second pulses, the reproducibility of their characteristics, realization of maximum radiation energy, and development of a method for controlling the instant of Q switching in a resonator with a plasma mirror.

[1)]The stage-pump energies U are indicated here and below as fractions of the maximum value. Thus, the $U_2 = 0.40$ means that the pump energy of the second stage is 32 kJ.

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Translated by J. G. Adashko

Resonant ionization of atoms under conditions of adiabatic level inversion

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(Submitted 27 April 1979)
Zh. Eksp. Teor. Fiz. **77**, 2200–2210 (December 1979)

Analytic expressions are obtained for the structure and the line shape of the dispersion dependence of the probability of the resonant polarization w on the radiation frequency ω under conditions of adiabatic level inversion due to the dynamic Stark effect in a field of variable amplitude. It is shown that under certain conditions the $w(\omega)$ dispersion curve is not characterized by a single parameter (width). A narrow and high principal maximum can appear against the background of the relatively broad $w(\omega)$ line. The parameters of the principal maximum and of the $w(\omega)$ as a whole are obtained. Conditions for the realization of the regime of adiabatic inversion of levels are investigated, particularly the conditions for the onset of the narrow principal maximum. The effect of spatial inhomogeneity of the field on the probability of resonant ionization under conditions of adiabatic level inversion is investigated.

PACS numbers: 32.70.Jz, 32.60. + i

1. Resonant multiphoton ionization of atoms has recently attracted considerable attention both experimentally[1-9] and theoretically.[10-18] In the experiment it is customary to investigate either the dependence of the ionization probability w on the intensity of the radiation at a given frequency ω, or the $w(\omega)$ dependence at a fixed field intensity. The experiments of the latter type are apparently most suitable for the determination of the physical nature of the resonant-ionization process, since they yield direct information on the width and position of the maximum and in general on the shape of the $w(\omega)$ dispersion curve. It is this which governs the formulation of the principal problems faced in the theory of resonant ionization: the determination of the shape of the dispersion curve and the elucidation of the physical mechanisms of its formation. From this point of view, an investigation of the excitation of the resonant level is insufficient for a complete solution of the problem of resonant ionization.

One of the principal parameters that characterize the $w(\omega)$ dispersion curve is its width Γ. Various physical

0038-5646/79/121056-06$02.40

mechanisms of the broadening of the dispersion curve in a strong external field have been investigated.[11-15] It is convenient to characterize each of these mechanisms by a corresponding parameter that plays the role of the width of the dispersion curve, i.e., the width of the resonance.

According to Refs. 14 and 15, in the model in which the interaction is turned on instantaneously in a spatially homogeneous field the resonance width Γ is defined as

$$\Gamma=\max\{\hbar\Delta\omega, \Gamma_f, \Gamma_i, \Gamma_f(\Gamma_i\tau/\hbar)^{1/2}\}. \quad (1)$$

Here $\Delta\omega$ is the spectral width of the radiation and is equal to $1/\tau$ in the case of a single-mode laser, where τ is the pulse duration; $\Gamma_f=4|V_{01}^{(k_1)}|$ is the field width due to the mixing of the ground and resonant states, k_1 is the multiplicity of the resonance and of the composite matrix element of the resonant $0\to 1$ transition, $v=-(1/2)\mathbf{d}\cdot\mathbf{F}_0$, f_0 is the amplitude of the wave field intensity; $\Gamma_i=2\pi|V_{1E}^{(k_2)}|^2$ is the ionization width, k_2 is the minimal number of photons necessary for the transition from the resonant state to the continuum,

$$E=E_0+(k_1+k_2)\hbar\omega\approx E_1+k_2\hbar\omega,$$

and $E_{0,1}$ are the energies of the ground and resonant states. The parameter $\Gamma_f(\Gamma_i\tau/\hbar)^{1/2}$ in (1) is connected with the possibility of 100% ionization of the atom during the time of the pulse under conditions close to exact resonance.

