Thermoelectromagnetic waves in conductors in a strong magnetic field

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We study electromagnetic waves in metals, semimetals, and semiconductors in the presence of a strong magnetic field and a temperature gradient at different relative directions of the magnetic field, the temperature gradient, and the wave vector. Our study is based on a microscopic theory of the conductivity and the thermoelectric current. In a number of cases the waves are thermomagnetic in the following sense: a) the frequency (or the damping) is proportional to the temperature gradient; b) a growth of the wave (i.e., self-excitation) occurs which is proportional also to the temperature gradient; c) under certain conditions, depending on the temperature gradient, cyclotron resonance occurs and the cyclotron waves may also grow with time. We also consider the case where there is no magnetic field present. We give a linear theory of these effects in the present paper.

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1. INTRODUCTION

The concept of thermo-magnetic waves was introduced in Refs. 1, 2 and they were studied for a frequency ω much smaller than the collision frequency ν. Kobylov[3] observed these waves experimentally.

In the present paper we generalize the concept of thermo-magnetic waves: we study both low-frequency and high-frequency ($\omega \gg \nu$) electromagnetic waves when there is a strong (but not quantized) magnetic field $\mathbf{B}_0$ present in a conducting medium in which there is a temperature gradient ∇T (T is measured in energy units). We call these waves thermo-electromagnetic, if the frequency ω or its imaginary part depends strongly on ∇T, for instance, is proportional to it. We consider the dispersive properties of the thermo-electromagnetic waves and also the way the frequency of the wave and its damping (or growth) depend both on ∇T and on the constant magnetic field $\mathbf{B}_0$.

In contrast to the earlier papers[1,2] where we introduced phenomenological coefficients, we determine the current density $\mathbf{j}$ from the solution of the kinetic equation; we estimate the collision integral in the approximation of an average collision frequency ν averaged over the electron momenta.

We assume an isotropic quadratic electron spectrum. We evaluate the current density in the approximation linear in ∇T (which is sufficient for solids) and also linear in the alternating electric field $\mathbf{E}'$ and magnetic field $\mathbf{B}'$. We restrict ourselves to the case of carriers of a single sign and of a single energy band.

2. KINETIC EQUATION AND CURRENT DENSITY

The kinetic equation for the current-carrier distribution function $f(\mathbf{p})$ has the form

$$\frac{\partial f(\mathbf{p})}{\partial t}+\mathbf{v}\nabla f(\mathbf{p})+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\frac{\partial f(\mathbf{p})}{\partial \mathbf{p}}+\nu[f(\mathbf{p})-f_0(\mathbf{p})]=0, \quad (1)$$

where $f_0(\mathbf{p})$ is the equilibrium distribution function in which, however, the temperature T varies slowly from point to point so that

$$\nabla f_0(\mathbf{p})=\frac{\partial f_0}{\partial T}\nabla T. \quad (2)$$

We shall assume that the inhomogeneity length $L=T/\nabla T$ is appreciably longer than the particle mean free path. The kinetic coefficients depend then on T as a parameter and we can neglect this dependence for the solution of the kinetic equation.

To simplify the formulae we introduce the cyclotron frequencies

$$\boldsymbol{\Omega}_0=e\mathbf{B}_0/mc, \quad \boldsymbol{\Omega}'=e\mathbf{B}'/mc, \quad (3)$$

and put $\mathbf{E}'\propto\boldsymbol{\Omega}'\propto\exp[i(\mathbf{k}\cdot\mathbf{r}-\tilde{\omega}t)]$ where

$$\tilde{\omega}=\omega+i\gamma. \quad (4)$$

We put further

$$\mathbf{B}=\mathbf{B}_0+\mathbf{B}', \quad \mathbf{E}=\mathbf{E}_0+\mathbf{E}', \quad (5)$$

where $\mathbf{E}_0$ is the thermo-electric field which has a component parallel to ∇T and also components $\mathbf{E}_0^{\parallel}\propto\boldsymbol{\Omega}_0(\boldsymbol{\Omega}_0\nabla T)$ and $\mathbf{E}_0^{\perp}\propto[\nabla T\times\boldsymbol{\Omega}_0]$. We look for the distribution function in the form

$$f(\mathbf{p})=f_0(\mathbf{p})+f_1(\mathbf{p})+f_2(\mathbf{p})+f_3(\mathbf{p}), \quad (6)$$

where $f_1(\mathbf{p})\propto\nabla T$ is time-independent and satisfies the equation

$$(1+\xi)\mathbf{v}\nabla T\frac{\partial f_0}{\partial T}+\frac{e}{m}\mathbf{E}_0\frac{\partial f_0}{\partial \mathbf{v}}+[\mathbf{v}\times\boldsymbol{\Omega}_0]\frac{\partial f_1}{\partial \mathbf{v}}+\nu_1 f_1=0. \quad (7)$$

