layers. Moreover, the electron free path length should be small in comparison with the reciprocal of the momentum of the phonon.

The first condition means: $l < \xi_0 (T_c/w)^2 (T - T_c)/T_c$; satisfaction of the second condition depends on the direction of sound propagation. For sound propagating in the plane of the layers, the inequality $l < k_1^{-1}$ is always satisfied if the first condition is satisfied. In the case of sound propagation perpendicular to the plane of the layers, the same situation holds at sufficiently high probability of jumps between the layers $\omega > \varepsilon_F^{1/2} (T - T_c)^{1/2}$. In the opposite case, a more severe limitation on the free path length is necessary for satisfaction of Eq. (20) in any direction:

$$l < \xi_0 (w/v_F)^2 (T_c/(T - T_c)).$$

Thus, the temperature dependence of α_{fl}/α_n has the following form in a dirty layered metal. Upon approach to T_c such that $\delta_0^2 < (T - T_c)/T_c < 1$ (under the condition that the quasi-two-dimensionality parameter $\delta_0^2 = \pi^7 w^2/4T_c \ll 1$), the fluctuations have a two-dimensional character and correspond to sound scattering by the fluctuation pairs in each layer separately. The corresponding temperature dependence $\sim (1 - T/T_c)^{-2}$. Upon further decrease in the temperature: $(T - T_c)/T_c < \delta_0^2$, electrons from different layers take part in the fluctuation pairing, the fluctuation picture becomes three-dimensional and, although the closeness to the two-dimensional region causes the absolute value of α_{fl}/α_n to exceed the corresponding quantity in two-dimensional region, the growth of α_{fl}/α_n now takes place according to the much slower temperature law: $\sim (1 - T_c/T)^{-3/2}$.

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Domain structure of ferroelectric materials

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It is pointed out that in uniaxial ferroelectric materials the domain structure should be branched. The parameters of the simplest structure are determined.

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For uniaxial ferroelectric materials, the domain structure usually discussed in the literature is that proposed by Mitsui and Furuichi[1] (see Fig. 1). It is essentially the usual Kittel structure, in which, however, allowance is made for the difference of the polarization P in this case from its nominal value as a result of polarizability in the field produced by the structure near the exit of domains to the surface of the crystal. In Ref. 1 the susceptibility α_{ik} ($P_i = \alpha_{ik} E_k$) was regarded as constant, independent of the electric field $\mathbf{E}$. This limitation actually produces results, obtained in Ref. 1, that are inapplicable to ferroelectric materials. In fact, the depolarizing field near the exit of domains to the crystal surface is $4\pi P$ in order of magnitude; and since the susceptibility of ferroelectric materials is large, the polarization P here will differ significantly from the nominal value P_0. Under such conditions, there is no reason why α should not depend on $\mathbf{E}$. Furthermore, according to Ginzburg's theory[2] states with $P < P_0/3^{1/2}$ are completely unstable; and they necessarily occur, for a large value of the susceptibility, even in a small depolarizing field.

This situation leads to the impossibility of existence of the domain structure of Fig. 1 and of other similar structures, with a large distance between the places of exit of domain boundaries to the surface, in ferroelectric materials. Avoidance of a large value (of order $4\pi P$) of the depolarizing field is possible only in a branched structure. But the structure proposed in Ref. 1 can exist in pyroelectric materials in which the susceptibility is small.

We shall consider the simplest branched structure (Fig. 2). Such a branching scheme was proposed by

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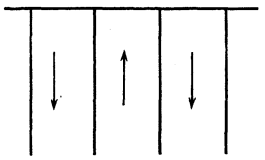

FIG. 1.

Privorotskiĭ[3] for the case of ferromagnetic materials. Here we shall determine the parameters of this structure.

