that the results of Ref. 8, when expressed in terms of the amplitudes P_i, give Eq. (19) for the $P_{2,3,as}$; however, they do not contain the logarithmic term, which is small as compared with $m^2\chi^{2/3}$, and therefore give $P^{(1)}_{1,as} = 0$ instead of Eq. (18).

3)In deriving (32) from (31) we used the condition $(\partial^2 L/\partial\mathfrak{F}\partial G)|_{G\to 0} = 0$; this condition is easy to understand since the quantity concerned is a pseudoscalar expressed in terms of G and, since it is finite, it must vanish as $G \to 0$.

4)In the second of Refs. 12, the PO in a magnetic field was expressed in terms of scalar functions χ_i related to the P_i by the equations $\alpha\chi_1 = P_1$ and $\alpha\chi_{2,3} = P_1 + P_{2,3}$. From (36) and (37) we obtain the asymptotic formulas

$$\chi^{(1)}_{1,as} \approx \chi^{(1)}_{3,as} \approx \frac{\alpha k^2}{3\pi} \ln \frac{eH}{m^2},$$

which are accurate to terms that are constant in H and agree with the results obtained in the usual manner in Ref. 12.

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Nonlinear interaction between waves in strongly inhomogeneous media

B. S. Abramovich and V. V. Tamoĭkin

Gorkiĭ Radiophysics Research Institute

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Nonlinear three-wave interaction between waves in a randomly inhomogeneous dispersive medium is considered. A kinetic equation for the probability distribution of the second-harmonic intensity is obtained in the diffusion approximation. It is shown on the basis of the diffusion equation that the random mismatching of the wave phases, violating the phase synchronism condition, results in a weakening of the nonlinear interaction, which goes over to a stationary regime corresponding to the limiting efficiency value of an optical frequency doubler (50%). The dynamics of the transition of the nonlinear interaction to the stationary conditions is analyzed. An extension to the case of nondegenerate three-frequency nonlinear interaction is presented.

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1. INTRODUCTION

Significant attention has been paid in recent years to the analysis of resonance interaction of waves in dispersive inhomogeneous media. Here the nonlinear interaction process is accompanied by regular or random mismatchings of the phases, the conditions of phase synchronism to which the resonance interaction of the waves is extremely sensitive are violated, and this leads to a weakening of the nonlinear interaction.

The greatest number of researches has been devoted to the multiplication of laser frequencies in nonlinear crystals with inhomogeneities. The interest in this problem in laser physics is due to the fact that the effect of generation of light harmonics allows one to estimate very simply, by experiment, the effect of the nonlinear inhomogeneities on the nonlinear properties of the crystal. However, because of the difficulties of theoretical analysis, the analysis of the basic question of the effectiveness of the conversion of the basic radiation into subharmonics is carried out either in the approximation of the given pump field,[1-3] or in the approximation of the given pump intensity,[4,5] which essentially reduce the considered problems to linear ones. The parametric instabilities in an inhomogeneous plasma are considered in similar fashion.[6-8]

For a systematic analytic consideration of the effect of the inhomogeneities of the medium on the process of nonlinear interaction, it is necessary to reject the approximations usually employed. An attempt to go beyond the framework of the parametric approximation has been undertaken in the recent papers of Filonenko

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and Mel'nik,[9,10] where a method of obtaining solutions of one-dimensional truncated equations for the quasistatic interaction of waves in a weakly nonlinear, weakly inhomogeneous medium has been developed. In particular, the problem is analyzed in Ref. 10 of the limiting efficiency of parametric frequency doublers under conditions of random mismatching of the phases of the waves. However, the method is proposed by the authors is based on the Born approximation of perturbation theory for the scattering of a nonlinear second-harmonic wave by specified inhomogeneities. At the same time, it is assumed that the effect of random or regular phase dis-synchronism on the process of the nonlinear interaction is small, which significantly restricts the region of applicability of the results obtained in Ref. 10 to the case of optically thin media. Thus, only the initial stage of the nonlinear interaction is described in Refs. 9 and 10, and on the basis of this stage it is impossible to follow completely the dynamics of the nonlinear interaction of waves in a sufficiently long inhomogeneous medium and, in particular, to answer the important question as to the effect of multiple scattering of nonlinear waves of the fundamental frequency and the harmonic on the effectiveness of the generation.