The solution of this equation is

$$f_1(\mathbf{p})=\frac{\mathbf{v}}{\nu_1(\nu_1^2+\Omega_0^2)}\frac{\partial f_0}{\partial\varepsilon_p}\{\nu_1^2((1+\xi)x-\alpha)\nabla T$$
$$+\nu_1((1+\xi_\perp)x-\alpha_\perp)[\nabla T\times\boldsymbol{\Omega}_0]+((1+\xi_\parallel)x-\alpha_\parallel)(\boldsymbol{\Omega}_0\nabla T)\boldsymbol{\Omega}_0\}=\mathbf{v}g_1(\varepsilon_p) \quad (8)$$

($x=(\varepsilon_p-\mu)/T$, ε_p is the energy and μ the chemical potential). Here ξ, $\xi_\perp$, and $\xi_\parallel$ take qualitatively into account the drag of the electrons by the phonons, while α, $\alpha_\perp$, and $\alpha_\parallel$ are thermo-electric coefficients determined by the condition that the corresponding components of the constant current $\mathbf{j}_1$ vanish.

The function $f_2(\mathbf{p})\propto\mathbf{E}'$ is independent of ∇T and determined by the equation

$$\frac{\partial f_2}{\partial t}+\mathbf{v}\nabla f_2+\frac{e}{m}\mathbf{E}'\frac{\partial f_0}{\partial \mathbf{v}}+[\mathbf{v}\times\boldsymbol{\Omega}_0]\frac{\partial f_2}{\partial \mathbf{v}}+\nu_2 f_2=0. \quad (9)$$

Its solution is

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$$[\text{illegible}] = -\frac{e\mathbf{v}}{(\nu_2 - i\omega + i\mathbf{kv})((\nu_2 - i\omega + i\mathbf{kv})^2 + \Omega_0^2)} \frac{\partial f_0}{\partial \varepsilon_p} \cdot \{(\nu_2 - i\omega + i\mathbf{kv})^2 \mathbf{E}' + (\nu_2 - i\omega + i\mathbf{kv})[\mathbf{E}' \times \boldsymbol{\Omega}_0] + \boldsymbol{\Omega}_0(\mathbf{E}'\boldsymbol{\Omega}_0)\}. \tag{10}$$

We shall in what follows consider waves for which $\mathbf{k}\cdot\mathbf{v} \ll \omega$; estimates show that this is feasible both for high-frequency ($\omega \gg \nu, \mathbf{k}\cdot\mathbf{v}$) and for low-frequency ($\nu \gg \omega, \mathbf{k}\cdot\mathbf{v}$) waves. This means neglect of Landau damping and when $\boldsymbol{\Omega}_0 \parallel \nabla T$ also neglect of spatial dispersion (articles B and D below).

Finally, the function $f_3(\mathbf{p})$ is proportional to $\boldsymbol{\Omega}'$ and ∇T; the corresponding equation has the form

$$\frac{\partial f_3}{\partial t} + \mathbf{v}\nabla f_3 + [\mathbf{v} \times \boldsymbol{\Omega}_0] \frac{\partial f_3}{\partial \mathbf{v}} + [\mathbf{v} \times \boldsymbol{\Omega}'] \frac{\partial f_1}{\partial \mathbf{v}} + \nu_3 f_3 = 0, \tag{11}$$

and its solution is

$$f_3(\mathbf{p}) = -\frac{\mathbf{v}}{\tilde{\nu}(\tilde{\nu}^2 + \Omega_0^2)} \{\tilde{\nu}^2 [\boldsymbol{\Omega}' \times \mathbf{g}_1] + \tilde{\nu}[[\boldsymbol{\Omega}' \times \mathbf{g}_1]\boldsymbol{\Omega}_0] + \boldsymbol{\Omega}_0([\boldsymbol{\Omega}' \times \mathbf{g}_1]\boldsymbol{\Omega}_0)\}, \tag{12}$$

