Fiz. 27, 181 (1978) [JETP Lett. 21, 169 (1978)].

[9]R. I. Dzhioev, I. A. Merkulov, V. A. Novikov, and V. G. Fleĭsher, Fiz. Tverd. Tela (Leningrad) 21, 1321 (1979) [Sov. Phys. Solid State 21, 764 (1979)].

[10]M. I. D'yakonov, I. A. Merkulov, and V. I. Perel', Zh. Eksp. Teor. Fiz. 76, 314 (1979) [Sov. Phys. JETP 49, 160 (1979)].

[11]M. I. D'yakonov and V. G. Fleĭsher, Zh. Eksp. Teor. Fiz. 68, 1514 (1975) [Sov. Phys. JETP 41, 759 (1975)].

[12]V. A. Novikov and V. G. Fleisher, Zh. Eksp. Teor. Fiz. 71, 778 (1976) [Sov. Phys. JETP 44, 410 (1976)].

[13]R. I. Dzhioev, B. P. Zakharchenya, and V. G. Fleĭsher, Pis'ma Zh. Tekh. Fiz. 2, 193 (1976) [Sov. Tech. Phys. Lett. 2, 73 (1976)].

Translated by J. G. Adashko

(He³)₂ van der Waals molecular dimers in solutions of the quantum liquids He³–HeII

E. P. Bashkin

Institute of Physics Problems, USSR Academy of Sciences
(Submitted 27 July 1979)
Zh. Eksp. Teor. Fiz. 78, 360–375 (January 1980)

Bound states of He³ impurity quasiparticle pairs, produced as a result of van der Waals attraction on the free surface in thin films, in narrow capillaries, and on vortex filaments in superfluid He⁴ are investigated. With decreasing temperature, the contribution of the bound states to the thermodynamics of the solution becomes decisive. The coefficient of inelastic absorption of first sound due to the bound state in the field of the acoustic wave is calculated. It is predicted that a system of impurity excitations in narrow capillaries or on vortex filaments can have a resonant singularity (at a frequency corresponding to the threshold of the splitting of the bound state) wherein the absorption coefficient for monochromatic sound becomes infinite. The temperature of the gas-liquid phase transition and the superfluid transition temperature are calculated for a Bose system of van der Waals pairs (He³)₂ on a surface and in thin HeII films. It is shown that the superfluid transition temperature can be of the order of 35 mK.

PACS numbers: 67.60.Fp, 67.70. + n

It is known that He³ atoms dissolved in superfluid He⁴ form, at sufficiently low concentrations, a weakly non-ideal gas with attraction between the quasiparticles.[1-3] The presence of arbitrarily weak attraction between fermions leads at exponentially low temperatures to the formation of Cooper pairs and to a phase transition of the dissolved He³ into the superfluid state (see, e.g., Refs. 2, 4, 5). This attraction between the impurity excitations turns out, however, to be too weak to produce Van der Waals bound states of He³ particles within the volume of the solution. It appears therefore that no Van der Waals dimer (He³)₂ are produced under ordinary conditions. (We note that generally speaking at high pressures, when the effective mass and the radius of the interaction of the He³ quasiparticles increase, these bound states might appear.)

On the other hand, in a two-dimensional attraction field a discrete s-level corresponding to a bound state of particles is always present. One can regard as two-dimensional, for example, a system of impurity excitations of He³ in a sufficiently thin film of He II, bounded on one or both sides by a solid surface. The He II film can then turn out to be fully macroscopic, i.e., its thickness can greatly exceed atomic dimensions. In fact, let the attraction between the dissolved He³ atoms lead in the two-dimensional case to formation of a bound-state (He³)₂ with characteristic dimension r_0. It is then clear that the interaction of the impurity excitations in the He II film can be regarded as two-dimensional if the film thickness d is much less than the dimer dimension, i.e., $d \ll r_0$. The exponential smallness of the binding energy Δ of the dimer compared with the characteristic kinetic energy $\hbar^2/ma^2$ (m is the effective mass of the He³ quasiparticle and a is the atomic dimension) means that $r_0 \gg a$, i.e., there exists a region of macroscopic thickness of the film $a \ll d \ll r_0$. Since we are confining ourselves only to pair interaction of particles, a procedure valid only for sufficiently low concentrations, such that $r_0 \ll l$ (l is the average distance between the dissolved He³ atoms), we are dealing in fact with a rarefied monolayer of impurity quasiparticles in a macroscopic film of superfluid He⁴.

It should also be noted that under the influence of the Van der Waals forces it is He⁴ which crystallizes predominantly on the solid surface that bounds the solution film, owing to the difference between the molar volumes of the helium isotopes. According to the phase diagram of the solid solutions at temperatures $T \lesssim 0.318$ K, the He³ atoms are practically not dissolved in crystalline He⁴ (see, e.g., Ref. 6). This makes it possible to explain qualitatively why the dissolved He³ atoms are not localized on a solid wall, and they can be regarded as a gas of excitations with two-dimensional interaction between them (in this sense, the interaction of the impurity quasiparticles with the solid wall remains three-dimensional).

A model-based calculation of the dependence of the concentration of the dissolved He³ atoms on the distance to the solid surface, in a wide temperature interval, was carried out by Peshkov.[7]

Surface impurity He³ states on the free surface of a

0038-5646/80/010181-08$02.40
181

superfluid solution also constitute a strictly two-dimensional system of Fermi particles. The existence of a two-dimensional gas of surface He^3 impurities was first predicted by Andreev[8] and experimentally confirmed by Zinov'eva and Boldarev[9] and by Edwards et al. (see the review[10]). Model-based estimates of the s-scattering amplitude in the Fermi-liquid region lead to contradictory results concerning the sign of the interaction (attraction or repulsion) in a system of surface levels.[10,11] But even in the case of repulsion, formation of $(He^3)_2$ bound Van der Waals pairs with nonzero values of the orbital angular momentum is possible in s scattering in principle. A more detailed discussion of the experimental data on surface levels in connection with the proposed theory is contained in the Conclusion.

Dimerization of He^3 in superfluid mixtures occurs, of course, also in one-dimensional systems, since a one-dimensional potential well always has a bound level. A one-dimensional system of He^3 impurities is realized in sufficiently narrow capillaries whose diameter d satisfies the condition $a \ll d \ll r_0 \ll l$ considered above (rarefied monatomic chain of He^3 quasiparticles in a macroscopic channel).

If the He II contains in addition quantized vortex filaments, then the atomic impurities He^3 are localized with decreasing temperature near the cores of these filaments[12] and form one-dimensional linear chains of impurity excitations with attraction between them.

Thus, under definite conditions, even weak attraction between the impurity quasiparticles leads to the onset of bound states of dissolved He^3 atoms and to formation of a Bose system of $(He^3)_2$ Van der Waals dimers. With decreasing temperature, the number of dimers increases, thus altering substantially the thermodynamics and the kinetics of the system. At sufficiently low temperature, phase transitions due to condensation of the dimer gas into a liquid, as well as the Kosterlitz and Thouless[13,14] transition of the $(He^3)_2$ impurity into the superfluid state are possible in a two-dimensional Bose system of $(He^3)_2$ dimers.

1. DIMER BINDING ENERGY

The dispersion law of a solitary He^3 impurity quasiparticle in an immobile superfluid He^4 background, at small wave vectors $\mathbf{k}$, can be written in the form[1,8]

$$\mathscr{E}_1(\mathbf{k}) = \varepsilon_0 + \hbar^2 k^2 / 2m, \tag{1.1}$$

where ε_0 is the finding energy of the impurity at rest, and m is the effective mass of the excitation. The energy spectrum of the $(He^3)_2$ bound pair is then defined by

$$\mathscr{E}_2(\mathbf{k}) = 2\varepsilon_0 - \Delta + \hbar^2 k^2 / 4m. \tag{1.2}$$

The quantity Δ in (1.2) sets the binding energy of the $(He^3)_2$ dimer.

