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The acoustoelectric current in a metal for an arbitrary conduction-electron dispersion law

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Expressions for the acoustoelectric current and the acousto-emf are derived without any assumptions concerning the conduction-electron collision mechanism or dispersion law. The calculation is carried through for the case of a sound wave whose wavelength is considerably smaller than the electron mean free path. The resulting formulas permit the effect of a magnetic field to be taken into account.

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When sound is absorbed by a metal, the transfer of momentum from the sound wave to the conduction electrons may give rise to a current (called the acoustoelectric current) or, in the case of an open circuit, to a potential difference between the ends of the specimen (the acousto-emf).[1]

When the wavelength $\lambda = 2\pi/q$ of the sound is considerably shorter than the electron mean free path l (when $ql \gg 1$), the sound wave can be treated as a packet of coherent phonons having a δ-function distribution $N(\mathbf{k})$ in wave-vector space[1)]:

$$N(\mathbf{k}) = \frac{(2\pi)^3}{\hbar\omega_q s_q}\Phi\delta(\mathbf{k}-\mathbf{q}), \tag{1}$$

where $\mathbf{k}$ is the current phonon wave vector, Φ is the sound energy flux density, and ω_q and s_q are the frequency and group velocity of a sound wave with the wave vector $\mathbf{q}$ (if the elastic anisotropy of the metal can be neglected, the group and phase velocities s_q and $\tilde{s}_q$ of the sound will be equal).

To calculate the dragging force exerted by the phonons on the electrons one can use the usual collision integral for the electron-phonon interaction,[5] which also takes the presence of nonequilibrium phonons into account.[3] Such an approach was used in Ref. 6. The expression obtained in Ref. 6 for the acoustoelectric current density $\mathbf{j}^A$ (see (7)) is applicable for an arbitrary conduction-electron dispersion law, but, as is shown in this paper, it is limited to the τ approximation. The problem addressed here is to construct a theory of the acoustoelectric effect that would be free of this approximation. In addition, the formulas obtained here make it possible to take account of the part played by an external magnetic field (within the limitations of the theory of galvanomagnetic phenomena[7]).

We shall use the operator $\hat{W}$ to describe the collisions of the electrons with thermal phonons and lattice defects. The kinetic equation for the addition $f(p)$ to the equilibrium Fermi distribution function $F(\varepsilon_p)$ for the conduction electrons, linearized in Φ, has the form (see Refs. 8 and 9)

$$\hat{\mathcal{L}}_H\{f\} \equiv \frac{e}{c}[\mathbf{v}\times\mathbf{H}]\frac{\partial f}{\partial \mathbf{p}} + \hat{W}\{f\} = U, \qquad U = U^A + U^c, \tag{2}$$

where

$$U^c = -\frac{\partial F}{\partial \varepsilon}e\mathbf{E}\mathbf{v}, \tag{3}$$

$$U^A = \frac{2\pi}{\hbar}\frac{\Phi}{\hbar\omega_q s_q}\{|g_{p-\hbar q,p}|^2[F(\varepsilon_{p-\hbar q})-F(\varepsilon_p)]\delta(\varepsilon_{p-\hbar q}-\varepsilon_p+\hbar\omega_q) + |g_{p+\hbar q,p}|^2[F(\varepsilon_{p+\hbar q})-F(\varepsilon_p)]\delta(\varepsilon_{p+\hbar q}-\varepsilon_p-\hbar\omega_q)\},$$

and $g_{p',p}$ is the electron-phonon interaction matrix element. The first term on the left in Eq. (2) was introduced to describe the part played by an external magnetic field H. The electric field E may be applied to the conductor independently, but it may also be induced by the sound flux, and in that case it is determined from the condition $\mathbf{j} = 0$ and represents the acousto-emf (the equation has been linearized in E as well as in Φ). We neglect the term $\mathbf{v}\cdot\partial f/\partial \mathbf{r}$ that represents the space dispersion of the electronic properties since the sound attenuation length s/Γ is usually considerably longer than the electron mean free path l.

We note that the collision operator $\hat{W}$ may also describe the dragging of phonons by electrons.[3] In this case, by $\hat{W}$ we must understand the operator that is obtained after eliminating the nonequilibrium addition to the Bose distribution for the thermal phonons from the two kinetic equations (those for the electrons and for the phonons—see Ref. 8, Sec. 25). In what follows we shall not consider specific collision mechanisms, but shall only make use of very general properties of $\hat{W}$.

