Quantum-electrodynamic effects inside a charged black hole and the problem of Cauchy horizons

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The evolution of the space-time metric and physical fields inside a charged black hole is considered with allowance for the quantum-electrodynamic production of electron-positron pairs in a strong electric field. A consistent treatment of the processes leads to the conclusion that inside the black hole there is no nonsingular Cauchy horizon (in contrast to the classical unperturbed Reissner-Nordström solution), and the structure of space-time is analogous to the structure of an uncharged (Schwarzschild) black hole.

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§1. INTRODUCTION

Just as the collapse of a nonrotating, electrically neutral star leads to the formation of an uncharged black hole with Schwarzschild metric, the collapse of a charged nonrotating spherical star leads to the formation of a charged black hole with Reissner-Nordström metric and electric field E:

$$ds^2=\left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)dt^2-\left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta\, d\varphi^2),$$
$$E=Q/r^2, \tag{1.1}$$

where M is the mass of the black hole, Q is its electric charge, and $Q^2 \leq GM^2$ (units are chosen for which $c=1$). In contrast to the Schwarzschild metric, the metric (1.1) has two horizons (which are determined by the condition $g^{11}=g_{00}=0$):

$$r=r_\pm=GM\pm(G^2M^2-GQ^2)^{1/2}. \tag{1.2}$$

The outer horizon $r=r_+$ is an event horizon and has the same properties as the horizon of the Schwarzschild metric. The inner horizon $r=r_-$ is a Cauchy horizon, i.e., if prior to the start of the collapse of the star initial Cauchy data are specified at some instant in the whole of space, they determine the evolution only for the region $r>r_-$. In the region between the outer and inner horizons ($r_-<r<r_+$), the metric (1.1) can be conveniently rewritten in synchronous form by introducing the proper time in the T frame of reference[1]:

$$\tau=-\int_{r_-}^{r}dr\frac{r}{[(r_+-r)(r-r_-)]^{1/2}}=-\int_{r_-}^{r}\frac{dr}{a(r)} \tag{1.3}$$

and, to avoid confusion, replacing the spacelike coordinate t by x. Then (1.1) takes the form

$$ds^2=d\tau^2-a^2(\tau)\,dx^2-r^2(\tau)\,(d\theta^2+\sin^2\theta d\varphi^2), \tag{1.4}$$

where the dependences $r(\tau)$ and $a(\tau)$ are given in explicit form by (1.3). Note that in the synchronous frame of reference the inner and outer horizons correspond to a fictitious Kasner singularity of the form (1, 0, 0).

It has been shown in a number of papers that if a body collapses with small departures from spherical symmetry and without rotation, then in the space exterior to r_+ in the limit $t\to\infty$ the metric tends to a static metric (Schwarzschild for neutral bodies[2,3] or Reissner-Nordström for charged bodies[4,5]). It can be assumed that the same is true for the formation of a black hole from an initially strongly asymmetric body. Outside the black hole, all radiative modes of physical fields that have their sources on the collapsing body are also damped as $t\to\infty$.

Finally, the evolution of the metric of the black hole and the behavior of physical fields within the event horizon r_+ have recently been analyzed. In Ref. 6, the following result was proved for the case of a Schwarzschild black hole: If the deviations from spherical symmetry are small on r_+, then inside the black hole the metric tends in the limit $t\to\infty$ to exact spherical symmetry except for an ever contracting zone near the true singularity $r=0$, where the perturbations grow.

For the investigation of the internal structure of a charged black hole, the presence of the Cauchy horizon r_- is of the greatest importance. The stability of r_- against small perturbations of different types has been investigated in a number of papers.[7-9] It was shown that this horizon is unstable against small perturbations that arise outside and on the boundary of the black hole.[1)]

The aim of the present paper is to investigate the real structure of space-time that arises inside a black hole as a result of the collapse of a charged body when allowance is made for quantum processes of particle production in strong fields.

In Sec. 2, we consider the process of external discharging of a charged black hole and determine the range of variation of Q and M for which the problem of the existence of a Cauchy horizon is nontrivial and requires study of concrete physical processes. In Sec. 3, we investigate the evolution of the electric field and the metric inside the black hole due to pair production in the case when without allowance for pair production the electric field would be greater than the critical $E_c=\pi m^2/e\hbar$ (when the pair production process is strong). Section 4 is devoted to the opposite case of a weak field $E<E_c$. In Sec. 5, we compare the magnitudes of the quantum and classical effects that lead to instability of the Cauchy horizon.

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§2. EXTERIOR CHARGE OF A BLACK HOLE

The evolution of the metric and the electric field inside a charged black hole depends on the ratio of Q to M. We here establish the range within the exterior charge Q of a black hole formed as a result of collapse can vary.

First, as we have already said, a black hole can arise only if $Q \leq \sqrt{G}M$ or $Q/e \leq 5.4 \times 10^5 M$ (g), where $-e$ is the charge of the electron (the region below the line 1 in Fig. 1). If it is assumed that the electric charge of the black hole is of the order of the electric charge of the star from which it was formed by collapse, then

$$Q/e \sim GMm/e^2 = 2.6\cdot10^{-16}\ M\ (\text{g}), \tag{2.1}$$

where m is the mass of the electron (see, for example, Ref. 10). However, there exist mechanisms for increasing the charge of the black hole by accretion.[11]

The main process that limits Q above is the quantum-electrodynamic process of e^+e^- pair production from the vaccum by a strong electric field. In the case of black holes, this process was considered by Markov and Frolov,[12] and also in Refs. 13 and 14. In this process, one of the particles of the pair, which has the same charge as the black hole, escapes to infinity, and the second is captured by the black hole. Energetically, this process is possible if

$$e\varphi(r_+) = eQ/r_+ \geqslant mc^2. \tag{2.2}$$

As we shall see below, the problem of the existence of the inner Cauchy horizon is nontrivial only for black holes with $M \gtrsim 10^{28}$ g. Therefore, in the present paper we restrict ourselves to considering black holes whose gravitational radius $r_+ \approx 2GM/c^2$ appreciably exceeds the electron Compton wavelength $\lambda\!\!\!^{-} = \hbar/mc$, which corresponds to the condition $M \gg 3 \times 10^{17}$ g.[2)]

Note that electron-positron pairs (and other particles) could be produced by the gravitational field of the black hole by itself (Hawking effect). However, this last effect can be ignored if

$$e\varphi(r_+) = \frac{eQ}{r_+} \gg T_{BH} \approx \frac{\hbar c}{4\pi r_+}, \quad \frac{Q}{e} \gg \frac{\hbar c}{4\pi e^2} \approx 11. \tag{2.3}$$

We calculate the rate of electric discharge of the charged black hole through the process of electron-positron pair production. We consider the case when the inequality (2.2) is satisfied with a wide margin, i.e., $eQ/r_+ \gg mc^2$ (otherwise, discharging hardly occurs in practice and Q remains constant). Then at infinity the produced electrons and positrons are ultrarelativistic. In this case, in the calculation of the number of pairs produced in the electric field of the black hole we can use the approximation of an electric field that is constant in time and space.