In the two-dimensional case, the Schrödinger equation in the c.m.s. of the two interacting He^3 quasiparticles, which determines the value of Δ, takes in the $\mathbf{k}$ representation the form

$$\left(\frac{\hbar^2 k^2}{m} + \Delta \right) \psi(\mathbf{k}) + \frac{1}{(2\pi)^2} \int u(\mathbf{k} - \mathbf{k}') \psi(\mathbf{k}') d^2 k' = 0. \tag{1.3}$$

In (1.3), the Fourier components of the interaction potential $u(r)$ and of the coordinate wave functions $\psi(\mathbf{r})$ are determined in the usual manner from the formula

$$F(\mathbf{k}) = \int F(\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}} d^2 r.$$

At not too high temperatures and impurity concentrations $T < \hbar^2 / ma^2$, $k_F a \ll 1$ (k_F is the Fermi wave vector) the quasiparticle interaction potential $u(\mathbf{k} - \mathbf{k}')$ can always be represented as an expansion in partial amplitudes

$$u(q) = \sum_{m=0}^{\infty} (-1)^m u_m q^{2m},$$

$$u_m = \frac{\pi}{2^{2m-1}(m!)^2} \int_0^\infty u(r) r^{2m+1} dr, \qquad q = \mathbf{k} - \mathbf{k}'. \tag{1.4}$$

The expansion (1.4) is valid, generally speaking, for a field $u(r)$ that decreases exponentially at infinity. On the other hand, if the interaction potential has a power law decrease $u(r) \propto r^{-n}$ as $r \to \infty$ then the results obtained below are valid for such dimer orbital-momentum values at which the integral equations (1.4) for u_l do not begin to diverge, i.e., at $n > 2(l + 1)$.

For the scattering of the He^3 quasiparticles at low energies, the integral part of the Schrödinger equation (1.3) must be cut off at a certain small wave vector k_c such that $k \lesssim k_c \ll 1/a$ (this corresponds to the condition of "joining together" the wave functions in the coordinate representation). Let Δ_l be the depth of the level of the bound state of a pair of He^3 quasiparticles with relative-motion orbital angular momentum l. The corresponding wave functions in the polar system of coordinates $(\mathbf{k}, \varphi)$ take the following form:

$$\psi_l(\mathbf{k}) = \chi_l(\varepsilon) e^{il\varphi}, \qquad \varepsilon = \hbar^2 k^2 / 2m. \tag{1.5}$$

We substitute expressions (1.4) and (1.5) in (1.3). Recognizing that $q^2 = k^2 + k'^2 - 2kk' \cos\theta$, where $\theta = \varphi - \varphi'$ is the scattering angle we can easily note that when a dimer with a given value of l is made up, the only terms of the expansion of the interaction potential $u(\mathbf{q})$ (1.4) in the Schrödinger equation (1.3) that make a contribution are those of the sum with $m = l + 2n$, $n = 0, 1, 2 \ldots$ In turn, only the terms that contain $\cos^l \theta$, with $m - l = 2n$, will enter in Eq. (1.3). Since the series expansion in (1.4) is with respect to small momenta $ka \ll 1$, in the principal approximation it suffices to retain in the Schrödinger equation only one fundamental term with $m = l$ (for partial scattering with given l):

$$u(\mathbf{q}) = (-1)^l (kk')^l e^{il(\varphi - \varphi')} u_l. \tag{1.6}$$

In this approximation the use of the local potential $u(r)$, neglecting retardation effects in the interaction, is justified because the dependence of the interaction of the quasiparticles He^3 on their velocities in each of the partial scattering amplitudes is of the same order of smallness as the terms discarded in the Schrödinger equation.

Following the indicated transformation, Eq. (1.3) in a specified partial channel acquires the form

$$(2\varepsilon + \Delta_l) \chi_l(\varepsilon) + \frac{1}{2} g_0 \mu_l \frac{\varepsilon^l}{\varepsilon^{1/2}} \int_0^{\varepsilon_c^{(l)}} \chi_l(\varepsilon') \varepsilon'^{l/2} d\varepsilon' = 0 \tag{1.7}$$

and determines the relation between the level depth Δ_l and the wave function of the bound state

$$\chi_l(\varepsilon) = \text{const} \frac{\varepsilon^{l/2}}{2\varepsilon + \Delta_l}. \qquad (1.8)$$

In (1.7), $\varkappa = 2m\varepsilon/\hbar^2$, $\varepsilon_c^{(l)} = \hbar^2 k_c^2/2m$ is the energy cutoff parameter for the bound state with angular momentum l, and $g_0 = m/\pi\hbar^2$ is the two-dimensional state density.

In interaction with zero angular momentum, the binding energy of the s-dimer $(He^3)_2$ can be expressed with logarithmic accuracy in terms of the zeroth term in the expansion of the interaction potential (1.4) in the momenta u_0. This is possible because of the logarithmic divergence on the lower limit of integration in (1.7) with the wave function (1.8) at $l = 0$. Under these conditions, at arbitrarily small attraction $u_0 < 0$, the solution of Eq. (1.7) leads to the known expression[15] for the s-dimer binding energy:

$$\Delta_0 \approx \frac{2\hbar^2}{ma^2} \exp\left(-\frac{4}{g_0|u_0|}\right). \qquad (1.9)$$

(The number 2 in the pre-exponential factor of (1.9) is retained, since the Schrödinger equation contains the reduced mass of two colliding particles, which in this case is equal to $m/2$.)

In scattering with larger angular momenta $l > 0$, the existence of shallow levels Δ_l already requires satisfaction of several threshold conditions for attraction between the He^3 quasiparticles. Unfortunately, at $l > 0$ it is impossible to exclude from the dispersion equations the cutoff parameter $\varepsilon_c^{(l)}$ (not even with logarithmic accuracy as in the case of s scattering), and by the same token express Δ_l in terms of u_l. The reason is that there are no divergences in (1.7) at $l > 0$. To determine the energy of the bound states with zero orbital angular momenta it is necessary to have information on the concrete form of the interaction potential $u(r)$. With the aid of (1.7) it is possible to obtain only qualitative estimates of the minimal threshold values u_l:

$$|u_l|_{\min} \sim \frac{l}{g_0} a^{2l}, \quad u_l < 0.$$

At $|u_l| < |u_l|_{\min}$, the onset of bound states with small level depth is more readily impossible. We emphasize that estimates of $|u_l|_{\min}$ are quite approximate, since they are obtained by extrapolation to the region $ka \sim 1$.

We shall henceforth confine ourselves almost always to the study of the properties of s-dimers $(He^3)_2$, since they can appear without satisfying threshold conditions on the attraction between the particles. The normalized wave function (1.5), (1.8) for the s-dimer takes in the coordinate representation

$$\psi_0(\mathbf{r}) = \frac{\nu_0}{\pi^{1/2}} K_0(\nu_0 r), \qquad \nu_0 = \frac{(m\Delta)^{1/2}}{\hbar}, \qquad (1.10)$$

where $K_0(x)$ is a MacDonald function. The calculation of the dimer dimensions with the aid of the wave function (1.10) is standard and leads to the result $r_0 = \pi^2/16\nu_0$.