Because of the δ-function factors in the quantity U^A on the right-hand side of the kinetic equation (2), it might seem that in calculating the acoustoelectric effects one could ignore the integral character of the operator $\hat{W}$ i.e. that one could neglect the arrival term in $\hat{W}$ and (in analogy with the theory of the anomalous skin effect[10]) introduce a relaxation time τ_p that depends on the quasimomentum p [see formula (5) of Ref. 6]. However, this is not the case. Although electrons of the $\mathbf{q}\cdot\mathbf{v} = \omega_q$ strip actually contribute to the acoustoelectric current, to the extent that the operator W is of integral charac-

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ter all the electrons at the Fermi surface take part in forming the distribution function for the strip electrons.[2)]

Using the formal solution of Eq. (2),

$$f=\hat{\mathscr{L}}_H^{-1}\{U\}=\hat{\mathscr{L}}_H^{-1}\{(U^A+U^C)\}=\hat{\mathscr{L}}_H^{-1}\{U^A\}+\hat{\mathscr{L}}_H^{-1}\{U^C\}, \tag{4}$$

we can calculate the current density $\mathbf{j}=\mathbf{j}^A+\mathbf{j}^C$:

$$j_i^A=\frac{2e}{(2\pi\hbar)^3}\int v_i\hat{\mathscr{L}}_H^{-1}\{U^A\}\,d^3p, \tag{5}$$

$$j_i^C=\sigma_{ik}(\mathbf{H})E_k, \quad \sigma_{ik}(\mathbf{H})=-\frac{2e^2}{(2\pi\hbar)^3}\int\frac{\partial F}{\partial\varepsilon}v_i\psi_k(\mathbf{p},\mathbf{H})\,d^3p, \tag{6}$$

where $\psi(\mathbf{p},\mathbf{H})$ is a solution of the kinetic equation

$$\frac{e}{c}[\mathbf{vH}]\frac{\partial\psi_i}{\partial\mathbf{p}}+\hat{W}_p\{\psi_i\}=v_i, \quad \hat{W}_p\{\ldots\}=\left(\frac{\partial F}{\partial\varepsilon}\right)^{-1}\hat{W}\left\{\frac{\partial F}{\partial\varepsilon}\ldots\right\}, \tag{7}$$

used in the theory of the galvanomagnetic properties of metals,[7] and when $H=0$, in the theory of electrical conductivity.

Regardless of the electron-scattering mechanism, the electrical conductivity tensor satisfies the Onsager relations:

$$\sigma_{ik}(\mathbf{H})=\sigma_{ki}(-\mathbf{H}). \tag{8}$$

As a rule, it is Eq. (8), and not the specific structure of Eq. (6), that assures the hermiticity of $\hat{W}$, and in what follows we shall assume that $\hat{W}$ is hermitian.[3)] This makes it possible to express the acoustoelectric current in the form

$$j_i^A=\frac{2e}{(2\pi\hbar)^3}\int U^A\psi_i(\mathbf{p},-\mathbf{H})\,d^3p. \tag{9}$$

According to Eqs. (9) and (3), the acoustoelectric current density $\mathbf{j}^A$ is the sum of two integrals. If we introduce the new variable $\mathbf{p}'=\mathbf{p}-\hbar\mathbf{q}$ into the first of these integrals and take account of the fact that the matrix $g_{\mathbf{p},\mathbf{p}'}$ is hermitian (so that $|g_{\mathbf{p},\mathbf{p}'}|^2=|g_{\mathbf{p}',\mathbf{p}}|^2$) we can express j_i^A in the following compact form:

$$j_i^A=-\frac{e}{2\pi^2\hbar^4}\frac{\Phi}{\hbar\omega_q s_q}\int|g_{\mathbf{p}+\hbar\mathbf{q},\mathbf{p}}|^2[F(\varepsilon_{\mathbf{p}+\hbar\mathbf{q}})-F(\varepsilon_{\mathbf{p}})][\psi_i(\mathbf{p}+\hbar\mathbf{q},-\mathbf{H})-\psi_i(\mathbf{p},-\mathbf{H})]\delta(\varepsilon_{\mathbf{p}+\hbar\mathbf{q}}-\varepsilon_{\mathbf{p}}-\hbar\omega_q)\,d^3p. \tag{10}$$

