Phase transition in a spin glass

M. V. Feĭgel'man and A. M. Tsvelik

L. D. Landau Institute for Theoretical Physics, Academy of Sciences of the USSR
(Submitted 16 July 1979)
Zh. Eksp. Teor. Phys. 77, 2524–2538 (December 1979)

We construct a diagram technique for a spin glass in the vicinity of its transition without using the replica method. We observe a strong interaction of the large-scale longitudinal gapless modes in the low-temperature phase. The lower critical dimensionality of the theory is $d = 4$. When $4 < d < 6$ the critical indices determine the magnetic susceptibility and the specific heat [A. B. Harris, T. C. Lubensky, and J. H. Chen, Phys. Rev. Lett. 36, 415 (1976)]. We show that there is no transverse spin stiffness in a Heisenberg spin glass.

PACS numbers: 75.40.Fa

1. INTRODUCTION

The problem of the existence and the properties of a phase transition in the spin glass state has in recent years been studied intensively both theoretically and experimentally. Recent experiments[1] and computer simulations[2] show that the kink in the magnetic susceptibility observed by Canella and Mydosh[3] depends on the time of observation and does, therefore, not correspond, apparently, to a phase transition being present. Nonetheless, something like a phase transition is going on in the systems considered and a theoretical study of phase transitions in model spin glasses is therefore of interest, at least as a first approximation to an understanding of the crux of the matter.

We give a survey of the main theoretical papers on phase transitions in a spin glass. Edwards and Anderson[4] observed a phase transition in the framework of a self-consistent field theory in a system of spins with random alternating exchange interactions. The phase transition is connected with the occurrence of average coordinate-dependent spin values $\langle \mathbf{S}_i \rangle \neq 0$ while the average magnetic moment vanishes: $\overline{\langle \mathbf{S}_i \rangle} = 0$ (here and henceforth pointed brackets indicate thermodynamics averages and a bar averaging over inhomogeneities). They proposed as an order parameter, characterizing the transition, the quantity

$$\lim_{t\to\infty} \overline{\langle \mathbf{S}_i(0)\mathbf{S}_i(t)\rangle} = Q = \overline{\langle \mathbf{S}_i \rangle^2},$$

which characterizes the extent to which the spins are frozen in. Edwards and Anderson used the so-called replica method which makes it possible formally to average over random parameters of the system before taking the thermodynamic average. Using the same method, Harris, Lubensky, and Chen[5] showed that the mean field theory for a spin glass is valid only when the spatial dimensionality $d > 6$, and they evaluated critical indices in a $6-\epsilon$-expansion. Harris and Fish,[6] and also Reed[7] using an analysis of high-temperature series expansions, showed that the phase transition in the Edwards-Anderson model with nearest-neighbor interactions vanishes when $d < 4$, i.e., the lower critical (marginal) dimensionality of the theory is $d_c = 4$. Unfortunately, this method does not enable us to explain the physical reason for the disappearance of the phase transition; in particular, the magnitude of d_c remains unknown for a real spin glass with RKKY exchange. To elucidate these problems it is necessary to study the behavior of the correlation functions in the low-temperature phase, in a similar way as was done for ordered systems.[8]

A study of the low-temperature phase in the framework of the replica method has met with serious difficulties; it turned out that the equation of state obtained by Sherrington and Kirkpatrick[9] which corresponds to the mean field approximation is unstable[10,11] [one of the correlation functions in their solution[9] has a negative gap $\propto (T-T_c)^2$]; moreover, the instability occurs already in first order in $|T-T_c|$ when fluctuations are taken into account.[12] Bray and Moore[11,12] observed that the Hamiltonian of the replica method has a solution different from the one given in Ref. 9, and obtained a solution stable to order $(T-T_c)^2$,[11] and in first order in the fluctuations.[12] The most important singularity of this solution is the presence of a *longitudinal* gapless mode of fluctations which leads to a divergence of the first correction to the order parameter when $d \leqslant 4$.[12]

It is apparently extremely difficult to obtain an exact proof of the existence of a gapless mode to all orders in $|T-T_c|$ and in the fluctuations in the framework of the replica method. Bray and Moore[13] used the non-averaged self-consistent field equations of Thouless, Anderson, and Palmer (TAP)[14] to show that the presence of local soft modes in an Ising spin glass is uniquely connected with the existence of the gapless correlation function found in Ref. 11. Using a numerical solution of the TAP equations they showed in the same paper the existence of the soft modes. The existence of gapless longitudinal fluctuations in a spin glass is thus firmly established (at least in the self-consistent field approximation).

In the present paper we consider the Anderson-Edwards model with a Gaussian random distribution of the exchange integrals. We construct (without using the replica method) a diagram technique which describes an Ising spin glass in the vicinity of a phase transition—section 2 of the paper is devoted to an exposition of this technique. Section 2 also contains an evaluation of the critical indices in the paramagnetic phase. The results are the same as the ones obtained in Ref. 5 by the replica method.

In section 3 we consider in the self-consistent field method framework the low-temperature phase. We

 0038-5646/79/121222-07$02.40

show that the standard method of introducing condensate averages leads to the same difficulties as in Refs. 9–11; we propose a method which gives a correct expansion of the Green function in terms of the order parameter Q. Together with the condition $G^{-1}(q=0)=0$ for there being no gap this expansion determines the equation of state which is the same as the one obtained by Bray and Moore.[13]

In section 4 we study the effect of fluctuations on the behavior of the low-temperature phase. The advantage of the proposed diagram technique manifests itself in that case in the fact that taking fluctuations into account does not lead in the leading order in $|T-T_c|$ to the appearance of a negative gap, even if we use the standard method for introducing the condensate. The lower critical dimensionality of the theory turns out to equal $d_c=4$, as in Bray and Moore's theory.[12]

We observe in 6-dimensional space a strong dependence of the long-wavelength ($q^2 \ll Q$) fluctuations leading to an increase of the effective charge at large distances ("asymptotic freedom"). Because of that we cannot determine exactly the form of the correlation functions at large distances. It turns out, however, that the temperature dependence of the order parameter $Q(\tau)$ and the specific heat are determined by the momenta $q^2 \sim Q$ and can thus be obtained from the critical indices found in the parametric phase.