Indeed, if

$$E \ll E_c = \pi m^2c^3/e\hbar \approx 4.2\cdot10^{16}\ \text{V/cm},$$

the condition of applicability of the approximation of a constant field is[3)] $E'/E \ll eE/mc^2$, and for $E \gg E_c$ (Ref. 16) it is the condition $E'/E \ll (eE/\hbar c)^{1/2}$. In the region $r \sim r_+$, which makes the main contribution to the creation effect, the first inequality is always satisfied under the adopted condition $eQ/r_+ \gg mc^2$, and the second is if

$$Q/e \gg \hbar c/e^2 \approx 137, \tag{2.4}$$

which is identical to (2.3) apart from the numerical coefficient.

It is well known that the specific rate of e^-e^+ pair creation in a constant electric field is given by the expression (see, for example, Ref. 17)

$$\frac{dN}{dt\,dV} = \frac{1}{4\pi^3}\frac{e^2E^2}{\hbar^2c}\exp\left(-\frac{E_c}{E}\right). \tag{2.5}$$

This expression is also true for $E > E_c$ if N is the mean number of created pairs. Then the rate at which the black hole loses electric charge is

$$\frac{dQ}{dt} = -\frac{e^3Q^2}{\pi^2\hbar^2c}\int_{r_+}^{\infty}\frac{dr}{r^2}\exp\left(-\frac{E_cr^2}{Q}\right). \tag{2.6}$$

If $Q/r_+^2 \gg E_c$, integrating (2.6), we find that, irrespective of Q_0, the value of Q decreases to the value $\sim Q_1 = \pi m^2c^3r_+^2/e\hbar$ in the very short time

$$t \sim t_1 = \pi\frac{\hbar c}{e^2}\left(\frac{\lambda\!\!\!^{-}}{r_+}\right)^2\frac{r_+}{c} < \frac{r_+}{c}, \tag{2.7}$$

if $M > 5 \times 10^{18}$ g.[4)] In reality, the discharge time will be longer, since in Eq. (2.6) we have not taken into account the time needed for the produced particle to escape from the black hole or fall into it. The precise determination of p_1 requires specification of the concrete mechanism of formation of the initial charge Q_0 of the black hole.

There follows then a stage of slow discharge ($E < E_c$ everywhere, $Q < Q_1$). In the limit $t \to \infty$, $Q \propto 1/\ln t$.

Since the characteristic time of the process of collapse of the star and formation of the black hole is of order r_+/c, in this time Q decreases to the value

$$Q_2 = \frac{\pi m^2 r_+^2 c^3}{e\hbar}(\ln A - \ln\ln A)^{-1}, \tag{2.8}$$

where $A = e^2(2\pi\hbar c)^{-1}(r_+/\lambda\!\!\!^{-})^2 \gg 1$. For $Q \ll \sqrt{G}M$, we have numerically

$$\frac{Q_2}{e} = \frac{1.4\cdot10^{-33}M^2}{\lg M - 19 - 0.5\lg(\lg M - 19)}, \tag{2.9}$$

where M is measured in grams (line 2 in Fig. 1). Lines 1 and 2 intersect at the point $M \approx 3 \times 10^{40}$ g $\approx 10^7 M_\odot$ and $Q/e \approx 2 \times 10^{46}$.

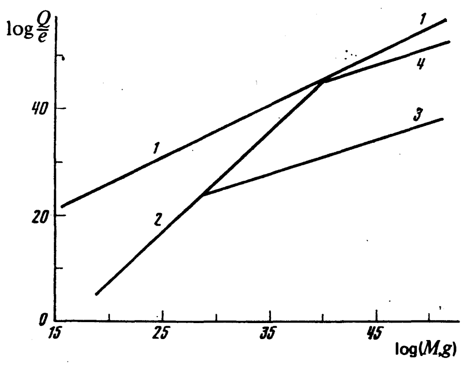

FIG. 1. Different regions of values of the change Q and mass M of the black hole. For the boundaries of the region, see the text.

The further decrease of Q for an exterior observer is so slow that it can be ignored and Q regarded as constant. However, an observer falling into the black hole and moving in the T region could see (and indeed will see, as we shall show below) how the value of Q decreases as a result of quantum processes with decreasing coordinate r, which in the T region is timelike. For such an observer, the surface of the horizon $r=r_+$ remains in the past.

The main aim of the present work is to ascertain whether there exists a nonsingular inner Cauchy horizon in a charged black hole. In the allowed region below the lines 1 and 2 in Fig. 1, there is a large region in which a negative answer (i.e., absence of a Cauchy horizon) is clear from the very beginning. Indeed, from the physical point of view the singularity must be regarded as being not where the curvature invariants become infinite but where they are still finite but so large that the classical theory of gravitation becomes invalid because of quantum-gravitational effects (namely, $|R_{iklm}R^{iklm}| \sim l_g^{-4}$, $l_g=(G\hbar/c^3)^{1/2} \approx 1.6\times 10^{-33}$ cm). It is natural to refer to such a singularity as physical, in contrast to the mathematical singularity determined by the condition that the invariants or other physical characteristics of the gravitational field are infinite.

In the required region, $Q \ll \sqrt{G}M$. At the same time

$$r_- \approx \frac{Q^2}{2Mc^2} \ll r_+ \approx \frac{2GM}{c^2}.$$

In the region $r_- \ll r \ll r_+$, the metric (1.4) is approximately a vacuum Kasner metric with exponents $(-1/3, 2/3, 2/3)$ and the electric field does not influence the evolution of the metric. We also have

$$R_{iklm}R^{iklm} \approx \frac{12r_+^2}{r^6} = \frac{64}{27(c\tau)^4} \gg R_{ik}R^{ik} = \frac{4G^2Q^4}{c^8r^8}. \qquad (2.10)$$

A nonsingular Cauchy horizon exists if $|\tau(r_-)| \gg t_g = l_g/c$, i.e., if the influence of the electric field on the metric becomes significant before a physical singularity is reached. In Fig. 1, we have drawn the line

$$|\tau(r_-)| = \frac{2}{3c}\frac{r_-^{3/2}}{r_+^{1/2}} = t_g,$$

$$\frac{Q}{e} = 6^{1/3}G^{1/3}(\hbar c)^{1/6}\frac{M^{2/3}}{e} \approx 2.7\cdot 10^4 M^{2/3}\,(\mathrm{g}) \qquad (2.11)$$

(line 3).[5] It is obvious that as a result of the influence of quantum, and also dissipative processes the charge $Q(r)$ can only decrease with decreasing r (i.e., with increasing proper time τ). Therefore, if the external charge Q of the black hole lies in the region below the line 3, we can immediately say, without going into a detailed consideration of the processes taking place inside the black hole, that a nonsingular inner Cauchy horizon does not exist, but instead there arises a vacuum singularity similar to the one in the Schwarzschild metric in the limit $r \to 0$.