where

$$\tilde{\nu} = \nu_3 - i\omega. \tag{13}$$

As all three functions $f_1(\mathbf{p})$, $f_2(\mathbf{p})$, and $f_3(\mathbf{p})$ are proportional to the first Legendre polynomial, the frequencies $\nu_1 \approx \nu_2 \approx \nu_3 = \nu$ and in what follows we shall therefore not distinguish between them. Simple calculations then lead to the following expression for the variable part of the electrical current:

$$j = j_2 + j_3.$$

where

$$\mathbf{j}_2 = -\frac{e^2}{3} \frac{J_1}{\tilde{\nu}(\tilde{\nu}^2 + \Omega_0^2)} [\tilde{\nu}^2 \mathbf{E}' + \tilde{\nu}[\mathbf{E}' \times \boldsymbol{\Omega}_0] + \boldsymbol{\Omega}_0(\mathbf{E}'\boldsymbol{\Omega}_0)], \tag{14}$$

$$\begin{aligned} \mathbf{j}_3 = -\frac{e}{3} \frac{J_2}{\nu\tilde{\nu}(\nu^2 + \Omega_0^2)(\tilde{\nu}^2 + \Omega_0^2)} \{&\nu^2\tilde{\nu}^2 [\nabla T \times \boldsymbol{\Omega}'] \\ &+ \nu\nu^2 [[\nabla T \times \boldsymbol{\Omega}_0] \times \boldsymbol{\Omega}'] + \tilde{\nu}^2 (\boldsymbol{\Omega}_0 \nabla T)[\boldsymbol{\Omega}_0 \times \boldsymbol{\Omega}'] + \nu^2\tilde{\nu}[[\nabla T \times \boldsymbol{\Omega}'] \times \boldsymbol{\Omega}_0] \\ &+ \nu\tilde{\nu}[[[\nabla T \times \boldsymbol{\Omega}_0] \times \boldsymbol{\Omega}'] \times \boldsymbol{\Omega}_0] + \tilde{\nu}(\boldsymbol{\Omega}_0 \nabla T)[[\boldsymbol{\Omega}_0 \times \boldsymbol{\Omega}'] \times \boldsymbol{\Omega}_0] \\ &+ \nu^2 \boldsymbol{\Omega}_0([\nabla T \times \boldsymbol{\Omega}']\boldsymbol{\Omega}_0) + \nu\boldsymbol{\Omega}_0([[\nabla T \times \boldsymbol{\Omega}_0] \times \boldsymbol{\Omega}'] \times \boldsymbol{\Omega}_0)\}; \end{aligned} \tag{15}$$

here

$$J_1 \approx \int v^2 \frac{\partial f_0}{\partial \varepsilon_p} (d\mathbf{p}) = -\frac{3n}{m} \tag{16}$$

both when there is and when there is no degeneracy, while

$$J_2 \approx \int v^2 \frac{\partial f_0}{\partial T} (d\mathbf{p}) = \frac{3n}{m}, \frac{p_F T}{18\hbar^3} \tag{17}$$

respectively, when there is no degeneracy and when there is degeneracy; $(d\mathbf{p}) = 2d^3p/(2\pi\hbar)^3$. In the last case we used the fact that when there is degeneracy

$$\mu(T) = \mu_0 \left[1 + \frac{\pi^2}{12}\left(\frac{T}{\mu_0}\right)^2\right]. \tag{18}$$

The expression for $\mathbf{j}_3$ needs an explanation. The current $\mathbf{j}_3$ is non-vanishing only when ν depends on the electron energy and on average because the frequency ν depends on the relative directions of the electron and phonon currents. The frequencies ν_1 and ν_3 enter into the expressions for $\mathbf{j}_1$ and $\mathbf{j}_3$ in a different way and therefore $\mathbf{j}_3$ does not vanish, although $\mathbf{j}_1$ does. Averaging the frequency ν makes the expressions for the currents $\mathbf{j}_2$ and $\mathbf{j}_3$ correct apart from numerical coefficients of order unity.

It is convenient to introduce the dimensionless quantities

$$\nabla\tilde{T} = \nabla T/mc v, \quad \nabla\tilde{T} = p_F T \nabla T/nvc\hbar^3$$

respectively, for the non-degenerate and the degenerate gas; we also introduce the notation $\varkappa = ck$.