We show in section 5 that there is no transverse spin stiffness in a Heisenberg spin glass (in the self-consistent field approximation); a detailed analysis of the Heisenberg glass will be given in subsequent papers. Section 6 contains a discussion of the results; we show that the limitation to the Edwards-Anderson model is unimportant and all qualitative conclusions are retained for a real spin glass with RKKY interaction.

2. SPIN GLASS IN THE PARAMAGNETIC REGION

In this section we describe a diagram technique for a spin glass at $T>T_c$ (T_c is the transition temperature) and we evaluate scaling dimensionalities using a $6-\varepsilon$ expansion.

The partition function of the Anderson-Edwards model has the form

$$Z=\sum_{S_i=\pm 1}\exp\left(\frac{1}{2}\beta\sum_{\langle ij\rangle}J_{ij}S_iS_j+\beta\sum_i S_ih_i\right), \tag{1}$$

where i and j are sites of the regular lattice on which Ising spins are positioned, $J_{ij}\neq 0$ only for nearest neighbors. The quantities J_{ij} are random quantities and their distribution is Gaussian:

$$P\{J_{ij}\}=\prod_{ji}(2\pi J^2)^{-1/2}\exp(-J_{ij}^2/2J^2). \tag{2}$$

To describe the properties of our model we use an Ising model diagram technique.[5] The zero Green function is in that technique a chain of exchange integrals:

$$g_{ij}=\sum_{n=0}\beta^n(J_{ik_1}J_{k_1k_2}\dots J_{k_nj}).$$

This chain is depicted in Fig. 1. When this is necessary we shall put points on the lines corresponding to sites k_i which are connected by the exchange integrals.

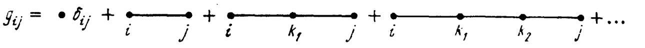

FIG. 1.

In the proposed diagram technique there is an infinite set of vertices and the n-th order vertex, into which n lines g converge is

$$\frac{1}{n!}\frac{\partial^{(n)}}{\partial x^n}\ln(\operatorname{ch} x)|_{x=0}.$$

We shall evaluate the functions

$$G_{ik}=\overline{\langle\langle S_iS_k\rangle\rangle^2},\quad K_{ik}=\overline{\langle\langle S_i^2S_k^2\rangle\rangle}$$

(where the symbol $\langle\langle\dots\rangle\rangle$ indicates irreducible correlators). The singularity corresponding to a phase transition manifests itself just in these. In the paramagnetic region $\langle S_i\rangle=0$ and by virtue of the definition of the irreducible correlators

$$K_{ik}=-2G_{ik}. \tag{3}$$

We must write down an expansion of the correlation functions for a given realization $\{J_{ij}\}$ and afterwards average them over the distribution (2). For a Gaussian distribution the average of any set of exchange integrals splits into a product of pair averages which will be indicated by a dash line in the diagrams.

The bare correlator G_0 is shown in Fig. 2—this is a double chain with successive averaging. Changing to the momentum representation we get

$$G_0(\mathbf{k})=\frac{1}{1-J^2(\mathbf{k})/T^2}\approx\frac{1}{\mathbf{k}^2+\tau}; \tag{4}$$
$$\tau=\frac{T^2}{zJ^2}-1=\frac{T^2}{T_c^2}-1,\quad J^2(\mathbf{k})=J^2\sum_{(\mathbf{n})}e^{i\mathbf{kn}}.$$

In the approximation $z\gg 1$ the transition temperature is thus $T_c=Jz^{1/2}$ (z is the number of nearest neighbors). The exact correlator $G(\mathbf{k})$ can be expressed in terms of the self-energy part $\Sigma(\mathbf{k})$ which is not cut along a pair of lines $\overline{J_{ij}^2}$:

$$G(\mathbf{k})=\Sigma(\mathbf{k})\,(1-J^2(\mathbf{k})\Sigma(\mathbf{k})/T^2)^{-1}. \tag{5}$$

The diagrammatic series for Σ when $T>T_c$ is shown in Fig. 3. The terms within the square brackets are of order of smallness $1/z$ and can be neglected in the self-consistent field approximation.

We note that we drop everywhere "finger" type diagrams—see Fig. 4, which cancel exactly when we sum. Diagrams a, b of Fig. 4 may serve as an illustration of this statement, and an exact proof of it is the following one: all fingers are renormalized points on the line $\langle\langle S_iS_k\rangle\rangle$, i.e., they are an expansion of the correlator $\langle\langle S_n^2\rangle\rangle=1-\langle S_n\rangle^2$. Above the transition point $\langle S_n\rangle=0$ and $\langle\langle S_n^2\rangle\rangle=1$, i.e., the point is not renormalized. This means also that all fingers cancel.

We note that in the diagram technique for a ferromagnetic in the renormalization of the transition point there are only graphs of the kind of Fig. 4b present so that the renormalized point is not $\langle\langle S_n^2\rangle\rangle$.

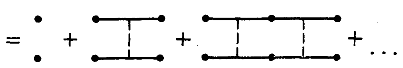

FIG. 2.