Note that as a consequence of this we do not have to consider the existence of an inner horizon for black holes with "natural" value of Q determined by (2.1). The point is that the straight line (2.1) lies below the line 3 up to masses so large ($M \approx 10^{60}$ g) that they are incompatible with the present size of the Universe ($M_{\mathrm{hor}} \approx 10^{57}$ g for $\Omega = 1$).

Further, lines 2 and 3 intersect at $M \approx 5\times 10^{28}$ g (≈ 10 Earth masses) and $Q/e \approx 4\times 10^{23}$. Therefore, in the region lying above line 3 the inequalities $r_+ \gg \lambda\!\!\!^{-}$, (2.3), and (2.4) are satisfied with a huge margin.

§3. DAMPING OF THE ELECTRIC FIELD BELOW THE EVENT HORIZON THROUGH THE PRODUCTION OF ELECTRON–POSITRON PAIRS

We now turn to the study of the physical processes inside the charged black hole. In the synchronous T frame of reference (1.4) the electromagnetic field remains, as before, purely electric and does not depend on the spatial coordinates. $E(\tau)$ increases during the contraction, i.e., with decreasing $r(\tau)$. In Fig. 1, we focus our attention on the range of Q and M values for which $E > E_c$ in the T region near the inner horizon and, therefore, e^+e^- pairs are produced rapidly there. For this, we draw in Fig. 1 the line $E(r=r_-) \equiv Q/r_-^2 = E_c$ (line 4). For $Q \ll \sqrt{G}M$, the equation of line 4 simplifies:

$$\frac{Q}{e} = \left(\frac{4}{\pi}\frac{\hbar M^2}{m^2e^2}\right)^{1/3} \approx 5.9\cdot 10^{18} M^{2/3}\,(\mathrm{g}). \qquad (3.1)$$

(In Secs. 3–5, $c=1$.) Lines 2 and 4 intersect at $M \approx 6\times 10^{39}$ g, $Q/e \approx 2\times 10^{45}$ (for which $Q/\sqrt{G}M \approx 0.6$). In Sec. 3, we shall consider the region bounded by the lines 2, 3, and 4, in which pairs are produced rapidly and the curvature for $r=r_-$ is less than the Planck curvature.

In the considered region, it may be assumed that $Q \ll \sqrt{G}M$. Then $r_- \ll r_+$, and for $r_- \ll r \ll r_+$

$$r(\tau) = \left(\frac{3}{2}\right)^{2/3} r_+^{1/3}|\tau|^{2/3}, \quad a(\tau) = \left(\frac{r_+}{r}\right)^{1/2} = \left(\frac{2}{3}\right)^{1/3}\left|\frac{r_+}{\tau}\right|^{1/3}. \qquad (3.2)$$

This is the vacuum Kasner regime, and in the first approximation the electric field does not influence the evolution of the metric. It follows from (2.8) that on line 2 and to the right of it $E(r=r_+) \ll E_c$, and therefore strong pair production begins for $|\tau| \ll r_+$ when E increases to E_c, i.e., in the stage (3.2).

The produced pairs are accelerated by the field, which leads to a current that decreases Q and E. We shall show below that the damping of the electric field does not occur monotonically but through weakly damped oscillations. It is important to emphasize that as a result of the considered processes the field cannot exceed the critical value E_c for more than a short time (this will be discussed below). To show this, we assume the opposite, that the field has increased to $E_0 > E_c$. In this case, as we shall show, the field amplitude would rapidly decrease to E_c during a characteristic time τ_0. The further decrease of the amplitude E occurs very slowly, and we shall ignore it. Deferring to the end of the section the calculation of τ_0 (it is of order 10^{-18}–10^{-19} sec), we consider the further evolution of the electric field and the space-time metric.

The energy density of the produced particles increases as $a^{-2}r^{-2} \propto |\tau|^{-2/3}$ until their relaxation, when they are two opposite fluxes of ultrarelativistic particles moving along the x axis, and as $(ar^2)^{-4/3} \propto |\tau|^{-4/3}$ after the relaxation of the particles, when they can be described by the equation of state $p = \varepsilon/3$. Therefore, the produced particles cannot change the asymptotic behavior (3.2), which leads to a true singularity. This could be done only by an electric field. The strong damping of E for $E>E_c$ has the consequence that E during the stage (3.2) does not exceed E_c for $|\tau|>\tau_0$ (Q decreasing). For $|\tau|<\tau_0$, the quantum effects do not succeed in significantly reducing Q during the hydrodynamic time $|\tau|$. Therefore, for $|\tau|<\tau_0$ the change in Q can be ignored, and the electric field increases, as in (1.1):

$$E\approx E_c(\tau_0/|\tau|)^{4/3}\propto r^{-2}. \tag{3.3}$$

We now find what condition τ_0 must satisfy if the electric field is not to influence the metric (3.2) down to times $|\tau|\sim t_g$, when the true singularity occurs (cf. the discussion in Sec. 2). This occurs if

$$8\pi G\varepsilon_{el}=GE^2\approx GE_c^2\left(\frac{\tau_0}{t_g}\right)^{8/3}\ll t_g^{-2}, \tag{3.4}$$

whence

$$\tau_0\ll t_g\left(\frac{\hbar}{Gm^2}\right)^{3/4}\left(\frac{e^2}{\hbar}\right)^{3/8}\approx 3\cdot 10^{-11}\ \text{sec}. \tag{3.5}$$

Therefore, if the condition (3.5) is satisfied, then, irrespective of the actual mechanism of relaxation of E to E_c, the electric field cannot influence the vacuum asymptotic behavior (3.2). As a result, a Cauchy horizon does not arise inside the charged black hole [as we have said above, it would correspond to an asymptotic behavior of the form $r(\tau)\approx\text{const}, a(\tau)\propto|\tau|$ as $|\tau|\to 0$], and there is instead a true singularity of the same form as in the case of an uncharged black hole.

We now show that the inequality (3.5) is indeed satisfied with a large margin. For this, it is necessary to solve the problem of the production of electron-positron pairs in an external homogeneous variable electric field with allowance for the back reaction of the produced pairs on the field. Problems of this kind have not hitherto been studied systematically; only simpler problems of pair production in a given external field without allowance for the back reaction have been solved.

