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Theory of pure short *S–c–S* and *S–c–N* microjunctions

A. V. Zaĭtsev

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A theory is developed for the nonstationary Josephson effect in pure S–c–S junctions (mean free path l much larger than the constriction radius a), and the current-voltage characteristic (CVC) of such a junction is obtained. The model considered for the construction was an opening of small radius $a \ll (\xi^{-1}(0) + l^{-1})^{-1}$ in a thin impermeable partition [Kulik and Omel'yanchuk, Sov. J. Low Temp. Phys. **3**, 459 (1977); Omel'yanchuk, Kulik, and Shekhter, JETP Lett. **25**, 437 (1977)]. A linear response from an S–c–S junction was obtained with a direct current smaller than the critical value, and the resistive regime was investigated in the voltage region $V \ll \Delta$ near T_c and in the region $V \gg \Delta$ at arbitrary temperatures. In the case of the S–c–N junction, the CVC was obtained for arbitrary V and T. The results differ not only quantitatively but also qualitatively from those obtained by Artemenko, Volkov, and Zaĭtsev [Soviet Phys. JETP **49**, 924 (1979); Solid State Commun. **30**, 771 (1979)] for dirty short constrictions.

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INTRODUCTION

It is known that weakly coupled structures of the S–c–S type (S—superconductor, c—geometrical constriction), which include point contacts, junctions of variable thickness, etc., are the most promising for applications.[1] Although much progress was made recently in the study of the properties of such systems, the results were obtained for the so-called "dirty" constrictions, i.e., those whose characteristic dimensions a and d (which characterize the respective parameters of the constriction in the plane normal to the current direction and in the same direction) greatly exceed the mean free path l.

The study of the Josephson effect in S–c–S was initiated by Aslamazov and Larkin.[2] It is based on the simplified nonstationary Ginzburg–Landau equations, which are generally speaking valid only for zero-gap superconductors. They have shown[2] that near T_c the Josephson effect in short dirty constrictions ($l \ll (a,d) \ll \xi T$) can be described within the framework of a simple resistive model, in which the current is the sum of an ohmic component and a Josephson component:

$$I = V(t)/R + I_c \sin\varphi, \quad V(t) = \dot{\varphi}/2, \tag{1}$$

where φ is the phase difference of the order parameter and $V(t)$ is the voltage on the junction (the electron charge is assumed equal to unity). It follows from (1) that the current–voltage characteristic (CVC), which is the dependence of the time-averaged voltage V on the direct current, is of the form

$$V = R(I^2 - I_c^2)^{1/2}. \tag{2}$$

In experiment, however, deviations are observed from the resistive model; this is not surprising, since the latter was obtained on the basis of simplified equations. In recent studies of the properties of S–c–S systems, microscopic equations have been used. Kulik and Omel'yanchuk[3], using the Eulenberger equations that describe equilibrium processes in superconductors, have constructed the theory of the stationary Josephson effect in short dirty microjunctions, where the following condition is valid:

$$l \ll (a, d) \ll (D/\Delta)^{1/2}. \tag{3}$$

Here $D = lv_F/3$ is the diffusion coefficient. It turned out that the connection between the current and the phase difference deviates from the Josephson relation when the temperature is lowered. The next step in the study of dirty short microjunctions[1)] [in the sense of condition (3)] was made by Artemenko, Volkov, and the author,[5] who constructed a theory of the nonstationary Josephson effect on the basis of microscopic equations. We considered limiting cases of small voltages $V \ll \Delta$ (near T_c) and large voltages $V \gg \Delta$ (at arbitrary temperatures). It was shown that near T_c the expression for the current depends substantially on the ratio of the characteristic voltage $V_c = I_c R$ to the reciprocal energy-relaxation time τ_ϵ^{-1}. In particular, Eq. (1) is valid only in the limit as $\tau_\epsilon V_c \to 0$. It turned out, however, that the CVC at $V \ll \Delta$ differs little from relation (2). The form of the CVC $I(V)$ in the voltage region $V \gg \Delta$ was obtained at arbitrary temperature in the following simple form:

$$I = \frac{V}{R} + \frac{\Delta}{R}\left(\frac{\pi^2}{4} - 1\right) \operatorname{th}\frac{V}{2T}. \tag{4}$$

Thus, at large V the CVC has an excess current, i.e., it takes on an asymptotic form shifted relative to the straight line $I = V/R$ by an amount independent of V. This effect, first observed in weakly coupled systems by Pankove,[6] is the most pronounced deviation from the predictions of the resistive model, according to which the CVC takes the form of Ohm's law at $I \gg I_c$.

In pure S–c–S junctions (whose parameters satisfy the condition $l \gg (a, d)$, only the stationary Josephson effect was investigated so far.[7] Kulik and Omel'yanchuk[7] used a constriction model in the form of a hole of radius a in an impermeable infinitesimally thin ($d \ll a$) partition. This model suits best junctions obtained by breakdown of a dielectric film, and can also describe short point contacts and variable-thickness junctions. They assumed[7] a sufficiently small radius of the hole:

$$a \ll \zeta, \quad \zeta^{-1} = \xi^{-1}(0) + l^{-1}. \tag{5}$$

The restriction (5) makes it possible to solve relatively simply the Eulenberger equation and to obtain the connection between the current and the phase difference. It turned out that when the temperature is decreased the function $I(\varphi)$ differs substantially from sinusoidal and this difference is more strongly pronounced than in dirty contacts. In the present paper we construct a theory of the nonstationary Josephson effect in pure microjunctions, for which the condition (5) is satisfied. Just as in Ref. 5, we obtain the linear response in a direct current smaller than the critical value, investigate the resistive regime in the region $V \ll \Delta$ near T_c, and in the region $V \gg \Delta$ at arbitrary temperatures. It turns out that the results differ not only quantitatively but also qualitatively from those obtained in Ref. 5 for dirty microjunctions.

We investigate also a pure S–c–N junction (N is normal metal), in which the hole radius also satisfies the condition (5) (where ξ is now the coherence length of the superconductor). S–c–N junctions have been intensively investigated experimentally of late.[8,9] Dirty S–c–N junctions were investigated theoretically by Artemenko, Volkov, and the author.[10] In the present paper we obtain the CVC of pure S–c–N systems at arbitrary voltages and temperatures. In this case the results differ not only quantitatively but also qualitatively from those obtained in Ref. 10 for dirty S–c–N junctions.