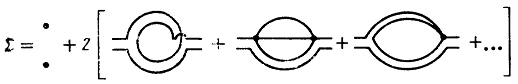

FIG. 3.

It is clear from Fig. 3 that in our diagram technique there are triple vertices—this means that the theory will be logarithmic in 6-dimensional space and we apply the self-consistent field method for $d > 6$. An arbitrary graph for Σ is obtained as follows: we select a line g_{kl} from one correlator $\langle\langle S_i S_j \rangle\rangle$ and pair it with the same line from another core. One can easily formalize this process by taking the variation

$$\delta\langle\langle S_i S_j\rangle\rangle/\delta(J_{kl}\beta)=\langle\langle S_i S_j S_k S_l\rangle\rangle+\langle S_k\rangle\langle\langle S_i S_j S_l\rangle\rangle+\langle S_l\rangle\langle\langle S_i S_j S_k\rangle\rangle$$
$$+\langle\langle S_i S_k\rangle\rangle\langle\langle S_j S_l\rangle\rangle+\langle\langle S_i S_l\rangle\rangle\langle\langle S_j S_k\rangle\rangle.$$

The quantity $\delta\langle\langle S_i S_j\rangle\rangle$ is graphically represented in Fig. 5. An open circle denotes an average spin and a hashed polygon irreducible correlators.

In the high-temperature phase $\langle S_n\rangle = 0$, the diagram of Fig. 5d goes into G_0 and, hence, there remain only the graphs of Fig. 5a, e which give the diagrams of Fig. 6. Since for $T > T_c$

$$K=\langle\langle S_i^2 S_k^2\rangle\rangle=-2G$$

(the hashed block in Fig. 6) Σ reduces to the diagram of Fig. 6a with a minus sign.

For $T > T_c$ the diagram technique contains one vertex

$$W=G^{-3}\overline{\langle\langle S_i S_j\rangle\rangle\langle\langle S_i S_k\rangle\rangle\langle\langle S_j S_k\rangle\rangle}$$

(the triply connected vertex occurs in the diagram of Fig. 6a). The doubly connected vertex

$$\Gamma=G^{-2}K^{-1}\overline{\langle\langle S_i S_k^2 S_j\rangle\rangle\langle\langle S_i S_j\rangle\rangle}$$

(see the diagram of Fig. 6b) is expressed at $T > T_c$ in terms of W: $\Gamma = W$; we have used here the identity

$$\langle\langle S_i S_j S_k^2\rangle\rangle=-2\langle\langle S_i S_k\rangle\rangle\langle\langle S_j S_k\rangle\rangle$$

(for $T > T_c$). The diagrams giving the parquet corrections to the vertex W are shown in Fig. 7.

In the 6-dimensional space the equations at the vertex have the following form:

$$dW/d\xi=-2W^3Z^3,\quad d\ln Z/d\xi=\tfrac{2}{3}W^2Z^3,\quad d\ln\widetilde{\mathscr{T}}/d\xi=-4W^2Z^3. \qquad (6)$$

We have used here the notation

$$\xi=\ln\frac{1}{q},\quad G(q)=\frac{Z(q,\tau_R)}{q^2+\tau_R},$$
$$\tilde{\mathscr{T}}=\frac{d\tau_R}{d\tau}Z^{-1}=\frac{\partial G^{-1}(0)}{\partial\tau},$$

which is a standard one for the theory of phase transitions. Performing the $6-\epsilon$-expansion we get the critical indices which are the same as those found in Ref. 5.

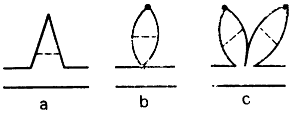

FIG. 4.

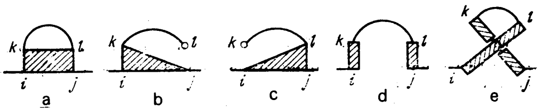

FIG. 5.

3. LOW-TEMPERATURE PHASE IN THE SELF-CONSISTENT FIELD APPROXIMATION

In the low-temperature phase the magnetic moments acquire average values $\langle S_k\rangle$ which depend on the lattice site k. Our main problem consists in evaluating the order parameter $Q = \overline{\langle S_k\rangle^2}$ in terms of which we can express measurable physical quantities (magnetic susceptibility, neutron elastic scattering cross section, and so on). In order to use the diagram technique to evaluate the order parameter we must determine the external field associated with it. It seems natural to do this as follows: we apply to the system a weak magnetic field h_k depending on the site k with a Gaussian correlation law:

$$\overline{h_k h_n}=H\delta_{kn},\quad \bar{h}_k=0;$$

such a field leads to the appearance of randomly directed magnetic moments with a vanishing average moment while we have for its mean square $Q = \overline{\langle S_k\rangle^2}$

$$\left.\frac{\partial Q}{\partial H}\right|_{H=0}=\sum_m\overline{\langle\langle S_k S_m\rangle\rangle^2}=G(q=0). \qquad (7)$$

The diagrammatic series for Q in the self-consistent field approximation is shown in Fig. 8 (the field h is indicated by a cross, the average spin on a site by an open circle, and the order parameter Q by two circles averaged together).