We consider first the single-loop approximation, in which the evolution of the homogeneous electric field is described by one of the Maxwell equations,

$$dE^\alpha/d\tau=-4\pi\langle 0|j^\alpha|0\rangle,\quad \alpha=1,2,3, \tag{3.6}$$

where $j^\alpha=e\bar\psi\gamma^\alpha\psi$ is the Heisenberg current operator in the external electric field $E^\alpha(\tau)$ without allowance for radiative corrections, and $|0\rangle$ is the initial (vacuum) Heisenberg state vector (a magnetic field does not arise). In the metric (1.4), Eq. (3.6) takes the form

$$dQ/d\tau=-4\pi ar^2j^1=-4\pi ar^2e\langle 0|\bar\psi\gamma^1\psi|0\rangle. \tag{3.7}$$

The spinor ψ satisfies the equation

$$[i\gamma^\mu(\hbar D_\mu+ieA_\mu)-m]\psi=0, \tag{3.8}$$

where γ^μ are the covariant γ matrices, D_μ is the generalized covariant derivative of a spinor, and

$$A_\mu=(0,A_1,0,0),\quad A_1=\int\frac{Qa}{r^2}d\tau. \tag{3.9}$$

Separating the variables in the standard manner as in flat space-time (see, for example, Ref. 17), and squaring, we obtain an equation for the time-dependent part of the spinor:[6)]

$$\frac{1}{ar^2}\frac{d}{d\tau}\left(ar^2\frac{d\psi}{d\tau}\right)+\left[\frac{m^2}{\hbar^2}+\frac{j(j+1)}{r^2}+\frac{1}{a^2\hbar^2}(k+eA_1)^2+\frac{ie}{\hbar}\frac{d}{d\tau}\left(\frac{A_1}{a}\right)\right]\psi=0. \tag{3.10}$$

A necessary condition for the applicability of the single-loop approximation is the possibility of describing the electric field classically. The criterion for this is (see, for example, Ref. 18)

$$E\gg\sqrt{\hbar}/T^2, \tag{3.11}$$

where T is the characteristic time of variation of the field.

If the inequality (2.4) is satisfied, the value of the angular momentum of the particles characteristic for the problem will be

$$j=\frac{p_\perp r}{\hbar}\sim(e\hbar E)^{1/2}\frac{r}{\hbar}=\left(\frac{eQ}{\hbar}\right)^{1/2}\gg 1 \tag{3.12}$$

($p_\perp$ is the physical momentum of the particle perpendicular to the direction of the electric field), and in what follows we shall therefore regard j as a continuous variable. In the region above line 3, for $|\tau|>t_g(\hbar/e^2)^{3/4}\approx 2\cdot 10^{-42}$ sec and when the inequality (2.2) is satisfied, the characteristic time required by a particle to acquire in the field E a relativistic velocity parallel to the field,

$$\tau_1=\max\left(\frac{m}{eE},\frac{|p_\perp|}{eE}\right),$$

is much shorter than the characteristic time of variation of the metric $|\tau|$, and therefore the motion of the electrons and the positrons is basically determined by the electric field. This also enables us in (3.10) to take into account the dependence on the time τ in r and a (but not in E and Q) adiabatically. If

$$|p_\parallel|\equiv\frac{1}{a}\left|k+e\int\frac{Qa}{r^2}d\tau\right|\gg|p_\perp|,m, \tag{3.13}$$

where $p_\parallel$ is the physical momentum parallel to the electric field, then in (3.10) the WKB approximation is valid, and

$$\psi_{k,j}=\frac{1}{(ar^2)^{1/2}}\left[\alpha_{kj}\exp\left\{-i\int\frac{1}{\hbar a}\left(k+e\int\frac{Qa}{r^2}d\tau_1\right)d\tau\right\}+\beta_{kj}\exp\left\{i\int\frac{1}{\hbar a}\left(k-e\int\frac{Qa}{r^2}d\tau_1\right)d\tau\right\}\right], \tag{3.14}$$

where α_{kj} and β_{kj} are constants satisfying $|\alpha_{kj}|^2+|\beta_{kj}|^2=1$. Note that in the case of an electric field in flat space-time ($\dot a=\dot r=0$) (3.14) is an exact solution of (3.10) for $p_\perp=m=0$. The quantity $|\beta_{kj}|^2=n(k,j)$ is the mean number of pairs in the considered mode. Because of the charge symmetry of the initial state and the production process, the population numbers in each mode satisfy

$$n_{e^-}(k,j)=n_{e^+}(k,j)=n(k,j). \tag{3.15}$$

We assume that $\tau_1\ll T$ (this will be verified later). Then the evolution of each mode will consist of long ($\sim T$) quasiclassical stages (3.14), during which the contribution of the mode to the total current ($J_{kj}=2en_{kj}$) is virtually constant because of the fact that in the ultrarelativistic regime the change in the velocity is small, and of short stages, when (3.13) is violated and, on the one hand, the previously produced particles will change the direction of their motion along the x axis, and, on the other, new particles will be produced.

We derive an equation that will describe the evolution of E and Q on time scales much longer than τ_1 (but, naturally, much shorter than T and $|\tau|$). In such an approach, the short stages will be assumed to be instantaneous. In calculating the mean value of the current, we can in the first approximation ignore the local polarization of the vacuum (i.e., the difference between the effective permittivity of the vacuum and unity), since it is known[16,19] that this correction becomes of order unity only in exponentially large fields

$$E/E_c \sim \exp\{3\pi\hbar/e^2\},$$

which we do not consider. But in (3.7) we must retain the contribution from pair production. Although the instantaneous contribution from production is less than the contribution from vacuum polarization [by $\ln(E/E_c)$ times for $E \gg E_c$], the former can, in contrast to the latter, accumulate with the course of time, and therefore the integrated contribution from the production effect over a long interval of time $\Delta\tau \gg \tau_1$ is not small.

The upshot is that (3.7) can be represented in the quasiclassical form

$$Q = -\frac{4e}{\pi a\hbar}\int_0^{\infty} j\,dj \int_{-\infty}^{\infty} dk\, n(k,j,\tau)\left(k+e\int d\tau\frac{Qa}{r^2}\right)a^{-1}\mathscr{E}^{-1}, \tag{3.16}$$

$$\mathscr{E} = \left[m^2 + \frac{(j+{}^1/_2)^2\hbar^2}{r^2} + \frac{1}{a^2}\left(k+e\int d\tau\frac{Qa}{r^2}d\tau\right)^2\right]^{1/2};$$

the dot denotes differentiation with respect to τ. In (3.16), the contribution from the two spin states is taken into account; n is determined in (3.15).