Since the thickness of the He II film with dissolved He^3 atoms, as noted in the Introduction, may turn out to be fully macroscopic, it is reasonable to make a rough estimate of the values of Δ and r_0 from the experimental

data on the properties of impurity excitations in a volume: $m = 2.33m_3$ (m_3 is the mass of the He^3 atom), $a \sim 2$ Å, and $\bar{u} \sim 1$ K. The value of Δ_0 is then not too small: $\Delta_0 \sim 0.1$ K, $r_0 \sim 5$ Å, so that we can study dimerization by using solutions with concentrations available for experiment. It must be emphasized, however, that since the binding energy Δ_0 (1.9) depends on $g_0|u_0|$ exponentially, the accuracy of the estimates of Δ_0 and r_0 also depends very strongly on the employed values of m, a, and $\bar{u}$.

In the case when He^3 impurity quasiparticles are arranged in one-dimensional chains, it suffices to retain in the Schrödinger equation only the zeroth term of the expansion of the Fourier component of the interaction potential $u(r)$ in small momenta:

$$\left(\frac{\hbar^2 k^2}{m} + \Delta\right)\psi(k) + \frac{V_0}{2\pi} \int \psi(k') dk' = 0,$$

$$V_0 = \int_{-\infty}^{\infty} u(r) dr. \qquad (1.11)$$

Equation (1.11) is equivalent to the following dispersion equation:

$$1 + \frac{mV_0}{2\pi\hbar^2} \int_{-\infty}^{\infty} \frac{dk}{k^2 + \nu^2} = 0, \qquad \nu^2 = \frac{m\Delta}{\hbar^2}, \qquad (1.12)$$

which has, for any attraction $V_0 < 0$, a unique solution[15] that determines the depth of the level of the bound state of the pair of He^3 impurity quasiparticles:

$$\Delta = mV_0^2/4\hbar^2. \qquad (1.13)$$

The wave function of this state decreases with distance exponentially, with a decrement $1/\nu$:

$$\psi(r) = \nu^{1/2} e^{-\nu|r|}, \quad \nu = mV_0/2\hbar^2. \qquad (1.14)$$

The dimer dimension corresponding to the wave function (1.14) is $r_0 = \frac{1}{2}\nu = \hbar^2/mV_0$. Simple estimates with the aid of (1.13) and (1.14) and the numerical data indicated above yield $\Delta \sim 0.1$ K and $r_0 \sim 4$ Å. We emphasize that the rough estimates obtained here for Δ and r_0 are, however, more accurate than in the two-dimensional case, since the initial equations do not contain an exponential dependence.

2. THERMODYNAMICS OF DIMERS

The thermodynamics of a dimerized system of impurity atoms is determined by the number of produced bound pairs of He^3 quasiparticles. Thus, from the thermodynamic point of view we are dealing with chemical equilibrium of the form $2He^3 \rightleftarrows (He^3)_2$. We introduce the degree of dimerization, defined as the ratio of the number of paired atoms to the total number N_0 of the He^3 atoms in the system:

$$\alpha = 2N_2/N_0, \quad N_1 + 2N_2 = N_0.$$

Here N_2 is the number of the produced $(He^3)_2$ dimers and N_1 is the number of unpaired atoms He^3. Since the number of dimers is not constant but depends on the temperature T, the equilibrium value of α is defined by the equation $\partial F/\partial N_2 = 0$ (where F is the total free energy of the system); this is equivalent to the chemical-equilibrium condition

$$\mu_2 = 2\mu_1 \qquad (2.1)$$

[μ_1 and μ_2 are the chemical potentials for each of the components of the mixture of He^3 with (He$^3)_2$].

The free energy F of the mixture is determined by the known statistical-physics formula[16]

$$F=F_4-T\sum_i N_i\ln\left[\frac{e}{N_i}Z_i\int e^{-\mathscr{E}_i/T}\frac{dk_i\,dr_i}{(2\pi)^s}\right];\qquad i=1,2. \tag{2.2}$$

Here the $\mathscr{E}_i$ are given by (1.1) and (1.2), s is the number of degrees of freedom of the particle in the impurity, Z_i is the partition function corresponding to the internal state of the particle, and F_4 is the free energy of the superfluid He^4 background. In the calculations that follow we shall assume throughout Boltzmann statistics for the He^3 quasiparticles. Indeed, the condition $r_0 \ll l$ for the applicability of the paired interaction approximation (see the Introduction) means that $T_0 \ll \Delta$, where T_0 is the Fermi degeneracy temperature of the He^3 excitations, which is calculated for the total number of the impurity atoms N_0, i.e., as if there were no dimerized He^3 atoms ($\alpha=0$). Thus, in the case of sufficiently low concentrations and high temperatures $T_0 \ll \Delta \ll T$, when there are practically no dimers in the system, the impurity excitations form a nondegenerate Boltzmann gas. With decreasing temperature to $T \ll \Delta$ the number of unpaired He^3 quasiparticles decreases rapidly exponentially, $\alpha \to 1$ (see below), so that the degeneracy temperature of single He^3 impurities always remains lower than T. Effects connected with quantum degeneracy of a dimerized Bose system (He$^3)_2$ will be discussed later.

In the two-dimensional case, calculations by formula (2.2) (with allowance for the fact that no change of the internal states of the interacting particles takes place under Van der Waals dimerization) lead to the obvious expression

$$F=F_4+N_1\mu_1+N_2\mu_2-T(N_1+N_2), \tag{2.3}$$

$$\mu_1=\varepsilon_0+T\ln\left(\frac{\mathfrak{N}_1}{g_0T}\right),\qquad \mu_2=2\varepsilon_0-\Delta+T\ln\left(\frac{\mathfrak{N}_2}{g_0T}\right),$$

where $\mathfrak{N}_1$ and $\mathfrak{N}_2$ are respectively the numbers of single impurity He^3 atoms and dimers (He$^3)_2$ per unit area. The chemical equilibrium condition (2.1) determines with the aid of (2.3) the equilibrium degree of dimerization

$$\alpha=1+X-(X^2+2X)^{1/2}, \tag{2.4}$$

where

$$X=\frac{T}{4T_0}e^{-\Delta/T},\qquad T_0=\frac{\mathfrak{N}_0}{g_0},$$

$\mathfrak{N}_0$ is the total number N_0 of impurity atoms per unit area. In the case of high temperatures $T\gtrsim\Delta$ we have

$$\alpha\approx\frac{1}{2X}=2\frac{T_0}{T}e^{\Delta/T}$$

with decreasing temperature, $T\ll\Delta$, the degree of dimerization tends exponentially to unity:

$$\alpha\approx1-(2X)^{1/2}=1-(T/2T_0)^{1/2}\exp(-\Delta/2T_0).$$

The paramagnetic susceptibility χ of the system is determined by the number N_1 of the unpaired impurity He^3 atoms:

$$\chi=\frac{\beta^2}{T}N_0(1-\alpha), \tag{2.5}$$

where β is the nuclear magnetic moment of He^3. Inas-

much as in singlet dimerization there are produced spinless quasimolecules (He$^3)_2$, the observed deviations in the temperature dependence of the magnetic susceptibility χ from the Curie-Weiss law enable us to study directly the dynamics of the onset of bound Van der Waals pairs.