If the interaction is not turned on instantaneously, then all the resonance mechanisms determined by the parameters of Eq. (1) can be realized as before.[14,15] However, new mechanisms come into play, since the positions of the ground and resonant levels depend on the time because of the dynamic Stark effect in a field of definite amplitude $F_0f(t)$ ($f_{\max}=f(0)=1$):

$$E_{0,1}(t)=E_{0,1}-\tfrac{1}{4}\alpha_{0,1}F_0^2f^2(t),$$

where $\alpha_{0,1}(\omega)$ are the dynamic polarizabilities of the ground and resonant levels. The condition for the appearance of new mechanisms of formation of the dispersion curve takes the form $\Gamma<\alpha F_0^2$, where $\alpha=1/4|\alpha_1-\alpha_0|$, and the width Γ is determined by Eq. (1). For the case of weak mixing of the ground and resonant states, when $\Gamma_f\ll\hbar/\tau$, this mechanism was investigated in Refs. 13 and 15 (see also Ref. 16). The main result[13] is that a new width parameter appears

$$\Gamma_{st}=(\alpha F_0^2)^{1/3}(\hbar/\tau)^{2/3},$$

and can compete with all the remaining parameters. This conclusion is valid, however, only if the field has a high spatial homogeneity. For pulses with a smooth spatial-envelope profile, the averaging of the equations of Ref. 13 over the spatial distribution of the field at $\alpha F_0^2\gg\hbar/\tau$ leads to a resonance curve of width $\sim\alpha F_0^2$.[17] The parameter Γ_{st} in these fields cannot determine the width of the dispersion curve, since it is certainly less than either $\hbar/\tau$ or αF_0^2.

The case of a strong mixing of the levels $\Gamma_f\gg\hbar/\tau$ with a considerable Stark shift $\alpha F_0^2>\Gamma$ in fields with a smooth temporal envelope $f(t)$ was considered in Refs. 16 and 18. Principal attention, however, was paid to the atom-excitation dynamics. No analytic expressions were obtained in the general case for the shape and width of the dispersion curve, since no complete analysis was made of the conditions under which the corresponding resonant-ionization mechanism can be realized. These problems are the topic of the present article.

2. In accord with the problem posed above, we consider the case when the Stark shift is large, $\alpha F_0^2\gg\Gamma_f$, and strong mixing of the ground and resonant levels takes place, $\Gamma_f\gg\hbar/\tau$. These inequalities can be satisfied simultaneously only for a special choice of the experimental conditions. In fact, the simplest qualitative estimate of the parameters Γ_f and αF_0^2 yields

$$\Gamma_f\sim E_{at}(F_0/F_{at})^{k_1},\quad \alpha F_0^2\sim E_{at}(F_0/F_{at})^2, \quad (2)$$

where E_{at} and F_{at} are respectively the characteristic atomic energy and intra-atomic field. According to (2), the condition $\alpha F_0^2\gg\Gamma_f$ at $F_0\ll F_{at}$ can be satisfied only at $k_1\geq 3$. Actually, this relation between αF_0^2 and Γ_f can be satisfied also if they have the same functional dependence on F_0, i.e., at $k_1=2$, provided that the composite matrix element $V_{01}^{(2)}$ is numerically anomalously small. In this case, however, one can hardly satisfy the condition $\Gamma_f\tau/\hbar\gg 1$ for the strong mixing of the levels. At $k_1\geq 3$ the condition $\alpha F_0^2\gg\Gamma_f$ is satisfied automatically, but difficulties can likewise arise with the requirement $\Gamma_f\tau/\hbar\gg 1$. To satisfy this inequality in the case of $k_1\geq 3$, on the contrary, it is necessary that the matrix element $V_{01}^{(k_1)}$ be numerically anomalously large. Examples of this kind are known. According to Ref. 18, in the case of the three-photon $6S$–$6F$ transition in the Cs atom we have

$$\Gamma_f\approx 10^5E_{at}(F_0/F_{at})^3,$$

whereas

$$\alpha F_0^2\approx(10^2-10^3)E_{at}(F_0/F_{at})^2.$$

Since the numerical value of Γ exceeds by five orders of magnitude the value expected from the qualitative estimate (2), the inequalities

$$\alpha F_0^2\gg\Gamma_f\gg\hbar/\tau$$

are satisfied simultaneously under reasonable conditions—for example at $\tau=10^{-8}$ sec and $F_0=5\cdot 10^5$ V/cm. The equations for the probability amplitudes of excitation of the atom are solved at $\Gamma_f\gg\hbar/\tau$ in the quasiclassical approximation (in time) and according to Refs. 12 and 19 they yield the following expression for the ionization probability:

$$w=\frac{\Gamma_i}{2\hbar}\int_{-\infty}^{+\infty}dt\left(1-\frac{\Delta(t)\operatorname{sgn}\Delta}{[\Delta^2(t)+\frac{1}{4}\Gamma_f^2f^{2k_1}(t)]^{1/2}}\right)f^{2k_2}(t), \quad (3)$$

where

$$\Delta(t)=E_1(t)-E_0(t)-k_1\hbar\omega,\quad \Delta=\Delta(t=\pm\infty)=E_1-E_0-k_1\hbar\omega.$$