To close the system of equations, we must obtain an equation for $n(k,j,\tau)$. In our approximation, the variation of n (production or annihilation of particles) occurs almost instantaneously at the instant of time[7)] when

$$k+e\int d\tau\frac{Qa}{r^2} \approx 0.$$

The change in n can be calculated in accordance with the formulas for particle production in a constant electric field if the inequalities considered in Sec. 2 before (2.4) are satisfied, it being necessary to replace the differentiation with respect to the time. These inequalities are equivalent to the previously adopted condition $\tau_1 \ll T$, $|\tau|$.

It is known that the mean number of pairs produced from the vacuum in a constant electric field in each mode is[17]

$$n = \exp\left[-\frac{\pi(m^2+p_\perp^2)}{e|E|\hbar}\right]. \tag{3.17}$$

Hence, we have the equation

$$\dot n(k,j,\tau) = \delta\left(k+e\int\frac{Qa}{r^2}d\tau\right)\frac{ea|Q|}{r^2} \times\exp\left\{-\frac{\pi(m^2r^2+(j+{}^1/_2)^2\hbar^2)}{e|Q|\hbar}\right\}[1-2n(k,j,\tau)]. \tag{3.18}$$

The second term in the last factor in (3.18) describes induced production or annihilation.

It is convenient to represent (3.16) in a different form, which makes it possible to integrate over k. We multiply (3.16) by a and differentiate with respect to τ. Since the function $n(k,j,\tau)$ depends weakly on k in intervals $\Delta k \sim eQ\tau_1 a/r^2$, it can be taken in front of the integral over k. Using the condition $\tau_1 \ll |\tau|$ to ignore all terms that contain $\dot a$ and $\dot r$, and making a number of transformations, we obtain the equation

$$\frac{1}{a}(aQ)^{\cdot} + \frac{2}{\pi^2}\frac{e^3Q^2\,\mathrm{sgn}\,Q}{\hbar^2 r^2}\exp\left(-\frac{\pi m^2r^2}{e\hbar|Q|}\right) + \frac{8e^2Q}{\pi r^2\hbar}\int_0^{\infty} j\,dj\, n\left(k=-e\int\frac{Qa}{r^2}d\tau, j, \tau\right) \times\left[1-\exp\left\{-\frac{\pi(m^2r^2+(j+{}^1/_2)^2\hbar^2)}{e|Q|\hbar}\right\}\right]=0. \tag{3.19}$$

The second term in (3.19) gives the change in the current due to spontaneous production, and the terms in the integrand in the square brackets describe the change in the current due, respectively, to the turning of the already produced particles and to the induced production.

Equations (3.18)–(3.19) form a complete system of equations that describe the evolution of the electric field in the single-loop approximation. Note that in their derivation we have not made any assumptions concerning the values of the ratios $T/|\tau|$ and E/E_c.

Suppose $T/|\tau| \ll 1$ and that at some time $E = Q/r^2 = E_0$ and produced particles are absent. Then in (3.19) there remain only the two first terms, and this relation can be readily integrated (with the dependence of r and a on τ ignored). The electric field vanishes over an interval of time T given by

$$T = \frac{\pi^{2/3}\Gamma(1/3)}{2\sqrt{3}\,\Gamma(5/6)}\left(\frac{\hbar^2}{e^3E_0}\right)^{1/2} \approx 3.8\left(\frac{\hbar^2}{e^3E_0}\right)^{1/2}, \quad E_0 \gg E_c, \tag{3.20}$$

$$T = \frac{\sqrt{\pi}}{2}\left(\frac{\hbar^3}{m^2e^2}\right)^{1/2}\frac{E_c}{E_0}\exp\left(\frac{E_c}{2E_0}\right), \quad E_0 \ll E_c. \tag{3.21}$$

The result (3.20) differs only by the coefficient from the result obtained qualitatively by Zel'dovich in Ref. 20; (3.21) agrees with the result of Parker and Tiomno.[21]

The electric field then changes sign and increases in modulus, and nonsinusoidal oscillations arise. The further evolution of the field was not considered in Ref. 20, and in Ref. 21 no allowance was made for the integral term in (3.19), which leads to a change in T in the following cycles. During the first cycles, the numerical coefficient in T changes in a fairly complicated manner, but after a large number N of oscillations a smooth asymptotic behavior with slow variation with N is established. It can be shown that for $E_0 \gg E_c$, $N \gg 1$, and $NT \ll |\tau|$ the period T and the amplitude E of the electric field and the current density J depend on N as follows:

$$T \propto (\ln N)^{-3/4}, \quad E \propto (\ln N)^{-1/4}, \quad J \propto (\ln N)^{1/4}. \tag{3.22}$$

In the case of the production of Bose particles in an electric field for $E > E_c(m)$, the time of significant reduction of the electric field is $\sim T$; therefore, the oscillations are immediately damped.[8)] The damping of oscillations with increasing N in the case of electrons is not due to the unlimited growth of the population numbers in each mode (as in the case of bosons) but to the fact that for a large number of modes the condition

$n \sim \frac{1}{2}$ begins to be satisfied; the main contribution to the integral in (3.19) is made by modes with

$$j^2 \sim \frac{e|Q|}{\hbar} \ln N, \quad n \sim 1/2.$$

Using the expressions (3.20)–(3.21) as estimates, we can verify that $\tau_1 \ll T$ (the conditions under which $\tau_1 \ll |\tau|$ were elucidated earlier). Then the conditions of applicability of the approximation of a homogeneous field for calculating the pair production and the weaker condition (3.11) are satisfied. Thus, all the assumptions made in the derivation of (3.18)–(3.19) are justified.

We now make a semiquantitative allowance for the effects that arise in the higher perturbation orders in $\alpha = e^2/\hbar$. The main processes that lead to enhanced damping of the weakly damped oscillations of E and Q found above are the following: Coulomb collisions of the electrons and positrons and their annihilation into photons, and bremsstrahlung of the electrons and positrons in the external electric field. In addition, in the considered system plasma (two-stream) instabilities may arise. We consider these processes separately.