A similar diagram expression exists for the self-energy Σ (see Fig. 9). Putting $H = \overline{h_k^2}$, the field associated with the order parameter, we get for Q and Σ equations which are the same as those found in Refs. 9–11. Their graphical descriptions are given, respectively, in Figs. 10 and 11. The corresponding analytical expressions have the form

$$Q=h^2+\frac{T_c^2}{T^2}Q-2\left(\frac{T_c^2}{T^2}Q\right)^2+\frac{2}{3}\left(\frac{T_c^2}{T^2}Q\right)^3+\ldots, \qquad (8)$$

$$\Sigma=1-2\frac{T_c^2}{T^2}Q+2\left(\frac{T_c^2}{T^2}Q\right)^2+\ldots \qquad (9)$$

and lead to the meaningless result $G^{-1}(q = 0) < 0$. (We remind ourselves that $G^{-1} = \Sigma - T_c^2/T^2$.) The instability occurs in the second order in τ: $G^{-1}(q = 0) \approx -\frac{1}{3}\tau^2$. As was to be expected in the main order of magnitude $G^{-1}(q = 0) = 0$.

The physical reason for this consists, apparently, in the fact that the mean square H of the magnetic field is in actual fact not a true variable associated with the order parameter in the low-temperature phase. To check this we imagine a small fluctuation of the spins around

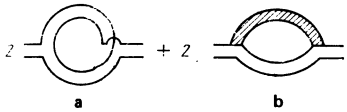

FIG. 6.

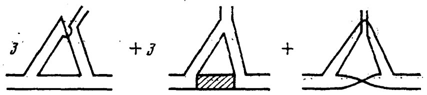

FIG. 7.

their average values: $\delta S_k \ll \langle S_k \rangle$; it is clear that the change in the free energy for such a fluctuation depends on the correlation between $\langle S_k \rangle$ and δS_k while our random field h_k is not at all correlated with the quantities $\langle S_k \rangle$. In other words, the following happens: the free energy and the parameter Q are complicated functionals of the distribution of the average moments $\langle S_k \rangle$ so that the change in the free energy depends not only on the magnitude of the change δQ but also on the direction in space of the moment configurations $\langle S_k \rangle$ through which this change was reached. When we apply a random field h_k to the system which is uncorrelated with the averages $\langle S_k \rangle$ we impose on the system a not completely correct direction of the change in the spins in configuration space; the deviation of this direction from the correct one is small when the order parameter Q is much less than the equilibrium value $Q(\tau)$ for a given temperature, but becomes appreciable when $Q \approx Q(\tau)$. It is thus completely natural that the formalism for introducing the condensate discussed here leads to an equation of state which is valid only as far as the main terms in the expansion in the order parameter are concerned. We note also that in the replica method $\overline{h_i^2} = H$ is the exact field associated with the field variable $Q_{\alpha\beta}$. In principle it must thus be possible to give a correct evaluation with such a field which was demonstrated to lowest orders by Bray and Moore.[11,12]

In actual fact one can assume that, as was mentioned in the Introduction, the absence of a gap in the correlation function G is proven. It seems to us that one should take just the condition $G^{-1}(q=0)=0$ as the basic one for the description of a spin glass. On the other hand, Eqs. (8) and (9) must be considered to be expansions of Q and Σ in series in $T_c^2\tilde{Q}/T^2$, where $\tilde{Q}$ in actual fact is not the same as Q because of the inexact choice of the associated field. To find the correct dependence $\Sigma(Q)$ we must thus eliminate the parameter $\tilde{Q}$ from Eqs. (8) and (9) after which we get

$$\Sigma = 1 - 2Q + 3Q^2 - 8Q^3 + \dots \quad (10)$$

The terms in (10) which are written down are the same as those obtained by Bray and Moore[13] from other considerations. Together with the condition $G^{-1}(q=0)=0$ Eq. (10) determines the equation of state $Q(\tau)$:

$$Q = {}^1/_2|\tau| + {}^3/_8\tau^2 + {}^1/_{16}|\tau|^3 + \dots \quad (11)$$

[in Ref. 13 the same equation is written in terms of the variable $t = T/T_c - 1$, $\tau = t(2+t)$].

At low temperatures we get

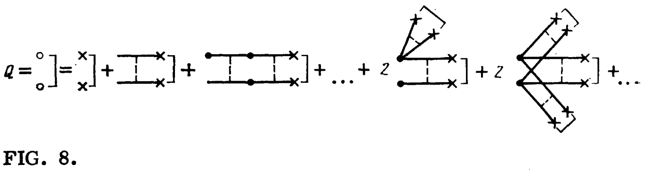

FIG. 8.

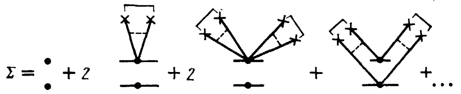

FIG. 9.

$$Q = \int_{-\infty}^{+\infty} \mathrm{th}^2\left(\frac{J(z\tilde{Q})^{1/2}}{T}x\right)\exp\left(-\frac{x^2}{2}\right)\frac{dx}{(2\pi)^{1/2}} = 1 - 2\frac{T}{T_c}(2\pi\tilde{Q})^{-1/2} + \dots,$$

$$\Sigma = \int_{-\infty}^{+\infty} \mathrm{ch}^{-4}\left(\frac{J(z\tilde{Q})^{1/2}}{T}x\right)\exp\left(-\frac{x^2}{2}\right)\frac{dx}{(2\pi)^{1/2}} = \frac{4}{3}\frac{T}{T_c}(2\pi\tilde{Q})^{-1/2} + \dots,$$

so that the condition $G^{-1}(q=0)=0$ gives $1 - Q = 3T^2/2T_c^2$ which is practically the same as the results of Refs. 13 and 14.

