Fluctuation sound absorption in layered superconductors

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It is shown that fluctuations contribute significantly to the sound absorption in dirty superconductors in a rather broad temperature range near the superconducting transition temperature. The character of the temperature dependence of the fluctuation absorption changes from two-dimensional to three-dimensional upon approach to the critical temperature.

1. Fluctuation electron pairing in a normal metal near the superconducting transition temperature manifests itself in the characteristic temperature dependence of a number of physical quantities.[1] Scattering of the electrons by the superconducting fluctuations leads to an increase of the sound absorption upon approach to the critical temperature.[2,3] Although the fluctuation corrections to the sound absorption coefficient depend on the temperature more strongly than other quantities, they become significant in pure metals only in a rather narrow range of temperatures near the superconducting transition temperature.[2]

The presence in the metal of impurities leads to a decrease in the correlation distance (pair size) and an increase in the role of the fluctuations. As is shown in the present work, in such a case, near T_c, the effects that manifest themselves most strongly in the fluctuation sound absorption are those connected with the dependence of the density of states on the energy. The fluctuation pairing of the electrons does not follow the change in the critical temperature in the compression and rarefraction regions and the arising disequilibrium leads to additional sound absorption in the metals.

In anisotropic metals, such as layered superconductors, there is an additional enhancement of the fluctuations as a consequence of the two-dimensional character of the electron motion.[4] As a result, for layered superconductors with a significant concentration of impurities, the fluctuation contribution to the sound absorption coefficient becomes of the order of the value of the ordinary absorption over a wide temperature range.

2. The sound absorption is determined by the imaginary part of the polarization operator, the fluctuation contributions to which, in first order perturbation theory and with account of electron scattering by the impurities, are represented by the diagrams shown in the drawing. The wavy lines indicate the fluctuation propagator, the solid lines the Green's functions of the normal metal averaged over the position of the impurities; the shaded blocks corresponds to sums of the "ladder" diagrams.[5,6]

Similar diagrams determine the fluctuation conductivity of the metal, while close to the critical temperature the most significant diagrams are the first, which contains twice the fluctuation propagator singular with respect to proximity to the transition temperature,[2] and the second, which makes the anomalous Maki-Thompson contribution.[7,8] The contributions of the remaining diagrams depend weakly on the temperature and their account is necessary only for complete compensation of all contributions at the frequency of the external field $\omega = 0$.

In the calculation of the fluctuation sound absorption, the quantity that is to be correlated with the vertices corresponding to the introduction of the external phonon field is no longer the velocity vector of the electron (as was the case for the vertex of interaction of the electron with the external electromagnetic field in the calculation of the conductivity), but the electron-phonon interaction constant. The scalar nature of such a vertex leads to the result that the contribution of the first diagram turns out to be different from zero only when account is taken of the dependence of the density of states ρ on the energy (it turns out to be proportional to $(\partial\rho/\partial\varepsilon)_{\varepsilon_F}$). However, the corresponding expression for the sound absorption coefficient depends more strongly on the closeness to the critical temperature than in the case of the conductivity. At high concentra-
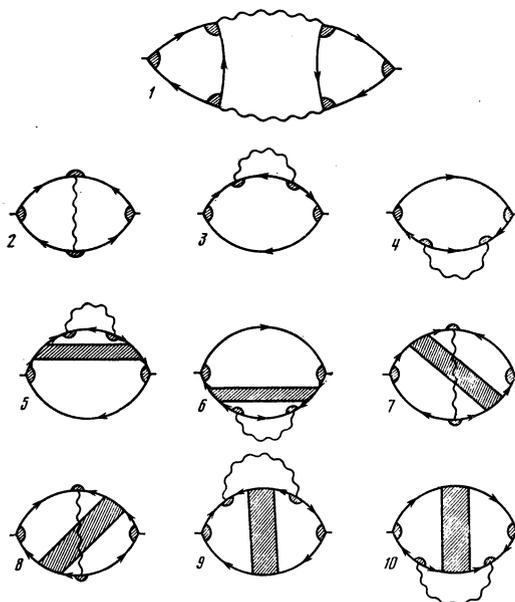

FIG. 1. Diagrams for the polarization operator when account is taken of the fluctuation pairing of electrons and of the scattering from impurities.

tion of the impurities (the free path length l of the electrons is small in comparison with the pair dimension ξ_0), the contribution of the first diagram to the sound absorption increases significantly.

