INTRABAND ABSORPTION

We shall now consider the intraband absorption results. An analysis of the dispersion of the real $\varepsilon_1 = n^2 - k^2$ and imaginary $\varepsilon_2 = 2nk$ parts of the complex permittivity in the wavelength range $17-7\mu$ shows that the situation is simplest in the martensitic phase.

In fact, the experimental dependence of $(1-\varepsilon_1)/\lambda^2$ on λ^2 is a straight line whose intercept gives the plasma frequency of the conduction electrons $\Omega^2 = 4.4 \times 10^{30}$ sec^{-2}. The experimental dependence $\sigma = \varphi(\varepsilon_1)$ gives $\gamma = 0.37 \times 10^{14}$ sec^{-1}.

The absence of frequency dispersion of $(1-\varepsilon_1)/\lambda^2$ shows that carriers in the martensitic phase are practically indistinguishable in respect of their relaxation frequency, forming a single hybridized conduction band. Consequently, in the case of the martensitic phase the dispersion of ε_1 and ε_2 can be described by simple Drude formulas.

The Drude-Zener formulas are completely inapplicable for the description of the dispersion of ε_1 and ε_2 of the austenitic phase of TiNi. The values of ε_1 and ε_2 for the austenitic phase behave typically like the majority of pure transition metals, exhibiting particularly a strong quadratic frequency dependence of the effective plasma $\Omega_{\text{eff}}^2 = (\varepsilon_1^2 + \varepsilon_2^2)\omega^2/\varepsilon_1$ and relaxation $\gamma = \varepsilon_2\omega/\varepsilon_1$ frequencies. This dependence may be due to two factors. One of them is the presence of several groups of carriers belonging to different sheets of the Fermi surface and characterized by very different relaxation frequencies; the other factor is a large contribution of the low-energy (and possibly zero-gap) intraband transitions. In the case under discussion, both factors are important.

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Translated by A. Tybulewicz

Optical properties of a semiconductor with an electron temperature superlattice

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(Submitted 28 April 1979)
Zh. Eksp. Teor. Fiz. 77, 2351–2357 (December 1979)

The optical properties of a semiconductor in which an electron temperature superlattice has been produced are investigated. It is shown that under these conditions a crystal having a cubic lattice becomes, generally speaking, a biaxial crystal. The phase change on reflection of a normally incident wave is calculated for various experimental geometries. The shape of the superlattice is examined in an appendix. It is shown that within the limitations of the assumptions made in the calculations, the solutions represent "billows" such as are familiar in the Bénard hydrodynamical problem.

PACS numbers: 64.70.Kb, 61.50.Ks, 78.20.Bh

1. INTRODUCTION

A theory of the electron-temperature superlattice produced in a monopolar semiconductor when the electron gas is sufficiently heated by intraband absorption of electromagnetic radiation has been developed in Refs. 1–6. According to Refs. 2–6, this superlattice arises in the presence of band bending at the irradiated surface of the specimen when the intensity I_m of the radiation at that surface exceeds a certain critical value I_{cr}. Then

0038-5646/79/121132-05$02.40

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the variations of the electron temperature T are accompanied by variations of the electric field strength and (to a considerably lesser degree) by variations of the free carrier concentration. The spatial period a of the stationary superlattice is given by the formula

$$a=2\pi k^{-1}\lambda_0^{-1},$$

in which $\lambda_0^{-1}=(2\varkappa_0\tau_0/3)^{1/2}$ is the cooling length, $\varkappa_0$ and τ_0 are the electronic heat conductivity and the energy relaxation time calculated for the case in which the electron temperature T is equal to the lattice temperature T_0, and k is a dimensionless wave number. According to Ref. 6, when $\varepsilon^2\equiv(I_m-I_{cr})/I_{cr}\ll 1$ we have

$$k=k_m\varepsilon^2, \tag{1}$$

where the quantity k_m depends on the form of the electron dispersion law and on the energy- and quasimomentum-scattering mechanisms, and can be easily determined from formulas (41)–(44) of Ref. 6. In particular, for the case that we shall consider in what follows, namely that of a simple parabolic dispersion law and scattering of energy and quasimomentum from piezoelectric acoustic phonons and charged impurities, respectively, we obtain[1)]

$$k_m=\psi_1(g)\left[1+\frac{\psi_2(g)}{F_0+3\ln(1/x)}\right],\quad x=\frac{1}{2}[(g^2+4)^{1/2}-g]. \tag{2}$$

