Pair production in the scattering of a photon by an intense electromagnetic wave in a uniform magnetic field

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The photoproduction of electron-positron pairs is calculated, with the interaction of charged particles with the electromagnetic field taken into account exactly, in the case of a field consisting of a uniform magnetic field and the field of a circularly polarized plane electromagnetic wave. An expression for the probability of pair production which is exact in the framework of the method of quasienergy states (QES) is derived in the form of an expansion in partial contributions corresponding to different numbers of wave photons which take part in the reaction and to the set of discrete states of the electron and positron in the magnetic field. The character of the occurrence of the process in the below-threshold range of energies of the external photon, $\omega' < m$, is analyzed relative to the analogous condition in a pure magnetic field with no actual absorption of photons of the wave. It is shown that, for a selected direction of propagation of the external photon, helicity conservation in the system leads to the existence of a rigid connection between the number l of photons absorbed from the wave and the discrete levels which characterize the transverse excitations of the electron and positron, $l+n-n'\pm 1$. Near-threshold effects in magnetic fields $H\sim H_0=m^2c^3/e\hbar$ are investigated, and also various polarization effects.

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1. INTRODUCTION

It has recently been noted[1] that intense magnetic fields, comparable with characteristic quantum-electrodynamic field $H_0=4.41\cdot 10^{13}$ G, can exist in the neighborhood of a pulsar, and such intense fields have been observed experimentally.[2] The study of various mechanisms for processes of accretion near neutron stars is closely related to the investigation of quantum-electrodynamic processes occurring in the presence of intense electromagnetic fields.

Among various methods for studying these processes, particular importance attaches to those that do not involve the assumption that the external fields are small in comparison with H_0, since the situation with which we are concerned does not permit a useful description of the phenomena in question unless the interaction of charged particles with these fields is treated exactly. Thus it is interesting to investigate, in the framework of quantum electrodynamics with an external field, a process which may be important both from the point of view of the astrophysical context we have mentioned, and also in connection with possible laboratory researches using intense sources of electromagnetic fields; the process to be considered is that of photoproduction of electron-positron pairs.

The external field we shall consider is one of the most general configurations, including both a constant uniform magnetic field H and the field of a plane electromagnetic wave propagated along the direction of the magnetic field-strength vector

$$\mathbf{A}=\mathbf{A}_1+\mathbf{A}_2,\quad \mathbf{A}_1=(0,xH,0),\quad \mathbf{A}_2=-\frac{\xi m}{e}(\mathbf{e}_x\sin\varphi-g\mathbf{e}_y\cos\varphi),\tag{1}$$

where the vector potential A_2 corresponds to a wave with circular polarization $g=\pm 1$, frequency ω, and amplitude E; k_μ is the wave four-vector, $\xi=eE/m\omega$ is the invariant intensity parameter of the wave, $\varphi=(kx)=\omega(t-z)$; a system of units in which $\hbar=c=1$ is used. A characteristic feature of this field is that it cannot by itself produce pairs, whatever may be the intensities of its components, so that increasing the intensities of the fields cannot take us beyond the framework of the one-particle problem. Also it is well known that in an electric field of this configuration an exact solution of the Dirac equation can be found,[3] and by its use one can in principle take into account exactly the interaction with such a field of arbitrary intensity.

In the present paper we consider the process of electron-positron pair production when an external (in relation to the field) photon is propagated along the direction of the magnetic field vector and opposite to the propagation of the field's "photons." A study of the pair-production process in this type of given field with a proton propagating in a direction perpendicular to the direction of the magnetic field was made earlier,[4] with some restrictions on the parameters of the problem. The dependence of the total probability of pair production on the angle at which the external photon propagates is not trivial; the choice of a definite "geometry" of the system can lead either to complete absence of the effect (external photon propagating in the direction of propagation of the wave), or to the appearance of supplementary selection rules for the resulting states of the electron and positron, owing to the additional symmetry of the case considered here, in which a circularly polarized external photon interacts with the field $\mathbf{A}_2$, as they propagate in opposite directions along the magnetic field vector. The conservation of helicity in this system distinguishes it qualitatively from the case considered in Ref. 5, where approximate expressions were obtained for the probabilities of partial process of photoproduction of pairs in a field configuration rather similar to that considered here.

As the basis of our calculation of the process of e^+e^- production we take the "hole" version of the Dirac theory; the exact treatment of the interaction of the charged particles with the given field is carried out in the framework of the method of quasienergy states (QES),[6,7] which has been used in the solution of a num-

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ber of problems with fields of analogous nonstationary condigurations.[4,5,8,9]

The exact inclusion (within the framework of the QES method) of the "photons" of the wave, and also the postulate that the intensity of the constant magnetic field is arbitrary, enables us to trace the change of the way the process varies over a wide region of the initial parameters of the problem. In Sec. 2 we discuss the specific form of the QES in the electromagnetic field (1). In Sec. 3 we derive an expression for the total probability of pair production in the form of a double sum, which corresponds both to contributions of various partial processes involving definite numbers of photons of the wave, and to contributions of the various electron and positron states characterizing particular Landau levels. The dependence of the pair-production process on the relation between the polarization properties of the wave and of the external photon is investigated.