1. GENERAL RELATIONS

To construct the theory we start from the system of equations for Green's functions integrated with respect to $\zeta_p = v(p - p_F)$.[11-13] The matrix form of this system is[13] (the notation is the same as in Ref. 5):

$$\mathbf{v}\frac{\partial}{\partial \mathbf{R}}\check{G} + \hat{\tau}_z \frac{\partial}{\partial t}\check{G} + \frac{\partial}{\partial t'}\check{G}\hat{\tau}_z + \check{H}(t)\check{G} - \check{G}\check{H}(t') + \check{\Sigma}\check{G} - \check{G}\check{\Sigma} = \check{0},$$
$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G} \\ \hat{0} & \hat{G}^A \end{pmatrix}, \quad \check{\Sigma} = \check{\Sigma}_{ph} + \frac{1}{2\tau_{imp}}\check{G}_0, \ \check{G}_0 = \int \frac{d\Omega_p}{4\pi}\check{G}, \tag{6}$$
$$\check{H}(t) = -i\mathbf{v}\mathbf{A}(t)\hat{\tau}_z\check{1} - i\check{\Delta}(t) - i\Phi(t)\check{1}.$$

The function $\hat{G}$ satisfies the additional normalization condition[13]

$$\check{G}^2 = \check{1}\delta(t - t'). \tag{7}$$

The current density is expressed in terms of the matrix $\hat{G}$ by the formula

$$J = -\frac{p}{4\pi}\int \frac{d\Omega_p}{4\pi}\mathbf{p}\,\mathrm{Sp}\,\hat{\tau}_z\hat{G}(t, t). \tag{8}$$

As already noted, the model of the construction is a hole of radius a in an impermeable thin screen, assuming that the quantity a satisfies the condition (5). Proceeding to the solution of (6), we consider first the region far from the hole, for which the distane R, measured from its center, is large enough: $R \gg a$. We recognize further that in this region we can equate quantities that are independent of the direction v to their values at infinity (neglecting in this case, as can be easily shown, small corrections of order $(a/R)^2 \ll 1$), i.e., we can assume

$$\check{\Delta}(\mathbf{R}) = \begin{cases} \check{\Delta}_1, & z < 0 \\ \check{\Delta}_2, & z > 0 \end{cases}, \quad \check{G}_0(R) \equiv \int \check{G}(\mathbf{R}, \mathbf{p})\frac{d\Omega_p}{4\pi} = \begin{cases} \check{G}_1, & z < 0 \\ \check{G}_2, & z > 0 \end{cases}, \tag{9}$$

where z is a coordinate perpendicular to the hole ($z = 0$ corresponds to its center), $\check{\Delta}_j$ and $\check{G}_j$ are the equilibrium values corresponding to the phase shifts χ_j, and $j = 1$ or 2. Taking the foregoing into account in each of the two regions $z < 0$ and $z > 0$, Eq. (6) reduces at $R \gg a$ to

$$\mathbf{v}\frac{\partial}{\partial \mathbf{R}}\check{G} + \hat{\tau}_z\frac{\partial}{\partial t}\check{G} + \frac{\partial}{\partial t'}\check{G}\hat{\tau}_z + \check{\Lambda}_j\check{G} - \check{G}\check{\Lambda}_j = \check{0}, \tag{10}$$

where the matrix $\check{\Lambda}_j$ takes the form

$$\check{\Lambda}_j(t, t') = -i[\check{\Delta}_j(t) + \Phi_j(t)\check{1}]\delta(t - t') + \check{\Sigma}_j(t, t'). \tag{10'}$$

We now consider the region near the hole, where $R \ll \zeta$. In this case Eqs. (6) and (10) reduce to

$$\mathbf{v}\frac{\partial}{\partial \mathbf{R}}\check{G} = \check{0}.$$

Since the regions $R \ll \zeta$ and $R \gg a$ overlap by virtue of (5), we see that, neglecting small corrections, we can solve in all of space the simpler equations (10) in place of (6). The solutions obtained for $z < 0$ and $z > 0$ must be matched together at $z = 0$.

We write down Eq. (10), changing to the Fourier representation:

$$\mathbf{v}\frac{\partial}{\partial \mathbf{R}}\check{G} + \check{K}_j\check{G} - \check{G}\check{K}_j = \check{0}, \tag{11}$$

where the matrix $\check{K}_j$ is defined by the relation (we omit the subscript j)

$$\int \check{K}(\varepsilon,\varepsilon')e^{-i\varepsilon t+i\varepsilon' t'}\frac{d\varepsilon\, d\varepsilon'}{(2\pi)^2}=\hat{S}(\chi(t))\left(\int \check{k}(\varepsilon)e^{-i\varepsilon(t-t')}\frac{d\varepsilon}{2\pi}\right)\hat{S}^{+}(\chi(t')),$$

$$\check{k}(\varepsilon)=-i\begin{pmatrix} \xi^R(\varepsilon)\hat{g}^R(\varepsilon) & \operatorname{th}\frac{\varepsilon}{2T}[\xi^R(\varepsilon)\hat{g}^R(\varepsilon)-\hat{g}^A(\varepsilon)\xi^A(\varepsilon)] \\ \hat{0} & \xi^A(\varepsilon)\hat{g}^A(\varepsilon)\end{pmatrix}+\frac{1}{2\tau_{imp}}\check{g}(\varepsilon),$$

$$\check{g}(\varepsilon)=\begin{pmatrix} \hat{g}^R(\varepsilon) & \operatorname{th}\frac{\varepsilon}{2T}[\hat{g}^R(\varepsilon)-\hat{g}^A(\varepsilon)] \\ \hat{0} & \hat{g}^A(\varepsilon)\end{pmatrix},\quad \hat{S}(\chi)\begin{pmatrix} e^{i\chi/2} & 0 \\ 0 & e^{-i\chi/2}\end{pmatrix},$$

$$\xi^{R(A)}(\varepsilon)=([\varepsilon+i\gamma_1^{R(A)}]^2-[\Delta+i\gamma_2^{R(A)}]^2)^{1/2}, \tag{11'}$$

$$\hat{g}^{R(A)}(\varepsilon)=g^{R(A)}(\varepsilon)\hat{\tau}_z+f^{R(A)}(\varepsilon)i\hat{\tau}_y=\frac{[\varepsilon+i\gamma_1^{R(A)}]\hat{\tau}_z+[\Delta+i\gamma_2^{R(A)}]i\hat{\tau}_y}{\xi^{R(A)}(\varepsilon)},$$

where the matrices $\hat{\gamma}^R=\gamma_1^R\hat{\tau}_z+\gamma_2^R i\hat{\tau}_y$, and $\hat{\gamma}^A=-\hat{\gamma}^R$ (see expression (12′) in Ref. 5) determine the relaxation processes due to electron–phonon interaction.