In the present work, we consider in the diffusion approximation the process of degenerate three-particle interaction in a one-dimensional medium with large-scale random inhomogeneities. A closed stochastic integro-differential equation is introduced for the random intensity of the second harmonic. A diffusion Fokker-Einstein equation is obtained on the basis of the given equation for the probability density function of the second harmonic. It follows from analysis of the latter equation by the method of moments that the presence of a random mismatch of phases in the nonlinear medium leads to saturation of the nonlinear transformation of the fundamental radiation into the harmonic, while the limiting efficiency turns out to be equal to 50%. The expressions obtained for the moments of the intensity of the second harmonic in the case of a sufficiently extended nonlinear scattering medium correspond to an equilibrium stationary probability distribution of the intensity. A qualitative analysis is carried out of the dynamics of the settling of the generation in a stationary regime, which in particular, allows us to obtain the limits of applicability of the method of random phases in similar problems. The results are generalized to the case of nondegenerate nonlinear interaction of three waves.

2. DERIVATION OF THE BASIC RELATIONS

The quasistatic process of the generation of the second harmonic in a lossless nonlinear medium[1)] with linear large-scale inhomogeneities is described by the following set of truncated equations for the complex amplitudes of the first and second harmonics $A_{1,2}(z)$[1]:

$$dA_1/dz=-i\beta_1A_2A_1^*\exp\{i\psi(z)\},\qquad dA_2/dz=-i\beta_2A_1^2\exp\{-i\psi(z)\}. \tag{1}$$

Here $\beta_{1,2}$ are constant coefficients of the nonlinear interaction,[1]

$$\psi(z)=\int_0^z dz'\Delta k(z')$$

is the phase shift from random inhomogeneities of the medium that arise in the interaction of the waves of the fundamental frequency ω_1 and the harmonic $\omega_2=2\omega$, for which $\Delta k(z)=2k_1(z)-k_2(z)$ is the local wave detuning with characteristic spatial scale $l(k_{1,2}l\gg 1)$.

We transform from the set of equations (1) to an equation of second order for the complex amplitude of the second harmonic $A_2(z)$:

$$\frac{d^2A_2}{dz^2}+i\Delta k(z)\frac{dA_2}{dz}+2\beta_1\beta_2|A_1|^2A_2=0. \tag{2}$$

Hence the equation describing the generation of the second harmonic in the given-field approximation is obtained at $\beta_1=0$, while in the given-intensity approximation it is obtained at $|A_1|^2=\text{const}$. We now attempt to investigate the equation (2) in self-consistent fashion, without making any assumptions on the character of the nonlinear term in this equation.

For simplicity, we assume that the second harmonic is absent ($A_2(0)=0$) "at the input" to the nonlinear inhomogeneous medium ($z=0$), and the initial amplitude of the fundamental radiation ($A_1(0)=A_1^{(0)}$) is given. We further allow the condition of phase synchronism to be satisfied "in the mean," i.e., $\langle\Delta k(z)\rangle=0$. Then $\beta_1=\beta_2=\beta$ and, for the normal amplitude of the second harmonic $a=A_2/A_1$ with account of the Manley-Rowe relations

$$|a|^2+|A_1/A_1^{(0)}|^2=1 \tag{3}$$

it is easy to obtain the following nonlinear stochastic equation from (2):

$$\frac{d^2a}{d\zeta^2}+i\Delta(\zeta)\frac{da}{d\zeta}+(1-|a|^2)a=0,\qquad a(0)=0,\quad a'(0)=-i/\sqrt{2}, \tag{4}$$

where the dimensionless coordinate $\zeta=z/L_{NL}$ is introduced, $L_{NL}=(2^{1/2}\beta|A_1^0|)^{-1}$ is the characteristic length of the nonlinear interaction in the uniform medium, $\Delta=L_{NL}\Delta k$.

In what follows we shall be interested in the statistical characteristics of the intensity of the second harmonic; therefore it is convenient to transform to the equation for the random function:

$$I(\zeta)=|a(\zeta)|^2. \tag{5}$$

To obtain the latter, we introduce the function $P=i(a^*a'-aa^{*\prime})$, $Q=a'a^{*\prime}$ for which, in accord with the definition (5), it follows from Eq. (4) that

$$I''+\Delta(\zeta)P+2[(1-I)I-Q]=0,\quad P'-\Delta(\zeta)I'=0,\quad Q'+(1-I)I'=0,\qquad I(0)=I'(0)=P(0)=0,\quad Q(0)=1/2. \tag{6}$$