The entropy of a two-component mixture of single atoms and dimers in a superfluid He^4 background is obtained by differentiating the free energy (2.3):

$$S=S_4+N_1\left(2+\frac{\varepsilon_0-\mu_1}{T}\right)+N_2\left(2+\frac{2\varepsilon_0-\Delta-\mu_2}{T}\right), \tag{2.6}$$

where S_4 is the contribution of the pure He^4 to the entropy. After simple transformations we obtained with the aid of (2.1) and (2.2)

$$S=S_4+N_0\left\{2-\alpha-\frac{\alpha}{2}\frac{\Delta}{T}-\ln\left[\frac{T_0}{T}(1-\alpha)\right]\right\}. \tag{2.7}$$

We next obtained from (2.7), taking into account the equality

$$\frac{\partial\alpha}{\partial T}=-\alpha\frac{1-\alpha}{1+\alpha}\frac{1}{T}\left(1+\frac{\Delta}{T}\right)$$

the final expression for the heat capacity of the solution (at constant area)

$$C=C_4+N_0\left[1-\frac{\alpha}{2}+\frac{\alpha}{2}\frac{1-\alpha}{1+\alpha}\left(\frac{\Delta}{T}+1\right)^2\right]. \tag{2.8}$$

Here C_4 is the heat capacity of pure He^4.

We write down also the value of the thermodynamic potential Ω, which has the meaning of surface tension in the case of a free HeII surface with He^3 impurity levels:

$$\Omega=F-\mu_1N_1-\mu_2N_2=F_4-T(N_1+N_2), \tag{2.9}$$

or ultimately

$$\Omega=F_4-N_0T(1-{}^1\!/_2\alpha). \tag{2.10}$$

Equation (2.10) was written out for a system with a fixed number of surface excitations, i.e., at sufficiently low temperatures and concentrations, when it can be assumed that practically all the He^3 impurity quasiparticles have emerged from the interior of the solution to the surface.[8,10,11] In the general case the values of N_1 and N_2, which determine the surface tension (2.9), can be easily obtained from the equilibrium conditions

$$\mu_1={}^1\!/_2\mu_2=\mu \tag{2.11}$$

and from the constancy of the total number of impurity He^3 atoms in the solution

$$N_1+2N_2+\mathcal{N}=N_0. \tag{2.12}$$

In (2.11) and (2.12) the number $\mathcal{N}$ of the impurity quasiparticles in the volume of the solution and the chemical potential μ of the volume excitations are connected by the relation[6,8,16]

$$\mu=E_0+T\ln\left\{\frac{\mathcal{N}}{2V}\left(\frac{2\pi\hbar^2}{MT}\right)^{3/2}\right\}, \tag{2.13}$$

where M and E_0 are the effective mass and the zero-point binding energy of the He^3 quasiparticles in the volume, and V is the volume of the solution. After simple calculations we obtain with the aid of (2.3) and (2.11)–(2.13)

$$N_1 = \frac{2}{\Phi_2}[(\Phi_1{}^2+\Phi_2)^{1/2}-\Phi_1],$$

$$N_2 = \frac{1}{8}\Phi_2 N_1{}^2, \qquad \mathscr{N} = (\Phi_1-1)N_1,$$

$$\Phi_1(T) = 1 + \gamma \left(\frac{g_0}{2}\right)^{1/3}\left(\frac{M}{m}\right)^{1/2} T^{5/4}\exp\left(\frac{\varepsilon_0-E_0}{T}\right), \qquad \gamma = \frac{V}{S_\Delta},$$

$$\Phi_2(T) = \frac{8}{g_0 S_\Delta}\frac{1}{T}e^{\Delta/T},$$

where S_H is the area of the free surface of the solution.

In the one-dimensional case all the calculations of the thermodynamic functions are carried out in similar fashion. Thus, we have for the chemical potentials μ_1 and μ_2

$$\mu_1 = \varepsilon_0 + T\ln\left\{\frac{n_1}{2}\left(\frac{2\pi\hbar^2}{mT}\right)^{1/2}\right\}, \qquad \mu_2 = 2\varepsilon_0 - \Delta + T\ln\left\{n_2\left(\frac{\pi\hbar^2}{mT}\right)^{1/2}\right\}. \tag{2.14}$$

The degree of dimerization is determined here from the condition (2.1) by Eq. (2.4), the only difference being that X is given in the one-dimensional case by the expression

$$X = \frac{1}{2}\left(\frac{\pi}{2}\frac{T}{T_0{}^*}\right)^{1/2}e^{-\Delta/T}, \tag{2.15}$$

i.e., at high temperatures $T \gtrsim \Delta$ it has a weaker temperature dependence than in the two-dimensional case. In (2.15), $T_0{}^* = (\pi\hbar n_0)^2/2m$ is the "one-dimensional" degeneracy temperature for the total number of impurity atoms n_0 (per unit length). The paramagnetic susceptibility is in this case also determined by (2.5).

The entropy of the system of linear chains differs from the entropy in the two-dimensional case (2.6) because of the change of the heat capacity per dissolved He^3 atom as a result of the change in the number of degrees of freedom of the particle:

$$S = S_* + N_1\left(\frac{3}{2}+\frac{\varepsilon_0-\mu_1}{T}\right) + N_2\left(\frac{3}{2}+\frac{2\varepsilon_0-\Delta-\mu_2}{T}\right). \tag{2.16}$$

We find next with the aid of (2.14) and of the chemical-equilibrium condition (2.1)

$$S = S_* + N_0\left\{\frac{3}{2}-\frac{3}{4}\alpha-\frac{\alpha}{2}\frac{\Delta}{T}-\ln\left[(1-\alpha)\left(\frac{T_0{}^*}{\pi T}\right)^{1/2}\right]\right\}. \tag{2.17}$$

The rate of change of the equilibrium degree of dimerization α with changing temperature is now determined by the relation

$$\frac{\partial\alpha}{\partial T} = -\alpha\frac{1-\alpha}{1+\alpha}\frac{1}{T}\left(\frac{1}{2}+\frac{\Delta}{T}\right). \tag{2.18}$$

Differentiating (2.17) and taking (2.18) into account, we obtain ultimately

$$C = C_* + N_0\left\{\frac{1}{2}-\frac{\alpha}{4}+\frac{1}{2}\alpha\frac{1-\alpha}{1+\alpha}\left(\frac{\Delta}{T}+\frac{1}{2}\right)^2\right\}. \tag{2.19}$$

Expression (2.19) determines the temperature dependence of the heat capacity of a weakly concentrated He^3–HeII solution that fills a finely porous matrix, or in the presence of quantized vortex filaments.

3. THRESHOLD ABSORPTION OF SOUND

By way of example of nonstationary kinetic phenomena in a dimerized system of impurity He^3 atoms we consider the threshold absorption of first sound in a solution with a red boundary $\omega = \Delta/\hbar$. The mechanism of the threshold absorption is connected with the inelastic

single-phonon process of splitting of a bound Van der Waals pair (He^3)$_2$ by a sound wave of frequency $\omega \gtrsim \Delta/\hbar$. In the initial state there is a discrete level $E_i = -\Delta$ corresponding to the bound pair of (He^3)$_2$ quasiparticles, and an acoustic phonon with definite momentum $\hbar q$. It makes sense to consider the hydrodynamic acoustic oscillations only for wave numbers $q < 1/a$ or, equivalently, for frequencies $\omega < c/a \approx 10^{12}$ sec^{-1} (c is the speed of the first sound in He II). The final state corresponds to an infinite relative motion of the released single He^3 quasiparticles. Since we are interested in the decay of a two-dimensional (or one-dimensional) bound state in the field of a longitudinal sound wave, we shall henceforth take $\mathbf{q}$ to mean the projection of the wave vector of the oscillations on the corresponding plane (or on the line in the one-dimensional case) in which the relative motion of the interacting particles takes place.