Recognizing that $v\partial\tau/\partial\mathbf{R}=1$, where $\tau=\mathbf{R}\cdot\mathbf{v}/v^2$, we easily obtain the solution of (11):

$$\check{G}(\mathbf{R},\mathbf{p})=\exp[-\check{K}_j\tau]\check{C}_j(\mathbf{p})\exp(\check{K}_j\tau)+\check{G}_j. \tag{12}$$

It is assumed in (12) that the straight line passing through the end of the radius vector R in the p direction passes through the hole. If, however, this line intersects the screen, then $\check{G}(\mathbf{R},\mathbf{p})=\check{G}_j$. Here, as everywhere, we omit the dependences of the matrices on the energy units, to abbreviate the notation. Matching the solutions (12) at $z=0$ we obtain the first relation for the determination of the matrices $\check{C}_j(\mathbf{p})$:

$$\check{G}_1+\check{C}_1(\mathbf{p})=\check{G}_2+\check{C}_2(\mathbf{p}). \tag{13}$$

In addition, stipulating that the first term in (12) tends to zero as $\tau\to\pm\infty$, we obtain (see the Appendix)

$$\check{G}_j\check{C}_j(\mathbf{p})=(-1)^{j+1}\check{C}_j(\mathbf{p})\operatorname{sign}p_z, \tag{14a}$$
$$\check{C}_j(\mathbf{p})\check{G}_j=(-1)^j\check{C}_j(\mathbf{p})\operatorname{sign}p_z. \tag{14b}$$

It will be convenient next to introduce the matrix[2)]

$$\check{C}(\mathbf{p})={}^1/_2[\check{C}_1(\mathbf{p})-\check{C}_1(-\mathbf{p})]={}^1/_2[\check{C}_2(\mathbf{p})-\check{C}_2(-\mathbf{p})]. \tag{15}$$

Relation (15) follows from (13). It is convenient to calculate the current at $z=0$. As a result we obtain from (8), (12), and (15)

$$I=-\frac{pa^2}{4}\int\frac{d\Omega_p}{4\pi}p_z\operatorname{Sp}\hat{\tau}_z\hat{C}(\mathbf{p}). \tag{16}$$

We now proceed to determine the matrix $\check{C}(\mathbf{p})$. To this end, using (13), we express $\check{C}_2(\mathbf{p})$ in terms of $\check{C}_1(\mathbf{p})$ and substitute, for example, in (14b). As a result we obtain the system

$$\check{C}_1(\mathbf{p})\check{G}_1=-\check{C}_1(\mathbf{p})\operatorname{sign}p_z,$$
$$\check{C}_1(\mathbf{p})\check{G}_2=\check{C}_1(\mathbf{p})\operatorname{sign}p_z+(\check{G}_1-\check{G}_2)(\check{1}\operatorname{sign}p_z-\check{G}_2). \tag{17}$$

Adding the equations in (17), we readily obtain the relation

$$\check{C}(\mathbf{p})\check{G}_+=\check{G}_-\operatorname{sign}p_z, \tag{18}$$

in which we have introduced the matrices

$$\check{G}_\pm={}^1/_2[\check{G}_1\pm\check{G}_2].$$

It follows from (18) that $\check{C}(\mathbf{p})$ can be represented in the form

$$\check{C}(\mathbf{p})=\check{C}\operatorname{sign}p_z,$$

where the matrix $\check{C}$ does not depend on the direction of p. Ultimately we get from (18)

$$\check{C}=\check{G}_-\check{G}_+^{-1}. \tag{18'}$$

The components of the matrix $\check{G}_+^{-1}$ can be easily obtained:

$$\check{G}_+^{-1}=\begin{pmatrix}(\hat{G}_+^R)^{-1} & -(\hat{G}_+^R)^{-1}\hat{G}_+(\hat{G}_+^A)^{-1}\\ \hat{0} & (\hat{G}_+^A)^{-1}\end{pmatrix}, \tag{19}$$

where

$$\hat{G}_+=\hat{G}_+^R\hat{n}_+-\hat{n}_+\hat{G}_+^A+\hat{G}_-^R\hat{n}_--\hat{n}_-\hat{G}_-^A,$$
$$\hat{n}_\pm={}^1/_2(\hat{n}_1\pm\hat{n}_2),\quad \hat{n}_j(t,t')=\hat{S}(\chi_j(t))n(t-t')\hat{S}^+(\chi_j(t')), \tag{20}$$
$$n(t)=\int\operatorname{th}\frac{\varepsilon}{2T}e^{-i\varepsilon t}\frac{d\varepsilon}{2\pi}.$$

Substituting (19) in (18), and taking into account (20) and the properties of the matrices $\check{G}_\pm^{(R,A)}$ (see Ref. 5), we get

$$\hat{C}^R=\hat{G}_-^R(\hat{G}_+^R)^{-1},\quad \hat{C}^A=\hat{G}_-^A(\hat{G}_+^A)^{-1}, \tag{21}$$
$$\hat{C}=\hat{C}^r+\hat{C}^a,\quad \hat{C}^r=\hat{C}^R\hat{n}_+-\hat{n}_+\hat{C}^A,$$
$$\hat{C}^a=(\hat{G}_+^R)^{-1}\hat{n}_-(\hat{G}_+^A)^{-1}+\hat{C}^R\hat{n}_-\hat{C}^A-\hat{n}_-. \tag{22}$$

It follows from (18), (22), and (16) that the total current through the opening can be represented in the form

$$I=I^r+I^a=-\frac{1}{4R}\operatorname{Sp}\hat{\tau}_z[\hat{C}^r(t,t)+\hat{C}^a(t,t)], \tag{23}$$

where, as will be shown later, $R=\pi/(p_F a)^2$ is the resistance of the pure opening in the normal state (which was first calculated in Ref. 14). We proceed now to solve some concrete problems, using the results obtained in the present section.