2. THE QUASIENERGY STATES

Let the magnetic field be directed along the z axis, $\mathbf{H}=(0, 0, H)$. The wave functions of an electron in the field (1) are of the form[4]

$$\Psi = N_- \begin{pmatrix} A_1\psi_{n-1} + B_1R^*\psi_n \\ A_2\psi_n + B_2R\psi_{n-1} \\ A_3\psi_{n-1} + B_3R^*\psi_n \\ A_4\psi_n + B_4R\psi_{n-1} \end{pmatrix}. \tag{2}$$

Here $\psi_n = e^{-iS}\, U_n(\rho)$; the $U_n(\rho)$ are functions connected with the Hermite polynomials $H_n(\rho)$ by the relation

$$U_n(\rho) = (2^n n! \sqrt{\pi})^{-1/2} \exp(-\rho^2/2) H_n(\rho),$$

where

$$\rho = m\gamma^{1/2}(x - \xi\Delta_-^{-1}\cos\varphi) + p_2^-/m\gamma^{1/2}, \quad \Delta_- = \omega\alpha_- - g\omega_H,$$
$$\alpha_- = m^{-1}(p_0^- - p_3^-), \quad \gamma = H/H_0, \quad \omega_H = m\gamma, \quad p_0^- = m[1 + (p_3^-/m)^2 + 2\gamma n]^{1/2},$$

p_2^- and p_3^- are, respectively, the transverse and longitudinal compoments of the momentum of the particle in the magnetic field. The function S is

$$S = m\alpha_- t + p_3^-\omega^{-1}\varphi + g\xi(\Delta_-)^{-1}(\omega_H x + p_2^-)\sin\varphi$$
$$- p_2^- y + \xi^2 m\varphi(2\Delta_-)^{-1} - gm\xi^2\omega_H(2\Delta_-)^{-2}\sin 2\varphi.$$

The normalization constant N_- and the function R are given by the expressions

$$N_- = [1 + \xi^2\omega^2(\alpha_-/\Delta_-)^2(1 + \alpha_-^2 + 2\gamma n)^{-1}]^{-1/2},$$
$$R = 2^{-3/2} ig\omega\xi(\Delta_-)^{-1}\exp(ig\varphi),$$

and A_i and B_i are spin coefficients.[10] For the positron the state n', α^+, p_2^+, p_3^+ is characterized by the wave function charge conjugate to the function (2). In accordance with the definition given, for example, in Ref. 4, the states of the electron and positron in the external field (1) can be specified by the introduction of a quasienergy and a third component of quasimomentum

$$q_0^\mp = p_0^\mp + \xi^2 m\omega(2\Delta_\mp)^{-1}, \quad q_3^\mp = p_3^\mp + \xi^2 m\omega(2\Delta_\mp)^{-1}, \tag{3}$$

which in the nonstationary field play a role in the conservation laws analogous to that of the ordinary energy of a particle in a magnetic field and the component of its momentum along the magnetic field, being subject to the relation

$$q_0^\mp - q_3^\mp = p_0^\mp - p_3^\mp = m\alpha_\mp.$$

From this it follows, in particular, that the integrals of the motion $\alpha_\mp = (kp^\mp)/\omega m$ retain their meaning both in the pure magnetic field and also after the wave is turned on. This fact makes these variables convenient for the classification of the states of particles in the external field (1). It is easy to see that the choice of a definite circular polarization of the wave leads (for a fixed direction of the magnetic field) to the appearance of characteristic resonance singularities in the spectrum of an electron or a positron. The existence of these singularities has a radical effect on the character of the processes that occur in the external field (1). In particular, it leads to the possibility of pair production in the region below threshold ($\omega' \ll m$) relative to the analogous reaction in a pure magnetic field without real absorption of "photons" of the wave[5] (see also Ref. 4).

Qualitatively we can analyze this phenomenon by starting from the expressions (3) and noting that the conservation laws for the quasienergy and the z component of the quasimomentum in the field in question are of the form

$$k_0' + lk_0 = q_0^- + q_0^+, \quad k_3' + lk_3 = q_3^- + q_3^+. \tag{4}$$

Here $q_0^\mp$ are the quasienergies of the electron and positron, l is the number of quanta of the wave that are involved in the reaction, and the quantities k_0' and k_3' are components of the four vector of the external photon,

$$k_\mu' = (\omega', 0, 0, -\omega').$$

Subtracting one equation in (4) from the other, we find that the quantities $\alpha_\mp$ satisfy the equation

$$\alpha_- + \alpha_+ = 2\omega'/m. \tag{5}$$

For definiteness let us consider the polarization of the wave that leads to the case of resonance action of the field on the electron. From Fig. 1, which represents the dependence of the quasienergy q_0^- on the parameter α_- (solid lines) (and where the positron branch $q_0^+ = q_0^+(\alpha_+)$ is placed in the region of negative values of the parameter α_-, $\alpha_- < 0$, $\alpha_+ = -\alpha_-$), we see that the effect of the wave can be characterized by the magnitude of the deviation from the corresponding dependence in the pure magnetic field (dashed curve). Direct comparison shows that thereas in the pure magnetic field the electron and positron states are separated by an interval

$$\Delta\varepsilon = 2m,$$

pair production can occur in the field (1) for $\omega' < m$. For example, in the case of a relatively weak wave ($\xi^2 \ll 1 + 2\gamma n$) the region of "anomalous" behavior of the quasienergy (i.e., the region of marked deviation from the analogous dependence in a pure magnetic field) is very narrow:

$$\Delta\alpha_- \approx \frac{\omega_H}{\omega}\left[1 - \xi^2\left(1 + 2\gamma n + \frac{\omega_H^2}{\omega^2}\right)^{-1}\right]$$

and is close to the cyclotron resonance point $\alpha_- = \omega_H/\omega$.