Here $g=\gamma I_{cr}$, γ is the absorption coefficient for the heating light expressed in units of λ_0 (and is therefore dimensionless), and F_0 is the absolute energy difference between the edge of the conduction band (or the valence band in the case of holes) and the Fermi level, expressed in units of T_0 (as in Ref. 6, we consider a nondegenerate charge-carrier gas). Values of the functions ψ_1 and ψ_2 are given in Table I for values of g from 0.1 to 0.2. We see that when $g\sim 1$ in the vicinity of the threshold we have $a\approx(2\pi\times 10^2)\varepsilon^{-2}\lambda_0^{-1}$. For $\lambda_0=10^4$ cm^{-1} and $\varepsilon^2=0.1$, this gives $a=2\pi\times 10^{-1}$. In accordance with Ref. 6, when $g\ll 1$ we obtain[2)] $k_m\approx 2/3g$, i.e. $a\approx 3\pi g\lambda_0^{-1}\varepsilon^{-2}$. Here the quantity a may be of the order of a micron, but, as was shown in Ref. 6 by the methods of the nonlinear theory, it is very difficult to satisfy the condition for stability of the superlattice with small values of g. Values of a that are not too large can evidently be obtained only at a fairly high supercriticality.

The amplitude of the spatial oscillations of the electron temperature when $\varepsilon^2\ll 1$ can also be determined with the aid of the nonlinear theory.[6] We write (see Fig. 1)

$$(T-T_0)/T_0=\xi_s(z)+\varepsilon\xi_1(z)\cos\mathbf{kr}+o(\varepsilon^2), \tag{3}$$

where $\mathbf{k}=\{k_x,k_y\}$ and $\mathbf{r}=\{x,y\}$. Then for the scattering mechanisms mentioned above we obtain the following equation from formulas (10), (17), (23), (24), and (47)–(49) of Ref. 6:

$$\xi_1^2(0)=2^{5/2}g\varkappa^7/7(\nu_2-\nu_{2c})(1+\varkappa^2)^{1/2}. \tag{4}$$

The quantity ν_2, which is defined by formula (35) of Ref. 6, characterizes the rate of energy exchange between electrons in the crystal and in the surrounding medium as well as the change in this energy exchange rate on heating the electron gas. The superlattice is stable (at least in the small) so all the preceding formulas are valid as soon as ν_2 exceeds the critical value ν_{2c} given by

$$\nu_{2c}=m(g)/\gamma^2(F_0+3\ln 1/x)^2$$

[values of the function $m(g)$ are given in Table I].

The ratio of the components k_x and k_y is not determined by condition (1), since the condition involves only the modulus $k=(k_x^2+k_y^2)^{1/2}$. It follows from the nonlinear theory (see the Appendix) that one of the following possibilities is realized in the system under consideration: $k_y=0$ and $k_x=\pm k$, or $k_x=0$ and $k_y=\pm k$. In view of the physical equivalence of the x and y axes, it is sufficient to consider only one of these possibilities, say the first one.

TABLE I.

g	ψ_1	ψ_2	ψ_3	ψ_4	$\ln(1/x)$	m [6]
0.1	0.249	6.01	4.32	0.751	0.150	90
0.2	0.163	6.03	5.12	1.51	0.300	40
0.3	0.108	6.07	2.44	2.27	0.448	7.7
0.4	0.0710	6.12	1.25	3.04	0.596	1.8
0.5	0.0472	6.18	0.712	3.83	0.742	0.53
0.6	0.0315	6.26	0.414	4.65	0.887	0.17
0.7	0.0212	6.36	0.242	5.48	1.03	0.057
0.8	0.0144	6.46	0.161	6.34	1.17	0.022
0.9	0.00980	6.58	0.105	7.23	1.31	0.0075
1.0	0.00674	6.71	0.0694	8.15	1.44	0.0030
1.1	0.00467	6.85	0.0480	9.11	1.58	0.0012
1.2	0.00249	7.00	0.0324	10.1	1.71	$5.1\cdot10^{-4}$
1.3	0.00231	7.16	0.0222	11.1	1.83	$2.2\cdot10^{-4}$
1.4	0.00164	7.32	0.0168	12.2	1.96	$1.0\cdot10^{-4}$
1.5	0.00118	7.50	0.0114	13.3	2.08	$4.7\cdot10^{-5}$
1.6	0.000851	7.68	0.00872	14.5	2.20	$2.3\cdot10^{-5}$
1.7	0.000621	7.88	0.00602	15.7	2.31	$1.0\cdot10^{-5}$
1.8	0.000456	8.07	0.00490	17.0	2.43	$5.9\cdot10^{-6}$
1.9	0.000338	8.28	0.00374	18.3	2.54	$3.3\cdot10^{-6}$
2.0	0.000253	8.48	0.00286	19.6	2.64	$1.5\cdot10^{-6}$