The physical meaning of (3) is quite obvious. The coefficient of Z in the integrand of (3) coincides with the square of the modulus of the coefficient of the wave function of the resonant state in one of the quasienergy functions of a two-level system in a resonant field of constant amplitude[15]

$$\psi_{0,1}=\frac{1}{2^{1/2}}\left\{\left(1\mp\frac{\Delta}{(\Delta^2+\frac{1}{4}\Gamma_f^2)^{1/2}}\right)^{1/2}\varphi_1\pm\left(1\pm\frac{\Delta}{(\Delta^2+\frac{1}{4}\Gamma_f^2)^{1/2}}\right)^{1/2}\varphi_0\exp(ik_1\omega t)\right\},$$

where $\varphi_{0,1}$ are the wave functions of the ground and resonant states. This means that the employed quasiclassical approximation is equivalent to an adiabatic switching-on of the interaction, such that the quasienergy functions ψ_0 and ψ_1 are not intermixed. The dependence of the field amplitude on the time appears in this case only in the parametric dependence of the detuning $\Delta(t)$ and of the interaction energy $\Gamma_f f^{R_1}(t)$ on t.

We estimate now the conditions for the applicability of Eq. (3). The known conditions for the applicability of the quasiclassical approximation[20] can be written in the form[17]

$$\Phi \equiv \hbar |\dot{U}_{\text{eff}}| / |U_{\text{eff}}|^{3/2} \ll 1, \tag{4}$$

where $U_{\text{eff}} \approx 1/4(\Delta^2(t) + 1/4\Gamma_f^2 f^{2k_1}(t))$. By virtue of the condition $\alpha F_0^2 \gg \Gamma_f$, the scale of variation of the detuning $\Delta(t)$ is on the whole large compared with Γ_f. Therefore the most doubtful region from the point of view of the applicability of the quasiclassical approach is the vicinity of the points where $\Delta(t)$ vanishes. In the vicinity of these points the derivative of the function $U_{\text{eff}}(t)$ changes sign in a small interval, i.e., the function experiences a break. If $\Delta(t_0) = 0$, then we can use in the vicinity of t_0 the linear expansion $\Delta(t) \approx \varkappa(t - t_0)$, where $\varkappa \sim \alpha F_0^2/\tau$, and we can also assume $F(t) \equiv 1/4\Gamma_f f^{k_1}(t) \approx F(t_0)$, since $F(t)$ is a smooth function of t. The function $\Phi(t)$ takes under these approximations the form

$$\Phi(t) = \frac{8\hbar\varkappa^2 |t - t_0|}{[\varkappa^2 (t-t_0)^2 + 4F^2(t_0)]^{3/2}}. \tag{5}$$

The maximum value of $\Phi(t)$ is reached at

$$t - t_0 \approx \pm F(t_0)/\varkappa \sim \Gamma_f \tau/\alpha F_0^2$$

and its order of magnitude is

$$\Phi_{max} \sim \hbar \alpha F_0^2 / \tau \Gamma_f^2.$$

Consequently, the conditions for applicability of the quasiclassical approach and of Eq. (3) are determined in this case by the set of inequalities

$$\Gamma_i \ll \hbar/\tau \ll \Gamma_f \ll \alpha F_0^2 \ll \Gamma_f^2 \tau/\hbar. \tag{6}$$

Under these conditions, the quasiclassical approximation and Eq. (3) are valid at all values of the detuning $\Delta(0)$.

3. We consider now values of $\Delta(0)$ such that the function $\Delta(t)$ vanishes at points close to the inflection points of the $\Delta(t)$ curve, $t_{1,2} \approx \mp\tau/2$ (see Fig. 1). The probability of finding the atom in the excited state, which is equal to

$$w_1 = \frac{1}{2}\left(1 - \frac{\Delta(t)\,\mathrm{sgn}\,\Delta}{[\Delta^2(t) + 1/4\Gamma_f^2 f^{2k_1}(t)]^{1/2}}\right),$$

assumes in this case a peculiar form: it is small at $t < t_1$ and $t > t_2$ and is close to unity at $t_1 < t < t_2$ (Fig. 1b). The time Δt of the transitions from the value ≈ 0 to the value ≈ 1 can be easily estimated by expanding $\Delta(t)$ in the vicinity of the points $t_{1,2}$ in the series $\Delta(t) \approx \mp\varkappa(t - t_{1,2})$ and replacing $f^{2k_1}(t)$ by $f^{2k_1}(t_{1,2}) \sim 1$; this yields

$$\Delta t \approx \Gamma_f/\varkappa \sim \tau\Gamma_f/\alpha F_0^2.$$

By virtue of the inequalities (6) the transition time Δt is small compared with the pulse duration τ and is large compared with the mixing time of the ground and