The transition probability in the c.m.s. is determined by the usual quantum-mechanics formula

$$dw = \frac{2\pi}{\hbar}|\langle\psi_f|\hat{V}_{int}|\psi_i\rangle|^2\delta(-\Delta+\hbar\omega-E)\frac{dp}{(2\pi\hbar)^s}; \qquad s=1,2. \tag{3.1}$$

Here E and $\mathbf{p}$ are the energy and momentum of the relative motion of the particles in the final state. The operator $\hat{V}_{int}$ of the interaction with the phonon field is of the form[2,6]

$$\hat{V}_{int} = \frac{\partial\varepsilon_0}{\partial\rho}\delta\hat{\rho} + \frac{1}{2}\frac{\delta m}{m}(\hat{\mathbf{p}}\hat{\mathbf{v}}_s+\hat{\mathbf{v}}_s\hat{\mathbf{p}}), \tag{3.2}$$

where $\delta\hat{\rho}$ and $\hat{\mathbf{v}}_s$ are the operators of the perturbation of the density and velocity of the superfluid motion of He^4 in the acoustic wave, ρ is the density of the pure He^4, $\mathbf{p} = -i\hbar\nabla$, $\delta m = m - m_3$, and m_3 is the mass of the He^3 atom. We define the operators $\delta\hat{\rho}$ and $\hat{\mathbf{v}}_s$ by the equations

$$\delta\hat{\rho} = ae^{i(\mathbf{qr}-\omega t)}, \qquad \hat{\mathbf{v}}_s = \mathbf{e}be^{i(\mathbf{qr}-\omega t)}, \qquad \mathbf{e} = \mathbf{q}/q. \tag{3.3}$$

In the zeroth approximation in the He^3 concentration, the continuity equation

$$\partial\rho/\partial t + \mathrm{div}\,\rho\mathbf{v}_s = 0$$

yields the relation

$$a = \rho b/c. \tag{3.4}$$

The intensity I of the sound radiation is $\rho c\overline{v_s^2}$ $= \frac{1}{2}\rho c|b|^2$. From this we obtain directly the amplitude b normalized to one phonon:

$$|b|^2 = 2\hbar\omega/\rho c. \tag{3.5}$$

We consider the reaction of splitting of the dimer far from the red absorption boundary $\hbar\omega \gg \Delta$. In this case we can take the wave function ψ_f of the final state to be a plane wave, i.e., the wave function of the free motion

$$\psi_f = e^{i\mathbf{kr}}, \qquad \mathbf{k} = \mathbf{p}/\hbar. \tag{3.6}$$

Integrating in (3.1) the δ function with respect to dE with the aid of expressions (3.2)–(3.6), we obtain the differential cross section of the reaction (in the two-dimensional case)

$$do^{(2)} = \frac{1}{2}g_0\frac{\omega}{\rho c}\left[\frac{\delta m}{m}\left(\mathbf{ep}-\frac{1}{2}\hbar g\right)+\frac{\rho}{c}\frac{\partial\varepsilon_0}{\partial\rho}\right]^2\psi_i^2(\mathbf{k-q})\,d\theta, \tag{3.7}$$

where $\psi_i(\mathbf{k})$ is the Fourier component of the wave function of the initial state ψ_i. In the one-dimensional case

this procedure leads to the following result for the total cross section of the dimer decay reaction:

$$\sigma^{(1)} = \frac{m\omega}{\hbar p \rho c}\left[\frac{\delta m}{m}\left(p - \frac{1}{2}\hbar q\right) + \frac{\rho}{c}\frac{\partial \varepsilon_0}{\partial \rho}\right]^2 \psi_i^2(k-q). \qquad (3.8)$$

The wave functions of the initial ground state in the two-dimensional and one-dimensional cases are determined respectively by Eqs. (1.10) and (1.14). Their Fourier components are in this case

$$\psi_i(k) = 2\nu^{1/2}/(\nu^2 + k^2) \quad \text{in the one-dimensional case,}$$

$$\psi_i(\mathbf{k}) = 2\pi^{1/2}\nu/(\nu^2 + k^2) \text{ in the two-dimensional case,} \qquad (3.9)$$

where $\nu^2 = m\Delta/\hbar^2$. The cross section for the acoustic splitting (3.8) then takes the form[1]

$$\sigma^{(1)} = 4\frac{m\omega}{\hbar p \rho c}\nu^3\left[\frac{m^{-1}\delta m(p - \frac{1}{2}\hbar q) + \rho c^{-1}\partial\varepsilon_0/\partial\rho}{\nu^2 + (k-q)^2}\right]^2. \qquad (3.10)$$

The integration in (3.7) with respect to $d\theta$, with account taken of (3.9), is elementary, albeit cumbersome:

$$\sigma^{(2)} = g_0\frac{\omega}{\rho c}(2\pi\nu)^2\left\{\left(\frac{\hbar}{2q}\frac{\delta m}{m}\right)^2 F + 2(\hbar q)^2\frac{\delta m}{m}k^2\frac{B}{D^3}\right.$$
$$\left. + \frac{1}{4}(\hbar q)^2 B^2\frac{(D^2 + 4k^2q^2)^{1/2}}{D^3}\right\}; \qquad (3.11)$$

here the functions F, B, and D are defined by the expressions

$$D^2 = [(k-q)^2 + \nu^2][(k+q)^2 + \nu^2],$$

$$F = 1 + \frac{(4k^2q^2 + D^2)^{1/2}(4k^2q^2 - D^2)}{D^3},$$

$$B = 2\frac{\rho}{\hbar\omega}\frac{\partial\varepsilon_0}{\partial\rho} \cdot \frac{\delta m}{m}.$$

The sound absorption coefficient $\Gamma(\omega)$ is obtained by multiplying the decay cross section σ by the number of dimers per unit volume, which, obviously coincide with the value of $\mathfrak{N}_2$ (or n_2 in the one-dimensional case). To avoid cumbersome formulas, we confine ourselves to the expressions for $\Gamma(\omega)$ in the dipole approximation, i.e., neglecting the recoil momentum $\hbar q$ in the operators $\delta\hat{\rho}$ and $\hat{\mathbf{v}}_s$ (3.3) and in the cross sections (3.7) and (3.8). This is always justified, since the condition for the applicability of the dipole approximation $qa \ll 1$ coincides in this case with the condition for the hydrodynamic treatment of the oscillations of the liquid. Taking into account the energy conservation law

$$\Delta + p^2/m = \hbar\omega$$

we obtain ultimately from (3.7)–(3.9) for the sound absorption coefficient

$$\Gamma^{(1)}(\omega) = 2\left(\frac{\Delta}{\hbar\omega}\right)^{1/2}\frac{n_0\alpha}{\hbar\rho c}\left[\delta m\left(\frac{\hbar\omega}{m}\right)^{1/2} + \frac{\rho}{c}\frac{\partial\varepsilon_0}{\partial\rho}\right]^2, \quad s = 1,$$

$$\qquad (3.12)$$

$$\Gamma^{(2)}(\omega) = 4\pi\frac{\mathfrak{N}_0\alpha}{\hbar\rho c}\frac{\Delta}{\hbar\omega}\left[\frac{1}{2}(\delta m)^2\frac{\hbar\omega}{m} + \left(\frac{\rho}{c}\frac{\partial\varepsilon_0}{\partial\rho}\right)^2\right], \quad s = 2.$$

Numerical estimates by means of Eqs. (3.12) show that at $T \ll \Delta$ an experimental study of the sound damping $\Gamma^{(1)}/q$ or $\Gamma^{(2)}/q$ may turn out to be feasible. We note a curious fact: in the two-dimensional case the absorption coefficient $\Gamma^{(2)}(\omega)$ tends as $\omega \to \infty$ not to zero but to a constant value

$$\Gamma^{(2)}(\infty) = 2\pi\Delta\frac{(\delta m)^2}{m}\frac{\mathfrak{N}_0\alpha}{\hbar\rho c}.$$