Integrating the last two equations of the set (6) and substituting the resultant expression in the first of Eqs. (6), we finally obtain a closed nonlinear integro-differential stochastic equation for the random intensity $I(\zeta)$:

$$I''+\Delta(\zeta)\int_0^\zeta d\zeta'\Delta(\zeta')I'(\zeta')-3I^2+4I-1=0,\qquad I(0)=I'(0)=0,\quad I''(0)=1. \tag{7}$$

Equation (7) has the stochastic integral

$$I'^2+\left[\int_0^\zeta d\zeta'\Delta(\zeta')I'(\zeta')\right]^2=2I(I-1)^2, \qquad (8)$$

from which it follows that the intensity does not change too rapidly in each realization, since according to (8) $|I'|\leq(8/27)^{1/2}$ and

$$\left|\int_0^\zeta d\zeta'\Delta(\zeta')I'(\zeta')\right|\leq(^8/_{27})^{1/2}.$$

For the statistical analysis of Eq. (7), it is necessary to furnish the statistics of the process $\Delta k(z)$ and consequently, $\Delta(\zeta)$. We shall assume that the wave detuning $\Delta k(z)$ is a homogeneous Gaussian random process with zero mean ($\langle\Delta k\rangle=0$) and correlation function $\langle\Delta k(z)\Delta k(z')\rangle=B_{\Delta k}(z-z')$. Further, let the dimensions of the inhomogeneities l can be sufficiently small, i.e., $l\ll L_{NL}$ and $l\ll L_p$, where $L_p=2(\langle\Delta k^2\rangle l)^{-1}$ is the characteristic length of the linear multiple scattering of the waves in a randomly inhomogeneous medium. The latter assumption is equivalent to the replacement of the correlation function $B_{\Delta k}(z-z')$ by the effective correlation

$$B_{\Delta k}^{eff}(z-z')=2D\delta(z-z'),\quad D=\frac{1}{2}\int_{-\infty}^{\infty}dzB_{\Delta k}(z). \qquad (9)$$

Correspondingly, (9) and the function of a random variable $\Delta(\zeta)$ will be delta correlated, while $B_\Delta^{eff}(\zeta-\zeta')=2\gamma^{-1}\delta(\zeta-\zeta'),\gamma=L_p/L_{NL}$. That is, the quantity γ is the basic parameter of the given problem, characterizing the effectiveness of the nonlinear transformation in a weakly inhomogeneous medium.

Using the correlation properties of the function of a random variable $\Delta(\zeta)$ it is not difficult to determine the range of change of the limiting coefficient of the transformation of the fundamental radiation into the harmonic $\eta=\langle I(\zeta)\rangle$. Actually, averaging with the help of the Furutsu-Novikov equation (7) over the ensemble of realizations $\Delta(\zeta)$, we obtain

$$\langle I''\rangle+\gamma^{-1}\langle I'\rangle-3\langle I^2\rangle+4\langle I\rangle-1=0. \qquad (10)$$

The limiting efficiency is determined by the stable state of equilibrium of Eq. (10) at $\zeta=\infty$. Separating the regular component ($I=\langle I1\rangle+\Delta I$, $\langle\Delta I\rangle=0$) in $I(\zeta)$ we find the following state of equilibrium of Eq. (10) in the usual way:

$$\langle I_{1,2}^{(\infty)}\rangle={}^2/_3\pm({}^1/_9-\langle\Delta I^{(\infty)2}\rangle)^{1/2}, \qquad (11)$$

One of which is unstable (I_1^{∞}). Thus, even a trivial analysis of (10) and (11) shows that the random mismatching of the phase synchronism of the waves leads to a disruption of one-hundred-percent generation of the second harmonic, while "saturation" of the process of nonlinear interaction takes place in a sufficiently extended scattering medium; the efficiency of the transformation approaches the stationary value $\eta^{(\infty)}$, the limits of variation of which are determined by the expression (11):

$${}^1/_3\leq\eta^{(\infty)}\leq{}^2/_3. \qquad (12)$$

We note that the approximation of the given intensity of the fundamental radiation, which corresponds to the discarding of the nonlinear term in Eq. (10), leads to the limiting efficiency $\eta^{(\infty)}=\frac{1}{4}$.