Since the dispersion equation is very complicated it is necessary for us to carry out estimates to simplify it. We shall put $k \sim 10$ to 10^2 cm^{-1}, $\nu \sim 10^{10}$ to 10^{11} s^{-1} for metals and $\nu \sim 10^8$ to 10^{10} s^{-1} for bismuth,[4] $\nabla T \sim 10$ K/cm, i.e., $\nabla\tilde{T} \sim 10^{-8}$ to 10^{-10} in metals and $\nabla\tilde{T} \sim 10^{-5}, 10^{-7}$ in bismuth along the binary and the trigonal axes, respectively; furthermore, $\Omega_0 \sim 10^{11}$ to 10^{12} s^{-1}; as to the plasma frequency $\omega_p = (4\pi ne^2/m)^{1/2}$, most natural is the inequality $\omega_p^2 \gg \Omega_0^2$, but the opposite inequality is also possible for semiconductors. Finally, we note that in what follows the inequality $a^2 \ll b^2$ will indicate that we neglect a^2 as compared to b^2.

3. THERMO-ELECTROMAGNETIC WAVES

Substituting the expressions for the current density into the Maxwell equations we get the polarization of the waves and the equation connecting the complex frequency $\omega + i\gamma$ with the wave vector (we note that the relation

$$\operatorname{div} \mathbf{E}' = 4\pi e \int f(\mathbf{p})\, (d\mathbf{p})$$

shows that the electrical field can also have a longitudinal component). As it is extraordinarily complicated we study it in different particular cases, choosing well defined directions of the vectors $\boldsymbol{\Omega}_0$, ∇T, and $\boldsymbol{\varkappa}$.

A. All three vectors mutually perpendicular

The dispersion equation splits into equations for the ordinary and the extra-ordinary waves; the latter has also a longitudinal component. We have

$$\tilde{\nu}(\varepsilon\omega^2 - \varkappa^2) + i\omega\omega_p^2 = \frac{i\nu\omega_p^2([\widetilde{\nabla T} \times \boldsymbol{\Omega}_0]\boldsymbol{\varkappa})}{\nu^2 + \Omega_0^2}, \tag{19}$$

$$\varepsilon\omega(\varepsilon\omega^2 - \varkappa^2) + \frac{i\omega_p^2}{\tilde{\nu}^2 + \Omega_0^2}[i\omega\omega_p^2 + \tilde{\nu}(2\varepsilon\omega^2 - \varkappa^2)] = -\frac{i\nu\omega_p^2([\widetilde{\nabla T} \times \boldsymbol{\Omega}_0]\boldsymbol{\varkappa})}{(\nu^2 + \Omega_0^2)(\tilde{\nu}^2 + \Omega_0^2)}[i\omega_p^2 + \varepsilon\omega(\nu + \tilde{\nu})], \tag{20}$$

where ε is the dielectric permittivity of the lattice.

For the ordinary wave the case $\omega^2 \gg \nu^2$ is not realized.

A1) $\Omega_0^2 \gg \nu^2 \gg \omega^2$:

$$\omega = \frac{\nu\omega_p^2([\widetilde{\nabla T} \times \boldsymbol{\Omega}_0]\boldsymbol{\varkappa})}{\Omega_0^2(\varkappa^2 + \omega_p^2)}, \quad \gamma = -\frac{\nu\varkappa^2}{\varkappa^2 + \omega_p^2}. \tag{21}$$

The wave is linearly polarized along the magnetic field; it is weakly damped, if $\nabla\tilde{T} \gg \varkappa\Omega_0/\omega_p^2$. If (for metals) $\varkappa \sim 10^{11}$ s^{-1}, $\omega_p \sim 10^{16}$ s^{-1}, $\Omega_0 \sim 10^{11}$ s^{-1}, we must have $\nabla\tilde{T} \gg 10^{-10}$. We put $\nabla T \approx 10$ K/cm, and then $\nabla\tilde{T} \approx 10^{-8}$ to 10^{-9} and the inequality is satisfied.

We turn to the extraordinary wave:

A2) $\Omega_0^2 \gg \omega^2 \gg \nu^2$. In that case when $\omega_p^2 \ll \Omega_0^2$

$$\omega = \omega_p(\varepsilon\varkappa^2/\omega_p^2 + \omega_p^2/\Omega_0^2)^{1/2}, \qquad \gamma = \pm\frac{\nu^2([\widetilde{\nabla T} \times \boldsymbol{\Omega}_0]\boldsymbol{\varkappa})\omega_p^2(2\varkappa^2 + \Omega_0^2)}{\Omega_0^6 \qquad \omega}. \tag{22}$$

The wave is elliptically polarized. If $[\nabla\tilde{T} \times \boldsymbol{\Omega}_0]\boldsymbol{\varkappa} > 0$, a right-handedly polarized wave grows and a left-handedly polarized wave is damped; if the inequality has the opposite sign the branches change place. A weak growth and a weak damping are possible under a wide range of conditions.