The calculation of the sound absorption coefficient

near the threshold of the reaction $(\hbar\omega - \Delta)/\Delta \ll 1$ already calls for knowledge of the exact (or quasiclassical) wave functions ψ_f of the continuous spectrum, i.e., for knowledge of the concrete form of the interaction potential $u(r)$. However, in accordance with the general rules of the behavior of reaction cross sections near threshold,[15] the absorption coefficient (in accord with the detailed balancing principle) is proportional to the statistical weight of the final states $d\mathbf{p}/(2\pi\hbar)^s$. Taking the energy conservation law into account, we find that in the two-dimensional case the splitting cross section near the threshold does not depend on the frequency and tends to a constant limit $\sigma^{(2)} = C = \text{const}$. The value of the constant C can be determined quite approximately by extrapolating expression (3.12) to $\omega = \Delta/\hbar$:

$$C \approx \frac{8\pi}{\hbar p c}\left[\frac{1}{2}(\delta m)^2\frac{\Delta}{m} + \left(\frac{\rho}{c}\frac{\partial\varepsilon_0}{\partial\rho}\right)^2\right].$$

In the one-dimensional case the decay cross section $\sigma^{(1)}$ as $\hbar\omega \to \Delta + 0$ becomes infinite like $(\hbar\omega - \Delta)^{-1/2}$, i.e., it has a clearly pronounced resonant character.

In a real experiment, the nonmonochromaticity of the excited sound oscillations leads to a smoothing of the singularity in $\Gamma^{(1)}$ and of the kink in $\Gamma^{(2)}$ at $\hbar\omega = \Delta$.

We note in conclusion that a similar sound-absorption mechanism takes place when an impurity He^3 particle is torn by an acoustic wave away from the axis of a vortex filament into the volume of the liquid. The calculation of this process is quite similar, all the more since we know the exact wave functions of the initial and final states.[12] The absorption coefficient in this case, however, is proportional to the number of vortex filaments, which under real experimental conditions is always small. Therefore the observation of this effect is at present quite difficult.

4. PHASE TRANSITIONS IN TWO-DIMENSIONAL $(He^3)_2$

A. Gas-liquid transition

If the temperature is lowered enough, a phase transition becomes possible in the Bose system $(H^3)_2$, accompanied by condensation of the gas of Van der Waals dimers into a liquid. A characteristic feature of this liquid is that it is rarefied to a considerable degree. In fact, in any liquid the average distance between the particles that make up the liquid is determined by the dimensions of the particles themselves. In our case the characteristic length is the dimer dimension r_0, which greatly exceeds the atomic distances a. We emphasize that in the two-dimensional case, in contrast to the three-dimensional case, the condensation of a gas into a liquid (at sufficiently low temperatures) is always possible, even at arbitrarily low density of the gas. The reason is that in the three-dimensional case the expansion of the system energy in the interaction coincides with the expansion in the density, i.e., at low density the gas never condenses, as is in fact observed in He^3–$HeII$ solutions. In the two-dimensional case, on the other hand, the interaction may turn out to be very important even at quite small He^3 concentrations.

The two-dimensional phase transition in a system of

surface Fermi states of He^3 was first considered by Andreev and Kompaneets.[17] The condensation temperature of a Bose system of Van der Waals dimers $(He^3)_2$ is calculated in analogy with their work.[17] The chemical potential μ_2 of the dimer gas is determined from the normalization condition of the distribution function

$$\frac{1}{(2\pi)^2} \int_0^\infty \frac{d^2k}{\exp\{[\mathscr{E}_2(k)-\mu_2]/T\}-1} = \mathfrak{R}_2, \qquad (4.1)$$

where $\mathscr{E}_2(k)$ is given by (1.2). For any finite temperatures $T \neq 0$ we have

$$\mu_2 < 2\varepsilon_0 - \Delta,$$

which ensures the convergence of the integral in (4.1). A direct calculation yields

$$\mu_2 = 2\varepsilon_0 - \Delta + T \ln[1-\exp(-\mathfrak{R}_2/g_0 T)]. \qquad (4.2)$$

At high temperatures, expression (4.2) coincides with (2.3). The equation $\mu_L(T_c) = \mu_2(T_c)$, where μ_L is the chemical potential of the liquid phase, determines the condensation temperature T_c.

At low temperatures, the contribution of the compressibility and of other thermal degrees of freedom to μ_L can be neglected. If the dimer interaction is not too strong, then the gas–liquid phase transition takes place already in the region of the quantum degeneracy of $(He^3)_2$. In this case $(\mu_L - 2\varepsilon_0 + \Delta)/T_0 < 1$, and since the degeneracy temperature of the dimer Bose gas is $T_d \lesssim T_0 \ll \Delta$, it follows that $\mathfrak{R}_2 \approx \mathfrak{R}_0/2$. Under these conditions the temperature of the condensation of the dimer gas into a liquid is

$$T_c \approx \mathfrak{R}_2/g_0\lambda \approx T_0/2\lambda, \qquad (4.3)$$

where

$$e^{-\lambda z} + e^{-\lambda} = 1, \quad z = 2(\mu_L - 2\varepsilon_0 + \Delta)/T_0.$$

The thermodynamic characteristics of the system of impurity atoms (surface tension, second-sound velocity, and others) should exhibit singularities at the condensation point $T = T_c$ (see, e.g., Ref. 17). We note, however, that all the effects connected with the density discontinuity at the transition point $T = T_c$ may turn out to be quite negligible because of the rarefaction (see above) of the condensed phase. In a certain sense $(He^3)_2$ liquid is a tremendous disordered polymer molecule $(He^3)_n(n \to \infty)$ with large distance $r_0 \gg a$ between the monomirrors. Under conditions when Eq. (4.3) is valid, the dense condensed state of the He^3 impurity atoms (with distances a between particles) turns out to be metastable to a transition to a rarefied Van der Waals liquid.

B. Superfluidity of impurity $(He^3)_2$

Since the degeneracy temperature of the Bose liquid $(He^3)_2$ is high enough, $T_d \sim \Delta$, it follows that at temperatures $T \lesssim T_0 = \mathfrak{R}_0/g_0$ both the gas phase and all the more the condensed phase of the two-dimensional $(He^3)_2$ will exhibit properties of quantum liquids. Since there is no Bose-Einstein condensation in the two-dimensional case at finite temperatures, the ordinary superfluidity mechanism connected with the formation of a condensate of $(He^3)_2$ Bose particles is likewise nonexistent.

In the two-dimensional case, however, a peculiar phase transition into the superfluid state is possible and is accompanied by a universal jump of a density of the superfluid component at the transition point (Kosterlitz and Thouless[13], and also Ref. 14). The reason for this transition mechanism is that production of vortices with short core lengths is energywise unprofitable at sufficiently low temperatures. A very important role is played here by the condition that the superfluid motion of the quantum liquid be potential (since the correlator of the phase φ at short wave vectors diverges only logarithmically, it follows that one can always introduce a superfluid-motion velocity proportional to $\nabla\varphi$).

