. Grechukhin and A. A. Soldatov, Zh. Eksp. Teor. Fiz. **73**, 31 (1977) [Sov. Phys. JETP **46**, 15 (1977)].
[10]D. P. Grechukhin and A. A. Soldatov, Zh. Eksp. Teor. Fiz. **72**, 390 (1977) [Sov. Phys. JETP **45**, 205 (1977)].
[11]G. Backenstoss *et al.*, Phys. Lett. **B25**, 365, 547 (1967).
[12]H. A. Jahn, Proc. R. Soc. London Ser. A **205**, 192 (1951).
[13]I. M. Band, M. A. Listengarten, M. B. Trzhasnovskaya, and V. I. Fomichev, Preprint 289, Leningrad Inst. Nucl. Phys., 1976.

Translated by J. G. Adashko

Mean electromagnetic field in a randomly inhomogeneous medium

Yu. A. Ryzhov

Gor'kiĭ Radiophysics Research Institute
(Submitted 17 April 1979)
Zh. Eksp. Teor. Fiz. **77**, 2179–2185 (December 1979)

The mean electromagnetic field in a medium with large random fluctuations of the permittivity is considered. The problem of finding the mean field is equivalent to the determination of the effective permittivity of such a medium. This latter has been sufficiently well studied in the case of weak fluctuations, i.e., at $\sigma_\varepsilon/\langle\varepsilon\rangle \ll 1 (\sigma_\varepsilon = \langle \varepsilon - \langle\varepsilon\rangle)^2\rangle^{1/2}$ and $\langle\varepsilon\rangle$ is the mean permittivity of the medium. The limiting case $\langle\varepsilon\rangle \to 0$ corresponding to the case of large relative fluctuations, is studied. As an illustration, we have considered the problem of the effective permittivity of a cold plasma with fluctuations of the electron density.

PACS numbers: 77.20. + y, 52.25.Mq

1. INTRODUCTION

One of the fundamental problems of electrodynamics of randomly inhomogeneous media is the problem of finding the regular component (mean value) of the field of sources immersed in such a medium. In linear electrodynamics, this problem reduces to the calculation of the effective permittivity of the random medium. This permittivity of a randomly inhomogeneous medium is determined from the relation between the mean values of the electric field and the induction:

$$\langle D_i\rangle = \hat{\varepsilon}_{ij}^{eff}\langle E_j\rangle. \tag{1}$$

Averaging is carried out over the ensemble of realizations of the random medium.

In an unbounded statistically homogeneous and an isotropic medium, the operator $\hat{\varepsilon}_{ij}^{eff}$ for harmonic fields ($e^{-i\omega t}$) is a linear integral operator with a difference kernel:

$$\langle D_i(\mathbf{r})\rangle = \int \varepsilon_{ij}^{eff}(\mathbf{r}-\mathbf{r}_1)\langle E_j(\mathbf{r}_1)\rangle d\mathbf{r}_1. \tag{2}$$

The Fourier transform of the kernel $\varepsilon_{ij}^{eff}(\mathbf{r}, \omega)$

$$\varepsilon_{ij}^{eff}(\omega, \mathbf{k}) = \int \varepsilon_{ij}(\mathbf{r}, \omega) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r} \tag{3}$$

has the form

$$\varepsilon_{ij}^{eff}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega, k) + \frac{k_i k_j}{k^2}\varepsilon^{l}(\omega, k). \tag{4}$$