2. THE *S-c-S* JUNCTION

1. Linear response

In this subsection we investigate the case when direct current $I_0\ll I_c$ and a weak alternating current $I_1\ll I_c$ flow through the junction. The phase difference can be represented in this case in the form $\varphi=\varphi_0+\varphi_1$, with $\varphi_1\ll 1$. In the calculation of the current we confine ourselves to terms of order φ_1. Since $\hat{n}_-\sim\varphi_1\hat{\tau}_z$, it is clear that to determine the matrix $\hat{C}^a$ from (22) we can neglect the dependence of all the matrices in (22) (with the exception of $\hat{n}_-$) on φ_1. As a result we find that the Fourier component $\hat{C}^a(\varepsilon_+,\varepsilon_-)$ takes the form

$$\hat{C}^a(\varepsilon_+,\varepsilon_-)=[(\hat{G}_+^R(\varepsilon_+))^{-1}\hat{\tau}_z(\hat{G}_+^A(\varepsilon_-))^{-1}-\hat{\tau}_z$$
$$-\hat{C}_0^R(\varepsilon_+)\hat{\tau}_z\hat{C}_0^A(\varepsilon_-)]\varphi_1(\omega)\left(\operatorname{th}\frac{\varepsilon_+}{2T}-\operatorname{th}\frac{\varepsilon_-}{2T}\right), \tag{24}$$

where $\varepsilon_\pm=\varepsilon\pm\omega/2$,

$$\hat{C}_0^R(\varepsilon)=\hat{G}_-^R(\varepsilon)[\hat{G}_+^R(\varepsilon)]^{-1}, \tag{25}$$

and the matrices $\hat{G}_\pm^R$ are equal to[5]

$$\hat{G}_+^R(\varepsilon)=g^R(\varepsilon)\hat{\tau}_z+f^R(\varepsilon)\cos({}^1/_2\varphi_0)i\hat{\tau}_y,\quad \hat{G}_-^R(\varepsilon)=if^R(\varepsilon)\sin({}^1/_2\varphi_0)\hat{\tau}_x. \tag{26}$$

Using (24)–(26), we obtain the Fourier component of the current $I^a(\omega)$, which, when added to the current $I^r(a)$ [which can be easily calculated from (21)–(23)] can be expressed in the form

$$I(\omega)=\frac{\varphi_1(\omega)}{8R}\int d\varepsilon\left[\operatorname{th}\frac{\varepsilon_-}{2T}B^R(\varepsilon_+,\varepsilon_-)-\operatorname{th}\frac{\varepsilon_+}{2T}B^A(\varepsilon_+,\varepsilon_-)\right.$$
$$\left.+\left(\operatorname{th}\frac{\varepsilon_+}{2T}-\operatorname{th}\frac{\varepsilon_-}{2T}\right)B^a(\varepsilon_+,\varepsilon_-)\right]+2\pi I(\varphi_0)\delta(\omega),$$

where

$$B^R(\varepsilon_+,\varepsilon_-)=1-\frac{[g^R(\varepsilon_+)g^R(\varepsilon_-)+f^R(\varepsilon_+)f^R(\varepsilon_-)\cos^2(\varphi_0/2)][1-f^R(\varepsilon_+)f^R(\varepsilon_-)\sin^2(\varphi_0/2)]}{[1+(f^R(\varepsilon_+)\sin(\varphi_0/2))^2][1+(f^R(\varepsilon_-)\sin(\varphi_0/2))^2]}. \quad (27)$$

A similar expression is obtained for $B^A(\varepsilon_+,\varepsilon_-)$, while $B^a(\varepsilon_+,\varepsilon_-)$ is obtained from $B^R(\varepsilon_+,\varepsilon_-)$ by making the substitutions $g^R(\varepsilon_-)\to g^A(\varepsilon_-)$ and $f^R(\varepsilon_-)\to f^A(\varepsilon_-)$. The expression for the dc component of the current in (26) is

$$I(\varphi_0)=\frac{\pi\Delta}{R}\sin\frac{\varphi_0}{2}\,\mathrm{th}\left[\frac{\Delta\cos(\varphi_0/2)}{2T}\right]. \quad (28)$$

Equation (28) was obtained earlier in Ref. 7.

If there is no direct current in the system, Eq. (27) reduces to

$$I(\omega)=\frac{\varphi_1(\omega)}{8R}B(\omega),$$

where $B(\omega)$ is a function well known from the linear electrodynamics of the homogeneous superconductors (see, e.g., Ref. 15). The situation here is therefore similar to that in dirty junctions.[15,16]

Expression (27) takes near T_c the simpler form

$$I(\omega)=\frac{\dot\varphi_1(\omega)}{2R}+2\pi I_c\sin\varphi_0\delta(\omega)+I_c\varphi_1(\omega)\cos\varphi_0$$
$$+I_c\frac{\omega}{\omega+i\nu_\varepsilon}\varphi_1(\omega)\sin^2\frac{\varphi_0}{2},\qquad \omega\ll\Delta. \quad (29)$$

Here

$$\nu_\varepsilon=\tau_\varepsilon^{-1}=7\zeta(3)\zeta_{ph}T^3/(sp)^2,$$

and s is the speed of sound. Equation (29) is valid in the case when $\cos(\varphi_0/2)$ is not too small: $|\cos(\varphi_0/2|\gg(\tau_\varepsilon\Delta)^{-1},\ \omega/\Delta$. As follows from (29), at low frequencies $\omega\ll\nu_\varepsilon$ the alternating part I_1 of the current becomes

$$I_1=\tilde\sigma V_1+\alpha\tilde\sigma V_1\cos\varphi_0+I_c\varphi_1\cos\varphi_0,\qquad V_1={}^1/_2\dot\varphi_1, \quad (29')$$

where $\tilde\sigma=(1+\lambda)/2R$, $\alpha=-\lambda(1+\lambda)^{-1}$, $\lambda=V_c\tau_\varepsilon$. The second term in (29′) is the so-called interference component of the current. At large $\lambda(\tau_\varepsilon\Delta^2/T\gg1)$ we have $\alpha\approx-1$. We note that it was concluded in Ref. 17 from measurements of the impedance of a point contact that $|\alpha|\sim1$ and $\alpha<0$.

2. RESISTIVE REGIME NEAR T_c. LOW VOLTAGES ($V\ll\Delta$)

In the calculation of the matrix $\hat C^a$ it is more convenient in this case not to start from (22) but to use the following method.