In the linear approximation to which we restrict ourselves in the present paper it is impossible to determine completely the growth rate and we can merely say that self-excited waves will result for which $\gamma > 0$ or that waves leaving the crystal will be amplified compared to a wave incident on it from outside. Indeed, in the linear approximation an outgoing wave satisfies the equation $\tilde{\omega} = ck$ and hence, a positive imaginary part of $\tilde{\omega}$, i.e., growth in time, corresponds to a positive imaginary part of k, i.e., spatial damping. The kinetics of self-excitation can thus be studied solely in a nonlinear theory.

When $\omega_p^2 \gg \Omega_0^2$ the magnitudes of ω and γ are the same as for the wave A1).

A3) $\omega \sim \Omega_0 \gg \nu$, $\omega_p \gtrsim \Omega_0$ where the ~-sign indicates equality. We have

$$\omega=\frac{\Omega_0}{1+\varepsilon\Omega_0^2/\omega_p^2}, \quad \gamma=\pm\frac{\nu^2([\widetilde{\nabla T}\boldsymbol{\Omega}_0]\boldsymbol{\varkappa})}{\Omega_0^3(1+\varepsilon\Omega_0^2/\omega_p^2)}. \tag{23}$$

When $\omega_p^2 \gg \Omega_0^2$ cyclotron resonance occurs. If $[\nabla\tilde{T}\times\boldsymbol{\Omega}_0]\boldsymbol{\varkappa} > 0$ the left-handedly circularly polarized wave grows and the right-handedly polarized one is damped; for the other sign of the inequality the situation is the reverse.

B. All three vectors collinear

The dispersion equation splits into equations for the Langmuir oscillations which do not contain ∇T and an equation for a transverse wave:

$$(\varepsilon\omega^2-\varkappa^2)(\tilde{\nu}^2+\Omega_0^2)\left[(\varepsilon\omega^2-\varkappa^2)(\tilde{\nu}^2+\Omega_0^2)+2i\omega_p^2\tilde{\nu}\left(\omega+\frac{\omega_T^3}{\nu^2+\Omega_0^2}\right)\right]$$
$$=\omega\omega_p^4\left[\omega(\tilde{\nu}^2+\Omega_0^2)+2\omega_T^3\left(1+\frac{\tilde{\nu}^2}{\nu^2+\Omega_0^2}\right)\right], \tag{24}$$

where $\omega_T^3 = (\boldsymbol{\Omega}_0\cdot\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\cdot\nabla\tilde{T})$.

We start with the case

B1) $\Omega_0^2 \gg \omega^2\nu^2$, $\varkappa^2$ $\varkappa_0^2$, where $\varkappa_0 = \omega_p^2(\Omega_0\cdot\nabla\tilde{T})/\Omega_0^2$. We have

$$\omega=\Omega_0\varkappa^2/\omega_p^2,$$
$$\gamma=\pm\frac{\nu\varkappa(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\Omega_0^2}\frac{\omega_p^4+\varkappa^2(2\omega_p^2+\varepsilon\Omega_0^2)}{\omega_p^4}. \tag{25}$$

The waves are circularly polarized in both branches, and the right-handedly polarized wave grows while the left-handedly polarized wave is damped, if $\boldsymbol{\Omega}_0\nabla\tilde{T} > 0$; if the sign of the inequality is the opposite growth and damping change places.

When $\varkappa^2 \ll \varkappa_0^2$

$$\omega+i\gamma=\pm\frac{i\varkappa}{\Omega_0^2}\left[\Omega_0^2(\boldsymbol{\Omega}_0\widetilde{\nabla T})^2+\frac{i}{\omega_p^2}2\nu(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})(\boldsymbol{\Omega}_0^2-2\omega_p^2\widetilde{\nabla T}^2)\right]^{1/2}. \tag{26}$$

Here also the left-handedly polarized wave is weakly and damped and the right-handedly polarized one grows weakly.