The characteristic value of the total density $n_{e^+} = n_{e^-} = n_0$ in the oscillations satisfies $n_0 \sim E/eT$, where T is given by (3.20)–(3.21), E is the amplitude of the electric field, and the energy of each particle is of order $\mathscr{E} \sim eET$. If the Coulomb logarithm is ignored, the time of free flight τ_e of the particles until Coulomb collisions or two-photon annihilation is

$$\tau_e = \frac{1}{\sigma_{e^-e^+} n} \approx \frac{1}{\sigma_{\gamma\gamma} n} \sim \frac{\mathscr{E}^2}{e^4 n}. \tag{3.23}$$

Then (with allowance for the dispersion of the particles over the energies)

$$\frac{\tau_e}{T} = \begin{cases} \left(\frac{\hbar}{e^2}\right)^2 \left(\ln \frac{\hbar}{e^2}\right)^{-2} & \text{for} \quad E \gg E_c \\ \left(\frac{\hbar}{e^2}\right)^2 \frac{E}{E_c} \exp\left(\frac{E_c}{E}\right) & \text{for} \quad E \ll E_c \end{cases} . \tag{3.24}$$

The frequency of the plasma oscillations under these conditions is

$$\omega_{pl}^2 = 8\pi e^2 n_e \left\langle \frac{dv}{dp} \right\rangle = 8\pi e^2 n_e \left\langle \frac{m^2}{\mathscr{E}^3} \right\rangle. \tag{3.25}$$

The main contribution to (3.25) is made by particles with $\mathscr{E} \sim m$ (although there are few of them). The characteristic growth time of the two-stream instability is $\tau_{pl} \sim \omega_{pl}^{-1}$, and

$$\frac{\tau_{pl}}{T} = \begin{cases} (E/E_c)^{1/4} & \text{for} \quad E \gg E_c, \\ (E_c/E)^{1/4} & \text{for} \quad E \ll E_c. \end{cases} \tag{3.26}$$

To estimate the bremsstrahlung intensity, we must consider in more detail the motion of a particle. Its characteristic transverse momentum is $p_\perp \sim (eE\hbar)^{1/2}$, and the energy and longitudinal momentum change in accordance with the law

$$\mathscr{E} \approx p_\parallel \propto (eE\hbar)^{1/2} \tau/\tau_1, \tag{3.27}$$

where τ is the time of motion of the particle since production or from the time of last turning ($\tau \gg \tau_1$). The angle of turning of the direction of motion of the particle from the time τ to the end of the period $T > \tau$ is

$$\theta \sim p_\perp / p_\parallel \sim \tau_1 / \tau.$$

On the other hand, radiation is emitted in an angle $\theta_r \sim \gamma^{-1}$. For $E \gg E_c$,

$$\theta_r / \theta \sim (E_c/E)^{1/4} \ll 1,$$

and therefore to estimate the intensity of the radiation for $E > E_c$ we can use the formulas for synchroton radiation (the emission in a given direction is produced by a small section of the trajectory). The characteristic frequency ω_1 of the maximum in the spectrum of the classical radiation under the given conditions is such that

$$\omega_1 \sim \frac{eE_\perp}{m} \gamma^2 \sim \frac{Ee\theta}{m} \gamma^2 \sim \frac{m}{\hbar} \left(\frac{E}{E_c}\right)^2 \frac{\tau}{\tau_1} \gg \frac{\mathscr{E}}{\hbar}. \tag{3.28}$$

Therefore, we are in the ultraquantum situation, when the main fraction of the emitted energy (of the order of the particle energy) is given to a single photon.[9] Using the formula for the intensity of quantum synchroton radiation (see, for example, Ref. 18)

$$I \approx \mathscr{E}^{2/3} E_\perp^{2/3} (e^2/\hbar)^{4/3}, \tag{3.29}$$

we find that for $E \gg E_c$ the total energy emitted by one particle during the period T is

$$\Delta\mathscr{E} \sim \mathscr{E} e^2/\hbar. \tag{3.30}$$

It can be seen from this that for $E \gg E_c$ the amplitude of the oscillations of E decreases significantly during $\hbar/e^2 \approx 137$ oscillations. After averaging over times $\tau \gg T$, the rate of decrease in the amplitude of the oscillations of the electric field E is given by the equation

$$dE/d\tau \approx E^{3/2} e^{7/2} \hbar^2. \tag{3.31}$$

We here ignore the logarithmic damping of E (3.22) obtained in the single-loop approximation, since it is insignificant over an interval of 137 oscillations. Similarly, it can be found that for $E \ll E_c$ the amplitude E decreases appreciably during $137E_c/E$ oscillations.

Comparing (3.24), (3.26), and (3.31), we see that of the three considered processes the most important are radiative losses for $E/E_c > 10^4$ and plasma instabilities otherwise. For $E \sim E_c$, the time τ_r is approximately two orders of magnitude longer than τ_{pl}. Integrating (3.31), we obtain the time of damping of the field from the initial value $E_0 \gg E_c$ to E_c due to radiative losses:

$$\tau_0 \sim \frac{\hbar}{m} \left(\frac{\hbar}{e^2}\right)^{5/2} \approx 2 \cdot 10^{-16} \text{ sec} \tag{3.32}$$

(τ_0 does not depend on E_0).

It is possible that plasma instability will decrease τ_0 by a further one or two orders of magnitude. However, we shall not consider the plasma instability in detail, since the estimate (3.32) for τ_0 already shows that the inequality (3.5) is satisfied with a large margin.

Thus, we conclude that the damping of the electric field due to quantum-electrodynamic effects in the region bounded by the lines 2, 3, and 4 in Fig. 1 is sufficient to have the consequence that a true vacuum singularity of Schwarzschild type arises instead of a nonsingular Cauchy horizon.

§4. INFLUENCE OF PRODUCED PARTICLES ON THE EVOLUTION OF THE METRIC IN THE CASE WHEN THERE IS NO STRONG ELECTRIC DISCHARGING

We now consider the last uninvestigated region of Q and M lying between lines 1 and 4 in Fig. 1. In this region $E(r\sim r_-)\ll E_c$, and therefore pair production does not lead to an appreciable decrease of the electric field. However, there is formed a certain (in general, small) number of particles, which are accelerated by the electric field to ultrarelativistic energies, their momentum being directed virtually along the x axis. We show that the presence of even a small number of such particles has the consequence that a Cauchy horizon cannot be formed.

The evolution of the metric (1.4) with matter in the form of an electric field and ultrarelativistic particles is described by the system of equations

$$\begin{gathered} 2\frac{\dot a}{a}\frac{\dot r}{r}+\left(\frac{\dot r}{r}\right)^2+\frac{1}{r^2}=8\pi G T_0^{\,0}=\frac{GQ^2}{r^4}+\frac{16\pi G\mathscr{A}\mathscr{E}}{ar^2},\\ 2\frac{\ddot r}{r}+\left(\frac{\dot r}{r}\right)^2+\frac{1}{r^2}=8\pi G T_1^{\,1}=\frac{GQ^2}{r^4}-\frac{16\pi G\mathscr{A}\mathscr{E}}{ar^2},\\ \dot Q=-\frac{8\pi e\mathscr{A}}{a}\operatorname{sgn}Q_0\\ \frac{1}{a}(a\mathscr{E})^{\cdot}=\frac{eQ}{r^2}. \end{gathered} \tag{4.1}$$

Here, $\mathscr{E}$ is the mean energy of a particle, and $n=\mathscr{A}/ar^2$ is the density of the produced electrons, $\mathscr{A}=\text{const}$. An influence of the terms containing $\mathscr{A}$ is manifested only for r very close to r_-, and therefore in accordance with (2.5)

$$\mathscr{A}=\frac{e^2Q_0^2}{4\pi^3\hbar^2}\int_{r_-}^{r_+}\frac{dr}{r^2}\exp\left(-\frac{\pi m^2r^2}{e\hbar Q_0}\right)\approx\frac{1}{8\pi^4}\frac{e^3Q_0^3}{\hbar m^2r_-^3}\exp\left(-\frac{\pi m^2r_-^2}{e\hbar Q_0}\right), \tag{4.2}$$

where Q_0 is the external charge of the black hole.