An essential difference between the calculation of the sound absorption and the calculation of the conductivity is also that in the case of high impurity concentration the scalar nature of the external vertices leads to a canceling of the anomalous Maki-Thompson contribution (which stems from the second diagram) by the corresponding contributions of the third and fourth diagrams. Therefore, in the calculation of the fluctuation sound absorption in a dirty metal near the superconducting transition temperature it is sufficient to calculate only the first diagram.

In the limit of a pure metal, diagrams 3 and 4 are small in comparison with the second diagram; however, even in this case, the anomalous Maki-Thompson contribution is already absent. Therefore, in the pure limit we likewise calculate only the first diagram, which gives the strongest temperature dependence.

In the temperature technique[5] the first diagram corresponds to the expression

$$\Pi_{ll}(\mathbf{k}, \omega_v) = -4T \sum_{\Omega_k} \int \frac{d\mathbf{q}}{(2\pi)^3} B^2(\mathbf{k}, \mathbf{q}, \Omega_k, \omega_v) L(\mathbf{q}+\mathbf{k}, \omega_v+\Omega_k) L(\mathbf{q}, \Omega_k), \quad (1)$$

where $\mathbf{k}$ is the sound wave momentum and ω_v is the Matsubara sound frequency, in terms of which it is necessary to carry out analytic continuation into the region of real frequencies. The symbol $B(\mathbf{k}, \mathbf{q}, \Omega_k, \omega_v)$ designates the expression corresponding to a block of three Green's functions, averaged over the location of the impurities; $L(\mathbf{q}, \omega_v)$ is the fluctuation propagator.

In the "ladder" approximation (i.e., neglecting the contribution of diagrams with intersecting dashed lines, a procedure valid when the free path length of the electrons l is large in comparison with the reciprocal of the Fermi momentum p_F^{-1}) the averaging of the Green's functions over the location of the impurities reduces to the replacement of the frequency ε_n by the quantity $\bar{\varepsilon}_n$, which is defined by the expression:[5]

$$\bar{\varepsilon}_n = \varepsilon_n + \frac{1}{2\tau}\text{sign }\varepsilon_n, \quad \frac{1}{\tau} = n\rho\langle|u|^2\rangle, \quad (2)$$

where τ is the time between collisions in the Born approximation, n is the impurity concentration, ρ is the density of states, the brackets $\langle\cdots\rangle$ denote the angular averaging over the Fermi surface, $u(\mathbf{q})$ is the Fourier component of the interaction potential of the electron with the impurity. Renormalization of the vertices in the case of isotropic scattering leads to the following expressions for the vertices λ_1 and λ_2 (they are different in the direction of the arrows):

$$\lambda_1^{-1}(\mathbf{q}, \varepsilon_n, \varepsilon_{n+v}) = 1 + \tau^{-1}\theta(-\varepsilon_n\varepsilon_{n+v})\langle(\mathbf{vq} - i\omega_v - i/\tau)^{-1}\rangle,$$
$$\lambda_2^{-1}(\mathbf{q}, \varepsilon_{n+v}, \Omega_k-\varepsilon_n) = 1 - \frac{\theta[-\varepsilon_{n+v}(\Omega_k-\varepsilon_n)]}{\tau|\bar{\varepsilon}_{n+v} - (\widetilde{\Omega_k-\varepsilon_n})|}\left[1 - \frac{\langle(\mathbf{vq})^2\rangle}{|\bar{\varepsilon}_{n+v} - (\widetilde{\Omega_k-\varepsilon_n})|^2}\right], \quad (3)$$

where $\theta(x)$ is the Heaviside function, $\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}$ is the velocity on the Fermi surface, and the expression for λ^{-1} is written down with accuracy to within terms of order

$\langle(\mathbf{vq})^2\rangle|\bar{\varepsilon}_{n+v} - (\widetilde{\Omega_k-\varepsilon_n})|^{-2}$,

since the momenta that are significant in the subsequent integration satisfy the requirement that this expression be small.