The momentum $\hbar\mathbf{q}$ of the phonons responsible for the acoustoelectric effect is considerably smaller than the Fermi momentum of the conduction electrons. This allows us to expand Eq. (10) in powers of q_i and retain only the first nonvanishing term; this yields

$$j_i^A=-\frac{e}{4\pi^2\hbar^3}\frac{\Phi q^2}{\rho s_q\omega_q}\int|\Lambda_p|^2\frac{\partial F}{\partial\varepsilon}\frac{\partial\psi_i(\mathbf{p},-\mathbf{H})}{\partial p_q}\delta(v_q-\tilde{s}_q)\,d^3p. \tag{11}$$

Here p_q and v_q are the projections of the quasimomentum $\mathbf{p}$ and velocity $\mathbf{v}$ onto the direction of the sound wave vector[4)] $\mathbf{q}$, Λ_p is the corresponding component of the renormalized deformation potential determined by the matrix element $g_{\mathbf{p}',\mathbf{p}}$,

$$|g_{\mathbf{p}+\hbar\mathbf{q},\mathbf{p}}|^2\approx\frac{\hbar|\Lambda_p|^2q^2}{2\rho\omega_q}, \quad aq\ll1, \tag{12}$$

and a is the interatomic distance. Then if we make use of the fact that the energy dependence of $\partial F/\partial\varepsilon$ contains a δ-function factor, we reach the following final expression:

$$j_i^A=\frac{e}{4\pi^2\hbar^3}\frac{\Phi q^2}{\rho s_q\omega_q}\oint_F|\Lambda_p|^2\frac{\partial\psi_i(\mathbf{p},-\mathbf{H})}{\partial p_q}\delta(v_q-\tilde{s}_q)\frac{dS}{v}. \tag{13}$$

This formula shows that the acoustoelectric current $\mathbf{j}^A$ can be expressed in terms of the vector ψ, which is known for the case $H=0$ from the theory of electrical conductivity, and for the case $H\neq0$ from the theory of galvanomagnetic phenomena (see Refs. 7 and 8).

In the absence of a magnetic field ($H\neq0$) is equal to the vector mean free path $\mathbf{l}(\mathbf{p})$ ($l_i(\mathbf{p})=\hat{W}_p^{-1}\{v_i\}$), i.e.

$$j_i^A=\frac{e}{4\pi^2\hbar^3}\frac{\Phi q^2}{\rho s_q\omega_q}\oint_F|\Lambda_p|^2\frac{\partial l_i(\mathbf{p})}{\partial p_q}\delta(v_q-\tilde{s}_q)\frac{dS}{v}. \tag{14}$$

In the τ approximation $l_i=\tau v_i$ ($\tau\equiv$ const!) we obtain the expression

$$j_q^A=\frac{e}{4\pi^2\hbar^3}\frac{\Phi q^2}{\rho s_q\omega_q}\oint_F|\Lambda_p|^2l(\mathbf{p})\frac{\partial^2\varepsilon}{\partial p_q^2}\delta(v_q-\tilde{s}_q)\frac{dS}{v^2}, \tag{15}$$

for the projection of the acoustoelectric current in the direction of $\mathbf{q}$, which agrees with formula (7) of Ref. 6 and contains the component m_q^* of the effective mass in the direction of $\mathbf{q}$ ($\partial^2\varepsilon/\partial p_q^2=1/m_q^*$).

Formula (15) is also valid, to terms of the order of $(s/v_F)j^A$, for an arbitrary collision operator $\hat{W}$, but only if, the strip $v_n=0$ (or, what is the same thing, $\mathbf{q}\cdot\mathbf{v}=0$) lies in the symmetry plane of the Fermi-surface cavity along which it passes.[5)]

The acousto-emf is determined by the condition that the total current $\mathbf{j}$ vanish [see Eqs. (5) and (6)]:

$$E_i^A=-\rho_{ik}(\mathbf{H})j_k^A, \quad \rho_{ik}(\mathbf{H})=\sigma_{ik}^{-1}(\mathbf{H}). \tag{16}$$

To analyze the above equations for specific dispersion laws and collision mechanisms and to compare their predictions with experimental data are beyond the scope of this paper. We note only that a comparison of formulas (14) and (16) with the expression

$$\Gamma(\mathbf{q})=\frac{\omega_q}{4\pi^2\hbar^3\rho s^2}\oint_F|\Lambda_p|^2\delta(v_q-\tilde{s}_q)\frac{dS}{v} \tag{17}$$

for the absorption coefficient for short-wave sound[2] that shows that the anisotropy need not be the same for sound absorption as the the acoustoelectric effect.[6,12]