When $\Omega_0^2 = 2\omega_p^2\nabla\tilde{T}^2$ the frequency turns out to be purely imaginary, i.e., in that approximation it is possible to excite a magnetic field at right angles to the field $\mathbf{B}_0$. The oscillation period of that field is larger than the growth time by a factor $\varkappa/\varkappa_0$. If $\varkappa \to 0$, the generated field will be not only nearly constant in time, but also nearly uniform in space.

B2) $\Omega_0^2 \gg \nu^2 \gg \omega^2$. In that case

$$\omega=\frac{\varkappa^2\Omega_0}{(2\varepsilon\varkappa^2\Omega_0^2+\omega_p^4)^{1/2}},$$
$$\gamma=\pm\frac{\nu\varkappa\omega_p^4(\omega_p^2-2\varkappa^2)(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\Omega_0^2(2\varepsilon\varkappa^2\Omega_0^2+\omega_p^4)^{3/2}}. \tag{27}$$

the situation is here analogous to B1) and is the same as the one considered in Ref. 2.

B3) $\omega \sim \Omega_0 \gg \nu$. If then $\omega_p^2 \gg \Omega_0^2$, we have

$$\omega=\frac{\Omega_0}{1+\varepsilon\Omega_0^2/\omega_p^2},$$
$$\gamma=\pm\frac{2\nu(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\Omega_0^3(1+\varepsilon\Omega_0^2/\omega_p^2)^2}. \tag{28}$$

The left-handedly circularly polarized wave grows and the right-handedly polarized one is damped, if $(\boldsymbol{\Omega}_0\cdot\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\cdot\nabla\tilde{T}) > 0$; for the opposite sign the branches change place.

If, however, $\omega_p^2 < \Omega_0^2$, we have

$$\omega=(\Omega_0^2+\omega_p^2/\varepsilon)^{1/2},$$
$$\gamma=\pm\frac{\omega_p}{\Omega_0}\left[\frac{\nu(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\varepsilon\Omega_0^2+\omega_p^2}\right]^{1/2}. \tag{29}$$

The polarization of the waves and the conditions for growth and damping are the same as in the preceding case.

B4) Cyclotron resonance: $\tilde{\omega} = \Omega_0(1+\beta)$, $\beta \ll 1$. We study it for the case when $\varkappa^2 \ll \Omega_0^2$, which simplifies the calculation and holds true in a strong magnetic field. We have

$$\omega-\Omega_0+i\gamma=\Omega_0(\beta'+i\beta''). \tag{30}$$

When $\nabla T = 0$ the quantity $\gamma < 0$, as it should be. Hence

$$\omega-\Omega_0+i\gamma=\frac{A}{B^2+C^2}(B+iC), \tag{31}$$

where

$$A=2\nu\omega_p^2[\Omega_0^2(2\varepsilon\Omega_0^2-\omega_p^2)+2(4\varepsilon\Omega_0-\omega_p^2/\Omega_0)(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})], \tag{32}$$
$$B=4\nu[2\varepsilon^2\Omega_0^6+\omega_p^2\Omega_0^2(\omega_p^2-4\varepsilon\Omega_0^2)+(\omega_p^4/\Omega_0)(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})], \tag{33}$$
$$C=2\omega_p^2[\Omega_0^3(\omega_p^2-2\varepsilon\Omega_0^2)+2\omega_p^2(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})]. \tag{34}$$

Complete cyclotron resonance starts when $A = 0$ or when $B = C = 0$, and both these equalities are fully realizable. If $A = 0$, in the approximation taken here $\gamma = 0$ also; if, however, $A \neq 0$, γ can change sign so that both damping and growth of the cyclotron thermo-magnetic waves is possible.

The quantity $A = 0$ if

$$\frac{\omega_p^2}{2\varepsilon}-\Omega_0^2=4\left(\Omega_0^2-\frac{\omega_p^2}{4\varepsilon}\right)\frac{(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\Omega_0^3}. \tag{35}$$

Hence, if $(\boldsymbol{\Omega}_0\cdot\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\cdot\nabla\tilde{T}) < 0$, either $\omega_0^2 > \omega_p^2/2\varepsilon$, or $\Omega_0^2 < \rho_p^2/4\varepsilon$; if, however, $(\boldsymbol{\Omega}_0\cdot\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\cdot\nabla\tilde{T}) > 0$, we have

$$\omega_p^2/4\varepsilon<\Omega_0^2<\omega_p^2/2\varepsilon. \tag{36}$$

Moreover, $C = 0$ if

$$\frac{2\varepsilon\Omega_0^2}{\omega_p^2}-1=\frac{2(\boldsymbol{\Omega}_0\boldsymbol{\varkappa})(\boldsymbol{\Omega}_0\widetilde{\nabla T})}{\Omega_0^3}.$$