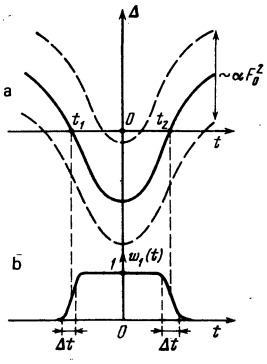

FIG. 1. Time dependence of resonance detuning. The dashed curves correspond to the extermal values $\Delta(0)$ at which adiabatic level inversion is possible. b) Probability $w_1(t)$ of exciting an atom in the adiabatic inversion regime.

resonant states $\hbar\Gamma_f^{-1}$ (the period of the Rabi oscillations).

Taking into account the indicated $w_1(t)$ dependence, Eq. (3) becomes

$$w = \frac{\Gamma_i}{\hbar}\int_{t_1}^{t_2} f^{2k_1}(t)\,dt \approx \frac{\Gamma_i(t_2 - t_1)}{\hbar} \sim \frac{\Gamma_i \tau}{\hbar}. \tag{7}$$

According to (7), the probability w decreases if the detuning $\Delta(0)$ changes in such a way that the points t_1 and t_2 come closer together. If $\Delta(0)$ changes in the opposite direction, then the points $t_{1,2}$ go over into the region of weaker fields, as a result of which the inequalities (6) are violated, especially the condition $\Gamma_f \gg \hbar/\tau$ of strong level mixing, which can be more accurately written in the form

$$\Gamma_f f^{k_1}(t_{1,2}) \gg \hbar/\tau.$$

If the last inequality is violated, then no substantial transitions to the excited state take place, i.e., the probability of ionization also decreases. Thus, the characteristic scale of variation of $\Delta(0)$ is the interval that corresponds to the change of the positions of the points $t_{1,2}$ on Fig. 1a from values $\sim \mp\tau$ to zero. Obviously, this detuning scale, and consequently also the resonance width Γ, are of the order of αF_0^2.

Equation (3) makes it possible not only to estimate the width Γ and the value of w (7), but also allows us to trace the detailed structure of the wing of the dispersion curve in the region of detunings $\Delta(0)$ at which the points t_1 and t_2 approach each other. Obviously, at $|t_{1,2}| \ll \tau$ the values of t_1 and t_2 can be obtained explicitly by using the expansion of the function $\Delta(t)$ near its extremal value

$$\Delta(t) \approx \Delta(0) + \tilde{\varkappa} t^2, \tag{8}$$

where $\tilde{\varkappa} \sim \alpha F_0^2/\tau^2$, and $\tilde{\varkappa} t^2$ was chosen positive, for the sake of argument, to correspond to Fig. 1a. The positions of the point $t_{1,2}$ can be easily determined with the aid of (8):

$$t_{1,2} = \mp(-\Delta(0)/\tilde{\varkappa})^{1/2}$$

at $\Delta(0) < 0$. Near one of these intersection points, say t_1,

$$\Delta(t) \approx -2(|\Delta(0)|\tilde{\varkappa})^{1/2}(t - t_1),$$

which is equivalent to the linear expansion used above far from $t=0$, with $\varkappa$ replaced by $2(|\Delta(0)|\tilde{\varkappa})^{1/2}$. The expansion used here is preferable at $|t_{1,2}| \ll \tau$ in the sense that it contains information on the dependence of $t_{1,2}$ on the detuning $\Delta(0)$. The transition time is given as before by

$$\Delta t \sim \Gamma_f/\varkappa \sim \Gamma_f/(|\Delta(0)|\tilde{\varkappa})^{1/2} \sim \tau\Gamma_f(|\Delta(0)|\alpha F_0^2)^{-1/2}.$$