We can obtain the following expression for the fluctuation propagator with account of averaging over the location of the impurities, near T_c:

$$L^R(\mathbf{q}, \omega) = -\rho^{-1}\left[\frac{T-T_c}{T_c} + \frac{3\eta}{v_0^2}\langle(\mathbf{vq})^2\rangle - \frac{i\pi\omega}{8T_c}\right]^{-1},$$
$$\eta = -\frac{v_0^2\tau^2}{3}\left\{\left[\psi\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2}\right)\right] - \frac{1}{4\pi T\tau}\psi'\left(\frac{1}{2}\right)\right\}, \quad (4)$$

which is analytic in the upper half-plane of the complex variable ω; the parameter η is of the order of the square of the dimension of the fluctuation Cooper pair ξ_0, and $\psi(x)$ is the logarithmic derivative of the gamma function.

We calculate the expression for the block of three Green's functions $B(\mathbf{k}, \mathbf{q}, \Omega_k, \omega_v)$, which enters in (1)

$$B(\mathbf{k}, \mathbf{q}, \Omega_k, \omega_v) = gT\sum_{\varepsilon_n}\lambda_1(\mathbf{k}, \varepsilon_{n+v}, \varepsilon_n)\lambda_2(\mathbf{q}+\mathbf{k}, \varepsilon_{n+v}, \Omega_k-\varepsilon_n)\lambda_2(\mathbf{q}, \varepsilon_n, \Omega_k-\varepsilon_n)$$
$$\times\int\frac{d\mathbf{p}}{(2\pi)^3}G(\mathbf{p}+\mathbf{k}, \bar{\varepsilon}_{n+v})G(\mathbf{q}-\mathbf{p}, \widetilde{\Omega_k-\varepsilon_n})G(\mathbf{p}, \bar{\varepsilon}_n), \quad (5)$$

where the electron-phonon interaction constant is denoted by g. Transforming in (5) to integration with respect to the variable $\xi(\mathbf{p})$ and calculating the sum over ε_n, we find that, just as in the pure case,[2] the expression (5) differs from zero only when account is taken of the energy dependence of the density of states:

$$B(\mathbf{k}, 0, \Omega_k, \omega_v) = B_0 + B_1(\mathbf{k}, \Omega_k, \omega_v), \quad B_0 = -\frac{g}{2}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}\left(\ln\frac{\omega_D}{2\pi T} + \frac{\pi}{8T\tau}\right),$$

$$\quad (6)$$

$$B_1(\mathbf{k}, \Omega_k, \omega_v) = -\frac{g}{4}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}\{1 - \langle[1+\tau^2(\mathbf{vk})^2]^{-1}\rangle\}^{-1}$$
$$\times\left\{2\omega_v\tau + \theta(\Omega_k)(1-\Omega_k\tau)\left[\psi\left(\frac{1}{2} + \frac{\Omega_k}{4\pi T} + \frac{\omega_v}{2\pi T}\right) - \psi\left(\frac{1}{2} + \frac{\Omega_k}{4\pi T}\right)\right]\right.$$
$$+ \theta(-\Omega_k)\theta(\Omega_k+\omega_v)\left[\psi\left(\frac{1}{2} + \frac{\Omega_k}{4\pi T} + \frac{\omega_v}{2\pi T}\right) - \psi\left(\frac{1}{2} + \frac{\Omega_k}{4\pi T}\right)\right.$$
$$+ \left.\psi\left(\frac{1}{2} + \frac{\Omega_k}{4\pi T} - \frac{\omega_v}{4\pi T}\right) - \psi\left(\frac{1}{2} - \frac{\Omega_k}{4\pi T} + \frac{\omega_v}{4\pi T}\right)\right] - \theta(-\omega_v-\Omega_k)$$
$$\times(1+\Omega_k\tau)\left[\psi\left(\frac{1}{2} - \frac{\Omega_k}{4\pi T} - \frac{\omega_v}{4\pi T}\right) - \psi\left(\frac{1}{2} - \frac{\Omega_k}{4\pi T} + \frac{\omega_v}{4\pi T}\right)\right]\right\}.$$