Adding relations (14) we get

$$\check G_j\check C+\check C\check G_j=\check 0. \quad (30)$$

A similar system of equations was investigated in Ref. 5. From (30), in particular, it follows that

$$\hat G_j^R\hat C+\hat C\hat G_j^A+\hat C^R\hat G_j+\hat G_j\hat C^A=\hat 0. \quad (30')$$

Next, following Ref. 5, we represent the matrix $\hat C^a$ in the form

$$\hat C^a\equiv\hat C-\hat C^r=\hat C^R\hat F^a-\hat F^a\hat C^A. \quad (31)$$

Taking into account the expression for $\hat C^r$, we can obtain from (30′) an equation for $\hat F^a$ (Ref. 5):

$$\hat G_\pm^R\hat F^a-\hat F^a\hat G_\pm^A=\hat G_\pm^R\hat n_--\hat n_-\hat G_\pm^A. \quad (32)$$

A solution of (32) was obtained in Ref. 5 for the case $V\ll\Delta\ll T$. Using the expression obtained in that reference for $\hat F^a(\varepsilon,t)$ (see Sec. 3 in Ref. 5), and recognizing that the matrices $\hat C^{R(A)}(\varepsilon,t)$ can be calculated using (25) and (26) [where φ_0 must be replaced by $\varphi(t)$], we get from (22) and (23) the current

$$I=\frac{\dot\varphi}{2R}+I_c\sin\varphi+I_cP\{\varphi\}, \quad (33)$$

$$P\{\varphi\}=\sin\frac{\varphi}{2}\,\mathrm{sign}\left(\cos\frac{\varphi}{2}\right)\int_{-\infty}^{t}\exp[-\nu_\varepsilon(t-t_1)]\dot\varphi(t_1)\times\sin\frac{\varphi(t_1)}{2}\,\mathrm{sign}\left(\cos\frac{\varphi(t_1)}{2}\right)dt_1.$$

The expression obtained for $P\{\varphi\}$ is valid for all t, with the exception of small (compared with the period $T=\pi/V$ of the Josephson oscillations) time intervals $\delta t\sim\Delta^{-1}$ near the instants of time t^* defined by the relation[3] $\cos[\varphi(t^*)/2]=0$. Actually the exact expression for $P\{\varphi\}$ will contain, in place of the discontinuous function sign $\cos(\varphi/2)$ a continuous function near the instants t^*, over times of the order of Δ^{-1}, in the range from -1 to $+1$. Since we are not interested in the details of these changes, and also since the refinement of $P(\varphi)$ near t^* introduces into the CVC corrections of the order of $(V/\Delta)\ll1$, we shall henceforth use (33).

We proceed now to analyze some limiting cases:

A. $\lambda=V_c\tau_\varepsilon\ll1$. In this case, in the region of small voltages $V\ll\nu_\varepsilon$, Eq. (33) reduces to

$$I=\frac{\dot\varphi}{2R}(1+\lambda-\lambda\cos\varphi)+I_c\sin\varphi. \quad (34)$$

From (34) we easily obtain the CVC:

$$V=R(I^2-I_c^2)^{1/2}/(1+\lambda),\qquad V\ll\nu_\varepsilon. \quad (35)$$

This expression differs from (2) in that R is replaced by $\tilde R=R(1+\lambda)^{-1}$. To calculate the CVC at $V\gg V_c$ we substitute $\varphi=2Vt$ in the functional (33) and average over the time. As a result we get

$$I=\frac{V}{R}+I_c\varkappa\left(\frac{\nu_\varepsilon}{V}\right), \quad (36)$$

where

$$\varkappa(y)=\left(\frac{4}{\pi}\right)^2\sum_{k=1}^{\infty}\left(\frac{2k}{4k^2-1}\right)^2\frac{y}{4k^2+y^2}\approx\begin{cases}y^{-1}, & y\gg1\\ y\left(1-\frac{8}{\pi^2}\right), & y\ll1\end{cases}.$$

Thus, even at $V\sim\nu_\varepsilon\gg V_c$ the quantity $I-V/R$ can be of the order of I_c.

B. $\lambda\gg1$. In this case at $V\gg\nu_\varepsilon$ it follows from (33) that in the principal approximation in the parameter $(\tau_\varepsilon V)^{-1}$ we have

$$I=\frac{\dot\varphi}{2R}+2bI_c\sin\frac{\varphi}{2}\,\mathrm{sign}\left(\cos\frac{\varphi}{2}\right), \quad (37)$$

where

$$b=T^{-1}\int_0^T\left|\cos\frac{\varphi(t)}{2}\right|dt,\qquad T=\frac{\pi}{V}.$$

Taking into account the relations

$$V=\left[\int_{-\pi}^{\pi}\frac{d\varphi}{\dot\varphi(\varphi)}\right]^{-1}\pi,\qquad b=\frac{V}{\pi}\int_{-\pi}^{\pi}\frac{\cos(\varphi/2)}{\dot\varphi(\varphi)}d\varphi$$

and substituting in them $\dot\varphi(\varphi)$ from (37), we find that the CVC is determined by the following parametric relation:

$$I=2I_cJb(J),\quad V=2V_\varepsilon(J^2-1)^{1/2}b(J),$$
$$b(J)=\frac{(J^2-1)^{1/2}}{\pi}\ln\frac{J+1}{J-1}. \tag{38}$$

As follows from (38), at $\nu_\varepsilon < V \ll V_c$ the current can become less than critical. In particular, $I=0.79I_c$ already at $V=0.24V_c$ ($J=1.05$). The CVC thus exhibits hysteresis in the case $\lambda \gg 1$.

In the voltage region $V>V_c$ it follows from (38) that

$$I=\frac{V}{R}+\frac{8}{\pi^2}I_c\left(\frac{V_c}{V}\right).$$

We note that Eq. (37) can be written in the form

$$I=\frac{\dot\varphi}{2R}+I_cb\sum_{n=1}^{\infty}a_n\sin(n\varphi),\qquad a_n=\frac{16}{\pi}\frac{(-1)^{n-1}n}{4n^2-1}.$$

The coefficients a_n of the higher harmonics $\sin(n\varphi)$ determine the heights of the subharmonic steps that will appear on the CVC of the irradiated contact at a voltage $V=\omega/2n$ (ω is the radiation frequency).