In concluding this section we show that there exists in the low-temperature phase still one gapless correlation function which is connected with the function G and defined as follows:

$$D_{ik} = \overline{\langle S_i \rangle \langle S_k \rangle \langle\langle S_i S_k \rangle\rangle}. \quad (12)$$

This correlator corresponds to small spin fluctuations matched to their average values

$$\delta S_i = \langle S_i \rangle \delta\varphi_i,$$

where $\delta\varphi_i$ is a smooth function of the coordinates. To evaluate the function D it is convenient to use the identity

$$-2D_{ik} = \overline{\langle S_i \rangle \langle\langle S_i S_k^2 \rangle\rangle},$$

which is graphically represented in Fig. 12. It is clear from the figure that

$$-2D(\mathbf{q}) = QG(\mathbf{q})K(\mathbf{q})\Gamma(\mathbf{q}) \quad (13)$$

(the vertex Γ is defined in section 2).

Using (13) and the correlator identity

$$K = \overline{\langle\langle S_i^2 S_k^2 \rangle\rangle} = -2G_{ik} + 4D_{ik},$$

we get

$$D(\mathbf{q}) = \frac{QG(\mathbf{q})\Gamma(\mathbf{q})}{G^{-1}(\mathbf{q}) + 2Q\Gamma(\mathbf{q})}, \quad (14)$$

$$K(\mathbf{q}) = -\frac{2}{G^{-1}(\mathbf{q}) + 2Q\Gamma(\mathbf{q})}. \quad (15)$$

For small momenta $(\mathbf{q}^2 \ll Q) D(\mathbf{q}) \approx \frac{1}{2}G(\mathbf{q})$; the correlator K has a finite gap: $-K^{-1}(\mathbf{q}=0) = Q\Gamma(0)$. We note that the functions G and $-\frac{1}{2}K$ correspond to the functions G_R and G_B of Refs. 11 to 13.

The correlation function K has the meaning of a susceptibility in relation to the field $\overline{h_k^2} = H$:

$$\partial Q/\partial H = \sum_{ik} \left(\overline{\langle\langle S_i S_k \rangle\rangle^2} - 2\overline{\langle S_i \rangle \langle S_k \rangle \langle\langle S_i S_k \rangle\rangle}\right) = -{}^1/_2 K(0).$$

4. FLUCTUATIONS IN THE LOW-TEMPERATURE PHASE

In this section we consider fluctuations in the low-temperature phase of a spin glass. We shall then have in mind the immediate vicinity of the phase transition

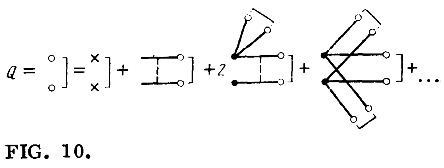

FIG. 10.

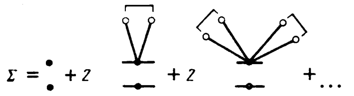

FIG. 11.

($|\tau|\ll 1$) and therefore we shall perform the calculations in the main order in Q and τ. In that approximation $Q=\tilde{Q}$ and Eqs. (8) and (9) are

$$Q=\frac{zJ^2}{T^2}Q-2\left(\frac{zJ^2}{T^2}Q\right)^2,\quad \Sigma=1-2\left(\frac{zJ^2}{T^2}Q\right), \tag{16}$$

so that the equation of state can be written in the form

$$QG^{-1}(q=0)=0 \tag{17}$$

and leads to the correct result.

We shall now show that the equation of state retains its form (17) also when fluctuations are taken into account. We show in Fig. 13 the characteristic diagrams which represent the corrections to the equation of state (16). One sees easily that each of these diagrams differs from the corresponding diagram for Σ only by a factor $(zJ^2/T^2)Q$. The equation of state thus has the form $Q=Q(zJ^2/T^2)\Sigma$ which is equivalent to (17). In our formalism the correlation function G (and, hence, also D) is thus automatically gapless. It is clear from Fig. 13a that the first correction to the equation of state contains a diagram which diverges logarithmically when $d=4$ (all such diagrams will be found below) so that there is no phase transition for $d\leqslant d_c=4$.

It is convenient to use the equation $G^{-1}(q=0)=0$ for the derivation of the equation of state $Q(\tau)$. To do this we take its total differential along the trajectory $Q=Q(\tau)$:

$$dG^{-1}=\frac{\partial G^{-1}(q=0)}{\partial\tau}\bigg|_{Q=Q(\tau)}d\tau+\frac{\partial G^{-1}(q=0)}{\partial Q}\bigg|_{Q=Q(\tau)}dQ=0.$$

There are Ward identities for the partial derivatives of G^{-1}

$$\partial G^{-1}/\partial Q=2\Gamma,\quad \partial G^{-1}/\partial\tau=\tilde{\mathcal{T}}. \tag{18}$$

We get thus

$$dQ/d|\tau|=\tilde{\mathcal{T}}(0)/2\Gamma(0), \tag{19}$$

where $\tilde{\mathcal{T}}(0)$, $\Gamma(0)$ are renormalized vertices for zero momentum evaluated for $Q=Q(\tau)$, i.e., with the gapless functions G and D.

The renormalizations of the vertices are, as usual, in the region of momenta $|\tau|\ll q^2\ll 1$ the same as those found in the high-temperature phase; however, in the small momentum region $q^2\ll|\tau|$ there occur very unusual situations. In contrast to the usual Goldstone modes in degenerate systems the vertices of the interaction of gapless modes in a spin glass do not contain momenta so that already in a 6-dimensional space there is a strong interaction of long-wavelength fluctuations. The parquet diagrams for small momenta for Σ are shown in Fig. 14. The diagrams are constructed from the elements shown in Fig. 5. The graph of Fig. 5e gives the diagram of Fig. 14a; the diagrams of Fig. 14b, c, d are obtained from the elements of Fig. 5b, c, and the diagram of Fig. 14e from the element of 5d. The vertices W enter into the diagram of Fig. 14a. A new vertex X, which we now define, participates in the other graphs.