The term $B_1(k, \Omega_k, \omega_v)$ takes into account the dependence of the block on the external frequency ω_v. The momentum $\mathbf{q}$ is assumed to be equal to zero, since in the subsequent integration in (1) the following quantities are significant:

$$|\mathbf{q}| \sim \eta^{-1/2}[(T-T_c)/T_c]^{1/2} \ll p_F.$$

As is seen from Eq. (6), in the limit of a large number of impurities ($T_c\tau \ll 1$) the quantity B can become large. This is the difference between the contributions of the considered diagram to sound absorption and to the fluctuation conductivity, where the Green's-function block that determines the transport properties decreases with increase in the impurity concentration.[2]

3. To find the fluctuation sound absorption, we must calculate Eq. (1) for real frequencies ω. Following

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Éliashberg,[9] we replace the sum over Ω_k in (1) by the integral along the corresponding contour. Here the complex plane is divided into three regions, in each of which the analytic properties of the integrand in (1) are different. Transforming to integration along the real axis with the help of a shift in the variable, we obtain

$$\Pi_{fl} = -\frac{2B_0^2}{\pi}\int\frac{dq}{(2\pi)^3}\int_{-\infty}^{\infty}\operatorname{cth}\frac{z}{2T}[L^R(z+i\omega_\nu,q)$$

$$+L^A(z-i\omega_\nu,q)]\operatorname{Im}L^R(z,q)\,dz-\frac{2B_0}{\pi i}\int\frac{dq}{(2\pi)^3}\int_{-\infty}^{\infty}\operatorname{cth}\frac{z}{2T}$$

$$\times\Big\{B_1^R(k,z,\omega_\nu)L^R(z+i\omega_\nu,q+k)L^R(z,q)$$

$$-B_1^A(k,z-i\omega_\nu,\omega_\nu)L^A(z,q+k)L^A(z-i\omega_\nu,q)\Big\}dz. \quad (7)$$

The analytic continuation of the expression (7) is accomplished by the substitution $i\omega_\nu\to\omega$. Using the expressions (4) for the fluctuation propagator and formula (6) for $B_1(k,\Omega_k,\omega_\nu)$ (a term with the corresponding θ function is chosen in each region of analyticity), after expansion in ω we have

$$\operatorname{Im}\Pi_{fl} = -\frac{\omega B_0^2}{2T\rho^2}\int\frac{dq}{(2\pi)^3}\int_{-\infty}^{\infty}\operatorname{cth}\frac{z}{2T}\frac{[\beta^2(q)-\pi^2z^2/64T_c^2]}{[\beta^2(q)+\pi^2z^2/64T_c^2]^3}$$

$$\times\frac{\pi z}{8T_c}dz+\frac{\omega B_0}{4T\rho^2}g\Big(\frac{\partial\rho}{\partial\varepsilon}\Big)_{\varepsilon_F}\frac{3\eta\langle(vk)^2\rangle}{v_0^2}[1-\langle[1+\tau^2(vk)^2]^{-1}\rangle]^{-1}$$

$$\times\int\frac{dq}{(2\pi)^3}\int_{-\infty}^{\infty}\operatorname{cth}\frac{z}{2T}\frac{[3\beta^2(q)-\pi^2z^2/64T_c^2]}{[\beta^2(q)+\pi^2z^2/64T_c^2]^3}\frac{\pi z}{8T_c}dz,$$

where $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (8)$

$$\beta(q)=\frac{T-T_c}{T_c}+\frac{3\eta}{v_0^2}\langle(vq)^2\rangle.$$

The integrands in (8) have a pole at $|z|\ll1$. Using this, it is easy to carry out the integration over z:

$$\operatorname{Im}\Pi_{fl}=-\frac{\pi\omega}{16\rho^2}g^2\Big(\frac{\partial\rho}{\partial\varepsilon}\Big)_{\varepsilon_F}^2\gamma(\tau)\int\beta^{-3}(q)\frac{dq}{(2\pi)^3}, \quad (9)$$

where

$$\gamma(\tau)=\Big(\ln\frac{\omega_D}{2\pi T_c}+\frac{\pi}{8T_c\tau}\Big)\Big(\ln\frac{\omega_D}{2\pi T_c}+\frac{5\pi}{8T_c\tau}\Big).$$

Further calculation of the fluctuation sound absorption coefficient according to formula (9) depends on the specific form of the electron spectrum. Layered metals usually consist of metallic layers of thickness of several angstroms each, separated by layers of organic materials of approximately the same thickness. If the probability of electron jumps between the layers is not too great, then the Fermi surface is a corrugated cylinder:

$$\varepsilon_p-\varepsilon_F=v_0(|p_\perp|-p_F)+w\cos(p_za), \quad (10)$$

where $p_\perp$ is the projection of the momentum of the electron in the plane of the layer, v_0 and p_F are the velocity and Fermi momentum in this direction, p_z is the momentum of the electron in the perpendicular direction, a is the distance between the layers, and w/ε_F is the small corrugation parameter (however, w should be larger than T_c in order that the expression (4) for the fluctuation propagator, calculated in the self-consistent field approximation, be valid). In such a case,

$$\langle(vq)^2\rangle={}^1\!/_2v_0^2q_\perp^2+2w^2\sin^2(q_za/2). \quad (11)$$

Carrying out the integration in the expression (9) with account taken of the spectrum (10), we get

$$\operatorname{Im}\Pi_{fl}=-\frac{\omega g^2}{3\cdot2^7a\eta}\frac{1}{\rho^2}\Big(\frac{\partial\rho}{\partial\varepsilon}\Big)_{\varepsilon_F}^2\gamma(\tau)\Big[\Big(\frac{T-T_c}{T_c}\Big)^{-\frac{1}{2}}\Big(\frac{T-T_c}{T_c}+\frac{6\eta w^2}{v_0^2}\Big)^{-\frac{1}{2}}$$

$$+\Big(\frac{T-T_c}{T_c}\Big)^{-\frac{1}{2}}\Big(\frac{T-T_c}{T_c}+\frac{6\eta w^2}{v_0^2}\Big)^{-\frac{1}{2}}\Big]. \quad (12)$$

The dependence of the fluctuation sound absorption on the phonon momentum k is not taken into account in (12), a procedure valid if the temperature is not too near the transition temperature:

$$(T-T_c)/T_c\gg(\eta/v_0^2)\langle(vk)^2\rangle.$$

We also note that the effects of dragging of the impurities by the sound wave[10] appear in the terms having a weaker temperature singularity upon approach to T_c.

It is easy to see from the expression (12) that different temperature regimes in the fluctuation sound absorption are obtained, depending on the relation between the parameters $(T-T_c)/T_c$ and $6\eta w^2/v_0^2$. The parameter $\delta_0^2=6\eta w^2/v_0^2$ thus characterizes the "dimensionality" of the fluctuations. The two-dimensional behavior of the fluctuations:

$$\operatorname{Im}\Pi_{fl}\sim\Big(\frac{T_c}{T-T_c}\Big)^2$$

corresponds to the case $(T-T_c)/T_c\gg\delta_0^2$ (under the condition that $\delta_0^2\ll1$, otherwise we go outside the framework of the assumptions made on the closeness to the transition point). Here the dimension of the pair in the direction perpendicular to the layers,

$$\xi_z=\frac{wa}{v_0}\xi_\perp\sim\frac{wa}{v_0}\Big(\eta\frac{T_c}{T-T_c}\Big)^{\frac{1}{2}}$$