In the presence of two potential superfluid motions for HeII and the $(He^3)_2$ dissolved in it, an important role is assumed by the effect of dragging of the two components of the solution by each of the superfluid motions.[18] When the dragging effect is taken into account, the change of the density of the total energy of the solution as a result of the superfluid motions of both components can be represented in the form[18]

$$\delta E = \tfrac{1}{2}\rho_{11}^{(s)} v_1^2 + \rho_{12}^{(s)} v_1 v_2 + \tfrac{1}{2}\rho_{22}^{(s)} v_2^2, \qquad (4.4)$$

where $v_1 = (\hbar/2m_3)\nabla\varphi_1$ is the superfluid velocity of the $(He^3)_2$, $v_2 = (\hbar/m_4)\nabla\varphi_2$ is the superfluid velocity of the He^4, m is the mass of the He^4 atom, and $\rho_{\alpha\beta}^{(s)}(\alpha,\beta=1,2)$ is the matrix of the superfluid densities of the solution. We neglect for simplicity the superfluid motion of HeII, i.e., we put $v_2 = 0$. Although the values of the elements of the matrix $\rho_{\alpha\beta}^{(s)}$ were calculated in Ref. 18 for BCS pairing of impurity He^3 quasiparticles, a process that cannot occur in the two-dimensional case, nonetheless the expressions obtained for $\rho_{\alpha\beta}^{(s)}$ remain valid also for superfluid motion of Van der Waals $(He^3)_2$ pairs. In this case $\rho_{11}^{(s)} = m_3^2 \mathfrak{R}_s/m$, where $\mathfrak{R}_s$ is the number of "superfluid" He^3 atoms per unit area (the formula for $\rho_{11}^{(s)}$ can be regarded as a phenomenological definition of the quantity $\mathfrak{R}_s$).

The velocity field of a vortex filament of unit circulation is defined by the relation

$$v_1 = \frac{\hbar}{2m_3}\frac{1}{r}, \qquad (4.5)$$

where r is the distance from the vortex axis. With the aid of (4.4) and the expression for $\rho_{11}^{(s)}$ (see above) we obtain the change of the free energy $\delta F = \delta E - T\delta S$ (S is the entropy) due to vortex formation:

$$\delta F = 2\left(\frac{\pi\hbar^2\mathfrak{R}_s}{8m} - T\right)\ln\frac{b}{r}, \qquad (4.6)$$

where b is the radius of the vortex core. Equation (4.6) determines the temperature T_s of the two-dimensional superfluid transition in $(He^3)_2$ (cf. Refs. 13 and 14):

$$T_s = (\pi\hbar^2/8m)\mathfrak{R}_s. \qquad (4.7)$$

The form of Eq. (4.7) for T_s does not depend on the phase, gas or condensed, at which the Kosterlitz-Thouless transition takes place. Therefore if the temperature of the superfluid transition in the gas phase $T_s^{(G)}$ is less than T_s (4.3) (i.e., at $\mu_L - 2\varepsilon_0 + \Delta \sim T_0/2$), then superfluid two-dimensional drops of $(He^3)_2$ will be

187   Sov. Phys. JETP 51(1), Jan. 1980

E. P. Bashkin   187

produced even at the condensation point $T = T_c$ (since the temperature of the superfluid transition in the liquid phase $T_s^{(L)} \lesssim \Delta$ exceeds T_c). A region where a two-dimensional nonsuperfluid $(He^3)_2$ can exist would occur only in the case of an anomalously small jump of the superfluid density at $T = T_s^{(L)}$. On the other hand, if $T_s^{(G)} > T_c$ i.e., at $(\mu_L - 2\varepsilon_0 + \Delta)/T_0 \ll 1$, then a two-dimensional superfluid gas of $(He^3)_2$ is produced at the phase-transition point $T = T_s^{(G)}$.

The temperature of the superfluid transition in two-dimensional $(He^3)_2$ may turn out to be quite high. Thus, for a solution with surface concentration 0.2×10^{14} cm^{-2}, which corresponds to a volume concentration 0.5% (in which case $T_0 \ll \Delta$). The temperature of the superfluid transition is of the order of $T_s = T_c \lesssim 35$ mK (if $T_s^{(G)} < T_c$) and $T_s = T_s^{(G)} \lesssim 6$ mK (if $T_s^{(G)} > T_c$).

5. CONCLUSION

The principal experimental difficulty in the investigation of the dimerized state of He^3 is that the number of impurity atoms contained in the rarefied two-dimensional monolayer or linear chain is too small. Therefore, to observe the phenomena considered in this paper, it is necessary to use systems containing a large number of narrow channels or capillaries (for example, a finely porous material such as Vycor glass). The maximum effect due to dimerization corresponds to a decrease in the number of particles of the dissolved matter by a factor of two.

The presence of a clearly pronounced singularity at $\hbar\omega = \Delta$ in the absorption coefficient of the first sound by $(He^3)_2$ dimers localized on quantized vortex filaments makes it possible in principle to measure Δ at any He^3 concentration (such as resonant peak exists even in pure He^4 in the presence of vortex filaments, inasmuch it always contains up to 10^8 cm^{-3} of He^3 impurity atoms.) The small number of dimers leads, however, to an appreciable narrowing of the resonance region, another substantial obstacle to experimental observation.

Very sensitive to even a small amount of He^3 impurities is the surface tension on a free He II surface. At sufficiently low surface-impurity concentrations, attraction between the He^3 quasiparticles would lower the temperature and produce an additional change of the surface tension by dimerization. An experimental confirmation of the presence or absence of dimerization of the surface He^3 at very low concentrations would identify unequivocally the character of the interaction (attraction or repulsion) between the quasiparticles. The existing experimental data, although very well described by the equations for an interacting Fermi gas,[10] cannot deny completely the possibility of dimerization, since they have been obtained for relatively high concentrations of the surface impurities in the Fermi-degeneracy region. (We note also that these data do not contradict the dimerization in that region where it is feasible.) In addition one cannot fail to disregard the possibility of formation of Van der Waals pairs with higher moments.

The experimental data of Edwards *et al.*[19] point to a linear dependence of the impurity part of the surface tension on the chemical potential as $T \to 0$, a property possessed by a two-dimensional condensed liquid.[17] Extrapolation of the results of Ref. 19 makes it possible to determine the surface density of the impurities in the liquid at $T = 0$, which turns out to be $\mathfrak{N} = 1.3 \cdot 10^{14}$ cm^{-2}.[11,17] So small a surface density, compared with a dense monolayer, is a direct indication that a rarefied Van der Waals $(He^3)_2$ liquid exists. In this case $\Delta_0 \sim 30$ mK, and the condensation temperature T_c (4.3) turns out to be less than 30 mK.

We have spoken so far only of $(He^3)_2$ Van der Waals dimers. In the presence of attraction between the corresponding particles in the solution, there can exist also polymer Van der Waals $(He^3)_n$ formations with not too high values of n. (At large n it is energywise more profitable for the polymers to condense into liquid drops in the two-dimensional case or to break up into smaller fragments in the one-dimensional case.) Since we neglect the probability of collisions between three and more particles, the process of formation of the $(He^3)_n$ polymers goes through a stage of production of bound $(He^3)_2$ pairs. If the corresponding binding energies turn out to be close to Δ, then a noticeable number of polymers can exist in the solution at low temperatures in addition to the $(He^3)_2$ dimers.

[1)]The cross sections $\sigma^{(1)}$ and $\sigma^{(2)}$ were calculated for the problem of the motion of one particle in a Van der Waals attraction field, i.e., in fact without allowance for the identity of the He^3 quasiparticles.

[1]L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **59**, 669 (1948).

[2]J. Bardeen, G. Baym, and D. Pines, Phys. Rev. **156**, 207 (1967).

[3]E. P. Bashkin, Zh. Eksp. Teor. Fiz. **73**, 1849 (1977) [Sov. Phys. JETP **46**, 972 (1977)].