The condition $\Delta t \ll t_2 - t_1$ is satisfied at $|\Delta(0)| \gg \Gamma_f$, and the condition $|t_1 - t_2| \ll \tau$ at $|\Delta(0)| \ll \alpha F_0^2$. Within the framework of these limitations, the probability is given by

$$w = \frac{\Gamma_i}{\hbar}\int_{t_1}^{t_2} f^{2k_2}(t)\,dt \approx \frac{\Gamma_i(t_2-t_1)}{\hbar} = 2\frac{\Gamma_i}{\hbar}\left(\frac{|\Delta(0)|}{\tilde{\varkappa}}\right)^{1/2} \sim \frac{\Gamma_i\tau}{\hbar}\left(\frac{|\Delta(0)|}{\alpha F_0^2}\right)^{1/2}. \tag{9}$$

Thus, in the detuning interval $\alpha F_0^2 \gg |\Delta(0)| \gg \Gamma_f$ the probability w decreases with decreasing $|\Delta(0)|$ like $(|\Delta(0)|)^{1/2}$ (Fig. 2).

The minimal value of w reached on the boundary of this region at $\Delta(0) = -\Gamma_f$ is of the order of

$$\hbar^{-1}\Gamma_i\tau(\Gamma_f/\alpha F_0^2)^{1/2}.$$

With further increase of $\Delta(0)$ (in the region of positive values), the character of the dispersion curve changes substantially. The reason is that the function $\Delta(t)$ at $\Delta(0) > 0$ does not vanish, and the probability of finding the atom in the excited state at all values of t is small. At $\Delta(0) \gg \Gamma_f$ we have

$$\Delta(t) \gg \Gamma_f f^{2k_1}(t),$$

which makes it possible to simplify Eq. (3):

$$w = \frac{\Gamma_i\Gamma_f^2}{16\hbar}\int_{-\infty}^{+\infty} dt\,\frac{f^{2(k_1+k_2)}(t)}{\Delta^2(t)} \approx \frac{\Gamma_i\Gamma_f^2}{16\hbar}\int_{-\infty}^{+\infty} dt\,\frac{f^{2(k_1+k_2)}(t)}{(\Delta(0)+\tilde{\varkappa}t^2)^2}. \tag{10}$$

At

$$\Gamma_f \ll \Delta(0) \ll \alpha F_0^2$$

the maximum of the function $\Delta^{-2}(t)$ is much narrower than the maximum of the numerator in the integrand of (10), so that we get

$$w = \frac{\pi\Gamma_i\Gamma_f^2}{32\hbar[\Delta(0)]^{-3/2}\tilde{\varkappa}^{1/2}} \sim \frac{\pi\tau\Gamma_i\Gamma_f^2}{32\hbar[\Delta(0)]^{1/2}(\alpha F_0^2)^{1/2}}. \tag{11}$$

On the contrary, at $\Delta(0) > \alpha F_0^2$ we can put $\Delta(t) \approx \Delta(0)$ in (10), so that this expression goes over into the standard perturbation-theory equation

$$w = \frac{\Gamma_i\Gamma_f^2\tau_{k_1+k_2}}{16\hbar[\Delta(0)]^2}, \tag{12}$$

where

$$\tau_{k_1+k_2} = \int_{-\infty}^{+\infty} f^{2(k_1+k_2)}(t)\,dt$$

is the effective pulse duration for a process with multiplicity k_1+k_2.

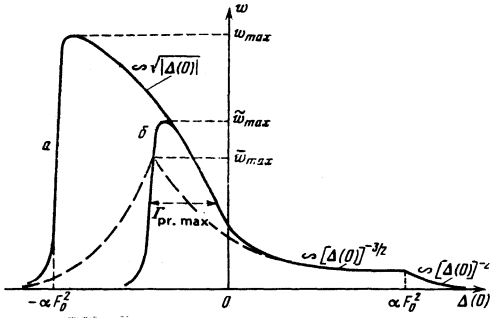

FIG. 2. Structure of dispersion curve $w(\omega) \equiv w(\Delta(0))$ in the adiabatic level-inversion regime: a) under conditions defined by inequalities (6), b) under conditions when the principal maximum takes place [inequalities (15)]. The dashed curve corresponds to the conditions defined by the inequalities (5) upon averaging of the probability w over the spatial distribution of the field: $w_{max} = \Gamma_i\tau_{k_2}/2\hbar$.

Thus, the detuning interval on which the probability w can be large and differ substantially from the perturbation-theory equation is determined in our case by the parameter αF_0^2 for both positive and negative values of $\Delta(0)$.