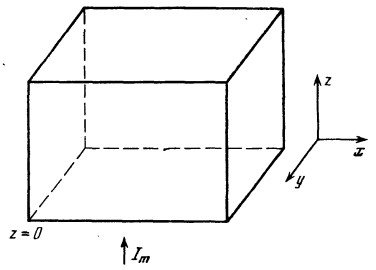

FIG. 1.

2. OPTICAL EFFECTS

It is natural to suppose that electromagnetic waves of appropriate wavelength λ_L (of the order of a) will be diffracted by the superlattice that we are considering provided the components of the dielectric tensor depend on T or on n.[3)] The situation here is the same as for the diffraction of light by ultrasonic waves, and we can immediately use the well known formulas (see, e.g., Ref. 7) merely by setting the frequency of the ultrasound equal to zero (but keeping the wave vector of the sound wave different from zero). We note, however, that according to what was said in Section 1, such an attempt in the infrared region can evidently be successful only at a fairly high supercriticality.

A feature of the system we are considering that distinguishes it from ordinary static lattices is that the electron temperature superlattice is made up of the same charge carriers as are accelerated in the field of

the electromagnetic wave. Diffraction is therefore not the only optical effect to be expected here.

As in Ref. 5, it is convenient to express the current density **j** produced by the electromagnetic wave of frequency ω and fieldstrength E (not to be confused with the the heating wave!) in the form

$$J_\alpha=\sigma(\xi_0)\left\{\delta_{\alpha\beta}+a_{\alpha\beta\mu\nu}\frac{u_\mu u_\nu}{v_0^2}\right\}E_\beta\equiv\sigma_{\alpha\beta}(x,y,z)E_\beta. \tag{5}$$

Here the Greek subscripts are vector indices, u_μ and u_ν are components of the convection velocity, and v_0 is the thermal velocity (or the Fermi velocity in the case of a degenerate gas) of the charge carriers. It remains only to note that in the present case (but not in that of Ref. 5) σ and $a_{\alpha\beta\mu\nu}$ are complex functions of the frequency. For the case of a classically high frequency, when $\omega\tau_p\gg1$ (but still $\hbar\omega<T$), where τ_p is the average momentum relaxation time, we easily find, with the aid of the kinetic equation, that

$$\operatorname{Re}\sigma_{\alpha\beta}=\frac{\sigma^{st}}{\omega^2\tau_p^2}\left(\delta_{\alpha\beta}+\frac{a^{st}_{\alpha\beta\mu\nu}u_\mu u_\nu}{v_0^2\omega^2\tau_p^2}\right),$$

$$\operatorname{Im}\sigma_{\alpha\beta}=\frac{\sigma^{st}}{\omega\tau_p}\left(\delta_{\alpha\beta}+\frac{4a^{st}_{\alpha\beta\mu\nu}u_\mu u_\nu}{v_0^2\omega^4\tau_p^4}\right). \tag{6}$$