C. The case $\Omega_0 \perp \nabla T \parallel \varkappa$

Two waves are possible here. The wave which is polarized along the magnetic field satisfies the equation

$$\tilde{\nu}(\varepsilon\omega^2-\varkappa^2)+i\omega\omega_p^2=\frac{i\nu^2\omega_p^2(\varkappa\widetilde{\nabla T})}{\nu^2+\Omega_0^2}. \quad (37)$$

The inequality $\omega^2 \gg \nu^2$ cannot be realized.

C1) $\Omega_0^2 \gg \nu^2 \gg \omega^2$. If we have the inequality $\nu\omega \ll \varkappa^2, \omega_p^2$ which is then the obvious inequality

$$\omega=\nu\frac{\omega_p^2}{\varkappa^2+\omega_p^2}\frac{\nu(\varkappa\widetilde{\nabla T})}{\Omega_0^2}, \quad \gamma=-\nu\frac{\varkappa^2}{\varkappa^2+\omega_p^2}. \quad (38)$$

The wave is linearly polarized along the magnetic field; it is weakly damped when $\nabla\tilde{T} \gg \varkappa\Omega_0^2/\nu\omega_p^2$, which can be realized.

Waves with electrical field components parallel to B_0 and $\varkappa$. The dispersion equation has the form

$$\varepsilon\omega(\varepsilon\omega^2-\varkappa^2)(\tilde{\nu}^2+\Omega_0^2)+i\omega_p^2[i\omega\omega_p^2+\tilde{\nu}(2\varepsilon\omega^2-\varkappa^2)]$$
$$=-\frac{i\nu\omega_p^2}{\nu^2+\Omega_0^2}(\varkappa\widetilde{\nabla T})[i\nu\omega_p^2-\varepsilon\omega(\nu\tilde{\nu}-\Omega_0^2)]. \quad (39)$$

C2) $\Omega_0^2 \gg \omega^2 \gg \nu^2$. This is possible only when $\Omega_0^2 \gg \omega_p^2$. In that case

$$\omega=\Omega_0\left(\frac{\varkappa^2}{\varepsilon\Omega_0^2}+\frac{\omega_p^4}{\varepsilon^2\Omega_0^4}\right)^{1/2},$$
$$\gamma=\pm\frac{\nu\omega_p^2(\varkappa\widetilde{\nabla T})}{2\Omega_0(\varepsilon\varkappa^2\Omega_0^2+\omega_p^4)^{1/2}}. \quad (40)$$

The wave is elliptically polarized. When $\varkappa > \omega\varepsilon^{1/2}$ and $\varkappa\cdot\nabla\tilde{T}>0$ the right-handedly polarized wave grows and the left-handedly polarized wave is damped. If one of these inequalities changes to the opposite one, the branches change place.

C3) $\Omega_0^2 \gg \nu^2 \gg \omega^2$. The frequency and damping of the wave are the same as in case C1), only the polarization is different.

C4) $\omega \sim \Omega_0 \gg \nu$. To simplify the calculations we restrict ourselves to the case $\omega_p^2 \gg \Omega_0^2$:

$$\omega=\Omega_0\frac{[\omega_p^4+\varkappa^2(\varepsilon\Omega_0^2+\omega_p^2)]^{1/2}}{\varepsilon\Omega_0^2+\omega_p^2},$$
$$\gamma=\pm\frac{\varepsilon\nu\Omega_0(\varkappa\widetilde{\nabla T})}{2\omega_p^6}[\varepsilon\Omega_0^2\omega_p^2+(\varepsilon\Omega_0^2+\omega_p^2)(\varepsilon\Omega_0^2-\varkappa^2)]. \quad (41)$$

When $\omega_p^2 \gg \Omega_0^2\varepsilon$ the situation is close to cyclotron resonance. If $\varkappa\cdot\nabla\tilde{T}>0$, the left-handedly circularly polarized wave grows and the right-handed one is damped; when the inequality changes sign, the position is the reverse.