Thus, just as in dirty constrictions,[5] the expression for the current near T_c is determined by the value of the parameter λ. However, the very form of the functional $I\{\varphi\}$, and consequently of the CVC, can vary strongly with the mean free path, a fact most clearly pronounced at $\lambda>1$.

3. REGION OF HIGH VOLTAGES $V\gg\Delta$

In the calculation of the current in this case we start with Eq. (22). Since the relations $V\gg\Delta$ means that $I \gg I_c$, the phase difference can be taken in the principal approximation to be equal to $\varphi=2Vt$.

The main complexity lies in the calculation of the matrices $[\hat G_+^{R(A)}]^{-1}$. The latter can be obtained with the aid of the equation

$$(\hat G_+^R)^{-1}=[\hat 1-(\hat G_-^R)^2]^{-1}\hat G_+^R=\left[\sum_{k=0}^{\infty}(\hat G_-^R)^{2k}\right]\hat G_+^R, \tag{39}$$

which follows from the relation[5] $(\hat G_+^R)^2+(\hat G_-^R)^2=\hat 1\delta(t-t')$. We now sum the series (39) by changing over to the Fourier representation and separate the principal aggregate of terms in which singularities accumulate near energies satisfying the condition $|\varepsilon\pm V/2|=0$. Recognizing that in the principal approximation in the parameter Δ/V we have

$$(\hat G_-^R)^{2k}(\varepsilon,\varepsilon')=\frac{1}{2^{2k}}[1-g^R(\varepsilon+\tilde V)g^R(\varepsilon-\tilde V)]^{2k}\delta(\varepsilon-\varepsilon')\hat 1,\qquad \tilde V=\frac{V}{2},$$

we obtain from (39)

$$(\hat G_+^R)^{-1}(\varepsilon,\varepsilon')=\frac{2}{g^R(\varepsilon+\tilde V)g^R(\varepsilon-\tilde V)+1}\hat G_+^R(\varepsilon,\varepsilon'). \tag{40}$$

From (21) and (40) we easily obtain the matrices $\hat C^{R(A)}$ and after simple calculations we get from (22) and (23)

$$I=\frac{1}{4R}\int_{-\infty}^{\infty}d\varepsilon\left(\text{th}\frac{\varepsilon+\tilde V}{2T}-\text{th}\frac{\varepsilon-\tilde V}{2T}\right)\Big\{1-\frac{1}{[1+g^R(\varepsilon+\tilde V)g^R(\varepsilon-\tilde V)][1+g^A(\varepsilon+\tilde V)g^A(\varepsilon-\tilde V)]}$$
$$\cdot[(g^R(\varepsilon+\tilde V)+g^R(\varepsilon-\tilde V))(g^A(\varepsilon+\tilde V)+g^A(\varepsilon-\tilde V))$$
$$+2f^R(\varepsilon+\tilde V)f^A(\varepsilon+\tilde V)+2f^R(\varepsilon-\tilde V)f^A(\varepsilon-\tilde V)]\Big\}$$
$$\approx\frac{1}{2R}\int_{-\infty}^{\infty}\left(\text{th}\frac{\varepsilon}{2T}-\text{th}\frac{\varepsilon-V}{2T}\right)\left[1-\frac{2f^R(\varepsilon)f^A(\varepsilon)}{(1+g^R(\varepsilon))(1-g^A(\varepsilon))}\right]d\varepsilon. \tag{41}$$

It was recognized in (41) that $V\gg\Delta$. Calculating the integral, we ultimately get

$$I=\frac{V}{R}+I_{\text{exc}}\text{th}\frac{V}{2T},\qquad I_{\text{exc}}=\frac{16}{3}\frac{\Delta}{R}. \tag{42}$$

Thus, at $V\gg\Delta$ the character of the dependence of $I(V)$ in pure contacts is the same as in dirty ones,[5] and in both cases the excess current varies with temperature in proportion to $\Delta(T)$. We note, however, that the value of RI_{exc} depends substantially on the mean free path.

3. THE *S-c-N* JUNCTION

The general expression (22) allows us to find the matrix C also in this case. It is convenient here to assume that the potential of the superconductor is equal to zero and that the potential of the normal metal N is equal to V. Taking the foregoing into account, we get

$$\hat G_1^{R(A)}(\varepsilon,\varepsilon')=[g^{R(A)}(\varepsilon)\hat\tau_z+f^{R(A)}(\varepsilon)i\hat\tau_y]\delta(\varepsilon-\varepsilon'), \tag{43}$$
$$\hat G_2^{R(A)}(\varepsilon,\varepsilon')=\pm\hat\tau_z\delta(\varepsilon-\varepsilon').$$

Therefore

$$\hat G_+^{R(A)}\hat G_-^{R(A)}=\pm{}^1/_2f^{R(A)}(\varepsilon)\hat\tau_x\delta(\varepsilon-\varepsilon'). \tag{44}$$

From (43) and (44) we easily obtain the matrices $[\hat G_+^{R(A)}]^{-1}$ and $\hat C^{R(A)}$, and after substituting in (22) and (23) we get

$$I=\frac{1}{2R}\int_{-\infty}^{\infty}\left[\text{th}\frac{\varepsilon}{2T}-\text{th}\frac{\varepsilon-V}{2T}\right]\left[1-\frac{f^R(\varepsilon)f^A(\varepsilon)}{(1+g^R(\varepsilon))(1-g^A(\varepsilon))}\right]d\varepsilon$$
$$=\frac{V}{R}+\frac{4T}{R}\text{arth}\left[\text{th}\left(\frac{\Delta}{2T}\right)\text{th}\left(\frac{V}{2T}\right)\right]+\frac{1}{R}\int_{\Delta}^{\infty}\left(\text{th}\frac{\varepsilon+V}{2T}\right.$$
$$\left.-\text{th}\frac{\varepsilon-V}{2T}\right)\frac{(\varepsilon-(\varepsilon^2-\Delta^2)^{1/2})^2}{\Delta^2}d\varepsilon. \tag{45}$$

From (45), in particular, we find that

$$I=\frac{1}{R}\left[V+2V\theta(\Delta-|V|)+2\Delta\left\{\frac{2}{3}\left(\frac{V}{\Delta}\right)^2-\frac{2}{3}\text{sign}\,V\left[\left(\frac{V}{\Delta}\right)^2-1\right]^{1/2}\right.\right.$$
$$\left.\left.-\frac{V}{\Delta}+\frac{4}{3}\text{sign}\,V\right\}\theta(|V|-\Delta)\right],\qquad T\ll\Delta,\quad |V-\Delta|\gg T, \tag{45'}$$
$$I=\frac{1}{R}\left(V+\frac{8}{3}\Delta\,\text{th}\frac{V}{2T}\right) \tag{45''}$$

at $V\gg\Delta$ or arbitrary V at $\Delta\ll T$.