Here the superscript st denotes the static value of the corresponding quantity. The components of the tensor $a^{st}_{\alpha\beta\mu\nu}$ are treated as phenomenological parameters. For the cubic crystals we are considering, this tensor has three independent components, which, in the principal axis system (we assume that the principal axes coincide with the x, y, and z axes of Fig. 1), are

$$a_{xxxx}=a_{yyyy}=\ldots=a_1,\ a_{xxyy}=\ldots=a_2,\ a_{xyxy}=\ldots=a_3.$$

The quantities u_μ were calculated earlier[2,5]; they are proportional to $\xi_1(0)$ and depend on x and y (through the scalar product $\mathbf{k}\cdot\mathbf{r}$), as well as on z (beyond the space charge layer at the illuminated surface as e^{-kz}). We see that under these conditions the cubic crystal is, generally speaking, transformed into a biaxial crystal, the components of the dielectric tensor depending on the intensity of the heating light and on the coordinates, and the two tensors $\operatorname{Re}\sigma_{\alpha\beta}$ and $\operatorname{Im}\sigma_{\alpha\beta}$ having the same principal axes at each point. We write

$$a^{st}_{\alpha\beta\mu\nu}u_\mu u_\nu=A_{\alpha\beta}.$$

Assuming in accordance with what was said above that $k_x\neq0$ and $k_y=0$ and taking the principal axes of the crystal as the x, y, and z coordinate axes, we find

$$\begin{aligned}&A_{xx}=a_1u_x^2+a_3u_z^2,\quad A_{yy}=a_2(u_x^2+u_z^2),\\&A_{zz}=a_2u_x^2+a_1u_z^2,\quad A_{xz}=2a_3u_xu_z=A_{zx},\\&A_{xy}=A_{yx}=A_{zy}=A_{yz}=0.\end{aligned} \tag{7}$$

It should be noted, however, that, as is evident from (6), in this case the absorption anisotropy is more significant than the refraction anisotropy; experiments with the reflection of light would therefore seem the more interesting.

We shall examine the case of normal incidence. The possible geometric conditions of the experiment are given in Table II.

Here there are two limiting cases, depending on the ratios of the period of the superlattice to the wavelength and absorption coefficient of the second (nonheating)

TABLE II.

	a	b	c
Incident wave propagation direction	Along the y axis	Along the x axis	Along the z axis
Reflecting surface	$y=0$	$x=0$	$z=0$
Incident wave polarization direction	Along the x axis (1) Along the z axis (2)	Along the y axis (1) Along the z axis (2)	Along the y axis (1) Along the x axis (2)

wave, λ_L and γ_L.

1). $\lambda_L\ll a$ and $\gamma_L\gg k\lambda_0$ (but $\gamma_L<r_0^{-1}$, where r_0 is the screening length that determines the thickness of the space charge layer at the illuminated surface). In this case the wave is almost entirely absorbed in a region within which the electron temperature remains practically constant, and one can associate different (locally constant) values of the components $\sigma_{\alpha\beta}$ with different regions of the surface having linear dimensions of the order of $(k\lambda_0)^{-1}$. In other words, here we are dealing with reflection from a "spotty" surface, the reflection taking place within each of the "spots", generally speaking, as from a biaxial crystal.

We denote the phase changes on reflection of a plane polarized wave by φ_1 and φ_2, the subscripts 1 and 2 corresponding to the possible geometric situations indicated in Table II. Using formulas (1)–(3), (6), and (7), we obtain the following equation after somewhat lengthy but elementary calculations:

$$|\varphi_1-\varphi_2|_\alpha=\frac{(\varepsilon_0\varepsilon_{ext})^{1/2}}{\varepsilon_0-\varepsilon_{ext}}\frac{\omega_p^2}{\omega^2}\frac{1}{\omega^2\tau_p^2}\left(\frac{T_0\tau_p}{mc^2\tau_0}\right)^{1/2}B_\alpha C(g). \tag{8}$$

Here ω_p is the plasma frequency of the charge carriers in the material being considered, ε_0 and ε_{ext} are the dielectric constants of the lattice and the surrounding medium (for simplicity we neglect the dispersion in both media), the subscript α takes the values a, b, and c in accordance with Table II, and

$$C(g)=\psi_3(g)/(F_0+3\ln 1/x)\,[F_0+\psi_4(g)]. \tag{9}$$

The values of the functions ψ_3 and ψ_4 are given in Table I, and the coefficients B_α are given by the expressions

$$B_a=4k\lambda_0\lambda_L e^{-2k\lambda_0 x}\times[(a_1-a_2)^2\cos^2(2k\lambda_0x)+4a_3^2\sin^2(2k\lambda_0x)]^{1/2}, \tag{10}$$

$$B_b=4k\lambda_0\lambda_L|a_1-a_2|e^{-2k\lambda_0 x}, \tag{11}$$

$$B_c=4k\lambda_0\lambda_L|a_1-a_2|\sin^2(k\lambda_0x). \tag{12}$$

Here k is the wave number defined, as before, by Eq. (1), and the coordinates x and z are expressed in ordinary units. We note that the quantities B_α do not vanish when $a_1-a_2=2a_3$, i.e. when the tensor $a_{\alpha\beta\mu\nu}$ is the same as for an isotropic medium. This should not be surprising: in this case the optical anisotropy is not due to the properties of that tensor, but is due to the presence of two special directions, fixed by the vector **k** and by the wave vector of the heating wave.