We turn our attention to a more detailed investigation of the statistics of the intensity $I(\zeta)$. Using the integral (8) we write out the Eq. (7) in the form of a set of two differential equations of first order:

$$p'=f(g)+\Delta(\zeta)[\varphi(g)-p^2]^{1/2},\quad g'=p,\quad p(0)=g(0)=0. \qquad (13)$$

Here the notation $g=I, p=I', f(g)=3g^2-4g+1, \varphi(g)=2g(g-1)^2$ has been introduced. By virtue of the degeneracy of the problem, (13) satisfies the condition of casuality and can be analyzed in the diffusion approximation. Actually, using the Gaussian nature and the delta-correlation of the process $\Delta(\zeta)$, we obtain from (13) by the standard procedure,[11] the following Einstein-Fokker equation for the probability distribution $W(g,p;\zeta)$:

$$\frac{\partial W}{\partial\tau}=-\gamma p\frac{\partial W}{\partial g}-\frac{\partial}{\partial p}[\gamma f(g)-p]W+\frac{\partial^2}{\partial p^2}[\varphi(g)-p^2]W,$$
$$W(g,p;\tau=0)=2\delta(g)\delta(p),\quad 0\leq g\leq1,\quad |p|\leq(8/27)^{1/2}, \qquad (14)$$

where $\tau=\gamma^{-1}\zeta=z/L_p$ is the optical path in the scattering medium.

The diffusion equation (14) is the starting point for the study of the statistical properties of the intensity of the second harmonic $I(\zeta)$. The statistics of $I(\zeta)$, in turn, completely characterize the process of nonlinear interaction of the waves in a randomly inhomogeneous medium.

3. LIMITING MOMENTS OF THE INTENSITY OF THE SECOND HARMONIC. STATIONARY SOLUTION OF THE DIFFUSION EQUATION

It is very difficult to obtain the solution of the boundary value problem (14). We therefore transform from Eq. (14) to the equations for the moments $q_{m,n}(\tau)=\langle p^m g^n\rangle$, $m,n=0,1,2\ldots$. Integrating (14), we find:

$$q'_{m,n}=-m^2q_{m,n}+2m(m-1)(q_{m-2,n+3}-2q_{m-2,n+2}$$
$$+q_{m-2,n+1})+\gamma nq_{m+1,n-1}+\gamma m(3q_{m-1,n+2}-4q_{m-1,n+1}+q_{m-1,n}), \qquad (15)$$
$$q_{0,0}(0)=1,\quad q_{m,n}(0)=0,\quad m+n\geq1.$$

The system (5) represents an infinite set of coupled equations, while the moments of the intensity that are of interest are determined in the following way:

$$\langle I^N(\tau)\rangle=q_{0,N},\quad N=1,2,\ldots\ . \qquad (16)$$

The kinetic equation (14), and the set of equations (15) following from it have the stationary solution

$$W(p,g;\tau=\infty)=W^{(\infty)}(p,g)\quad\text{and}\quad q_{m,n}(\tau=\infty)=q_{m,n}^{(\infty)}$$

which is independent of the initial conditions. We shall show that the infinite set (15) has a stationary solution

$$q_{0,N}^{(\infty)}=1/(N+1),\quad N=1,2,\ldots, \qquad (17)$$

that is consistent with the values $q_{3,n}^{(\infty)}=0$, $n=0,1,2\ldots$. For this, we must establish the fact that the last two equations do not contradict the stationary equations (15) and unambiguously determine all the remaining moments $q_{1,n}^{(\infty)}$ in this case. In fact, let us return, for example, to the two equations (15) at $m=1$ and $m=2$. With account of the obvious relation $q_{1,n}^{(\infty)}=0$, we have two independent equations for the determination of $q_{2,n}^{(\infty)}$:

$$nq^{(\infty)}_{2,n-1}+3q^{(\infty)}_{0,n+2}-4q^{(\infty)}_{0,n+1}+q^{(\infty)}_{0,n}=0, \tag{18}$$

$$-q^{(\infty)}_{2,n}+q^{(\infty)}_{0,n+3}-2q'^{(\infty)}_{0,n+2}+q^{(\infty)}_{0,n+1}=0. \tag{19}$$

Using (17), and after uncomplicated transformations, it is easy to establish the fact that the relations (18) and (19) lead independently to the same value of $q^{(\infty)}_{2,n}$

$$q^{(\infty)}_{2,n}=2/(n+2)(n+3)(n+4), \quad n=0,1,2,\ldots.$$

The consistency of the solution (17) with the other equations of the system (15) is shown in analogous fashion. In the stationary case, this system divides into a finite number of independent subsystems.