In the absence of produced particles ($\mathscr{A}=0$), the metric (1.4) would have the asymptotic behavior $a=a_0|\tau|$, $r\to r_-$ as $\tau\to 0$ ($\tau<0$). We show that for $\mathscr{A}\neq 0$ this is impossible; for if such an asymptotic behavior did hold,

$$\begin{gathered} Q=Q_0+\frac{8\pi e\mathscr{A}\operatorname{sgn}Q_0}{a_0}\ln|\tau|,\\ \mathscr{E}\propto 1/|\tau|,\quad \varepsilon_e\propto 1/\tau^2, \end{gathered} \tag{4.3}$$

where ε_e is the energy density of the electron-positron pairs, whereas the left-hand sides of the first two equations of the system (4.1) increase slower than τ^{-2}. Therefore, these equations cannot be satisfied for arbitrarily small τ.

A more careful investigation shows that during the stage (1, 0, 0)

$$\varepsilon_e a^2r^2=\mathscr{A}a\mathscr{E}\to\text{const}=\frac{1}{16\pi^5}\frac{e^5Q_0^5}{m^4r_-^6}\exp\left(-\frac{\pi m^2r_-^2}{e\hbar Q_0}\right) \tag{4.4}$$

and the subsequent variation of $\mathscr{E}a$ can be ignored. The change in Q is also small if we are in the region somewhat above line 3:

$$Q/e>3\cdot 10^{19}\,M^{1/3}\ (\text{g}), \tag{4.5}$$

which corresponds to the condition $E_c/E(r\sim r_-)>90$. The condition (4.5) simultaneously ensures the absence of oscillations of E.

During the (1, 0, 0) stage, $\varepsilon_e\sim\tau^{-2}$. When ε_e becomes of the order of the energy density of the electric field, $\varepsilon_{em}=E^2/8\pi$, there commences the stage

$$a\propto|\tau|,\quad r\propto\sqrt{|\ln|\tau||},\quad \varepsilon_e\propto 1/\tau^2|\ln|\tau||\gg\varepsilon_{em},$$

which, under the fulfillment of the condition (4.5), extends effectively to the time when $G\varepsilon_e\sim 1/l_g^2$, and the true singularity commences. Therefore, in this case too, a nonsingular Cauchy horizon cannot be formed.

§5. COMPARISON OF THE CLASSICAL GRAVITATIONAL AND QUANTUM-ELECTRODYNAMIC INSTABILITIES OF THE CAUCHY HORIZON IN A CHARGED BLACK HOLE

In the preceding sections, we have shown how pair production in the electric field under the event horizon of a black hole leads to the disappearance of the Cauchy horizon in a self-consistent solution. Here, we wish to note that instability of the Cauchy horizon already arises at the classical level, as was shown in Ref. 9. One can pose the problem of how nonspherical electromagnetic and gravitational perturbations change if as initial conditions on the event horizon one introduces the natural laws of damping of the fields outside the event horizon found in Ref. 3 in the case of an uncharged black hole and in Refs. 4 and 5 in the case of a charged black hole when these laws are continued analytically to the event horizon itself (this method was already used to analyze perturbations inside an uncharged black hole in Ref. 6). These laws of damping correspond to the assumption that the sources of the fields are on the collapsing star. We give here the answer for this problem (for more detail, see Ref. 22).

Let Φ be the characteristic function that describes a weak linear perturbation (for example, a model scalar perturbation) on the background of the Reissner-Nordström metric. Then, whereas on the outer horizon far from the surface of the collapsing body, i.e., for $u=t-r^*\to\infty$ and $v=t+r^*=\text{const}\gg GM$, where

$$r^*=\int dr\left[1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right]^{-1},$$

we have in accordance with Ref. 4

$$\Phi=\text{const}\cdot v^{-(2l+2)}, \tag{5.1}$$

where l is the multipolarity of the perturbation, in the region near the intersection of the inner and outer horizons in the unperturbed solution ($t\gg GM$, $r_-<r<r_+$, $|r^*|\ll t$)

$$\Phi=\text{const}\cdot\frac{1}{t^{2l+2}}P_l\left(\frac{2r-r_+-r_-}{r_+-r_-}\right)\left[1+O\left(\left|\frac{r^*}{t}\right|\right)\right], \tag{5.2}$$

and near the inner horizon ($v\to\infty$, $u=\text{const}\gg GM$)

$$\Phi=\text{const}\cdot\frac{r_+}{r_-}\ \frac{A}{(t+r^*)^{2l+2}}+\frac{B}{(t-r^*)^{2l+2}}, \tag{5.3}$$

where A and B are certain constants satisfying the condition

$$|A|^2-|B|^2=1, \tag{5.4}$$

their actual values depending on the type of the perturbation. For example, in the case of scalar perturbations[9]

$$A=\frac{1}{2}(-1)^l\left(\frac{r_+}{r_-}+\frac{r_-}{r_+}\right), \quad B=-\frac{1}{2}(-1)^l\left(\frac{r_+}{r_-}-\frac{r_-}{r_+}\right). \qquad (5.5)$$

The constant factor in Eqs. (5.1), (5.2), and (5.3) is the same.

On the inner horizon, Φ is finite, but invariants of the type $g^{ik}\Phi_{,i}\Phi_{,k}$, and also quantities such as the energy density of the perturbation measured by an observer in a freely falling frame of reference diverge. In particular,

$$g^{ik}\Phi_{,i}\Phi_{,k}\propto\frac{AB}{r-r_-}\frac{1}{(t^2-r^{*2})^{2l+3}}\propto\exp\left(r^*\frac{r_+-r_-}{r_-^2}\right)\frac{1}{(t^2-r^{*2})^{2l+3}}. \qquad (5.6)$$

However, in this case a self-consistent treatment of the problem is impossible, and one can speak strictly only of instability of the Cauchy horizon in the linear approximation.