Thus, in x cases a) and c) we should obtain a reflection pattern on moving the incident beam along the crystal surface that repeats itself periodically along the x axis with the period $\pi/k\lambda_0$ (in ordinary units). Moreover, on moving the beam along the y axis (in case c) the reflection pattern should not change. We note that these conclusions, like formulas (10)–(12) themselves,

are associated not only with assumption 1), but also with the above noted conclusion of the nonlinear theory concerning the ratio of the components k_x and k_y. Thus, experiments conducted under conditions 1) might give direct information on the structure of the electron temperature superlattice.

2). $\lambda_L \gg a$, $\gamma_L < k\lambda_0$ (and $\gamma_L < r_0^{-1}$). In this case the Maxwell equations for the propagation of the nonheating wave in the medium can be averaged over a volume whose linear dimensions are considerably larger than a and smaller than λ_L. This amounts to averaging the components of the tensor $\sigma_{\alpha\beta}$ in the corresponding way. The medium then assumes the symmetry of a uniaxial crystal in which the x principal optical axis is parallel to the y axis and the components of the effective tensor $\sigma_{\alpha\beta}$ are independent of the coordinates. The consequences are obvious from crystal optics (see, e.g., Ref. 8). Here we must expect weak absorption anisotropy in the propagation of a wave parallel or perpendicular to the heating light and, as in case a), we must also expect the phase change on reflection of a nonheating wave to depend on the polarization of the nonheating wave and on the intensity of the heating light. Meanwhile Eq. (8) remains valid, but in place of formulas (10)–(12) we obtain

$$B_a=0, \quad B_b=B_c=|a_1-a_2|. \tag{13}$$

APPENDIX

The shape of the electron temperature superlattice

The nonlinear theory of the electron temperature superlattice developed in Ref. 6 does not enable one to determine the ratio of the components k_x and k_y, i.e. the shape of the superlattice; one finds only the quantity $k^2=k_x^2+k_y^2$. The reason is that in a cubic crystal there is degeneracy in the directions of the x and y axes, and this degeneracy is not lifted by the nonlinear corrections calculated in Ref. 6. To determine the shape of the superlattice one could resort to higher approximations, much as is done in the problem of the Benard lattice in hydrodynamics (see, e.g., Refs. 9 and 10). This, however, involves rather cumbersome calculations. The method proposed by N. N. Bogolyubov,[11] which consists (in the present case) of lifting the degeneracy by introducing small anisotropic terms into the equations and subsequently letting them tend to zero (in the solution), is evidently more convenient here. These added terms can be regarded as independent of ξ. When the anisotropy of the kinetic coefficients in the (x, y) plane is taken into account, the condition for the existence of a superlattice determines not only the quantity k, but also the components k_x and k_y separately. The result we are interested in for the cubic crystal should remain valid when the above mentioned anisotropy is removed, and indeed, regardless of the order in which the anisotropic terms added to the different kinetic coefficients are made to tend to zero.[5)]

We introduce the mobility, heat conductivity, and thermoelectromotive force tensors, writing (the subscripts assume the values x, y, and z)

$$\mu_{ij}=\mu^0\delta_{ij}+\delta\mu_{ij}, \quad \varkappa_{ij}=\varkappa^0\delta_{ij}+\delta\varkappa_{ij}, \quad \alpha_{ij}=\alpha^0\delta_{ij}+\delta\alpha_{ij}. \tag{A1}$$