Since the quasienergy q_0^-, defined apart from a term which is a multiple of the energy of a "photon" of the wave,[6,7] can take negative values in the neighborhood of the point $\alpha_- = \omega_H/\omega$, it is easy to see that in the scale of α_- the electron and positron states are shifted apart by an amount

$$\Delta\varepsilon \approx m\omega_H/\omega,$$

which can be small, $\Delta\varepsilon \ll m$, for $\omega_H \ll \omega$.

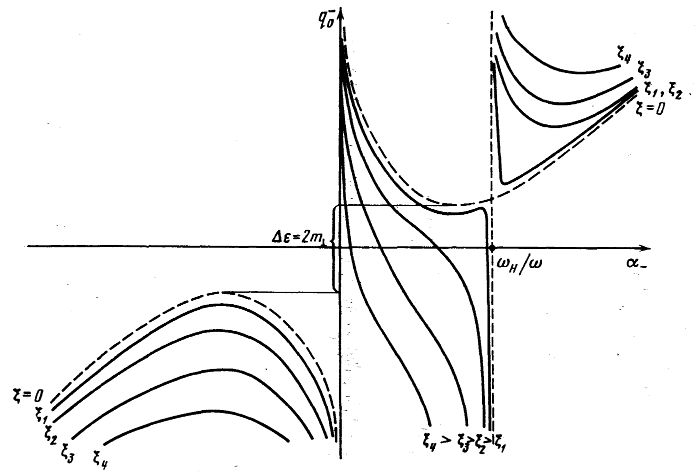

FIG. 1. Quasienergy branches of electron and positron as functions of the parameter $\alpha_\mp = m^{-1}(q_0 - q_3)$ for various values of the intensity of the wave, $\xi = 0$, $0 < \xi_1 < \xi_2 < \xi_3 < \xi_4$, with fixed quantum numbers n, n'; $m_\perp = m(1+2\gamma n)^{1/2}$.

With increasing intensity of the wave, the corridor of anomalous behavior becomes wider and wider and the point of intersection of q_0^- with the α_- axis ($q_0^-=0$) is shifted to the left, approaching zero:

$$\alpha_-^0 \approx \frac{1}{\xi^2}\frac{\omega_H}{\omega}(1+2\gamma n), \quad \xi^2 \gg 1+2\gamma n,$$

and thus the electron and positron states come even closer to each other. The situation is the same if the direction of the circular polarization is reversed, the only difference being that in this case it is the positron that is subject to a resonance action by the field.

3. PRODUCTION OF ELECTRON-POSITRON PAIRS

The probability of transitions between electron and positron states in the electromagnetic field given by the four-potential (1) can be written in a form analogous to the case of a pure magnetic field,[10] and differs from it only in the meanings of the variables that characterize the state of the final particles of the reaction[8]:

$$W_j = \frac{e^2 m^2 \gamma}{2k_0'} \sum_{n,n',l} \int dq_3^-\, dq_3^+\, \Phi_j \delta(k_0' + lk_0 - q_0^- - q_0^+)\, \delta(k_3' + lk_3 - q_3^- - q_3^+), \tag{6}$$

where $j = \pm 1$, 2, 3. The quantities Φ_j are expressed in terms of matrix elements of the Dirac matrices in the usual way (see, for example, Ref. 10) and for the direction of propagation of the external photon considered here can be found from the relations

$$\Phi_{\mp 1} = {}^1/_2(\Phi_2 + \Phi_3 \pm i\Phi_4), \quad \Phi_2 = (\bar{\alpha}_1^* \bar{\alpha}_1), \quad \Phi_3 = (\bar{\alpha}_2^* \bar{\alpha}_2),$$
$$\Phi_4 = (\bar{\alpha}_1^* \bar{\alpha}_2 - \bar{\alpha}_2^* \bar{\alpha}_1), \quad \bar{\alpha}_i = \int \bar{\Psi}' \alpha_i \exp(i\mathbf{k}'\mathbf{r}) \psi\, d^3x. \tag{7}$$

Here the functions $\Phi_{\mp 1}$ correspond to the process of pair production by a circularly polarized photon, and $\Phi_{2,3}$ to the case of a plane polarized external photon.