The problem of the calculation of $\varepsilon_{ij}^{eff}(\omega, k)$ was considered earlier in the theory of permittivity of gases[1,3] and the electrostatics of mixtures.[2,3] Similar problems are widely discussed at the present time in the theory of percolation and phase transitions.[4,5] In the theory of wave propagation in randomly inhomogeneous media, the effective permittivity has been considered in Refs. 6 and 7. In particular, for the case of a weakly inhomogeneous dielectric ($\sigma^2/\varepsilon_0^2 \ll 1$, $\sigma_\varepsilon^2 = \langle(\varepsilon-\varepsilon_0)^2\rangle$, $\langle\varepsilon\rangle = \varepsilon_0$, $\varepsilon(r) = \varepsilon_0 + \Delta\varepsilon(r)$) formulas were obtained in Ref. 7 for $\varepsilon^{eff}(\omega, 0)$ (without account of spatial dispersion due to the inhomogeneity of the medium). More general expressions for $\varepsilon^{eff}(\omega, \mathbf{k})$ were obtained in Refs. 8 and 9 by the self-consistent field method. In particular, the case of large fluctuations ($\sigma^2/\varepsilon_0^2 \gg 1$) was considered in Ref. 9. This case is physically very interesting, since it describes real situations that arise in the electrodynamics of strongly inhomogeneous mixtures and so on. In particular, an important example is an inhomogeneous plasma with macroscopic fluctuations of the electron density in which $\varepsilon_0(\omega) \to 0$ and $\sigma_\varepsilon/\varepsilon_0 \to \infty$ at the Langmuir frequency.

The following approximate expression was obtained earlier[9] for an inhomogeneous cold plasma:

$$\varepsilon^{eff}(\omega, k=0) = 0.56\varepsilon_0(\omega) + 0.5i\sigma_N/\langle N\rangle, \tag{5}$$

where $\sigma_N^2 = \langle\Delta N^2\rangle$ is the variance of the electron density fluctuations, which is assumed to be small ($\sigma_N/\langle N\rangle \ll 1$);

$$\varepsilon_0(\omega) = 1 - \omega_0^2/\omega^2, \quad \omega_0^2 = 4\pi e^2\langle N\rangle/m.$$

 0038-5646/79/121046-04$02.40

Equation (5) was obtained at $\varepsilon_0 \ll \sigma_N/\langle N\rangle$ (at frequencies close to the Langmuir frequency). It correctly reflects the physical picture of the behavior of the average field at resonance frequencies. The imaginary part of $\varepsilon^{eff}(\omega)$ describes in the given case the process of transformation of the energy of the regular (average) component of the field into the energy of the random plasma oscillations (the dispersion equation of which has the form $\varepsilon_0(\omega)=0$ in a cold plasma).

Far from the Langmuir frequency, the role of scattering in the plasma oscillations is small and the imaginary part of $\varepsilon_{ij}^{eff}(\omega, \mathbf{k})$ is determined by processes of scattering into electromagnetic waves.[7]

Equation (5) was obtained by the self-consistent field method. It is known that the latter gives the approximate value of the quasistatic permittivity $\varepsilon^{eff}(\omega, k=0)$ (or the effective conductivity), agreeing rather well with the experimental data in one-dimensional and two-dimensional inhomogeneous media. For three-dimensional inhomogeneities, some divergence from experiment is noted.[4] The situation is made worse by the fact that it is theoretically very difficult to estimate the degree of inaccuracy of Eq. (5) and to corroborate the self-consistent field method.

In the present work, we have obtained the value of $\varepsilon_{ij}^{eff}(\omega, \mathbf{k})$ of an arbitrary randomly inhomogeneous medium at $\varepsilon_0(\omega)\to 0$ by means of a different method. As an illustration, we consider a cold plasma with fluctuations of the electron density and find $\varepsilon_{ij}^{eff}(\omega_0, \mathbf{k})$ at the Langmuir frequency ω_0.

2. FUNDAMENTAL RELATIONS

We write down the vector wave equation in an arbitrary randomly inhomogeneous medium ($E \sim e^{-i\omega t}$);

$$\Delta E_i - \frac{\partial^2 E_i}{\partial x\,\partial x} + k_0^2\varepsilon(\omega, \mathbf{r}) E_i = -\frac{4\pi i\omega}{c^2} j_i, \tag{6}$$

where j_i is the current density of outside regular sources immersed in an inhomogeneous medium with dielectric permittivity $\varepsilon(\omega, \mathbf{r}) = \varepsilon_0(\omega) + \xi(\mathbf{r}, \omega)$ ($\varepsilon_0(\omega) = \langle\varepsilon(\mathbf{r}, \omega)\rangle$, $\xi(\mathbf{r}, \omega)$ is a random function of the point. We shall assume the random field to be statistically homogeneous and isotropic. We also represent the electric field E_i in the form of the sum of the mean (E_i^0) and scattered (e_i) fields:

$$E_i = \langle E_i\rangle + e_i = E_i^0 + e_i. \tag{7}$$

Averaging is carried out over the ensemble of realizations of the random field $\xi(\mathbf{r}, \omega)$.