Here and in what follows the superscript 0 denotes a quantity characterizing the cubic crystal, while $\delta\mu_{ij}$, $\delta\varkappa_{ij}$, and $\delta\alpha_{ij}$ are, generally speaking, small added terms (all of the same order). The nature of the problem is such that we are interested only in the anisotropy in the (x, y) plane. We shall accordingly assume that only the components $\delta\mu_{ij}$, $\delta\varkappa_{ij}$, and $\delta\alpha_{ij}$ for which $i\neq z$ and $j\neq z$ are different from zero.[6)] We use the same system of units as in the main text and, as in Refs. 1–6, we denote the potential of the electric field in the specimen (both the internal field and the field produced by external sources) by φ. We write

$$\varphi=\varphi^0+\delta\varphi, \quad \xi=\xi_0{}^0+\delta\xi, \quad \nabla\Phi=\nabla\Phi^0+\nabla\delta\Phi,$$

where $\nabla\Phi=\nabla\varphi+\alpha^0\nabla\xi$, and $\delta\varphi$, $\delta\xi$, and $\nabla\delta\Phi$ are small quantities of the same order as $\delta\mu_{ij}$, $\delta\varkappa_{ij}$, and $\delta\alpha_{ij}$. We consider only a nondegenerate electron gas and, as in Refs. 1–6, we assume that $\mu^0(\xi)^r$. Then we can write the expression for the velocity, the equation of continuity, and the energy transport equation as follows:

$$u_i=-\mu^0\frac{\partial\Phi}{\partial x_i}-\delta\mu_{ij}\frac{\partial\Phi^0}{\partial x_j}-\mu_0\delta\alpha_{ij}\frac{\partial\xi^0}{\partial x_j}\equiv u_i{}^0+\delta u_i, \tag{A3}$$

$$\operatorname{div}\mathbf{u}=0; \tag{A4}$$

$$\hat{L}[\xi^0,\Phi^0]-\operatorname{div}(\delta\xi\nabla\varkappa^0+\varkappa^0\nabla\delta\xi)-\frac{\partial}{\partial x_i}\left(\delta\varkappa_{ij}\frac{\partial\xi^0}{\partial x_j}\right)+\frac{5+2r}{3}[(\mathbf{u}^0,\nabla\delta\xi)+(\delta\mathbf{u},\nabla\xi^0)]+(\mathbf{u}^0,\nabla\delta\varphi)+(\delta\mathbf{u},\nabla\varphi^0)+\frac{d}{d\xi}(\xi\tau^{-1})_{\xi=\xi^0}\delta\xi=\gamma I(z). \tag{A5}$$

Here $\hat{L}$ denotes the operator that occurs in the theory for the case of isotropic kinetic coefficients[1-6]:

$$L[\xi^0,\Phi^0]=-\operatorname{div}(\varkappa^0\nabla\xi^0)+\frac{5+2r}{3}(\mathbf{u}^0,\nabla\xi^0)+(\mathbf{u}^0,\nabla\varphi^0)+\xi^0\tau^{-1}(\xi^0). \tag{A6}$$

We can also rewrite the boundary conditions in a similar way, but we do not need their explicit form here. We need only recall that the solution, regarded as a function of x and y, must satisfy periodicity conditions at the faces of the specimen.

We can determine the static solution under the above assumptions concerning the nature of the anisotropy:

$$\mathbf{u}_s=0, \quad \xi_s(z)=\xi_s{}^0(z)+\delta\xi_s(z), \quad \varphi_s(z)=\varphi_s{}^0(z)+\delta\varphi_s(z), \quad \nabla\Phi_s=0.$$

We also write

$$\xi=\xi_s+\xi_1(x, y, z), \quad \varphi=\varphi_s+\varphi_1(x, y, z), \nabla\Phi=\nabla\Phi_1(x, y, z), \tag{A7}$$

in which ξ_1, φ_1, and Φ_1 are quantities of the order of ε. It is convenient to express these quantities in the form

$$\xi_1=\sum_{k_x,k_y}{}' f_1(z)e^{i\mathbf{kr}}, \quad \Phi_1=\sum_{k_x,k_y}{}' \chi_1(z)e^{i\mathbf{kr}}. \tag{A8}$$

Here the dash on the summation sign means that the sum is to be taken for a fixed value of $k=(k_x^2+k_y^2)^{1/2}$; further,

$$f_1=f_1{}^0+\delta f_1, \quad \chi_1=\chi_1{}^0+\delta\chi_1, \quad k^2=k_0{}^2+\delta k^2.$$