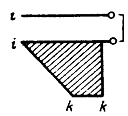

FIG. 12.

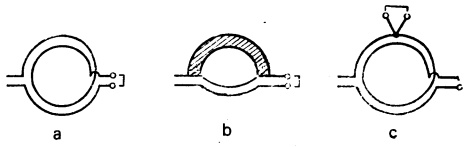

FIG. 13.

We consider the identity for the correlator

$$\overline{\langle\langle S_k^2 S_i S_j\rangle\rangle\langle\langle S_i S_j\rangle\rangle}=-2\overline{\langle\langle S_i S_k\rangle\rangle\langle\langle S_j S_k\rangle\rangle\langle\langle S_i S_j\rangle\rangle}-2\overline{\langle S_k\rangle\langle\langle S_i S_k S_j\rangle\rangle\langle\langle S_i S_j\rangle\rangle}. \tag{20}$$

Its diagrammatic expression is shown in Fig. 15. It is clear from the figure that the second term of the identity contains the vertex X which we need; together with the earlier determined vertices W and Γ we get (20) in the form

$$KG^2\Gamma=-2G^3W-2G^3X. \tag{21}$$

In the region $q^2\ll|\tau|$ we have $KG^{-1}\ll 1$ so that we get from (21) $X=-W$. After this we use (18) to get the parquet equations ($d=6$):

$$d\ln\Gamma/d\xi=8W^2Z^3, \tag{22}$$

$$d\ln\tilde{\mathcal{T}}/d\xi=8W^2Z^3,\quad \xi=\ln(\tau^{1/2}/q), \tag{23}$$

$$d\ln Z/d\xi=-\tfrac{4}{3}W^2Z^3. \tag{24}$$

The equation for the vertex W has the form

$$dW/d\xi=14W^3Z^3. \tag{25}$$

The corresponding graphs are shown in Fig. 16. It is clear from (22) to (25) that the effective charge $g=W^2Z^3$ increases when we go over to a large scale (so-called "asymptotic freedom"):

$$g=\frac{g_0}{1-24g_0\xi},\quad g_0=W^2(\tau)Z^3(\tau)\sim\ln^{-1}\frac{1}{|\tau|}.$$

We can therefore not determine the behavior of the correlation functions for the smallest momenta

$$q\leqslant q_c\approx|\tau|^{1/2}\exp(-1/24g_0)\approx|\tau|^{1/2}|\tau|^{1/\varkappa}.$$

However, this does not prevent us from determining the temperature-dependence of the order parameter $Q(\tau)$ and the specific heat $C(\tau)$ as we shall show that they are determined by momenta of dimensions $q^2\sim|\tau|$.

The fact is that Eqs. (22), (23) for the vertices Γ and

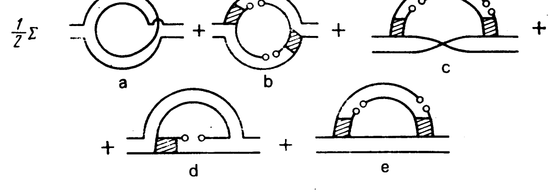

FIG. 14.

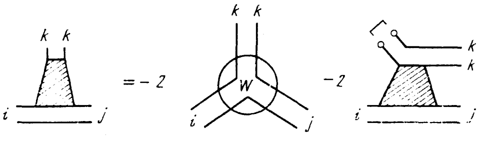

FIG. 15.

$\tilde{\mathcal{T}}$ are the same, i.e.,

$$d\ln\tilde{\mathcal{T}}/d\ln\Gamma=1,\quad \tilde{\mathcal{T}}(\xi)/\Gamma(\xi)=\tilde{\mathcal{T}}(\xi=0)/\Gamma(\xi=0). \tag{26}$$

Although Eqs. (22) to (25) are derived in the main parquet approximation, Eq. (26) is more general and remains the same in all orders. The fact is that in the region $q^2 \ll |\tau|$ differentiating Σ with respect to Q and τ leads to the same parquet diagrams, since only the denominators of the Green functions G are differentiated in which Q and τ occur additively. In other words, although the partial derivatives $\partial G^{-1}/\partial\tau$, $\partial G^{-1}/\partial Q$ are renormalized for $q^2 \lesssim |\tau|$, the total derivative along the trajectory $(dG^{-1}/d\tau)_{Q=Q(\tau)}$ does not contain integrals which are singular for small momenta. Thus,

$$Q={}^1/_2|\tau|\tilde{\mathcal{T}}(\tau)/\Gamma(\tau)\sim|\tau|^{\beta}, \tag{27}$$

where $\tilde{\mathcal{T}}(\tau)$ and $\Gamma(\tau)$ are the renormalized vertices at the momentum $q^2\sim|\tau|$, determined by scaling in the region $|\tau| \ll q^2 \ll 1$; the critical index is $\beta=1+\frac{1}{2}\varepsilon$ [see Ref. 5 and our Eq. (6)].

It is convenient for the determination of the specific heat to use the expression for the internal energy E:

$$E=-\frac{1}{2}\sum_{ij}\overline{J_{ij}\langle S_iS_j\rangle}=\frac{1}{2}\frac{zJ^2}{T}[-1+\overline{\langle\langle S_iS_j\rangle\rangle^2} + 2\overline{\langle S_i\rangle\langle S_j\rangle\langle\langle S_iS_j\rangle\rangle}+(\overline{\langle S_i\rangle^2\langle S_j\rangle^2}-Q^2)+Q^2]. \tag{28}$$

This equation is obtained by evaluating the variation

$$\delta\langle S_iS_j\rangle/\delta(J_{ij}\beta)=\langle S_i^2S_j^2\rangle-\langle S_iS_j\rangle^2.$$

The last term in (28) gives the self-consistent field result, the second, third, and fourth correspond to fluctuation corrections.