[4]D. Fay, Phys. Rev. B. **16**, 537 (1977).

[5]E. P. Bashkin, Phys. Lett. **69A**, 282 (1978).

[6]B. N. Esel'son, V. N. Grigor'ev, V. G. Ivantsov, E. Ya. Rudavskiĭ, D. G. Sanikidze, and I. A. Serbin. Rastvory kvantovykh zhidkosteĭ He^3—He^4 (Solutions of He^3—He^4 Quantum Liquids), Nauka, Moscow, 1973.

[7]V. P. Peshkov, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 356 (1975) [JETP Lett. **21**, 162 (1975)].

[8]A. F. Andreev, Zh. Eksp. Teor. Fiz. **50**, 1415 (1966) [Sov. Phys. JETP **23**, 939 (1966)].

[9]K. N. Zinov'eva and S. T. Boldarev, Zh. Eksp. Teor. Fiz. **56**, 1089 (1969) [Sov. Phys. JETP **29**, 585 (1969)].

[10]D. O. Edwards and W. F. Saam, Prog. Low Temp. Phys. **VIIA**, 285 (1978).

[11]S. Y. Shen, Dissertation (Ph.D.), Ohio State Univ., 1973.

[12]L. S. Reut and I. Z. Fisher, Zh. Eksp. Teor. Fiz. **55**, 722 (1968) [Sov. Phys. JETP **28**, 375 (1968)].

[13]J. M. Kosterlitz and D. J. Thouless, J. Phys. **6C**, 1181 (1973).

[14]D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).

[15]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, Moscow, 1974 [Pergamon].

[16]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika

(Statistical Physics), Nauka, Moscow, 1976, Vol. 1. [Pergamon].
[17]A. F. Andreev and D. A. Kompaneets, Pis'ma Zh. Eksp. Teor. Fiz. 17, 376 (1973) [JETP Lett. 17, 268 (1973)].
[18]A. F. Andreev and E. P. Bashkin, Zh. Eksp. Teor. Fiz. 69,
319 (1975) [Sov. Phys. JETP 42, 164 (1975)].
[19]H. M. Guo, D. O. Edwards, R. E. Sarwinski, and I. T. Tough, Phys. Rev. Lett. 27, 1259 (1971).

Translated by J. G. Adashko

The acoustoelectric current in a metal for an arbitrary conduction-electron dispersion law

M. I. Kaganov, Sh. T. Mevlyut, and I. M. Suslov

Institute of Physical Problems, USSR Academy of Sciences
(Submitted 31 July 1979)
Zh. Eksp. Teor. Fiz. **78**, 376–380 (January 1980)

Expressions for the acoustoelectric current and the acousto-emf are derived without any assumptions concerning the conduction-electron collision mechanism or dispersion law. The calculation is carried through for the case of a sound wave whose wavelength is considerably smaller than the electron mean free path. The resulting formulas permit the effect of a magnetic field to be taken into account.

PACS numbers: 72.50. + b

When sound is absorbed by a metal, the transfer of momentum from the sound wave to the conduction electrons may give rise to a current (called the acoustoelectric current) or, in the case of an open circuit, to a potential difference between the ends of the specimen (the acousto-emf).[1]

When the wavelength $\lambda = 2\pi/q$ of the sound is considerably shorter than the electron mean free path l (when $ql \gg 1$), the sound wave can be treated as a packet of coherent phonons having a δ-function distribution $N(\mathbf{k})$ in wave-vector space[1]:

$$N(\mathbf{k}) = \frac{(2\pi)^3}{\hbar \omega_q s_q} \Phi \delta(\mathbf{k} - \mathbf{q}), \tag{1}$$

where $\mathbf{k}$ is the current phonon wave vector, Φ is the sound energy flux density, and ω_q and s_q are the frequency and group velocity of a sound wave with the wave vector $\mathbf{q}$ (if the elastic anisotropy of the metal can be neglected, the group and phase velocities s_q and $\tilde{s}_q$ of the sound will be equal).

To calculate the dragging force exerted by the phonons on the electrons one can use the usual collision integral for the electron-phonon interaction,[5] which also takes the presence of nonequilibrium phonons into account.[3] Such an approach was used in Ref. 6. The expression obtained in Ref. 6 for the acoustoelectric current density j^A (see (7)) is applicable for an arbitrary conduction-electron dispersion law, but, as is shown in this paper, it is limited to the τ approximation. The problem addressed here is to construct a theory of the acoustoelectric effect that would be free of this approximation. In addition, the formulas obtained here make it possible to take account of the part played by an external magnetic field (within the limitations of the theory of galvanomagnetic phenomena[7]).

We shall use the operator $\hat{W}$ to describe the collisions of the electrons with thermal phonons and lattice defects. The kinetic equation for the addition $f(p)$ to the equilibrium Fermi distribution function $F(\varepsilon_p)$ for the conduction electrons, linearized in Φ, has the form (see Refs. 8 and 9)

$$\hat{\mathscr{L}}_H\{f\} = \frac{e}{c}[\mathbf{v} \times \mathbf{H}]\frac{\partial f}{\partial \mathbf{p}} + \hat{W}\{f\} = U, \tag{2}$$
$$U = U^A + U^c,$$

where

$$U^c = -\frac{\partial F}{\partial \underline{\varepsilon}} e \mathbf{E} \mathbf{v}, \tag{3}$$

$$U^A = \frac{2\pi}{\hbar} \frac{\Phi}{\hbar \omega_q s_q} \{|g_{p-\hbar q,p}|^2[F(\varepsilon_{p-\hbar q}) - F(\varepsilon_p)]\delta(\varepsilon_{p-\hbar q} - \varepsilon_p + \hbar\omega_q)$$
$$+ |g_{p+\hbar q,p}|^2[F(\varepsilon_{p+\hbar q}) - F(\varepsilon_p)]\delta(\varepsilon_{p+\hbar q} - \varepsilon_p - \hbar\omega_q)\},$$

and $g_{p',p}$ is the electron-phonon interaction matrix element. The first term on the left in Eq. (2) was introduced to describe the part played by an external magnetic field $\mathbf{H}$. The electric field $\mathbf{E}$ may be applied to the conductor independently, but it may also be induced by the sound flux, and in that case it is determined from the condition $j = 0$ and represents the acousto-emf (the equation has been linearized in $\mathbf{E}$ as well as in Φ). We neglect the term $\mathbf{v} \cdot \partial f/\partial \mathbf{r}$ that represents the space dispersion of the electronic properties since the sound attenuation length s/Γ is usually considerably longer than the electron mean free path l.

We note that the collision operator $\hat{W}$ may also describe the dragging of phonons by electrons.[3] In this case, by $\hat{W}$ we must understand the operator that is obtained after eliminating the nonequilibrium addition to the Bose distribution for the thermal phonons from the two kinetic equations (those for the electrons and for the phonons—see Ref. 8, Sec. 25). In what follows we shall not consider specific collision mechanisms, but shall only make use of very general properties of $\hat{W}$.

Because of the δ-function factors in the quantity U^A on the right-hand side of the kinetic equation (2), it might seem that in calculating the acoustoelectric effects one could ignore the integral character of the operator $\hat{W}$ i.e. that one could neglect the arrival term in $\hat{W}$ and (in analogy with the theory of the anomalous skin effect[10]) introduce a relaxation time τ_p that depends on the quasi-momentum p [see formula (5) of Ref. 6]. However, this is not the case. Although electrons of the $\mathbf{q} \cdot \mathbf{v} = \omega_q$ strip actually contribute to the acoustoelectric current, to the extent that the operator W is of integral charac-