Thus, at large voltages the CVC of an S–c–N junction has an excess current having half the value of I_{exc} in the S–c–S system. A similar situation takes place in dirty junctions.[5,10] It must be noted, however, that the considered pure case differs in some respects not only quantitatively but also qualitatively from that of dirty junctions. In particular, as follows from (45), ($\sigma=1/R$)

$$\sigma(V)=\frac{dI}{dV}$$
$$=\sigma\left\{3\theta(\Delta-|V|)+\left[4\left(\frac{V}{\Delta}\right)^2-4\frac{V}{\Delta}\left(\left(\frac{V}{\Delta}\right)^2-1\right)^{1/2}-1\right]\theta(|V|-\Delta)\right\}.$$

Thus, in contrast to the dirty case, the differential conductivity does not have a gap singularity at low temperatures: $\sigma(\Delta)=3(T=0)$. The singularity at $V=\Delta(T\ll\Delta)$ appears in the pure case only in the second derivative d^2I/dV^2. We note also that the function $\sigma_0(T)=(dI/dV)_{V=0}$ decreases with increasing T, with

$$\sigma_0(T)=\sigma\begin{cases}3, & T\ll\Delta\\ 1+{}^8/_3\Delta/T, & T\gg\Delta\end{cases},$$

whereas in dirty junctions $\sigma_0(T)$ is a nonmonotonic function and has a maximum at $T=0.7\Delta$.[10] In addition, in contrast to the pure case, the dirty model $\sigma_0(0)=\sigma$.

The current can be also easily determined in the case when the junction contact voltage has besides the dc component also an alternating component $V_\sim = V_1 \sin\omega t$. The expressions for the matrices $\hat{G}_\pm^{R(A)}$ and $\hat{C}^{R(A)}$ then remain the same as before, and only the matrices $\hat{n}_\pm$ change. Taking the foregoing into account, and calculating the direct current, we easily obtain

$$I=\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{V_1}{\omega}\right) I(V+n\omega),$$

where $J_n(x)$ is a Bessel function of order n and $I(V)$ is the CVC, determined by Eq. (45), in the absence of an alternating voltage. We note that an analogous expression was first obtained for tunnel junctions.[18] It is valid also in dirty constrictions.[10]

Thus, the developed theory shows that the properties of pure and dirty[5,10] constrictions have both like and unlike features. Common to both cases is, in particular, the fact that the functional $I\{\varphi\}$, which determines the current near T_c, depends essentially on the relation between V_c, V, and τ_ε^{-1}. The concrete form of $I\{\varphi\}$, however, varies rapidly with the mean free path, and the deviation of the current $I-\dot{\varphi}/2R$ from sinusoidal form (which is typical near T_c in the stationary case) is most strongly pronounced in pure constrictions. In the latter case, the hysteresis of the CVC (which is quite small in dirty junctions)[5] is likewise strongly pronounced and takes place at $\lambda \gg 1$. At high voltages $V \gg \Delta$, the CVC takes an asymptotic form which is similar in both pure and dirty constrictions, with an excess current I_{exc} whose value varies with temperature like $\Delta(T)$. The value of RI_{exc}, however, depends strongly on the mean free path.

A similar situation is typical also of S–c–N junctions at high voltages. The qualitative differences between pure and dirty constrictions manifest themselves in this case (just as in S–c–S systems) at $V \lesssim \Delta$. Whereas in the dirty limit the $\sigma(V)$ function has a gap singularity at low temperatures, in pure junctions there are no such singularities. In addition, the form of the function $\sigma_0(T)$, which is the differential conductivity at zero bias, changes qualitatively with changing mean free path.

We note in conclusion that by regulating the force that clamps the electrodes to the point contact (and by the same token varying the parameters of the short circuit) it is possible to trace the transition from the properties of the dirty constriction to that of the pure one.

APPENDIX

The condition that the first term of (12) must tend to zero as $\tau \to \pm\infty$ is equivalent to the condition

$$\exp(-\check{k}(\varepsilon)\tau)\check{C}(\varepsilon,\varepsilon')\exp(\check{k}(\varepsilon)\tau) \to \check{0}, \quad \tau \to \pm\infty, \qquad \text{(A.1)}$$

where $\check{C}=S+\check{C}S$. It follows from (11′) that the matrix $\check{k}(\varepsilon)$ can be rewritten in the form

$$\check{k}(\varepsilon)=k(\varepsilon)\check{g}(\varepsilon)+\tilde{k}(\varepsilon)\tilde{\check{g}}(\varepsilon),$$

$$k(\varepsilon)=-\frac{i}{2}[\xi^R(\varepsilon)+\xi^A(\varepsilon)]+\frac{1}{2\tau_{imp}}, \quad \tilde{k}(\varepsilon)=-\frac{i}{2}[\xi^R(\varepsilon)-\xi^A(\varepsilon)],$$

$$\tilde{\check{g}}(\varepsilon)=\begin{pmatrix} \hat{g}^R(\varepsilon) & \text{th}\frac{\varepsilon}{2T}[\hat{g}^R(\varepsilon)+\hat{g}^A(\varepsilon)] \\ \hat{0} & -\hat{g}^A(\varepsilon) \end{pmatrix}. \qquad \text{(A.2)}$$

It is easy to verify that

$$\tilde{\check{g}}\check{g}-\check{g}\tilde{\check{g}}=\check{0}, \quad \tilde{\check{g}}^2(\varepsilon)=\check{1}.$$

Therefore the following relations are valid

$$\exp(\check{k}\tau)=\exp(k\check{g}\tau)\exp(\tilde{k}\tilde{\check{g}}\tau), \quad \exp(\tilde{k}\tilde{\check{g}}\tau)=\check{1}\,\text{ch}\,\tilde{k}\tau+\tilde{\check{g}}\,\text{sh}\,\tilde{k}\tau. \qquad \text{(A.3)}$$