The singular part of the derivative of the specific heat $dC/d\tau=d^2E/d\tau^2$ contains parquet diagrams obtained by differentiating the Green functions in (28). As was explained above, the total temperature derivative does not contain diagrams which are singular at small momenta so that the singularity of $dC/d\tau$ is also determined by scaling:

$$(dC/d\tau)_{sing}\sim\tau^{-\alpha'},\quad \alpha'=-2\varepsilon. \tag{29}$$

It was noted by Pytte and Rudnick[16] that dC/dT contains a constant (non-singular) part so that the very weak singularity of (29) is practically unobservable.

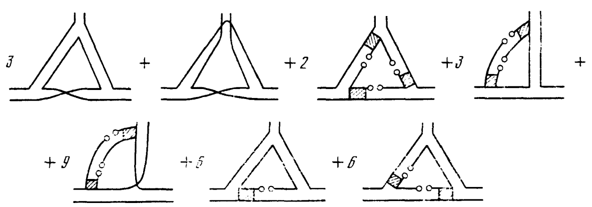

FIG. 16.

5. ABSENCE OF TRANSVERSE STIFFNESS IN A HEISENBERG SPIN GLASS

The diagram technique for a spin glass with multicomponent spins is constructed analogously to the one discussed above for the Ising model. We consider briefly the properties of the correlation function D of small fluctuations in the ordered phase:

$$D_{\gamma\delta}^{\alpha\beta}=\overline{\langle\langle S_1^\alpha S_2^\gamma\rangle\rangle\langle S_1^\beta\rangle\langle S_2^\delta\rangle}. \tag{30}$$

The diagram equation for $D_{\gamma\delta}^{\alpha\beta}$ has the form shown in Fig. 17. We restrict ourselves here to the mean field approximation for the correlation functions. In that approximation

$$G_{\gamma\delta}^{\alpha\beta}=\delta_{\alpha\gamma}\delta_{\beta\delta}G,$$

so that the analytical expression of Fig. 17 is

$$D_{\gamma\delta}^{\alpha\beta}(q)=\delta_{\alpha\gamma}\delta_{\beta\delta}QG^2+Q^2G^2K_{\gamma\delta}^{\alpha\beta} \quad (\overline{\langle S_\alpha\rangle\langle S_\beta\rangle}=\delta_{\alpha\beta}Q). \tag{31}$$

We note that $G(q)\sim 1/q^2$, as in the Ising model.

The fourth rank tensor $D_{\gamma\delta}^{\alpha\beta}$ has three linear invariants: $D_{\beta\beta}^{\alpha\alpha}, D_{\alpha\beta}^{\alpha\beta}, D_{\beta\alpha}^{\alpha\beta}$. The correlator corresponding to the transverse fluctuations is obtained by antisymmetrizing $D_{\gamma\delta}^{\alpha\beta}$ with respect to the indices pertaining to a single point:

$$D_\perp=\varepsilon_{\mu\nu\ldots\alpha\beta}\varepsilon_{\mu\nu\ldots\gamma\delta}D_{\gamma\delta}^{\alpha\beta}=D_{\alpha\beta}^{\alpha\beta}-D_{\beta\alpha}^{\alpha\beta}. \tag{32}$$

The irreducible correlator $K_{\gamma\delta}^{\alpha\beta}$ is symmetric in the indices pertaining to a single point so that in contrast to (32) the second term in (31) cancels out and we find

$$D_\perp(q)=m(m-1)QG^2(q)=Qm(m-1)/q^4, \tag{33}$$

where m is the number of spin components and $\frac{1}{2}m(m-1)$ the number of transverse modes. We have thus proved the absence of transverse spin stiffness in a spin glass. Taking the fluctuations completely into account can apparently not change this result.

6. DISCUSSION OF THE RESULTS

We have constructed a diagram technique for a spin glass in the vicinity of the transition in terms of the physical correlation functions without using the replica method. Together with the standard method of introducing the order parameter it allows us to take consistently into account fluctuations in the low-temperature phase in the main order in $|T-T_c|$.

We have detected a strong interaction of long-wavelength fluctuations leading to an increase in the effective charge in the large size region. The graphs which lead to the increase of the charge are, as follows from Fig. 16, due to the inhomogeneity of the low-temperature phase:

$$\overline{\langle S_i\rangle^2\langle S_k\rangle^2}\neq\overline{\langle S_i\rangle^2}\,\overline{\langle S_k\rangle^2}.$$

The correlation function G remains gapless by virtue

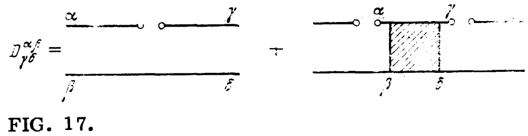

FIG. 17.

of the equation of state. The instability observed when one uses the replica method[12,16] originates, apparently, from the fact that the quantity Q is forced to play the role of a fluctuating variable which contradicts its physical meaning. The procedure of annihilating the "tadpoles" applied by Bray and Moore[12] leads to a result which is obtained automatically in our case. However, another kind of instability which arises in terms of higher order in $|T-T_c|^{10,11}$ exists also in our formalism. As explained above (section 3) it is connected with the inexact definition of the field associated with the order parameter Q. We have suggested a method of obtaining a correct expansion of the self-energy Σ in powers of Q. Together with the condition $G^{-1}(q=0)=0$ for the absence of a gap this enables us to determine the function $Q(\tau)$ to any order in $|T-T_c|$.