We substitute (7) in Eq. (6) and carry out the averaging:

$$\Delta E_i^0 - \frac{\partial^2 E_j^0}{\partial x_i\,\partial x_j} + k_0^2(\varepsilon_0 E_i^0 + \langle\xi e_i\rangle) = -\frac{4\pi i\omega}{c^2} j_i. \tag{8}$$

We subtract this equation from (6):

$$\Delta e_i - \frac{\partial^2 e_j}{\partial x_i\,\partial x_j} + k_0^2\varepsilon_0 e_i = -k_0^2\xi E_i^0 - k_0^2(\xi e_i - \langle\xi e_i\rangle). \tag{9}$$

The resultant equation is of course not closed relative to the random field e_i but this is not required for us to find $\varepsilon^{eff}(\omega, \mathbf{k})$. It is only sufficient to express the scattered field in terms of the field E_i^0, considering the latter as an external source.

For what follows, it is convenient to write (9) in the form of an integral equation

$$e_i = -k_0^2 \int G_{ij}^0(\mathbf{r}-\mathbf{r}_1)\xi(\mathbf{r}_1)E_j^0(\mathbf{r}_1)\,d\mathbf{r}_1 - k_0^2\int G_{ij}^0(\mathbf{r}-\mathbf{r}_1)(\xi e_j - \langle\xi e_j\rangle)\,d\mathbf{r}_1, \tag{10}$$

$$G_{ij}^0(\mathbf{r}) = -\left(\delta_{ij} + \frac{1}{k_0^2\varepsilon_0}\frac{\partial^2}{\partial x_i\,\partial x_j}\right)\frac{\exp(ik\varepsilon_0^{1/2} r)}{4\pi r}. \tag{11}$$

The merit of representing the singular Green's function $G_{ij}^0(r, r_1)$ in the form of the sum of two singular functions (isolating the singularity associated with the δ function) was noted in the work of Finkel'berg:[8]

$$G_{ij}^0(\mathbf{r}, \mathbf{r}_1) = \frac{1}{3k_0^2\varepsilon_0}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}_1) + G_{ij}^{01}(\mathbf{r}, \mathbf{r}_1), \tag{12}$$
$$G_{ij}^{01}(\mathbf{r}, \mathbf{r}_1) = \mathrm{PV}\,G_{ij}^0(\mathbf{r}, \mathbf{r}_1),$$

where the symbol PV indicates integration in the sense of the principal value. The function G_{ij}^{01} does not contain the δ singularity.

Substituting (12) in Eq. (10), we obtain the equation

$$f_i = -\frac{1}{3\varepsilon_0}\xi E_i^0 + \left\langle\frac{\xi f_i}{\xi+3\varepsilon_0}\right\rangle + \int G_{ij}(\mathbf{r}, \mathbf{r}_1)\xi(\mathbf{r}_1)E_j^0(\mathbf{r}_1)\,d\mathbf{r}_1$$
$$+3\varepsilon_0\int G_{ij}(\mathbf{r}, \mathbf{r}_1)\left[\frac{\xi(\mathbf{r}_1)f_j(\mathbf{r}_1)}{\xi(\mathbf{r}_1)+3\varepsilon_0} - \left\langle\frac{\xi(\mathbf{r}_1)f_j(\mathbf{r}_1)}{\xi(\mathbf{r}_1)+3\varepsilon_0}\right\rangle\right]d\mathbf{r}_1, \tag{13}$$

where $G_{ij} = -k_0^2 G_{ij}^{01}$, $f = e_i(1+\xi/3\varepsilon_0)$. We have transformed to a new variable f_i, the mean value of which also represents a quantity of interest to us. In fact, by definition,

$$\langle D_i\rangle = \hat{\varepsilon}_{ij}^{eff} E_i^0 = \langle\varepsilon E_i\rangle = \varepsilon_0 E_i^0 + \langle\xi e_i\rangle, \tag{14}$$
$$\langle\xi e_i\rangle = 3\varepsilon_0\langle f_i\rangle. \tag{15}$$