In carrying out the determination of these matrix elements with the functions (2), it must be noted that for the propagation of the external photon considered here the integration over the variable z, which by a simple substitution reduces to an integration over φ [$\varphi = \omega(t-z)$], can be performed trivially and brings in a Kronecker δ symbol,

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-igl\varphi} e^{i(n-n'\pm 1)\varphi} = \delta_{gl,n-n'\pm 1}.$$

Accordingly, the electron and positron transverse excitations, which are characterized by their Landau levels (n, n'), are rigidly connected with the number l of wave quanta involved in the reaction

$$n - n' - lg \pm 1 = 0.$$

The upper and lower signs in this formula correspond to the right and left circular polarizations of the external (non-wave) photon. It is easy to see that these specific selection rules for the states of the final particles are a manifestation of the conservation of the projection of the total angular momentum along the direction of the magnetic field (the z axis). From the conservation of helicity we have, for example, that in the given case the ground states of the electron and positron $n = n' = 0$, can be reached only when one photon of the wave is involved in the process, while this quantum can be either emitted ($l < 0$) or emitted ($l > 0$), by the system, depending on the relation of the directions of polarization of the wave and the external photon. The ground and first excited states of the electron and positron can be reached either without involving any real photon of the wave ($l = 0$), or in the interaction of the external photon with two photons of the wave ($l = \pm 2$), and so on. This effect leaves a definite imprint on the course of the entire process of electron-positron pair production in the field of a circularly polarized wave when the external photon has its direction of propagation opposite to that of the wave, and makes this process extremely sensitive to the directions of their polarizations.

It must be pointed our that this sort of effect does not occur in a linearly polarized wave,[5] nor with a different direction of propagation of the external photon.[4] We also note that the use of the selection rules makes it easy to perform one of the summations in the expression (5). Depending on which of the indices we sum over in this way, we can get different representations for the probability of the pair-production process. Using the explicit form of the normalization coefficients, and also changing the integration in (5) over the longitudinal components of the quasimomenta of the charged particles to an integration over the invariant variable α_- and α_+, according to the relation

$$\frac{1}{p_0^-p_0^+}N_-^2N_+^2\,dq_3^-\,dq_3^+\delta(\omega'+l\omega-q_0^--q_0^+)\delta(\omega'-l\omega+q_3^-+q_3^+)$$
$$=\frac{1}{m\alpha_-\alpha_+}d\alpha_-\,d\alpha_+\delta(\omega'+l\omega-q_0^--q_0^+)\delta\left(\alpha_-+\alpha_+-\frac{2\omega'}{m}\right),$$

we can write the total probability of pair production by an unpolarized photon in the form

$$W=\frac{e^2m^2\gamma}{\omega'}\sum_{n,n'}\int\frac{d\alpha_-}{\alpha_-\alpha_+}\{\Phi_-\delta_-+\Phi_+\delta_+\},$$
$$\Phi_-=\frac{1}{2\alpha_-\alpha_+}\left(\frac{2\omega'^2}{m^2}+\gamma n\alpha_+^2+\gamma n'\alpha_-^2\right)I^2_{n-1,n'}$$
$$+\sigma_2\left(\frac{1}{\Delta_-^2}I^2_{n,n'}+\frac{1}{\Delta_+^2}I^2_{n-1,n'-1}\right)-\sigma_3(\sigma_4I_{n,n'}+\sigma_5I_{n-1,n'-1})I_{n-1,n'},$$
$$\Phi_+=\frac{1}{2\alpha_-\alpha_+}\left(\frac{2\omega'^2}{m^2}+\gamma n'\alpha_+^2+\gamma n\alpha_-^2\right)I^2_{n,n'-1} \tag{8}$$
$$+\sigma_2\left(\frac{1}{\Delta_-^2}I^2_{n-1,n'-1}+\frac{1}{\Delta_+^2}I^2_{n,n'}\right)-\sigma_3(\sigma_4I_{n-1,n'-1}+\sigma_5I_{n,n'})I_{n,n'-1},$$
$$\sigma_2={}^1/_4\xi^2\omega^2\alpha_-\alpha_+,\quad \sigma_3={}^1/_2g\xi,\quad \sigma_4=\alpha_+\omega(2\gamma n)^{1/2}/\Delta_-,$$
$$\sigma_5=\alpha^-\omega(2\gamma n')^{1/2}\Delta_+,\ \delta_\mp=\delta(\omega'+(n-n'\mp1)\omega-q_0^--q_0^+),\ \alpha_+=2\omega'/m-\alpha_-.$$

Here $I_{n,n'}(x)$ is a Laguerre function, connected with the Laguerre polynomial $L_n^\alpha(x)$ by the well known relation

$$I_{n,n'}(x)=(n'!/n!)^{1/2}x^{(n-n')/2}e^{-x/2}L_{n'}^{n-n'}(x),$$

and the argument of the functions is

$$x={}^1/_2\xi^2m^2\gamma(1/\Delta_-+1/\Delta_+)^2.$$

It must be pointed out that the expression (8) also gives the probability of the process in the case when the external photon has linear polarization, and the contributions $\Phi_\mp\delta_\mp$ correspond to the probabilities of pair-production processes with a photon of definite circular polarization. The admissible values of the parameters $\alpha_\mp=2\omega'x_\mp/m$ are determined as solutions of the fourth-degree equation

$$\frac{\lambda n}{\tau}(1-x)(x\tau-1)+\frac{\lambda n'}{\tau}x(x\tau-\tau-1)\pm\lambda x(1-x)-1$$
$$+\xi^2\tau^2\frac{x(1-x)}{1-(2x-1)\tau-x(1-x)\tau^2},$$
$$\lambda=\frac{4\omega\omega'}{m^2},\quad \tau=\frac{\lambda}{2\gamma},\quad x=x_-. \tag{9}$$

The conditions for the existence of real solutions of this equation satisfying the condition $0<x<1$ with minimal values of the energy of the external photon determine the threshold of the pair-production reaction. Since the exact solution of Eq. (9) is very cumbersome, we shall carry through the analysis of the solution of this equation in a number of characteristic cases.

