Contribution to the theory of spin dynamics of superfluid ^{3}He–A

A. D. Gongadze, G. E. Gurgenishvili, and G. A. Kharadze

Physics Institute, Georgian Academy of Sciences
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We consider the singularities of spin dynamics of the superfluid phase A of liquid ^{3}He under conditions when at equilibrium the orbital-anisotropy axis **l** and the spin-quantization axis **d** make an angle $\alpha \neq 0$. It is shown that in this situation parametric excitation of longitudinal spin waves with $q \neq 0$ is made possible by homogeneous transverse pumping. The dynamic effects under conditions of pulsed NMR, which are characteristic of a spin-orbit configuration with $\alpha \neq 0$, are also considered.

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1. INTRODUCTION

Weak dipole-dipole interaction between the magnetic moments of nuclei plays a fundamental role in the properties of the superfluid phases A and B of liquid ^{3}He. In the superfluid state with triplet Cooper pairing the magnetic dipole-dipole interaction leads to establishment of a coherent "rigidity" between the spin and orbital degrees of freedom, and this rigidity influences the formation of an equilibrium spin-orbit configuration in ^{3}He–A and ^{3}He–B. The great variety of properties of these superfluid phases is the result of the joint action of the aforementioned spin-orbit forces and of various external factors, such as the orienting influence of the vessel walls, of the magnetic field, as well as of the superfluid flows.

When the spin degrees of freedom deviate from the equilibrium configuration, the coherent rigidity on the part of the orbital part of the order parameter manifests itself in the form of an additional torque that acts on the magnetic dipole moment of the liquid ^{3}He and influences by the same token the spectrum of the NMR frequencies of the superfluid phases.[1] Of course, the character of this spectrum depends essentially on the equilibrium spin-orbit configuration near which oscillation of the spin degrees of freedom of the condensate take place. For example, for the superfluid phase A, which is characterized by an orbital-anisotropy vector **l** and by a vector **d** along which the projection of the summary spin of the Cooper pair is equal to zero, the NMR frequencies depend essentially on the equilibrium angle between **d** and **l**. Far from the vessel walls and in the absence of superfluid flows we have $\mathbf{l} = \pm\mathbf{d}$, which corresponds to the minimum of the dipole-dipole part of the free energy. In this situation, as is well known, a positive shift of the transverse NMR frequency is observed. On the other hand, for ^{3}He–A contained in a narrow slit between two plane-parallel plates, conditions can be realized wherein (owing to the orienting influence of the walls) $\mathbf{d} \perp \mathbf{l}$ at equilibrium. In this case the dipole-dipole energy is maximal and a negative shift of the frequency of the transverse resonance is observed.[2,3] A more complicated situation develops in the presence of soliton textures (domain walls), within the limits of which a gradual change takes place in the relative orientation of the vectors **l** and **d**. The results are localized vibrational modes observed in the form of satellites in the NMR spectra of the superfluid ^{3}He–A.[4]

Violation of the dipole-dipole minimum with $\mathbf{l} = \pm\mathbf{d}$ can be realized dynamically when the magnetization (and with it also **d**) is deflected from the equilibrium orientation. By examining the frequency precession of the magnetization (before the direction of the latter relaxes towards the initial equilibrium orientation), we can investigate the dependence of the precession frequency shift on the deviation angle.[5]

The purpose of the present study was a theoretical investigation of some new effects involved in the spin dynamics of superfluid ^{3}He–A and coming into play under conditions when $\mathbf{l} = \pm\mathbf{d}$ at equilibrium. We consider below the case of a strong magnetic field ($H \gg 30$ G), in the presence of which we can assume with certainty that $\mathbf{d} \perp \mathbf{H}$ in the entire volume. Directing the z axis along **H** and the y axis along the equilibrium orientation **d**, we consider a configuration in which the orbital vector **l** lies in the yz plane and makes an angle $\alpha \neq 0$ with the y axis. We shall demonstrate below that under these conditions parametric excitation of the longitudinal spin waves with $q \neq 0$ becomes impossible under homogeneous transverse pumping. This will be followed by an investigation of the dependence of the shift of the magnetization precession frequency on the angle α under conditions of pulsed NMR. On the other hand, it will be shown that for a jumplike decrease of the intensity of the principal magnetic field by an amount $\Delta H = H_0/2$, which accompanies the transverse radio-frequency pulse, a unique "resonant" situation arises, which is typical of the case with $\alpha \neq 0$. Finally, we discuss briefly the possibility of realization of a sufficiently homogeneous phase A with $\alpha \neq 0$ in a relatively large volume of liquid ^{3}He.