We note that the following important relations hold:

$$\langle f_i\rangle = \langle\xi f_i/(\xi+3\varepsilon_0)\rangle, \quad \langle f_i/(\xi+3\varepsilon_0)\rangle = 0. \tag{16}$$

The second of these relations is a consequence of the first, while the first follows from (13).

We now proceed in the following fashion. We multiply Eq. (13) by $(\xi+3\varepsilon_0)^{-1}$ and average. With account of (16), we get

$$3\varepsilon_0\alpha\langle f_i\rangle = \beta E_i^0 - 3\varepsilon_0\int G_{ij}(\mathbf{r}, \mathbf{r}_1)\left\langle\frac{\xi(\mathbf{r}_1)}{\xi(\mathbf{r})+3\varepsilon_0}\right\rangle E_j^0\,d\mathbf{r}_1$$
$$-9\varepsilon_0^2\int G_{ij}(\mathbf{r}, \mathbf{r}_1)\left[\left\langle\frac{\xi(\mathbf{r}_1)f_j(\mathbf{r}_1)}{(\xi(\mathbf{r}_1)+3\varepsilon_0)(\xi(\mathbf{r})+3\varepsilon_0)}\right\rangle - \alpha\langle f_j(\mathbf{r}_1)\rangle\right]d\mathbf{r}_1, \tag{17}$$

where

$$\alpha = \left\langle\frac{1}{\xi+3\varepsilon_0}\right\rangle, \quad \beta = \left\langle\frac{\xi}{\xi+3\varepsilon_0}\right\rangle = 1 - 3\varepsilon_0\alpha,$$

$\varepsilon_0 = \varepsilon_0(\omega) + i0$ is the limiting value of the mean permittivity of the equilibrium medium at vanishingly small absorption. The corresponding limiting transition determines the circuiting rule in calculation of quantities of the type α.

The expression (17) can be transformed to the form

$$3\varepsilon_0\alpha\langle f_i\rangle=\beta E_i^0+\int \tilde{G}_{ij}(\mathbf{r},\mathbf{r}_1)R_1(\mathbf{r},\mathbf{r}_1)E_j^0(\mathbf{r}_1)d\mathbf{r}_1$$
$$-3\varepsilon_0\int \tilde{G}_{ij}(\mathbf{r},\mathbf{r}_1)\left[\left\langle\frac{\xi f_{j1}}{(\xi+3\varepsilon_0)(\xi_1+3\varepsilon_0)}\right\rangle+\left\langle\frac{f_{j1}}{\xi+3\xi_0}\right\rangle-\alpha\langle f_{j1}\rangle\right]d\mathbf{r}_1, \quad (18)$$

where

$$\tilde{G}_{ij}=3\varepsilon_0 G_{ij},\ \xi=\xi(\mathbf{r})\ \xi_1=\xi(\mathbf{r}_1),\ f_{j1}=f_j(\mathbf{r}_1),$$
$$R_1(\mathbf{r},\mathbf{r}_1)=-\langle\xi_1/(\xi+3\varepsilon_0)\rangle.$$

3. EFFECTIVE PERMITTIVITY OF RANDOMLY INHOMOGENEOUS MEDIA AS $\epsilon_0(\omega)\to 0$

In a plasma or plasma-like media, the average permittivity can vanish at a fixed frequency ω_0. As has already been pointed out above, the plasma oscillations that develop in the scattering process determine the principal part of the energy losses of an arbitrary regular electromagnetic field. A statistical transformation of the mean field energy into the energy of random plasma oscillations (or waves) results. The closer the frequency of the field is to the resonance frequency of the plasma, the more significant is the contribution of the scattering processes to the plasma mode. The contribution of the scattering to the electromagnetic wave can be neglected in this case.