The decrease of the probability w on the left wing of the resonance curve $w(\Delta(0))$ is due to the fact that when the points t_1 and t_2 move apart the field intensity at the term crossing points decreases. For this reason, starting with a certain limiting value $-\Delta(0)$, the adiabaticity conditions are not satisfied. If approximately we neglect the nonadiabatic transitions completely, then the probability w at this value of the detuning decreases sharply and vanishes (Fig. 2). The detuning at which this sharp decrease of the probability takes place is determined from the equations

$$\Delta(t) = 0, \quad \hbar\alpha F_0^2\left|\frac{d}{dt}f^2(t)\right| = \Gamma_f^2 f^{2k_1}(t).$$

The maximum value of the probability is $w_{max} = \Gamma_i\tau_{k_2}/2\hbar$, where

$$\tau_{k_2} = \int_{-\infty}^{+\infty} dt\, f^{2k_2}(t)$$

is the effective pulse duration for the process of multiplicity k_2.

The abrupt change of the atom excitation probability $w_1(t)$ from 0 to 1 in a short time interval Δt in the vicinity of the point where $\Delta(t) = 0$ corresponds to the well known effect of adiabatic level inversion.[21] In the usual formulation of the problem of adiabatic inversion, 100% excitation of the atom is due to the time variation of the resonant-field frequency. In the problem considered here, the radiation frequency is assumed constant, but the position of the atomic levels is itself shifted in the field with increasing (or decreasing) field amplitude. The possibility of adiabatic level inversion via a shift due to the quadratic Stark effect was apparently first noted in Ref. 22, and later also in Refs. 16, 18, and 23. Expressed in this language, the last inequality of (6) is equivalent to the ordinary adiabaticity condition when the frequency of the field changes with time.[21] Notice was also taken in Refs. 16 and 18 of the analogy between the equations that describe the resonant ionization and the equations used in the theory of atomic collisions.[24] From this point of view, the effect of adiabatic inversion is equivalent to one of the limiting cases of the Landau-Zener theory.[20] According to this approach, the atom is excited mainly at the crossing points of the terms $\Delta(t) = 0$. The probability of excitation of the atom after passing through the point t_1 (Fig. 1) can be written according to Ref. 20

in the form

$$w_1=1-\exp(-\Gamma_f^2\tau/\hbar\alpha F_0^2), \quad (13)$$

where the argument of the exponential is determined accurate to a coefficient ~1. Under the conditions when the inequalities (6) are valid, the exponential in (13) is small, so that $w_1 \approx 1$, i.e., adiabatic inversion takes place and all the conclusions drawn above concerning the structure of the dispersion curve are valid. The width Γ of the dispersion curve as a whole, just as the width of the principal maximum (see Fig. 2), is determined by the parameter $\Gamma_{st}=\alpha F_0^2$.

4. We discuss now the case when the Stark shift is so large that the adiabaticity condition [the last inequality of (6)] is violated, i.e., let $\alpha F_0^2 > \Gamma_f^2\tau/\hbar$. In this case the system goes through a resonance in too short a time to make a substantial mixing of the ground and resonant states possible. According to (13), the possibility of the excitation of the atom is small in this case, as is also the ionization probability. It is obvious, however, that the rate of passage through the resonance depends on the slope of the $\Delta(t)$ curve at the point $t_{1,2}$. This slope decreases as t_1 approaches t_2, i.e., with decreasing $|\Delta(0)|$. This means that while the adiabaticity condition is not satisfied at large detunings $\Delta(0)$ corresponding to the term crossing in the vicinity of the maximum slope of the $\Delta(t)$ curve, nonetheless the corresponding condition can be satisfied at smaller $|\Delta(0)|$ corresponding to the crossing of the terms near the extremal value of $\Delta(t)$. For a quantitative analysis of these conditions we can again use the expansion (8) and replace $\varkappa$ by $2(|\Delta(0)|\varkappa)^{1/2}$, from which it follows that the adiabaticity condition is satisfied at

$$|\Delta(0)| < \Gamma_f^4\tau^2/\alpha F_0^2\hbar^2 = \Gamma_{pr.max}, \quad (14)$$

where $\Gamma_{pr.max}$ is the maximum value of $|\Delta(0)|$ at which the condition (14) is still satisfied; in other words, it is the width of the principal maximum of the resonant ionization (Fig. 2). At