is less than the distance between the layers and the correlations of the electrons arise only within each layer. In the opposite limiting case $(T-T_c)/T_c\ll\delta_0^2$ the correlation length ξ_z becomes greater than the distance between the layers, and electrons from different layers can take part in the fluctuation pairings. The obtained temperature dependence is then the same as in the isotropic metal:

$$\operatorname{Im}\Pi_{fl}\sim(T_c/(T-T_c))^{\frac{3}{2}}.$$

The amplification of the absorption by a factor of $v_0/wa\sim\varepsilon_F/w$ in comparison with the isotropic case[2] is connected with the two-dimensional character of the motion of the electrons.

As is seen from formula (4), the parameter η falls off with increase in the impurity concentration and, consequently, the region of two-dimensional behavior is widened. The very value of the fluctuation sound absorption, which is determined by Eq. (4), increases in this case.

4. To find the relative contribution of the fluctuations, it is necessary to calculate the value of the sound absorption coefficient in the normal metal with spectrum (10). Following the work of Tsuneto,[10] we obtain

$$\Pi_n=-2i\omega\rho g^2\tau\Big[\frac{I_0}{1-I_0}-\frac{2}{(v_0k_z\tau)^2+(wak_z\tau)^2}\Big], \quad (13)$$

$$I_0=\langle[1+i\tau(vk)]^{-1}\rangle.$$

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Carrying out angular averaging over the Fermi surface, we can obtain an expression for I_0 under various limiting conditions. Thus, for a dirty metal ($\tau^{-1} \gg \max\{v_0 k_\perp, wak_z\}$) we have

$$\Pi_x = -\tfrac{1}{2}i\omega\rho g^2\tau. \qquad (14)$$

In the limit of a pure metal ($\tau^{-1} \ll \min\{v_0 k_\perp, wak_z\}$) we obtain

$$\Pi_x = -2i\omega\rho g^2(k_\perp^2 v_0^2 + w^2 a^2 k_z^2)^{-1/2}. \qquad (15)$$

Thus, in a pure layered metal, the sound absorption is anisotropic, while formula (15) is valid also in cases in which the momentum of the sound wave has a component only in the plane of the layers: $wak_z \ll \tau^{-1} \ll v_0 k_\perp$, and when the direction of propagation of the sound wave is perpendicular to the layers $v_0 k_\perp \ll \tau^{-1} \ll wak_z$.

Comparing Eqs. (12) and (13), we can obtain the expression for the relative contribution of the fluctuations to the sound absorption. In the pure metal $\eta = 7\zeta(3)v_0^2/48\pi^2 T_c^2$ and at not too low probability of electron jumps between layers ($w > T_c$), the fluctuation sound absorption near T_c is everywhere described by a three-dimensional law. For the ratio of the sound absorption coefficients in the pure case, we have

$$\frac{\alpha_{fl}^{III\,(p)}}{\alpha_n^{(p)}} = \frac{\pi^4}{2^{7/2}[7\zeta(3)]^{3/2}}\rho^{-2}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}^2 \ln^2\left(\frac{\omega_D}{2\pi T_c}\right)$$
$$\times \frac{T_c^3}{\varepsilon_F w}(v_0^2 k_\perp^2 + w^2 a^2 k_z^2)^{1/2}\left(\frac{T_c}{T-T_c}\right)^{1/2}. \qquad (16)$$

As is seen from (16), even at the maximum obtainable value of the phonon momentum ($\eta\langle(\mathbf{v}\cdot\mathbf{k})^2\rangle/v_0^2 \sim (T-T_c)/T_c$) the fluctuation correction becomes of the order of the value of the normal absorption only in the region of temperatures

$$\frac{T-T_c}{T_c} \sim \left(\frac{T_c}{\varepsilon_F}\right)^3\left(\frac{T_c}{w}\right)\ln^2\frac{\omega_D}{2\pi T_c}.$$

The two-dimensional character of the motion of the electrons leads as usual to a broadening of this region by a factor ε_F/w in comparison with the isotropic case.[2] Nevertheless, this region is very narrow in the case of a pure metal.