The first wedges originate, in each domain, at a certain definite distance (see below) from the crystal surface. The next branching occurs when the wedge attains a width equal to $\frac{1}{3}$ the period a of the structure. Thereafter, everything is repeated on a reduced scale, until the dimensions of the wedges become of the order of magnitude of the thickness of a domain wall, at the very surface of the crystal.

The wedges are extremely elongated; and if we neglect distortions in the regions where new wedges are being produced, we can easily find the form of the wedges from the requirement that the electric-field energy be a minimum. This problem was solved by Lifshitz,[4] who was investigating a domain structure with a single wedge in ferromagnets. We get for the electric-field energy, taken over the first branching in each domain,

$$\lambda P_0^2 \frac{a^3}{h_1}, \quad \lambda = \frac{2^6\pi}{h_1}\left(\frac{1}{2^{1/3}}-\frac{1}{3^{1/3}}\right)^2, \tag{1}$$

where h_1 is the height of the first wedge. The value of h_1 is determined by the condition that the sum of the energy (1) and of the energy of the domain boundaries must be a minimum. To each domain belong three boundaries (until the second branching); and if we neglect the small difference of the length of the wedge boundary from h_1 and also the small difference of the value of the surface tension of the slightly inclined boundaries from the energy Δ of ordinary boundaries, parallel to the spontaneous polarization, then the total energy of the boundaries for each domain is $3\Delta h_1$. The minimum is attained when

$$h_1 = (\lambda P_0^2 a^3/3\Delta)^{1/3}.$$

For the ith branching, the same formula is valid if a is replaced by $a_i = a/3^{i-1}$. Summation over all the branchings gives for the exit energy the value

$$\frac{6\lambda^{1/3}}{3^{1/3}-1} P_0 \Delta^{1/3} a^{1/3}. \tag{2}$$

The energy density of the domain structure in a plate of thickness L is

$$\left(1-2\frac{h}{L}\right)\frac{\Delta}{2a}+\frac{6\lambda^{1/3}}{3^{1/3}-1}\frac{P_0}{L}(\Delta a)^{1/3}, \tag{3}$$

where h is the depth of penetration of the branching into the domains:

$$h=\sum_{i=1}^{\infty} h_i = \frac{3}{3^{1/3}-1}\left(\frac{\lambda}{\Delta}\right)^{1/3} P_0 a^{1/3}.$$

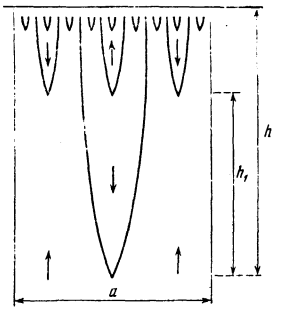

FIG. 2.

The first term in the expression (3) is the energy of the domain boundaries; the second, the exit energy. On minimizing with respect to a, we get

$$a=(\Delta L^2/\gamma^2 P_0^2)^{1/3} \approx 0{,}31\,(\Delta L^2/P_0^2)^{1/3}, \tag{4}$$

where

$$\gamma = 8\pi^{1/3}\left(\frac{1}{2^{1/3}}-\frac{1}{3^{1/3}}\right)\left(\frac{2}{3^{1/3}-1}-\frac{1}{3^{1/3}-1}\right).$$

The depth of penetration h is

$$h=\frac{3^{1/3}-1}{5\cdot 3^{1/3}-1} L \approx 0.096 L.$$

We have completely neglected conductivity, since only then can one speak of a thermodynamic-equilibrium domain structure. In the presence of finite conductivity, any domain structure must disappear in a state of complete equilibrium. The various structures that are usually observed are apparently metastable states. Thus, for example, if conductivity is "switched on" in the structure that we have considered, the presence of an electric field where domains exit to the surface of the crystal will produce a flow of charge, and this may lead to a displacement of the walls of the branching structure–to "erasure" of the branching; but the walls between the domains will remain, and the structure will ultimately have the form shown in Fig. 1 but with surface charge and without a depolarizing field.

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