As was shown in Ref. 6, in the case of a quadratic dispersion law the Weinreich relation[13] ($E_A=\pm\Gamma\Phi/|e|ns$) follows from formulas (15) and (16). Formulas (13) and (16) show that the acoustoelectric effect should depend substantially on the magnetic field $\mathbf{H}$, the quantity $\omega_c\tau$, where $\omega_c=eH/cm^*$ is the cyclotron frequency, serving as a measure of the magnetic field strength; further, all the results are very sensitive to the structure of the electron spectrum of the metal; to the topology of the Fermi surface (whether it is open or closed); and to the ratio of the number of electrons to the number of holes. We emphasize that the quantities $\mathbf{j}^A$ and $\mathbf{E}^A$ do not have the same field dependences here as in the theory of the galvanomagnetic phenomena. The study of these field dependences will be the subject of a separate paper.

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[1)]This approach has a rigorous basis for the case of longitudinal sound[2,3] but does not take into account the transformation of a transverse sound wave into an electromagnetic wave. This transformation leads to additional sound absorption owing to the evolution of Joule heat, and this substantially alters the frequency dependence of the absorption coefficient when $\lambda \approx \delta(\omega)$, where $\delta(\omega)$ is the skin penetration depth for electromagnetic wave of the sound frequency ω. The sound absorption coefficient resulting from this mechanism is of the same order as the absorption coefficient due to the deformation interaction.[4] The formulas for the acoustoelectric effect presented below are exact for longitudinal sound and are correct in order of magnitude for transverse sound [when $\lambda \neq \delta(\omega)$]. Strictly speaking, the formulas derived here describe the acoustoelectric effect for $ql \gg 1$, which is due to the deformation interaction.

[2)]We are indebted to P. E. Zil'berman for calling our attention to this fact.

[3)]The hermiticity of $\hat{W}$ for the case of electron scattering by impurities and phonons has been directly verified (see Ref. 11). The problem of the hermiticity of W and the possible consequences of its violation has not been thoroughly studied.

[4)]We note that if the sound does not propagate along one of the selected crystallographic directions, $\mathbf{q}$ will not be parallel to $\mathbf{s}_{\mathbf{q}}$, while the energy flux Φ will be parallel to $\mathbf{s}_{\mathbf{q}}$.

[5)]A crystal symmetry plane is always a symmetry plane of the Fermi surface, but such a plane need not intersect the Fermi surface. Not every symmetry plane of the gap in the Fermi surface is a crystal symmetry plane.

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Partial composition of a dense electron-hole system and exciton-plasma transition in uniaxially stressed silicon

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A high-density nonequilibrium electron-hole system is investigated in silicon crystals that are elastically deformed along the ⟨100⟩ axis, at temperatures $T \lesssim 20$ K. The two-phase gas + electron-hole liquid region is determined, and the critical temperature of the phase transition is estimated at $T_c = (14 \mp 1.5)$ K. Investigations of the photoconductivity and of the recombination radiation spectra, as well as of the kinetics of the spectra under pulsed excitation, are used to analyze the partial composition of the gas phase in a wide range of excitation densities, up to densities corresponding to the dimensionless parameter $r_s \sim 1.5$. It is established that when the density is increased to $r_s = 2.7 \mp 0.3$ and at $T = 12.5$ K the gas phase consists predominantly of excitons and excitonic molecules. An investigation of the transformation of a gas of excitons (biexcitons) into an electron-hole plasma at $T \gtrsim T_c$ has shown that the excitonic states disintegrate at densities corresponding to $r_s^* = 2.5$–2. The obtained value of r_s^* differs noticeably from the critical density estimated from Mott's criterion for an exciton-plasma transition, and comes close to the value of r_s^* calculated in the approximation of dielectric screening of the excitons.

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1. INTRODUCTION

An exciton gas of sufficiently high density can condense in semiconductors at low temperatures into an electron-hole liquid (EHL).[1,2] This condensation of the excitons is a first-order phase transition. The region of the coexistence of the gas and liquid phases in a nonequilibrium electron-hole (e-h) system is determined by a phase diagram usually plotted with the density and temperature as coordinates. The gas–EHL phase diagrams were investigated most thoroughly in the indirect semiconductors germanium[3,4] and silicon.[5,6] In these semiconductors, the binding energy in the liquid, relative to the excitonic term, turns out to be quite large (~0.4 Ry in Ge and 0.6 Ry in Si, where Ry the excitonic Rydberg), this being attributed to the strong degeneracy of the electron and hole bands.[7] The lifetimes in the liquid and gas phases in Ge and Si exceed the characteristic thermalization times. Therefore the condensation into drops of metallic EHL in these crystals

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