In order to understand the results of the limiting transition $\varepsilon_0(\omega)\to 0$ in Eq. (18), we note that the quantity $\langle\xi e_i\rangle=3\varepsilon_0\langle f_i\rangle$ is finite as $\varepsilon_0\to 0$. This follows from the results obtained by the effective field method (see (5) *I*). If $\lim\langle\xi e_i\rangle=F_i$ as $\varepsilon_0\to 0$, we then have the following expansion for $\langle f_i\rangle$ near the point $\varepsilon_0=0$:

$$\langle f_i\rangle=F_i/3\varepsilon_0+c_0+c_1\varepsilon_0+\dots\ . \quad (19)$$

We now consider the moment

$$B_1(\mathbf{r},\mathbf{r}_1)=\langle\xi f_{j1}/(\xi+3\varepsilon_0)(\xi_1+3\varepsilon_0)\rangle. \quad (20)$$

By virtue of the second of the relations (16), which is an expression for the condition $\langle e_i\rangle=0$, we have

$$\lim B_1(\mathbf{r},\mathbf{r}_1)=0,\ \varepsilon_0\to 0.$$

It is somewhat more complicated to treat with the moment

$$B_2(\mathbf{r},\mathbf{r}_1)=\langle f_{j1}/(\xi+3\varepsilon_0)\rangle. \quad (21)$$

The function B_2 can be regarded as the correlation functions of the processes $f_j(\mathbf{r})$ and $(\xi+3\varepsilon_0)^{-1}$. The process f_j depends on the scattered field, the correlation of which with ξ at the given point cannot be propagated over distances exceeding the quantity

$$L=v_{gr}\tau,$$

where τ is the characteristic time of exchange of realizations of the field $\xi(\mathbf{r})$, and v_{gr} is the group velocity of electromagnetic waves in the medium, determining the rate of rearrangement of the structure of the scattered field upon exchange of the realizations. For a plasma,

$$L=c\varepsilon_0^{1/2}\tau,$$

where c is the speed of light.

Thus, as $\varepsilon_0\to 0$, we have $L\to 0$ and the fields f_j and $(\xi+3\varepsilon_0)^{-1}$ depend statistically on one another only in the immediate vicinity of the argument $|\mathbf{r}=\mathbf{r}_1|$. But the Green's function $\tilde{G}(\mathbf{r},\mathbf{r}_1)$ in the integrals (18) includes by definition the immediate vicinity of the point r (integration in the sense of principal value). We then come to the conclusion that in the region that is important for the integration,

$$B_2(\mathbf{r},\mathbf{r}_1)=\alpha\langle f_{j1}\rangle \quad (22)$$

and consequently,

$$\lim 3\varepsilon_0\int \tilde{G}_{ij}\left[\left\langle\frac{f_{j1}}{\xi+3\varepsilon_0}\right\rangle-\alpha\langle f_{j1}\rangle\right]d\mathbf{r}_1=0, \quad \text{for} \quad \varepsilon_0\to 0. \quad (23)$$

We introduce the following notation:

$$\lim\tilde{G}_{ij}=\tilde{G}_{ij}^0,\quad \lim\alpha=\alpha_0,$$
$$\lim R_1(\mathbf{r},\mathbf{r}_1)=-\langle\xi_1/\xi\rangle=R(\mathbf{r},\mathbf{r}_1)=R(\mathbf{r}-\mathbf{r}_1) \quad (24)$$
$$\text{for}\ \varepsilon_0\to 0.$$

Carrying out the limiting transition $\varepsilon_0\to 0$, we obtain (with account of (23) and (24)), the relation

$$F_i=\frac{1}{\alpha_0}E_i^0+\frac{1}{\alpha_0}\int \tilde{G}_{ij}^0(\mathbf{r},\mathbf{r}_1)R(\mathbf{r},\mathbf{r}_1)E_j^0(\mathbf{r}_1)d\mathbf{r}_1, \quad (25)$$

which represents the desired linear connection between the vector $\langle D_i\rangle=\langle\xi e_i\rangle$ and the mean field E_j^0.