1. Weak magnetic field

In a weak magnetic field $H\ll H_0$ with $\tau\gg1$ it follows directly from Eq. (9) that

$$x_1=0,\quad x_2=1,\quad x_{3,4}={}^1/_2(1\pm(1-4(1+\xi^2)/\lambda l)^{1/2}),$$
$$l=n-n'\pm1. \tag{10}$$

In the present approximation the first two roots are nonphysical. The condition for the existence of the roots $x_{3,4}$ in a given case is essentially the same as the condition characteristic for the field of the wave, $\lambda_1>4(1+\xi^2)$, where $\lambda_1=\lambda(n-n'\pm1)$. In this limit there is a valid approximation of the Laguerre functions by Bessel functions for $n\gg1$, $n\gg l$:

$$I_{n,n-l}(z^2/4n)=J_l(z),$$

in which, far from the resonance point, $\alpha_-\omega\neq\omega_H$, the argument of the Bessel functions takes the form

$$z=2\xi\frac{u}{\lambda}(2\gamma n)^{1/2}\frac{\alpha^-\alpha^+\omega^2}{\Delta_-\Delta_+}=\frac{2\xi lu}{\lambda_l}\left(\frac{\lambda_l}{u}-1-\xi^2\omega^2\frac{\alpha^-\alpha^+}{\Delta_-\Delta_+}\right)^{1/2}.$$

Using this approximation for the Laguerre functions in Eq. (8), we readily obtain an expression for the spectral distribution of the probability of pair production in the quasiclassical limit ($H\ll H_0$):

$$dW^l=dW_1^l+dW_2^l,$$
$$dW_1^l=\frac{2e^2m^2du}{\omega'u[u(u-4)]^{1/2}}\left\{\left[\frac{\lambda_l}{2}-\frac{\lambda_l}{u}+1+\xi^2\omega^2\frac{\alpha_-\alpha_+}{\Delta_-\Delta_+}\left(1-\frac{u}{2}\right)\right]J_l^2\right.$$
$$+\frac{1}{4}\xi^2\omega^2\alpha_-\alpha_+\left(\frac{1}{\Delta_-^2}+\frac{1}{\Delta_+^2}\right)(J^2_{l-1}+J^2_{l+1})$$
$$\left.-\frac{1}{2}\xi\omega\left(\frac{\alpha_+}{\Delta_-}+\frac{\alpha_-}{\Delta_+}\right)\left(\frac{\lambda_l}{u}-1-\xi^2\omega^2\frac{\alpha_-\alpha_+}{\Delta_-\Delta_+}\right)^{1/2}(J_{l-1}+J_{l+1})J_l\right\}, \tag{11}$$
$$dW_2^l=\sigma g\xi\frac{\omega e^2m^2\,du}{\omega'u[u(u-4)]^{1/2}}\left\{\frac{1}{2}\xi\omega\alpha_-\alpha_+\left(\frac{1}{\Delta_+^2}+\frac{1}{\Delta_-^2}\right)(J_{l-1}+J_{l+1})\right.$$
$$\left.-\left(\frac{\alpha_+}{\Delta_-}+\frac{\alpha_-}{\Delta_+}\right)\left(\frac{\lambda_l}{u}-1-\xi^2\omega^2\frac{\alpha_-\alpha_+}{\Delta_-\Delta_+}\right)^{1/2}J_l\right\}(J_{l-1}-J_{l+1}),$$
$$\alpha^\mp=\frac{\omega'}{m}\left[1\pm\left(1-\frac{4}{u}\right)^{1/2}\right],$$

where dW_1^l is the differential partial probability for production of a pair by a linearly polarized photon, and dW_2^l is the term added if we change to the case of a circularly polarized external photon ($\sigma=\pm1$). In particular it follows from Eq. (11) that when the external photon and the wave are circularly polarized in the same direction ($\sigma g=1$) and when their polarization are opposite ($\sigma=-1$) there is an increase or a decrease, respectively, of the probability relative to the case of a linearly polarized photon.

It is easy to see that complete turning off of the magnetic field ($H\to0$) in the expression (11) leads to the well known result of Ref. 11, derived in calculating the pair production by a photon in the field of an electromagnetic wave.

2. The region of soft photons

In a different limiting case, $\tau\ll1$ ($\lambda\ll2\gamma$), Eq. (9) can again be solved easily. This region is interesting because here the process of producing an e^-e^+ pair is possible with a very weak restriction on the energy of the external photon ($\omega'\ll m$). The real roots of Eq. (9) that lie in the interval $0<x<1$ are determined by the expression

$$x_{1,2}=\frac{1}{2}\left\{1+\frac{2a}{b}\pm\left[\left(1+\frac{2a}{b}\right)^2-\frac{4}{b}\left(1+\frac{\lambda n}{\tau}\right)\right]^{1/2}\right\},$$
$$a=\gamma(n-n'),\quad b=\xi^2\tau^2+\lambda(n-n'\pm1). \tag{12}$$