As the free path length of the electrons l decreases it can become less than the reciprocal of the momentum of the phonon, but still remain larger than the dimension of the pair. Then the sound absorption in the normal metal is determined by Eq. (14), while the fluctuation absorption is still described by a three-dimensional law with the quantity η corresponding to the pure case:

$$\frac{\alpha_{fl}^{III\,(p)}}{\alpha_n^{(d)}} = \frac{\pi^4}{2^{7/2}[7\zeta(3)]^{1/2}}\rho^{-2}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}^2 \ln^2\frac{\omega_D}{2\pi T_c}\left(\frac{T_c^3}{\varepsilon_F w\tau}\right)\left(\frac{T_c}{T-T_c}\right)^{1/2}. \qquad (17)$$

We obtain here the result that the fluctuations become important over a much wider region of temperatures than in the previous case.

At electron free path lengths smaller than the dimensions of the pair, if the probability of electron jumps is not very small ($w > \tau^{-1/2}(T-T_c)^{1/2}$), the fluctuation absorption is described as before by a

three-dimensional law; however, now $\eta = \pi\tau v_0^2/24T_c$ corresponds to the dirty case

$$\frac{\alpha_{fl}^{III\,(d)}}{\alpha_n^{(d)}} = \frac{5\pi^{3/2}}{2^8}\rho^{-2}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}^2 \frac{1}{wT_c^{1/2}\varepsilon_F\tau^{5/2}}\left(\frac{T_c}{T-T_c}\right)^{1/2}. \qquad (18)$$

The fluctuation region is determined by the temperatures

$$\frac{T-T_c}{T_c} < \left(\frac{\varepsilon_F}{T_c}\right)\left(\frac{T_c}{w}\right)^{1/2}\frac{1}{(\varepsilon_F\tau)^3}$$

and becomes readily observable in experiment.

Finally, in the very dirty case $\tau < (T-T_c)w^{-2}$ the fluctuations become two-dimensional and the temperature dependence of the fluctuation sound absorption becomes stronger:

$$\frac{\alpha_{fl}^{II\,(d)}}{\alpha_n^{(d)}} = \frac{5\pi^2}{2^8}\rho^{-2}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}^2 \frac{1}{T_c\varepsilon_F\tau^4}\left(\frac{T_c}{T-T_c}\right)^2. \qquad (19)$$

In this case the fluctuation effects are important over a wide range of temperatures, which can even be of the order of T_c.

We now give the formula for the fluctuation sound absorption in a dirty metal with an isotropic spectrum:

$$\frac{\alpha_{fl}^{III}}{\alpha_n^{III}} = \frac{3^{1/2}5^2\pi^{5/2}}{2^{16}\cdot2^{1/2}}\left(\frac{\varepsilon_F}{T_c}\right)^{1/2}\frac{1}{(\varepsilon_F\tau)^{5/2}}\left(\frac{T_c}{T-T_c}\right)^{1/2}, \qquad (20)$$

where $\alpha_n^{III} = -\tfrac{4}{5}\pi^{-2}i\omega m p_F g^2\tau$ is the sound absorption coefficient in the normal metal with an isotropic spectrum in the limit of a large number of impurities.[11]

Thus, at a small mean free path length of the electrons, the fluctuation effects in sound absorption are significant over wide temperature range.

We also discuss the limitations on the possible momenta of the phonons, and the free path lengths necessary for validity of the obtained formulas (they arise because of failure to take into account the dependence of the fluctuation sound absorption on the momentum of the phonon: $(T-T_c)/T_c > \eta\langle(\mathbf{v}\cdot\mathbf{k})^2\rangle/v_0^2$). These conditions depend on the direction of propagation of the sound. In the case of sound propagation perpendicular to the plane of the layers, the limitation on the momentum of the phonon (or on the region of accessible temperatures) has the form

$$k_z < \left(\eta\frac{T_c}{T-T_c}\right)^{-1/2}\left(\frac{\varepsilon_F}{w}\right)$$

and turns out to be weaker than the similar condition for sound propagating along the layer:

$$k_\perp < \left(\eta\frac{T_c}{T-T_c}\right)^{-1/2}.$$

However, even the latter condition is satisfied in the experimentally accessible range of temperatures $(T-T_c)/T_c > 10^{-4}$ up to acoustic frequencies $\omega < 10$ MHz (i.e., throughout the entire ultrasonic region).