Since it follows from (A.3) that the matrix $\exp(\pm\tilde{k}\tilde{\check{g}}\tau)$ contains only terms that oscillate with increasing $|\tau|$, we can disregard this matrix in expression (A.1). Thus, the condition (A.1) is equivalent to

$$\begin{aligned}
&\exp[-k(\varepsilon)\check{g}(\varepsilon)\tau]\tilde{\check{C}}(\varepsilon,\varepsilon')\exp[k(\varepsilon')\check{g}(\varepsilon')\tau] \\
&=\tfrac{1}{4}[\check{1}-\check{g}(\varepsilon)]\tilde{\check{C}}(\varepsilon,\varepsilon')[\check{1}+\check{g}(\varepsilon')]\exp[k(\varepsilon)+k(\varepsilon')]\tau \\
&+\tfrac{1}{4}[\check{1}+\check{g}(\varepsilon)]\tilde{\check{C}}(\varepsilon,\varepsilon')[\check{1}-\check{g}(\varepsilon')]\exp[-k(\varepsilon)-k(\varepsilon')]\tau \\
&+\tfrac{1}{4}[\check{1}-\check{g}(\varepsilon)]\tilde{\check{C}}(\varepsilon,\varepsilon')[\check{1}-\check{g}(\varepsilon')]\exp[k(\varepsilon)-k(\varepsilon')]\tau \\
&+\tfrac{1}{4}[\check{1}+g(\varepsilon)]\tilde{\check{C}}(\varepsilon,\varepsilon')[\check{1}+\check{g}(\varepsilon')]\exp[k(\varepsilon')-k(\varepsilon)]\tau \to \check{0}.
\end{aligned}$$

For this expression to tend to zero with increasing $|\tau|$, it is necessary to satisfy the relations

$$\begin{aligned}
&[\check{1}-\check{g}(\varepsilon)]\check{C}(\varepsilon,\varepsilon')[\check{1}+\check{g}(\varepsilon')]=\check{0}, \quad \text{sign}(zp_z)>0, \\
&[\check{1}+\check{g}(\varepsilon)]\check{C}(\varepsilon,\varepsilon')[\check{1}-\check{g}(\varepsilon')]=\check{0}, \quad \text{sign}(zp_z)<0, \\
&[\check{1}\pm\check{g}(\varepsilon)]\check{C}(\varepsilon,\varepsilon')[\check{1}\pm\check{g}(\varepsilon')]=\check{0}.
\end{aligned} \qquad \text{(A.4)}$$

Relations (14) are the direct consequence of (A.4).

[1)]Junctions with lengths exceeding the characteristic dimension $(D/\Delta)^{1/2}$ were investigated (near T_c) in Refs. 4.

[2)]We note that the matrices $\check{C}_j(\mathbf{p})$ [in contrast to $\check{C}(\mathbf{p})$] have no inverses. For this reason the matrix $\check{C}_j(\mathbf{p})$ cannot be canceled out from the two sides of (14a) or (14b) (and the equations obtained by canceling them are incorrect).

[3)]The point is that when $|\cos(\varphi/2)| \lesssim V/\Delta$ the main contribution to the integral

$$\int_{-\Delta}^{\Delta} d\varepsilon\, \text{Sp}\,\hat{\tau}_z \hat{C}^a(\varepsilon,t)$$

is made by the region of low energies $|\varepsilon| \lesssim V$, at which the use of the expression for $\hat{F}^{(a)}(\varepsilon,t)$ [and hence $\hat{C}^a(\varepsilon,t)$] is incorrect.

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Stability of the Bose spectrum of superfluid systems of the He^3 type

P. N. Brusov

Leningrad Polytechnic Institute

V. N. Popov

Leningrad Division of the Mathematics Institute of the USSR Academy of Sciences

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We calculate the dispersion coefficient and investigate the stability to decays of the phonon modes of the Bose spectrum in the superfluid *A*, *B*, and *2D* phases of a model system of the He^3 type. All the modes in the *B* phase are stable, and in the *A* and *2D* phases the stability depends on the angle between the momentum of the collective excitation and the selected direction.

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1. INTRODUCTION

We have investigated, in a model Fermi system of the He^3 type, the stability of collective Bose excitations of the phonon type[1)] to decays of one excitation into several others. The Bose spectrum of He^3 was calculated in a number of studies.[1–5] Acoustic and spin waves were shown to exist in the *B* phase, and acoustic and orbital waves in the *A* and *2D* phases.

The stability of the Bose spectrum can be considered with respect to various processes: to pair decay into initial fermions (see Ref. 5 for orbital waves), and to decay of a collective Bose excitation or several Bose excitations of the same type or into several Bose excitations of different types corresponding to different energy-spectrum dispersion laws.

In the isotropic *B* phase, the decay of the phonon into individual fermions is forbidden, since the excitation energy is much lower than the binding energy 2Δ of the Cooper pair. The decay of an excitation into two or several excitations of the same type is kinematically forbidden if $d^2E/dk^2<0$, and the $E(\mathbf{k})$ curve bends downward away from the tangent uk (Fig. 1). This is equivalent to a positive dispersion coefficient γ in the dispersion law $E(\mathbf{k})=uk(1-\gamma k^2)$ (at small k). Therefore the question of the stability of phonon excitations is solved by calculating the corrections to the linear dispersion law. The calculation shows the stability of all the phonon branches in the *B* phase.

In the anisotropic phases (*A*, *2D*) the energy gap of the Fermi spectrum depends on the direction of the momentum and vanishes in the selected direction. Decay of the phonon into individual fermions is therefore energywise possible here. On the other hand, the question of the stability to decay into Bose excitations, just as in the *B* phase, reduces to finding the corrections to the linear dispersion law. A calculation shows that the excitation is stable if its momentum lies within certain cones described around a preferred direction, and is unstable in the opposite case.

2. THE He^3 MODEL AND THE HYDRODYNAMIC ACTION FUNCTIONAL

We consider the model system of the He^3 type proposed by Alonso and Popov.[4] The collective Bose excitations in the system are described by a functional of the hydrodynamic action S_h, obtained after functional integration over the Fermi fields. The functional takes the form

$$S_h=-\frac{1}{g}\sum_{p,i,\alpha}c_{i\alpha}^+(p)c_{i\alpha}(p)+\frac{1}{2}\ln\det\frac{\hat M(c,c^+)}{\hat M(0,0)}. \tag{2.1}$$

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