In the present limiting case the threshold condition for production of an e^-e^+ pair in the excited states n and n' is

$$\lambda>\frac{2\gamma}{\xi}\left\{\left[\left(\frac{\gamma l}{\xi}\right)^2+[(1+2\gamma n)^{1/2}+(1+2\gamma n')^{1/2}]^2\right]^{1/2}-\frac{\gamma l}{\xi}\right\}, \tag{13}$$
$$l=n-n'\pm1.$$

The function on the right side of the inequality (13) increases montonically with increase of the numbers n and n'. Accordingly, the lowest value of the parameter

$\lambda=2(kk')/m^2$ is reached when a physically obvious condition is satisfied; the particles of the pair must be produced in their ground states, with the absorption of one photon of the wave:

$$\lambda>\frac{2\gamma}{\xi}\left[\left(\frac{\gamma^2}{\xi^2}+4\right)^{1/2}-\frac{\gamma}{\xi}\right]. \tag{14}$$

A characteristic feature is that for large values of the wave intensity $\xi\gg 1$, $\xi\gg\gamma$, it follows from (14) that

$$\xi\tau>2, \tag{15}$$

and when the opposite condition $\gamma\gg 1$, $\gamma\gg\xi$ is satisfied we get

$$\gamma\tau>2. \tag{16}$$

The qualitative difference between these situations is that in the former case we are concerned essentially with the condition for one-photon pair production in a transformed reference system in which, owing to the complete predominance of the magnetic field in the given field configuration (1), there is no electric field. In this system the external photon appears as a photon of energy $m\xi\tau$. When the latter condition holds the nature of the process is different; in this case the "photon" of the wave and the external photon have a combined energy sufficient for the production of a pair, and consequently the threshold condition defines the threshold for the two-photon reaction in the presence of the external magnetic field. We note that for the electromagnetic field configuration (1) the second situation ($\tau<1$, $\gamma\tau>2$) can be realized only for ultrastrong magnetic fields $H>H_0$, since the following relations hold:

$$2\gamma\tau=\lambda, \quad \gamma=H/H_0.$$

3. Strong magnetic field

Magnetic field values $H\gtrsim H_0$ are interesting in connection with possible astrophysical applications.[1,2] In this case the expression (8) for the total probability of the process contains a finite number of terms in the sums; according to the conditions (13) each contribution has its own point at which it is "turned on." In the range of magnetic fields $H<H_0$, but still with the condition $\tau<1$ satisfied, these threshold limits are closely spaced. For example, solving the inequality (13) for the dynamical parameter τ in the region of low values of the quantum numbers n and n', we find that when the conditions

$$\frac{\gamma}{\xi^2}\left[\left(1+4\frac{\xi^2}{\gamma^2}\right)^{1/2}\mp 1\right]<\tau<\frac{1}{\xi}[1+(1+2\gamma)^{1/2}] \tag{17}$$

are satisfied the electrons and positrons are produced exclusively in the ground states $n=n'=0$. It follows immediately from (17) that in the case $2\xi\gg\gamma$ the threshold conditions for processes of pair production by external photons with different circular polarizations differ only by correction terms $\gamma^2/\xi^2\ll 1$:

$$4\frac{\gamma}{\xi}-2\frac{\gamma^2}{\xi^2}<\lambda<4\frac{\gamma}{\xi}+2\frac{\gamma^2}{\xi^2}.$$

If the opposite condition $2\xi\ll\gamma$ ($\tau\ll 1$) holds, the situation is very different for external photons with different circular polarizations. Whereas for photons with the same polarization as the wave photons the channel for pair production opens up for $\lambda>4(\gamma\gg 1)$, for external photons with the opposite polarization the threshold condition is much more rigid: $\lambda>4\gamma^2/\xi^2$ $(\gamma^2/\xi^2\gg 1)$.

It is interesting to analyze the expression (8) for the total probability of the process in the region of variation of the initial parameters near the threshold: in the present case this is possible because the interaction of the charged particles with the external field is taken into account exactly. Confining ourselves to the case $\tau\ll 1$, since it is in this limit that we can observe the absolutely lowest threshold for the pair production reaction, we shall assume that the values of the parameters of the problem ensure, according to Eq. (13), that production of an electron and a positron in the ground and first excited states $n, n'=0, 1$ is possible. Using the explicit form of the Laguerre functions

$$I_{0,0}(z)=e^{-z/2}, \quad I_{\substack{1,0\\0,1}}=\pm z^{1/2}e^{-z/2}$$

of the argument z, which in the limit now considered, $\tau\ll 1$, can be considered a constant

$$z=\frac{1}{2\gamma}\xi^2\tau^2=\frac{1}{(2\gamma)^3}\xi^2\lambda^2,$$

and performing the integration over the variable $\varkappa$ by using the properties of the δ function, after simple transformations we get

$$W_{l=0}^{\substack{n=1,\,n'=0\\n=0,\,n'=1}}=\frac{e^2m^2\gamma}{2\omega'}\frac{[1/\gamma+(3\mp 2)(1-1/z)]}{(1-A_-/z)^{1/2}(1-A_+/z)^{1/2}}e^{-z}, \tag{18}$$