The expression for $\varepsilon_{ij}^{eff}(\omega,\mathbf{k})$ at the frequency ω_0, which follows from (25) and which is a root of the equation $\varepsilon_0(\omega)=0$:

$$\varepsilon_{ij}^{eff}(\omega_0,\mathbf{k})=\frac{1}{\alpha_0}\delta_{ij}+\frac{1}{\alpha_0}L_{ij}(\omega_0,\mathbf{k}), \quad (26)$$

$$L_{ij}=\int \tilde{G}_{ij}^0(\mathbf{x})R(\mathbf{x})e^{-i\mathbf{k}\mathbf{x}}d\mathbf{x}. \quad (27)$$

We consider the special case in which ξ is the normal process:

$$\alpha=(2\pi\sigma\varepsilon)^{-1/2}I_+(3\varepsilon_0/\sigma_\varepsilon),\quad \alpha_0=-i(\pi/2)^{1/2}\sigma_\varepsilon,$$
$$R_1(\mathbf{r}-\mathbf{r}_1)=-\sigma_\varepsilon^{-2}\langle\xi\xi_1\rangle=R(\mathbf{r}-\mathbf{r}_1),\quad \sigma_\varepsilon=\langle\xi^2\rangle, \quad (28)$$
$$I_+(z)=\int_{-\infty}^{+\infty}\frac{\exp(-u^2/2)\,du}{u-z},\quad \operatorname{Im} z>0.$$

Substituting (28) in (26), we get the following expression for the effective permittivity under the assumption of the isotropicity of the fluctuations ξ:

$$\varepsilon_{ij}^{eff}(\omega,\mathbf{k})=\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\varepsilon_{eff}^{tr}(\omega_0,k)+\frac{k_ik_j}{k^2}\varepsilon_{eff}^{l}(\omega_0,k),$$
$$\varepsilon_{eff}^{tr}(\omega_0,k)=i(2/\pi)^{1/2}\sigma_\varepsilon[1-3q(p)],$$
$$\varepsilon_{eff}^{l}(\omega_0,k)=i(2/\pi)^{1/2}\sigma_\varepsilon[1+6q(p)], \quad (29)$$
$$q(p)=\left(\frac{\pi}{2}\right)^{1/2}\int_0^\infty\frac{1}{x}\Gamma(x)J_{5/2}(px)(px)^{1/2}dx,$$

$J_{5/2}(x)$ is the Bessel function, $p=kl$, $\Gamma(x)$ is the dimensionless, normalized correlation function of the fluctuations ε:

$$B_\xi(\mathbf{r})=\langle\xi(\boldsymbol{\rho}+\mathbf{r})\xi(\boldsymbol{\rho})\rangle=\sigma_\varepsilon^2\Gamma(r/l).$$

The function $q(p)$ increases from 0 (at $p=0$) to the value $q(\infty)=1/3$. The latter value is obtained asymptotically. Near $p=0$, the function $q(p)$ increases slowly and $q(p)\ll 1$ at $p\leqslant 1$, while at $p\leqslant 1$ we can use the first term of the expansion in p:

$$q(p) \approx \frac{1}{15} a_1 p^2 + \ldots; \quad a_1 = \int_0^\infty \Gamma(x) x \, dx \approx 1. \tag{30}$$

Using the formulas (29) and (30), we can find the absorption coefficients of the regular monochromatic plane waves at $\omega = \omega_0$ in a plasma with fluctuations of the electron density. Calculation shows that at the resonance frequency, the longitudinal waves are damped according to the law $e^{-r/l}$ and the electromagnetic waves (at $k_0 l \sigma_\varepsilon^{1/2} \ll 1$), are damped like $e^{\gamma r}$, where

$$\gamma = (2\pi)^{-1/4} k_0 \sigma_\varepsilon^{1/2}. \tag{31}$$

It is useful to give the results, which are easily obtained from (29), for $\varepsilon^l_{eff}(\omega_0, k=0)$ in the case of the Cauchy probability density distribution

$$W(\xi) = \frac{1}{\pi} \frac{\sigma_\varepsilon}{\sigma_\varepsilon^2 + \xi^2}. \tag{32}$$

The fact is that an exact value of the quasielectrostatic ε^l_{eef} for this distribution has been obtained in the work of Abramovich.[10] At $k = 0$ we have, from (29),

$$\varepsilon^l_{eff}(\omega_0, k=0) = 1/\alpha_0. \tag{33}$$

Finding α_0 with the help of (32), we get

$$\varepsilon^l_{eff}(\omega_0, k=0) = i\sigma_\varepsilon \tag{34}$$

—the value found by Abramovich.[10]

The author expresses his gratitude to V. I. Tatarskiĭ for significant remarks.