$$|\Delta(0)| > \Gamma_{pr.max}$$

the condition (14) is not satisfied, there is no adiabatic level inversion, and the ionization probability is small. With increasing αF_0^2 the width $\Gamma_{pr.max}$(14) decreases, i.e., the principal maximum of the resonant ionization becomes narrower (Fig. 2). The condition for the narrowing of the principal maximum with increasing αF_0^2 and the condition for the existence of an adiabatic-inversion regime takes respectively the form

$$\alpha F_0^2 \gg \Gamma_{pr.max}, \quad \Gamma_{pr.max} \gg \Gamma_f,$$

which yields in place of inequality (6)

$$\Gamma_i \ll \frac{\hbar}{\tau} \ll \Gamma_f \ll \frac{\Gamma_f^2\tau}{\hbar} \ll \alpha F_0^2 \ll \frac{\Gamma_f^3\tau^2}{\hbar^2}. \quad (15)$$

The maximum value of the probability w under these conditions can also be easily estimated

$$\tilde{w}_{max} = \frac{\Gamma_i(t_2-t_1)}{\hbar} = \frac{\Gamma_i}{\hbar}\left(\frac{\Gamma_{pr.max}}{\tilde{\varkappa}}\right)^{1/2} \sim \frac{\tau^2\Gamma_i\Gamma_f^2}{\hbar^2\alpha F_0^2}. \quad (16)$$

The structure of the dispersion curve at $\Delta(0)>0$ remains in this case the same as when the inequalities (6) are valid. It follows therefore that the maximum width that characterizes this curve under conditions (13) also remain the same as before, $\Gamma=\Gamma_{st}=\alpha F_0^2$.

A decrease of the slope of the $\Delta(t)$ curve takes place not only in the vicinity of its vertex, but also on the wings of the curve, i.e., at large values of $-\Delta(0)$. However, no possibility of adiabatic inversion of the levels in this region of detuning arises. The reason is that the resonance multiplicity k_1, as was already noted, must of necessity be large: $k_1 \geq 3$. This means that the dependence of the parameter $\Gamma_f^2\tau/\hbar$ on f_0 is much stronger ($\sim F_0^6$ at $k_1=3$) than the dependence of the Stark shift αF_0^2. Therefore if the condition $\alpha F_0^2 \ll \Gamma_f^2\tau/\hbar$ is not satisfied in the central part of the $\Delta(t)$ curve, it is all the more not satisfied on its wings. Although the slope of the $\Delta(t)$ curve (and the coefficient of αF_0^2) does decrease when the term crossing points t_1 and t_2 move apart, the parameter $\Gamma_f^2\tau/\hbar$ decreases even more rapidly with decreasing field intensity $F_0 f(t_{1,2})$ and the adiabaticity condition is not satisfied on the wings of the $\Delta(t)$ curve. With increasing parameter αF_0^2 the region of applicability of the adiabatic approximation becomes narrower and is localized in the vicinity of the vertex of the $\Delta(t)$ curve, a fact reflected in the narrowing of the principal maximum of the resonant function $w(\Delta(0))$ (Fig. 2).

We note finally that if we consider the process of adiabatic inversion in the resonant ionization as a function of the peak value of the intensity F_0, then when F_0 is increased a narrow principal maximum is first produced—conditions (5) are realized. With increasing field, the width of the principal maximum (14) increases to its maximum value $\sim\alpha F_0^2$. At these and larger values of the intensity F_0, the adiabaticity conditions are satisfied both near the vertex of the $\Delta(t)$ curve and in its central part, i.e., the inequalities (6) are valid.

5. We have assumed so far that the ionization width is so small that the maximum value of the probability is automatically less than unity. Under the conditions when the inequalities (14) are valid, this is equivalent to the initial assumption $\Gamma_i \ll \hbar/\tau$, and under conditions when a dispersion curve is produced with a narrow central maximum, this restriction on Γ_i becomes less stringent and takes the form

$$\Gamma_i \ll \alpha F_0^2\hbar^2/\Gamma_f^2\tau^2. \quad (17)$$

If these inequalities are violated, the principal maximum of the $w(\Delta(0))$ curve is cut off at the level $w=1$, since the probability w cannot exceed unity. This does not lead to an additional broadening of the dispersion curve so long as

$$w(\Delta(0)=\alpha F_0^2) \sim \frac{1}{16\hbar}\Gamma_i\Gamma_f^2\tau(\alpha F_0^2)^{-2} < 1,$$

i.e., so long as the width $\Gamma_{st}=\alpha F_0^2$, due to the adiabatic inversion, is larger than the width due to the total ionization of the atom $\Gamma_f(\Gamma_i\tau/\hbar)^{1/2}$.[13,15] In the opposite case, when

$$\Gamma_f(\Gamma_i\tau/\hbar)^{1/2} > \Gamma_{st}$$

the broadening due to 100% ionization of the atom is the predominant effect, and none of the singularities connected with the adiabatic level inversion manifest themselves.