$$A_\mp=\frac{1}{2\gamma}[1\mp(1+2\gamma)^{1/2}]^2;$$

$$W_{l=\pm 1}^{n=0,n'=0}=\frac{e^2m^2\gamma}{2\omega'\xi^2\tau^2}\left(1\pm\frac{\tau}{z}\right)^{-3/2}\left(1-\frac{B_\mp}{z}\right)^{-1/2}e^{-z},$$

$$B_\mp=\frac{4\mp\lambda}{2\gamma}=\frac{2}{\gamma}\mp\tau; \tag{19}$$

$$W_{l=\pm 2}^{\substack{n=1,\,n'=0\\n=0,\,n'=1}}=\frac{e^2m^2\gamma}{2\omega'}\frac{[1/\gamma\pm 2(1-1/(z\pm 2\tau))]}{(1\pm 2\tau/z)^2(1-A_-/(z\pm 2\tau))^{1/2}(1-A_+/(z\pm 2\tau))^{1/2}}e^{-z}. \tag{20}$$

Here we have displayed the contributions to the total probability (8) of pair production processes, which in the present case can be stated as a sum of the probabilities of reactions involving one [Eq. (18)], two [Eq.(19)], and three [Eq. (20)] photons:

$$W=\sum_{n=0}^{1}\sum_{n'=0}^{1}W_{nn'}.$$

The threshold singularity present in these formulas is a rather characteristic feature of processes in a magnetic field,[10] in which the particles produced "land" precisely on a discrete energy level in the magnetic field. As has been pointed out [see Eq. (13)], the "mechanism" of the two-photon reaction comes into play first, since it is the one that provides the energetically most favorable production of a pair in the ground state. With Eq. (19) as an example one can see that in the immediate neighborhood of the reaction threshold an exponential factor is imposed on the magnetic-field square-root singularity. For $\gamma\ll\xi^2\tau^2$ the resonance region is very narrow and is defined by the exponential e^{-z} with $z\gg 1$. When the strength of the magnetic field is increased the resonance region becomes wider. Generally speaking, an increase of the intensity of the wave, with the other parameters fixed, leads to a more rapid exponential decrease, but according to Eq. (13) new reaction channels begin to open up as the intensity of the wave is increased. A characteristic effect is that with increasing intensity of the wave the relative importance of the two-photon processes decreases:

$$\frac{W_{l=\pm1}}{W_{l=0}}=\frac{1}{2z}\frac{(1-A_-/z)^{1/2}(1-A_+/z)^{1/2}}{(1\pm\tau/z)^{1/2}(1-B_\mp/z)^{1/2}[1/\gamma+(3\mp2)(1-1/z)]}$$

$z>1$, since pair production into excited states $(n,n'\neq 0)$ is possible only for $2\gamma<\xi^2\tau^2$.

It is interesting to carry out an analysis of the probability of the process defined by Eq. (19) from the point of view of the consequences of Eqs. (13) and (14) which we have indicated. From the conditions (13) for the existence of processes with pair production in excited states it follows that in this case the inequality $\xi^2\tau^2>1+2\gamma$ must be satisfied.

Accordingly, for $\tau\ll 1$, $\gamma\ll 1$ the probability of each of the partial processes with fixed n and n' is always governed by an exponential factor with a large negative exponent. The exponent can be small in the case

$$\xi^2\tau^2\ll 2\gamma, \tag{21}$$

from which and Eq. (13) we see that only processes with the pair produced in the ground states $n=n'=0$ make an appreciable contribution to the total probability. For $\gamma<1$ this situation cannot be realized, since it leads to a contradiction between Eq. (21) and the inequality (14) that determines the threshold of the reaction in this case. For $\gamma\gg 1$ the inequalities (21) and (14) are consistent with each other, and as previously indicated [cf. Eqs. (15), (16)], there are two different mechanisms for the process. For the first of these possibilities, which is realized when the wave is very intense $(\xi\gg\gamma)$, for $2\gamma\tau<4$ we find that in this case photons with different circular polarizations have about the same probability for producing pairs,

$$W^\pm=\frac{2}{\xi\tau^2}\frac{e^2\omega}{(\xi^2\tau^2-4)^{1/2}}, \tag{22}$$

which decreases as the intensity of the wave increases. For the second possibility the probability and the cross section in the limit $2\gamma\gg\xi^2\tau$ are given by the expressions $(\lambda>4)$

$$W^+=\frac{2}{\lambda^2}e^2\omega\xi^2\tau,\quad \sigma^+=\frac{1}{\lambda\gamma}4\pi r_0^2,\quad r_0=\frac{e^2}{m}. \tag{23}$$

It follows from this that in this case the cross section does not depend on the intensity of the wave and decreases with increase of the magnetic field strength. If $\lambda<4\gamma^2/\xi^2$, photons with polarization opposite to that of the wave are propagated without absorption.

4. The quasiclassical region

As the value of the parameter $\chi=\xi\tau\gamma$ increases, higher and higher levels $n,n'\gg 1$ begin to be excited; this follows, for example, from the condition (13). Combining the expression for the argument of the Laguerre function with the condition (13), we find that in the limit $\xi\tau\gg 1$ the pair-production process becomes quasiclassical and we can approximate the Laguerre function with the Airy function $\Phi(t)$:

$$I_{n,n-l}(x)=\frac{(-1)^{n-l}}{\pi^{1/2}}[n(n-l)x_0^2]^{-1/6}\Phi(t),$$
$$t=[n(n-l)x_0^2]^{1/6}(x/x_0-1),\quad x_0\approx 4n.$$

It is not hard to show that in this limit with $\gamma\ll 1$, $\lambda\ll 1$ the expression (8) for the probability agrees with the well known classical result,[10,12] the corrections being of the order of γ^2 and λ^2.