Thus the limiting values of the moments of the intensity of the second harmonic are, in accord with (16) and (17), equal to

$$\langle I^{N(\infty)}\rangle=1/(N+1), \quad N=1,2,\ldots. \tag{20}$$

In particular, the stationary value of the efficiency of the nonlinear transformation of the fundamental radiation into the harmonic then follows:

$$\eta^{(\infty)}=\langle I^{(\infty)}\rangle=\tfrac{1}{2}. \tag{21}$$

Knowledge of all the moments (20) allows us to determine easily the stationary probability distribution of the intensity

$$w^{(\infty)}(g)=\int dpW^{(\infty)}(p,g),$$

which turns out to be uniform:

$$w^{(\infty)}(I)=\eta(I)-\eta(I-1), \tag{22}$$

where $\eta(x)$ is the Heaviside function.

Equations (15) enable us also to describe qualitatively the dynamics of the setting of the nonlinear interaction process in a stationary regime in two limiting cases: $\gamma \ll 1$ and $\gamma \gg 1$. Actually, let $\gamma \ll 1$; then $L_{NL} \gg L_p$ and, consequently, the shift of the relative phase difference of the waves because of the inhomogeneity of the medium over the length of the nonlinear interaction is large. In this case, the nonlinear interaction of the waves is greatly weakened and the average intensity of the harmonics approaches the stationary value in relaxational fashion (see the drawing, curve a). The latter corresponds essentially to the random phase approximation, since the coherence length is determined by the quantity L_p and the mismatch of phase takes place rather rapidly. Here the characteristic length over which the limiting value is reached is $z^* \sim \gamma^{-1}L_{NL}$. In the other limiting case, $\gamma \gg 1$, we have $L_{NL} \ll L_p$, the shift of the difference in phase of the waves over the nonlinear length is small and in the initial stage of the interaction we can expect a high effectiveness of the transformation of the fundamental radiation into the harmonic. However, as the wave progresses into the inhomogeneous nonlinear medium, multiple scattering begins to play an ever larger role, leading to a local development of the decay instability of the second harmonic; here the random phase dis-synchronism plays the role of a priming perturbation.[7] The oscillatory character of the establishment of the stationary regime is shown in curve b of the figure. Here the period of oscillation Λ is determined by the nonlinear length $\Lambda \sim L_{NL}$, while the characteristic establishment length is $z^* \sim \gamma L_{NL}$.

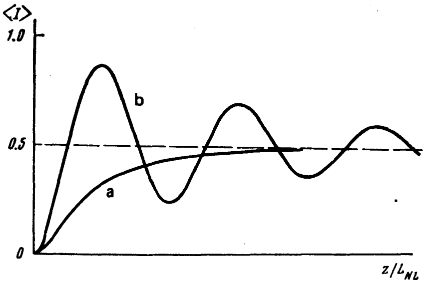

FIG. 1.

4. NONDEGENERATE NONLINEAR INTERACTION

We generalize the results obtained above to the process of nondegenerate resonance interaction of three waves in a randomly inhomogeneous medium. This problem, in particular, evokes special interest in connection with the problem of the heating of plasma by laser radiation.[12]

The equations of the three-particle quasistatic wave interaction $\omega_1+\omega_2=\omega_3$ in a medium with large-scale inhomogeneities have the form[1]

$$dA_{1,2}/dz=-i\beta_{1,2}A_3A^*_{2,1}e^{i\psi(z)}, \quad dA_3/dz=-i\beta_3A_1A_2e^{-i\psi(z)}, \tag{23}$$

where the phase shift

$$\psi(z)=\int_0^z dz'\Delta k(z'), \quad \Delta k(z)=k_3(z)-k_2(z)-k_1(z)$$

is the random wave detuning ($\langle\Delta k\rangle=0$) with characteristic scale $l \gg k^{-1}_{1,2,3}$.