The fluctuations are the strongest in the two-dimensional case in a dirty metal. Here the free path length should be such that the dimensions of the pair in the direction perpendicular to the plane of the layers are small in comparison with the distance between the

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layers. Moreover, the electron free path length should be small in comparison with the reciprocal of the momentum of the phonon.

The first condition means: $l < \xi_0 (T_c/w)^2 (T - T_c)/T_c$; satisfaction of the second condition depends on the direction of sound propagation. For sound propagating in the plane of the layers, the inequality $l < k_1^{-1}$ is always satisfied if the first condition is satisfied. In the case of sound propagation perpendicular to the plane of the layers, the same situation holds at sufficiently high probability of jumps between the layers $\omega > \varepsilon_F^{1/2}(T - T_c)^{1/2}$. In the opposite case, a more severe limitation on the free path length is necessary for satisfaction of Eq. (20) in any direction:

$$l < \xi_0 (w/\varepsilon_r)^2 (T_c/(T-T_c)).$$

Thus, the temperature dependence of α_{fi}/α_n has the following form in a dirty layered metal. Upon approach to T_c such that $\delta_0^2 < (T - T_c)/T_c < 1$ (under the condition that the quasi-two-dimensionality parameter $\delta_0^2 = \pi \tau w^2 / 4T_c \ll 1$), the fluctuations have a two-dimensional character and correspond to sound scattering by the fluctuation pairs in each layer separately. The corresponding temperature dependence $\sim (1 - T/T_c)^{-2}$. Upon further decrease in the temperature: $(T - T_c)/T_c < \delta_0^2$, electrons from different layers take part in the fluctuation pairing, the fluctuation picture becomes three-dimensional and, although the closeness to the two-dimensional region causes the absolute value of α_{fi}/α_n to exceed the corresponding quantity in two-dimensional region, the growth of α_{fi}/α_n now takes place according to the much slower temperature law: $\sim (1 - T_c/T)^{-3/2}$.

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Domain structure of ferroelectric materials

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It is pointed out that in uniaxial ferroelectric materials the domain structure should be branched. The parameters of the simplest structure are determined.

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For uniaxial ferroelectric materials, the domain structure usually discussed in the literature is that proposed by Mitsui and Furuichi[1] (see Fig. 1). It is essentially the usual Kittel structure, in which, however, allowance is made for the difference of the polarization P in this case from its nominal value as a result of polarizability in the field produced by the structure near the exit of domains to the surface of the crystal. In Ref. 1 the susceptibility α_{ik} ($P_i = \alpha_{ik} E_k$) was regarded as constant, independent of the electric field E. This limitation actually produces results, obtained in Ref. 1, that are inapplicable to ferroelectric materials. In fact, the depolarizing field near the exit of domains to the crystal surface is $4\pi P$ in order of magnitude; and since the susceptibility of ferroelectric materials is large, the polarization P here will differ significantly from the nominal value P_0. Under such conditions, there is no reason why α should not depend on E. Furthermore, according to Ginzburg's theory[2] states with $P < P_0/3^{1/2}$ are completely unstable; and they necessarily occur, for a large value of the susceptibility, even in a small depolarizing field.

This situation leads to the impossibility of existence of the domain structure of Fig. 1 and of other similar structures, with a large distance between the places of exit of domain boundaries to the surface, in ferroelectric materials. Avoidance of a large value (of order $4\pi P$) of the depolarizing field is possible only in a branched structure. But the structure proposed in Ref. 1 can exist in pyroelectric materials in which the susceptibility is small.

We shall consider the simplest branched structure (Fig. 2). Such a branching scheme was proposed by