It seems to us that the existence of a longitudinal gapless mode is a basic and most characteristic property of a spin glass. It would be extremely interesting to explain whether this mode be connected with some continuous symmetry group, as is usually the case. As mentioned in the Introduction, Bray and Moore[13] managed to show the connection between the soft local modes and the condition $G^{-1}(q=0)=0$. The existence of soft modes is connected with the "frustration" effect, i.e., with the fact that the ground state of the system is not well defined.[17] It seems therefore to us that longitudinal gapless fluctuations must exist in all systems with "frustration." We note that the conclusion reached in Ref. 13 can be repeated verbatim in the case where a uniform magnetic field is applied to the system. It is rather obvious that in a weak uniform field the local soft modes are conserved and hence that the function G remains gapless also in that case. The magnetic field is an external influence which lifts the degeneracy, if it is directed to match the average spins:

$$h_i=\langle S_i\rangle\mathscr{H}_i, \tag{34}$$

where $\mathscr{H}_i$ is a smooth function of the coordinates. Such a field displaces the density of eigenvalues $\rho(\lambda)$ found in Ref. 13:

$$\rho(\lambda,\mathscr{H})=\rho(\lambda-\mathscr{H}),$$

i.e., it destroys the soft modes. In our formalism

$$\frac{\partial Q}{\partial\mathscr{H}}=\sum_j\overline{\langle S_i\rangle\langle S_j\rangle\langle\langle S_iS_j\rangle\rangle}=D(q=0,\mathscr{H})=[2(Q-Q_0)\Gamma]^{-1}. \tag{35}$$

Integrating (35) we get

$$(Q-Q_0)^2\sim\mathscr{H},\quad F-F_0\sim|Q-Q_0|^3. \tag{36}$$

The behavior (36) of the free energy F was obtained by other means in Ref. 14.

We considered in this paper the Edwards-Anderson model with nearest-neighbor interactions. In a realistic spin glass the interaction has the form $J(r)\propto r^{-3}\cos 2p_Fr$ and the exchange integral distribution is, of course, not Gaussian. However, the fact that the distribution is not Gaussian leads only to a renormalization of the bare vertices which is well known to be unimportant for the properties of the phase transition. The existence of a small average value of the exchange integrals (which does not lead to the occurrence of an average magnetic moment) also does not change anything in our results. The important properties—a small average value of the exchange and uncorrelated exchange integrals for different spin pairs—are present both in a real spin glass and in the model considered. The rather slow decrease of the RKKY interaction is also unimportant as the correlation functions depend on the main term in the expansion of the quantity $\overline{J^2}(0)-\overline{J^2}(\mathbf{q})$ in terms of the momentum $\mathbf{q}$ which is the same as for the short range interaction:

$$\overline{J^2}(0)-\overline{J^2}(\mathbf{q})\sim\int\frac{d^3r}{r^6}(1-\cos\mathbf{qr})\sim q^2.$$

The main qualitative conclusions—the existence of a longitudinal gapless mode, the absence of transverse stiffness in a Heisenberg spin glass, and the absence of a phase transition in a three-dimensional system—are retained therefore for real spin glasses with an RKKY interaction. The "kink" in the magnetic susceptibility which is observed in spin glasses to depend on the time of observation is, apparently, connected with the very slow relaxation of metastable states. Of most interest would be to find an explanation of the logarithmic dependence[1] of the "transition" temperature on the time of observation.

In conclusion we express our gratitude for many discussions to S. L. Ginzburg, I. E. Dzyaloshinskiĭ, V. L. Pokrovskiĭ, E. F. Shender, and especially to D. E. Khmel'nitskiĭ, in discussions with whom the idea for the present paper arose.

[1]G. Eiselt, J. Kotzler, H. Maletta, D. Stauffer, and K. Binder, Phys. Rev. B19, 2664 (1979).

[2]A. J. Bray, M. A. Moore, and P. Reed, J. Phys. C 11, 1187 (1978).

[3]V. Canella and J. A. Mydosh, Phys. Rev. B6, 4220 (1972).

[4]S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975).

[5]A. B. Harris, T. C. Lubensky, and J. H. Chen, Phys. Rev. Lett. 36, 415 (1976).

[6]R. Fish and A. B. Harris, Phys. Rev. Lett. 38, 785 (1977).

[7]P. Reed, J. Phys. C 11, L976 (1978).

[8]A. M. Polyakov, Phys. Lett. 59B, 79 (1975).

[9]S. Kirkpatrick and D. Sherrington, Phys. Rev. Lett. 35, 1792 (1975); Phys. Rev. B17, 4384 (1978).

[10]J. R. L. de Almeida and D. J. Thouless, J. Phys. A 11, 983 (1978).

[11]A. J. Bray and M. A. Moore, Phys. Rev. Lett. 41, 1068 (1978).

[12]A. J. Bray and M. A. Moore, J. Phys. C 12, 79 (1979).

[13]A. J. Bray and M. A. Moore, J. Phys. C 12, L441 (1979).

[14]D. J. Thouless, P. W. Anderson, and R. G. Palmer, Phil. Mag. 35, 593 (1977).

[15]V. G. Vaks, A. I. Larkin, and S. A. Pikin, Zh. Eksp. Teor. Fiz. 51, 361 (1966) [Sov. Phys. JETP 24, 240 (1967)].

[16]E. Pytte and J. Rudnik, Phys. Rev. B19, 3603 (1979).

[17]D. C. Mattis, Phys. Lett. 56A, 421 (1976); G. Toulouse, Comm. Phys. 2, 115 (1977); J. Villain, J. Phys. C 10, 1717 (1977).

Translated by D. ter Haar