Thus, there exist two kinds of instability, quantum and classical, which lead to the disappearance of the Cauchy horizon. Which of them is the stronger? It is obvious that the quantum instability in the regime of strong discharging (see Sec. 3) is certainly stronger than the classical instability, since it arises far from the Cauchy horizon. In the case of the regime of weak discharging (Sec. 4), both instabilities (the quantum and the classical) become important in a narrow region near the Cauchy horizon (for $r-r_-\ll r_-$). The order of growth of the instabilities during the stage when they can be regarded as perturbations is the same (to logarithmic accuracy); as can be seen from (4.4) and (5.6), the contribution to the right-hand side of the Einstein equations is proportional to $(r-r_-)^{-1}$. The numerical coefficient in (5.6) depends on the initial conditions and is therefore somewhat uncertain, though there are no grounds for believing that it decreases rapidly with decreasing Q for given M. Since the coefficient A in (4.2) contains $\exp(-E_c/E(r_-))$, it can be seen that for sufficiently small $E(r_-)$ (i.e., significantly above line 4 in Fig. 1) the classical instability is in the general case stronger than the quantum instability.

§6. CONCLUSIONS

In the present paper we have shown that systematic study of quantum-electrodynamic processes below the event horizon of a charged black hole makes it possible to construct a self-consistent solution for this region of space-time. Using this solution, we have shown that these processes destroy the Cauchy horizon, which exists in the idealized classical Reissner-Nordström solution, and lead to the occurrence of a true singularity. On the other hand, classical (nonquantum) perturbations also lead to a similar situation (see Sec. 5). It is therefore to be expected that during the gravitational collapse of a real (with deviations from spherical symmetry) charged nonrotating star the structure of the space-time within the event horizon r_+ will be similar to the space-time structure of an uncharged (Schwarzschild) black hole.

A preliminary consideration also suggests that rotation of the collapsing star will not qualitatively change this conclusion.

Our conclusions concerning the instability of Cauchy horizons within black holes and that the space-time structure inside a real black hole is similar to the structure of the Schwarzschild space-time below the event horizon force a re-examination of possible ways in which the end of gravitational collapse could be avoided. One such possibility considered earlier was that after crossing the Cauchy horizon r_- the contracting matter could begin to re-expand into a different part of space-time, avoiding the true singularity. In this case, the space-time will have a complicated topology, and have so-called topological arms; see Refs. 23-25. In this connection, the possibility was considered of obtaining information from other regions of space-time and from the regions within such topological arms, this information arriving directly at the observer with the expanding matter or with particles that avoid passing through the true singularity. Such a possibility must now apparently be ruled out. Even if a replacement of contradiction by expansion within the black hole with the formation of complex topological structures is possible, it will still be necessary to pass through regions that from the classical (not quantum) point of view are true singularities of space-time. For the consistent study of processes in such singularities a quantum theory including gravitation is needed.

1) We use this opportunity to correct an inaccuracy in the expression on p. 414 of Ref. 9, in which we inaccurately spoke of instability of r_- with respect to perturbations specified within r_+, although instability of r_- with respect to a perturbation specified on r_+ can be proved.

2) The opposite case, which is of interest in connection with the problem of the evaporation of primordial electrically charged black holes of small mass, was considered by Page.[15]

3) This condition makes it possible to calculate the probability of pair production with exponential accuracy. If the constant-field approximation is also to give the correct pre-exponential factor, the stronger inequality $E'/E\ll\hbar e^2E^2/m^3c^5$ for $E\ll E_c$ must be satisfied. In the actual situations considered below, this inequality is satisfied.

4) The change in M during the process of discharging is small ($\Delta M\ll M$) if $Q_0\ll\sqrt{G}M_0$. If $Q_0\sim\sqrt{G}M_0$, then during the process of discharging M_0 may be reduced by more than a factor of two.

5) Note that on line 3 we have $r_-\gg l_g$.

6) In Eq. (3.10), we have omitted a number of terms that are small because of the conditions $|mr/\hbar r|\sim|ma/\hbar\dot{a}|\sim m|\tau|/\hbar\gg 1$ and $j\gg 1$.

7) We recall that the "instant of time" is in reality an entire interval $\Delta\tau\sim\tau_1$, $\Delta\tau\ll|\tau|$, T.

8) Therefore, in reality the oscillations of the electric field cannot occur when $E\gtrsim 7\times 10^4 E_c$ because of copious production of π-meson pairs.

9) Photons with frequency $\hbar\omega/(\xi-\hbar\omega)\sim 1$, which make the main contribution to the integrated intensity, are emitted under these conditions in the angle $\theta\sim\tau_1/\tau$.

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Isotropic cosmological models determined by vacuum quantum effects

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The evolution of cosmological models with scalar or spinor quantized fields is studied. In the class of spatially homogeneous isotropic models, all self-consistent models are found in which the metric is determined by vacuum quantum effects of massless fields. It is shown that the obtained results are also valid for massive fields.

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1. INTRODUCTION

One of the main problems of cosmology is the description of the evolution of the Universe near the cosmological singularity. As follows from the Penrose-Hawking theorems,[1] if the dominant energy conditions are satisfied for the matter that determines the metric it is impossible to avoid the occurrence of singularities in classical general relativity. At the same time, it is known that allowance for quantum effects leads to violation of the dominant energy conditions.[2] At the present time, a completed theory of quantization of the gravitational field does not yet exist, and it is therefore expedient to consider the part played by quantum effects in the framework of a semiclassical scheme, in which the gravitational field is classical but the fields of particles are second quantized. Such a scheme corresponds to the single-loop approximation to a fully quantized theory.

On dimensional grounds, one can conclude that the semiclassical approach is valid for a gravitational field characterized by a curvature that is small compared with the Planck curvature, $\rho \ll G^{-1/2} \sim 10^{33}$ cm^{-1} (G is the gravitational constant; we use a system of units in which $\hbar = c = 1$). If it is found that certain quantization effects of the matter fields are sufficient to eliminate the singularity, it can be assumed that this result is also true when quantization of the gravitational field is taken into account.

In the present paper, we solve the self-consistent problem of the evolution of isotropic cosmological models with quantized scalar or spinor fields. The condition of self-consistency takes the form that the external gravitational field produces a vacuum energy density and pressure of the quantized scalar or spinor fields that are required for the creation of this gravitational field in accordance with the Einstein equation. As will be shown below, in the class of homogeneous isotropic cosmological models there exist models that are self-consistent in this sense and do not possess singularities. This makes it possible to interpret the occurrence of the Universe as a manifestation of an instability of the vacuum state of a quantized field.

In Sec. 2 of the present paper, we formulate the equations of self-consistency of the cosmological models with scalar or spinor quantized fields. In Sec. 3, the self-consistency equations are solved in the case of massless fields, and we find all isotropic models determined by vacuum quantum effects. These models include de Sitter models of Planck dimensions and models that coincide asymptotically with Milne's model. In

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