D. The case $\Omega_0 \parallel \nabla T \perp \varkappa$

In this case the dispersion equation does not split up and is very complicated. It can be simplified when the condition $\Omega_0^2 \gg \nu^2$ is satisfied:

$$\varepsilon\omega(\tilde{\nu}^2+\Omega_0^2)[3\omega^2\Omega_0^2+\varkappa^2(\Omega_0\widetilde{\nabla T})^2]$$
$$=-i\omega_p^2[\omega^2\Omega_0^4/\tilde{\nu}-\varkappa^2(\Omega_0\widetilde{\nabla T})^2(\nu+\tilde{\nu}-\nu\omega^2/\Omega_0^2)]. \quad (42)$$

The case $\omega^2 \gg \nu^2$ is not realized; only low-frequency waves can be thermo-magnetic.

D1) $\Omega_0^2 \gg \nu^2 \gg \omega^2$:

$$\omega=\pm\frac{\sqrt{2}\,\nu\varkappa}{\Omega_0^2}(\Omega_0\widetilde{\nabla T}),$$
$$\gamma=\frac{\nu\varkappa^2(\Omega_0\widetilde{\nabla T})^2}{2\omega_p^2\Omega_0^4}(\varepsilon\Omega_0^2-3\omega_p^2). \quad (43)$$

The electrical field of the wave has components parallel to $\varkappa$(x-axis), Ω_0(z-axis), and a component E_y. In that case

$$E_z/E_y\sim\widetilde{\nabla T}. \quad (44)$$

Weak growth and weak damping occur when $\Omega_0 \gtrless (3/\varepsilon)^{1/2}\omega_p$.

4. THERMO-ELECTROMAGNETIC WAVES WHEN THERE IS NO CONSTANT MAGNETIC FIELD

Since Kopylov[3] has considered effects when there is no constant magnetic field present, we shall briefly discuss also that case. Three relative directions of the vectors $\varkappa$ and ∇T are here possible:

a) $\varkappa \parallel \nabla T$. The dispersion equation for the transverse wave has the form

$$\tilde{\nu}(\varepsilon\omega^2-\varkappa^2)+i\omega\omega_p^2+i\omega_p^2(\varkappa\widetilde{\nabla T})=0. \quad (45)$$

Putting $\omega^2 \ll \omega_p^2$ we have

$$\omega=\varkappa\tilde{\nabla}T \quad \gamma=-\nu\varkappa^2/\omega_p^2 \quad (46)$$

(since usually $\varkappa^2 \ll \omega_p^2$).

The longitudinal component is independent of ∇T.

b) $\varkappa \perp \nabla T$. The presence of ∇T manifests itself only in that for a wave with an electrical field which has components parallel to $\varkappa$ and ∇T, the ratio of the first to the second is $\propto\nabla\tilde{T}$ and equals

$$\pm\frac{\omega_p^2\varkappa\widetilde{\nabla T}}{\varepsilon\omega^2(\omega+i\nu)-\omega\omega_p^2}. \quad (47)$$

c) The vector $\varkappa$ has components $\varkappa_\parallel$ and $\varkappa_\perp$ (parallel and perpendicular to ∇T). Two thermo-electromagnetic branches are possible. Firstly, a branch for which the electrical field is at right angles to the ($\varkappa$, ∇T)-plane:

$$\omega=\varkappa\widehat{\nabla T}, \quad \gamma=-\nu\varkappa^2/\omega_p^2. \quad (48)$$

Secondly, a branch with an electrical field which has components parallel to $\varkappa$ and ∇T:

$$\omega=\varkappa\widehat{\nabla T}, \quad \gamma=-\nu\varkappa^2/\omega_p^2. \quad (49)$$

In all three cases the frequency and the damping are the same when $\omega^2 \ll \omega_p^2$: the waves are low-frequency waves and are weakly damped for the condition, which can be realized in metals and semi-metals, $\nabla\tilde{T} \gg \nu\varkappa/\omega_p^2$. If, however, $\omega^2 \gg \omega_p^2$, the frequency and damping are independent of ∇T.

When $\Omega_0=0$ there is a weakly damped longitudinal wave. It can leave the crystal only in the non-linear process of turning into a transverse wave[5] and we shall not consider this in the present paper.

[1] L. É. Gurevich, Zh. Eksp. Teor. Fiz. 44, 548 (1963) [Sov. Phys. JETP 17, 373 (1963)].
[2] L. É. Gurevich and B. L. Gel'mont, Zh. Eksp. Teor. Fiz. 47, 1806 (1964) [Sov. Phys. JETP 20, 1217 (1965)].