[1]W. F. Brown, Jr., J. Chem. Phys. 23, 1514 (1955).

[2]L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of continuous media) Fizmatgiz, 1957, Sec. 9 [Pergamon, 1959].

[3]V. M. Finkel'berg, Zh. Tekhn. Fiz. 34, 509 (1964) [Sov. Phys. Tech. Phys. 9, 396 (1964)].

[4]I. Webman and J. Jortnner, Phys. Rev. B16, 2593 (1977).

[5]L. P. Kadanoff *et al.*, Rev. Mod. Phys. 39, 395 (1967).

[6]I. M. Lifshitz, M. I. Kaganov and V. M. Tsukernik, Uchenye zapiski KhGU (Kharkov State University) 2, 41 (1950).

[7]E. A. Kaner, Izv. vuzov. Radiofizika 2, 827 (1959).

[8]V. M. Finkel'berg, Zh. Eksp. Teor. Fiz. 46, 725 (1964) [Sov. Phys. JETP 19, 494 (1964)].

[9]Yu. A. Ryzhov, V. V. Tamoĭkin and V. I. Tatarskiĭ, Zh. Eksp. Teor. Fiz. 48, 656 (1965) [Sov. Phys. JETP 21, 433 (1965)].

[10]B. S. Abramovich, Zh. ksp. Teor. Fiz. 77, 87 (1979) [Sov. Phys. JETP 50, 44 (1979)].

Translated by R. T. Beyer

Neodymium laser with a mirror plasma-optical *Q*-switch

V. A. Batanov, D. A. Dement'ev, A. N. Malkov, A. M. Prokhorov, and V. B. Fedorov

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted 24 April 1979)
Zh. Eksp. Teor. Fiz. 77, 2186–2199 (December 1979)

The use of laser radiation self-action in a plasma to develop plasma-optical elements for laser resonators is considered. Experiments with a neodymium laser show that a resonator plasma can simultaneously act as a resonator mirror, a laser Q-switch, and a nonlinear element for mode locking during giant pulse generation. The generation kinetics of a neodymium laser is investigated in detail. The possibility in principle of controlling the generation regime and varying the radiation characteristics over a broad range is demonstrated. A mean radiation power $P \approx 1$–10 GW is attained in a train of nanosecond radiation spikes; the power density at the plasma mirror is estimated at $I \approx 10^{13}$–10^{14} W/cm^2. There are indications that self-focusing of the radiation occurs at the plasma mirror. One manifestation of this is the selection of transverse modes in the laser resonator.

PACS numbers: 42.55.Px, 42.60.Da

1. INTERRELATED GENERATION OF HIGH-POWER LASER RADIATION AND OF A REFLECTING RESONATOR PLASMA

A new physical effect, the generation of high-power laser pulses in a resonator with plasma mirror and interrelated with high-temperature heating of the plasma mirror was reported earlier.[1] In the reported experiments the laser plasma produced in a gas or solid target produced in the focus of a lens performs two functions. First, it is the mirror of the resonator (resonator plasma) and produces positive feedback for the development of the lasing. The strong nonlinear increase of the plasma reflection in the course of the growth of the lasing amplitude ensures rapid switching-on of the Q of a resonator with such a plasma mirror and generation of giant laser pulses. In other words, during the course of generation the dense reflecting plasma serves as a mirror laser Q-switch. Second, the plasma mirror of the resonator serves also as a target for the laser radiation, whose absorption produces in the target a high temperature and a near-criti-

 0038-5646/79/121049-08$02.40