Mathematically, these conclusions follow formally from the fact that at $\Gamma_i > \hbar/\tau$ (but $\Gamma_i \ll \alpha F_0^2 \sim |\Delta(0)|$) Eq. (3) is replaced by[12,19]

$$w = 1 - \exp(-w_0), \tag{18}$$

where w_0 is given by Eq. (3).

6. Allowance for the spatial inhomogeneity of the field in the focal region leads to the need for averaging the probability w over the distribution of the intensity F_0. If the distribution of F_0 with respect to the transverse coordinates x and y is characterized by a smooth function $\varphi(x, y)$, such as a product of Gaussian curves $\varphi \sim \exp(-(x^2+y^2)/d^2)$, where d is the transverse dimension of the caustic, then averaging over $\varphi(x, y)$ influences quite significantly the shape of the principal maximum.[17] The results of these changes is shown qualitatively by the dashed curves in Fig. 2. The height of the principal maximum narrows down to the value

$$\bar{w}_{max} \approx \frac{\Gamma_f^4 \tau^2}{(\alpha F_0^2)^2 \hbar^2} \tilde{w}_{max} = \frac{\Gamma_{pr.\,max}}{\alpha F_0^2} \tilde{w}_{max} \ll \tilde{w}_{max}.$$

Its width increases and becomes approximately equal to[17]

$$\frac{\alpha F_0^2}{3(k_1-1)+k_2} \sim \Gamma_{st} = \alpha F_0^2.$$

Bearing these results in mind, we can present a general definition of the dispersion-curve width $w(\omega)$ with account taken of the adiabatic inversion process and with allowance for the spatial inhomogeneity of the field and of the probability averaging. In the general case, to generalize the definition (1) it is necessary to include among the competing parameters that can determine the value of Γ also the Stark width $\Gamma_{st} = \alpha F_0^2$:

$$\Gamma = \max\{\hbar/\tau, \Gamma_f, \Gamma_i, \Gamma_{st}, \Gamma_f(\Gamma_i\tau/\hbar)^{1/2}\}. \tag{19}$$

By virtue of the averaging over the spatial distribution of the field this equation does not contain the width parameter $\tilde{\Gamma}_{st} = (\alpha F_0^2)^{1/2}(\hbar/\tau)^{2/3}$.[13,17]

The parameter $\Gamma_{st} = \alpha F_0^2$ can determine the width of the resonance at two different physical broadening mechanisms: as a result of the spatial averaging of the probability, and directly as a result of the adiabatic inversion of the levels. This difference between the physical mechanisms is, of course, beyond the scope of Eq. (19), which is likewise not sensitive to the difference between the two adiabatic-inversion regimes characterized by the inequalities (15) and the inequalities (6). Nevertheless, the relation (19) can be useful, since it characterizes one of the possible parameters of the dispersion curve and also, albeit not fully unambiguously, it characterizes the broadening mechanism. The finer points of the detailed structure of the $w(\omega)$ curve and of the various methods of realizing the Stark broadening can, naturally, not be taken into account in a single formula (19). They were investigated for the most part earlier,[11-15] and some new aspects were considered in the present article and in recent work.[17,19] Within the framework of the considered general scheme, a particular place is occupied by the case of two-photon resonance ($k_1 = 2$) at arbitrary k_2 and especially at $k_2 = 1$. According to qualitative estimates of the type (2), in the latter case we have $\Gamma_i \sim \Gamma_f \sim \Gamma_{st}$. One of these parameters can prevail over the others only on account of anomalies of numerical character. If there are no such effects, then, just as in the case of instantaneous turning-on,[11] none of the intermediate broadening mechanisms—field, ionization, and Stark broadening—can occur. The width of the dispersion curve is determined by the relation

$$\Gamma = \max\{\hbar/\tau, \Gamma_f(\Gamma_i\tau/\hbar)^{1/2}\}.$$

With increasing field, a direct transition takes place from broadening due to the nonmonochromaticity of the radiation to a broadening due to 100% ionization of the atom during the pulse time.

The authors thanks N. B. Delone for discussions that prompted the present study.

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Translated by J. G. Adashko