The quantities δf_1, $\delta\chi_1$, and δk^2 are small of the same order as $\delta\mu_{ij}$, while the functions f_1^0 and χ_1^0 and the number k_0^2 are determined by solving the unperturbed (isotropic) problem. Substituting Eqs. (A7) and (A8) into Eqs. (A4) and (A5), making use of Eqs. (A3)–(A6), and retaining only terms of the first order in ξ_1, χ_1, φ_1, γ, and in the anisotropy, we obtain the following equa-

tions, in which the Greek subscripts assume the values x and y:

$$\hat{L}[\delta f_1, \delta\chi_1] = -(\delta\varkappa_{\beta\gamma} k_\beta^0 k_\gamma^0 + \varkappa^0(\xi_*^0)\delta k^2) f_1^0, \quad \text{(A9)}$$

and

$$-\frac{d^2\delta\chi_1}{dz^2} + k_0^2\delta\chi_1 = -\left(\frac{\delta\mu_{\beta\gamma}}{\mu^0(\xi_*^0)} k_\beta^0 k_\gamma^0 + \delta k^2\right)\chi_1^0 - \delta\alpha_{\beta\gamma} k_\beta^0 k_\gamma^0 f_1^0. \quad \text{(A10)}$$

The operator $\hat{L}$ occurring here is obtained from (A6) by replacing the derivatives with respect to x and y by the factors ik_x and ik_y.

The condition that Eqs. (A9) and (A10) be solvable simultaneously has the form

$$(A_{11}\delta\varkappa_{\beta\gamma} + A_{12}\delta\alpha_{\beta\gamma} + \bar{A}_{22}\delta\mu_{\beta\gamma}) k_\beta^0 k_\gamma^0 = -(\bar{A}_{11} + A_{22})\delta k^2, \quad \text{(A11)}$$

where

$$A_{11} = \int_0^l (f_1^0)^2 dz, \quad \bar{A}_{11} = \int_0^l \varkappa^0(\xi_*^0)(f_1^0)^2 dz, \quad A_{12} = \int_0^l f_1^0\chi_1^0 dz,$$

$$A_{22} = \int_0^l (\chi_1^0)^2 dz, \quad \bar{A}_{22} = \int_0^l (\chi_1^0)^2[\mu_*^0(\xi_*^0)]^{-1} dz.$$

These integrals (which depend on k_0^2) can be regarded as known since the functions f_1^0 and χ_1^0 are fully determined in the nonlinear theory.[6] For our purposes it is sufficient to know that the quantities $A_{11}, \ldots, \bar{A}_{22}$ are different from zero and bounded in absolute value.

As is evident from Eq. (A11), there are just two (equivalent) situations in which the ratio of the components k_x and k_y is not affected by the order in which the limits are taken in passing to the isotropic problem:

$$k_x = \pm k, \quad k_y = 0; \quad k_y = \pm k, \quad k_x = 0$$

[δk^2 is determined in an obvious way by Eq. (A11)]. The corresponding solutions are simply the "billows" familiar from the Bénard problem in hydrodynamics.[9]

[1)] Formula (2) is approximate: in its derivation terms that are small when $g \sim 1$, either numerically (with accuracy to 10%) or because of the smallness of the parameters indicated in Ref. 6, were dropped.

[2)] The symbol k_0 was used for k_m in Ref. 6.

[3)] Strictly speaking, the wave whose propagation we are now interested in could also heat the electron gas and thereby participate in the formation of the superlattice. For simplicity, however, we shall assume that the wave we are now considering is so weak that its effect on the heating of the electron gas may be neglected. There is another problem that is also of interest here—the essentially nonlinear problem of the effect of the superlattice on the propagation of the "heating" wave that produces it.

[4)] Most of the calculations in the Appendix were carried through with the collaboration of A. M. Temchin, for which I wish to thank him.

[5)] This method would probably turn out also to be useful in the analogous hydrodynamical problem.

[6)] We set aside by the same token the problem of the origin of the temperature superlattice in anisotropic crystals. Actually, this problem is by no means devoid of interest. Thus, we can convince ourselves that a static electron-temperature distribution can arise only if the condition $\alpha_{xz} = \alpha_{yz} = 0$ is satisfied. Otherwise the anisotropy of the crystal alone could give rise to a temperature superlattice.

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Translated by E. Brunner