As an example, we consider the process of generation of the field of the sum frequency, assuming that it did not exist on the boundary of the nonlinear medium, i.e., $A_3(0)=0, A_{1,2}(0)=A^{(0)}_{1,2}$. Similar to the above, we transform from Eq. (23) to the equation for the complex amplitude $A_3(z)$:

$$\frac{d^2A_3}{dz^2}+i\Delta k(z)\frac{dA_3}{dz}+\beta_3(\beta_1|A_2|^2+\beta_2|A_1|^2)A_3=0. \tag{24}$$

Further, using the Manley-Rowe relation

$$\frac{|A_1|^2}{\beta_1}+\frac{|A_3|^2}{\beta_3}=\frac{|A_1^0|^2}{\beta_1}, \quad \frac{|A_2|^2}{\beta_2}+\frac{|A_3|^2}{\beta_3}=\frac{|A_2^0|^2}{\beta_2}$$

and introducing the dimensionless variables

$$a=(2\beta_1\beta_2)^{1/2}L_{NL}A_3, \quad \Delta=L_{NL}\Delta k, \quad \zeta=z/L_{NL},$$
$$L_{NL}=[\beta_3(\beta_1|A_2^0|^2+\beta_2|A_1^0|^2)]^{-1/2}, \tag{25}$$

we can reduce Eq. (24) to the following form:

$$\frac{d^2a}{d\zeta^2}+i\Delta(\zeta)\frac{da}{d\zeta}+(1-|a|^2)a=0,$$
$$a(0)=0, \quad a'(0)=-i(2\beta_1\beta_2)^{1/2}\beta_3A_1^0A_2^0L_{NL}^2. \tag{26}$$

It is seen that the nonlinear stochastic equations (4) and (26) have different initial conditions. This circumstance leads to the result that the equation for the intensity of radiation of the sum frequency $I=|a|^2$ differs somewhat from Eq. (7) for the random intensity of the second harmonic. Actually, it follows from (26) that

$$I''+\Delta(\zeta)\int_0^\zeta d\zeta'\Delta(\zeta')I'(\zeta')-3I^2+4I-\Gamma=0,$$
$$I(0)=I'(0)=0, \quad I''(0)=\Gamma=2\beta_1\beta_2\beta_3^2|A_1^0|^2|A_2^0|^2L_{NL}^4. \tag{27}$$

It is easy to see that upon satisfaction of the relation $\beta_1|A_2^0|^2=\beta_2|A_1^0|^2$ Eq. (27) is identical with Eq. (7), since here $\Gamma=1$ and, consequently, the results obtained in the previous section are valid here. At $\Gamma\neq 1$, analysis of Eq. (27) in the diffusion approximation leads to the following stationary values of the moments of the intensity:

$$\langle I^{N(\infty)}\rangle=[1-(1-\Gamma)^{1/2}]^N/(N+1), \quad N=1, 2, \ldots, \tag{28}$$

where $0\leq\Gamma\leq 1$.

The result (28) shows up most graphically in the variables $\theta_{iz}=|A_i(z)|^2/\beta_i$, $i=1, 2, 3$ which determine the measure of the photon density of frequency ω_i in the stationary regime. In this notation, we get from (28), with account of (26)–(27):

$$\langle\theta_3^{N(\infty)}\rangle=\min(\theta_{10}{}^N, \theta_{20}{}^N)/(N+1), \quad N=1, 2, \ldots . \tag{29}$$

In particular we have for the "average number of quanta" of frequency $\omega_3(N=1)$:

$$\langle\theta_3^{(\infty)}\rangle=\min(\theta_{10}, \theta_{20})/2. \tag{30}$$

We note that, using the Manley-Rowe relations, it is trivial to determine the limiting values of the intensity of the radiation of the other frequencies with the aid of (30).

5. CONCLUSION

The method of analysis of processes of three-wave interaction in a nonlinear weakly inhomogeneous medium developed in the present work on the basis of the diffusion equation allows one to investigate arbitrary resonance interaction. Actually, only the specification of the initial conditions at the boundary of the nonlinear medium separates, within the framework of the general three-wave interaction described in the case of an inhomogeneous medium by the equations (23), the special types of interactions corresponding to different processes: upward transformation of frequency (coalescence), generation of difference frequencies (decay), parametric amplification. The presence here of random mismatch of the phases of the waves for any type of nonlinear interaction leads to the stationary regime of interaction independently of the initial conditions. The specific type of interaction determines only the character of the establishment of the limiting level of three-particle nonlinear interaction. An important feature of the stationary regime is that the steady-state values of the intensities of the interacting waves are independent of the value of the wave detuning fluctuations, which determines only the duration of the establishment process.

[1)]We note that, in the presence of absorption in the medium, the results are generalized in trivial fashion to the case of equal linear decrements of the attenuation of the first and second harmonics.[1]

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