We now point out that since Eq. (9) has a resonance denominator at $\tau>1$ it always has at least one real solution, and consequently, regardless of the (positive) value of the value of ξ, the energy of the external photon at which the pair-production process occurs is determined by the condition

$$\omega'>\frac{m}{2}\frac{\omega_H}{\omega}.$$

We also note that this same threshold condition was also indicated in the case of a linearly polarized wave.[5] The marked lowering of the threshold of the reaction, which is actually observed with a wave of very high intensity, is described by the inequalities (13), (14); in particular it follows from these conditions that in an intense wave $(\xi\gg 1)$ the process of production of an e^-e^+ pair by a photon with energy considerably smaller than the rest energy of the electron,

$$\omega'=\frac{1}{\xi}m\frac{\omega_H}{\omega},$$

can occur without real absorption of photons of the wave.

4. CONCLUSION

Detailed information about the character of the process of photoproduction of pairs in the external field (1) in the range of external-photon energies which is below-threshold as compared with the analogous reaction in a pure magnetic field $(\omega'\ll m)$[4,5] can be obtained from the exact expression (8). In the case when the external-photon energy $\omega'>\frac{1}{2}m\omega_H/\omega$ the pair-production process goes for arbitrary intensity of the wave $(\xi>0)$. Furthermore there are contributions to the total probability from all possible states of the electron and positron, characterized by their quantum numbers n and n'. In this case, if $\gamma\ll 1$, the reaction occurs in such a way that the main contribution to the total probability is from electron and positron states in high levels, $n,n'\gg 1$. If furthermore $\tau\gg 1$, then in this limit the expression (8) agrees with the known result[11] for the process of pair production in the field of a wave. If $\xi\tau\gg 1$ the main term of Eq. (8) agrees with the probability of the process in crossed fields.[10,12] In the region $\omega'\gg m\omega_H/\omega$ the dominant process involves lowering of the reaction threshold because of multiple absorption of photons of the wave, and the threshold conditions are essentially the same as the analogous conditions in the case of a pure wave: $l\omega\omega'>m^2(1+\xi^2)$.

If $\tau>1$ and $\gamma>1$ the threshold conditions on the energy of the external photon and on a wave photon are very rigid: $2\omega\omega'>\gamma m^2\gg m^2$; therefore in strong magnetic fields $(H>H_0)$ the main contribution to the probability comes from the region $\tau<1$, where fulfillment of the threshold conditions can be secured either owing to the large intensity of the wave, $\xi>\gamma$, or when the magnetic field is the controlling factor, $\gamma>\xi$.

A characteristic feature of the region $\omega'<\frac{1}{2}m\omega_H/\omega$ is that the partial processes that contribute to the total probability involve a limited number of the discrete levels of the electron and positron in the magnetic field [cf. Eq. (13)]. Furthermore the process of e^-e^+ production does not occur for all values of the intensity of the wave and the strength of the magnetic field. Owing to the conservation of helicity in the system, the number of photons of the wave that take part in the reaction is rigidly related to the transverse excitations of the electron and positron.

The first possibility for the process is realized when the conditions $\tau<1$, $\xi\tau>2$, and $\gamma\tau<2$ are satisfied. In this case the process in question is essentially equivalent to the process of one-photon pair production in a constant uniform magnetic field by an external photon with transverse energy $\omega'=m\xi\tau$. The invariant condition $\xi\tau>2$ assures the conservation of energy in the system.

The second mechanism for the reaction is important if $\tau<1$, $\xi\tau<2$, $\gamma\tau>2$. The realization of these conditions means that the only partial process that contribute to the total probability of the process are those that involve one photon of the wave; depending on the relation of the polarizations of the wave and of the external photon, this wave photon can be either emitted or absorbed in the interaction.

If $\xi\tau\gg\gamma^{\frac{1}{2}}$ the probability of partial processes with fixed values of n and n' is exponentially small. For the opposite condition, $\gamma^{1/2}\gg\xi\tau$, in a strong magnetic field ($\gamma>1$) processes are possible whose probability is not exponentially small, but falls [cf. Eq. (22)] or rises [cf. Eq. (23)] with increasing intensity of the wave. Here also the only states of the electron and positron that contribute are the ground states. The particles produced are strictly polarized relative to the direction of the magnetic field, owing to the specific properties of the ground states of the electron and positron in an electromagnetic field of the configuration (1) in question. The cross section for the two-photon reaction process does not depend on the intensity of the wave and falls off as the magnetic field strength increases [Eq. (22)], as compared with the analogous value in the free case (Breit-Wheeler cross section).

Accordingly, from the calculations made here for different regions of variation of the invariant parameters ξ, γ, λ we can draw the conclusion that the external field (1) leads to an extremely specific dependence of both the probability of the pair-production process and the threshold conditions for its occurrence on the polarization properties of the particles of the reaction.

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