2. PARAMETRIC EXCITATION OF LONGITUDINAL SPIN WAVES IN ^{3}He-A

The coupled oscillations of the magnetization **M** and of the vector **d** is superfluid ^{3}He–A are described by the Leggett system of equations

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$$\partial \mathbf{M}/\partial t=\gamma[\mathbf{M}\times\mathbf{H}]+\gamma\chi_s\Omega_A^2(\mathbf{dl})[\mathbf{d}\times\mathbf{l}]+\gamma\chi_s c_\parallel^2 a_{ij}[\mathbf{d}\times(\nabla_i\nabla_j\mathbf{d})], \quad (2.1)$$

$$\partial \mathbf{d}/\partial t=\gamma[\mathbf{d}\times(\mathbf{H}-\mathbf{M}/\gamma^2\chi_s)], \quad (2.2)$$

where γ is the gyromagnetic ratio for the ^{2}He nuclei, χ_s is the spin susceptibility of the normal phase, Ω_A is the dipole frequency of the superfluid phase A, $c_\parallel$ is the velocity of a spin wave with $\mathbf{q}\parallel\mathbf{l}$ and $a_{ij}=2\delta_{ij}-l_il_j$ is the anisotropy tensor. At equilibrium., i. e., at $H=H_0$ $=$const and $\mathbf{M}=\mathbf{d}=\nabla d_\mu\equiv 0$, we have

$$[\mathbf{M}_0\times\mathbf{H}_0]+\chi_s\Omega_A^2(\mathbf{d}_0\mathbf{l})[\mathbf{d}_0\times\mathbf{l}]=0,$$
$$[\mathbf{M}_0\times\mathbf{d}_0]+\gamma^2\chi_s[\mathbf{d}_0\times\mathbf{H}_0]=0.$$

Introducing the magnetic-field fluctuation $\mathbf{m}(t)=\mathbf{M}-\mathbf{M}_0$ and considering the case of a strong magnetic field, when $\mathbf{d}_0\perp\mathbf{H}_0\parallel\mathbf{z}$ regardless of the orientation of the orbital vector $\mathbf{l}=(0,\cos\alpha,\sin\alpha)$, we arrive at the following system of differential equations for $m(t)$ and $\delta\mathbf{d}(t)$ $=\mathbf{d}-\mathbf{d}_0$:

$$\partial\mathbf{m}/\partial t-\gamma[\mathbf{m}\times\mathbf{H}_0]+\gamma\chi_s\Omega_A^2\{(\mathbf{ld}_0)[\mathbf{l}\times\delta\mathbf{d}]+(\mathbf{l}\delta\mathbf{d})[\mathbf{l}\times\mathbf{d}]\}$$
$$-\gamma\chi_s c_\parallel^2 a_{ij}[\mathbf{d}\times(\nabla_i\nabla_j\delta\mathbf{d})]=\gamma[\mathbf{M}\times\mathbf{h}], \quad (2.3)$$

$$\frac{\partial\delta\mathbf{d}}{\partial t}+\frac{1}{\gamma\chi_s}[\mathbf{d}\times\mathbf{m}]=\gamma[\mathbf{d}\times\mathbf{h}], \quad (2.4)$$

where $\mathbf{h}(t)$ is the alternating part of the external magnetic field. We shall assume henceforth that $\mathbf{h}\perp\mathbf{H}_0$.

In the linear approximation, the natural oscillations of the longitudinal magnetization are described by the equation

$$(\partial^2/\partial t^2-c_\parallel^2 a_{ij}\nabla_i\nabla_j+\Omega_A^2\cos^2\alpha)m_z(\mathbf{r},t)=0,$$

which specifies the dispersion of the longitudinal spin waves:

$$\omega_l^2(\mathbf{q})=\Omega_A^2\cos^2\alpha+c_\parallel^2 a_{ij}q_iq_i.$$

It must be borne in mind that at $\alpha\neq 0$ the transverse components of the magnetization also oscillate at these frequencies, but under strong-field conditions ($\gamma H_0\gg\Omega_A$) their relative magnitude is small.

We turn now to the exact nonlinear equation that describes the dynamics of m_z. When the y axis is oriented along $\mathbf{d}_0$, this equation takes the form

$$\partial m_z/\partial t-\gamma\chi_s\Omega_A^2\cos^2\alpha(1+\delta d_y+\delta d_z\operatorname{tg}\alpha)\delta d_x$$
$$+\gamma\chi_s c_\parallel^2 a_{ij}\nabla_i\nabla_j\delta d_x-\gamma\chi_s c_\parallel^2 a_{ij}[\delta\mathbf{d}(\nabla_i\nabla_j\delta\mathbf{d})]_z=0\,. \quad (2.5)$$

It is easily seen that if the homogeneous transverse magnetic field $\mathbf{h}(t)=(0,h\cos\omega t,0)$ is made to build up oscillations of δd_z, then as a result of the nonlinear coupling between δd_x and δd_z, we can realize parametric excitation of oscillations of the longitudinal magnetization component with $q\neq 0$. A picture of similar character, that of parametric generation of spin-wave modes, takes place in antiferromagnets.[6] The specific feature of the situation considered by us is that the excitation of the longitudinal oscillations of the magnetization under conditions of transverse "pumping" is possible only at $\alpha\neq 0$.

It is easy to verify that in the approximation linear in h the induced homogeneous oscillations δd_z are given by

$$\delta d_z(t)=\frac{\gamma h\omega_0}{[(\omega^2-\omega_{tr}^2)^2+(\omega\Gamma_{tr})^2]^{1/2}}\cos(\omega t+\delta)=a(\omega)\cos(\omega t+\delta), \quad (2.6)$$

where $\omega_{tr}^2=\omega_0^2+\Omega_A^2\cos2\alpha$ is the square of the frequency of the transverse NMR ($\omega_0=\gamma H_0$), while Γ_{tr} describes the width of the resonance line. Substituting the presented expression for $\delta d_z(t)$ in (2.5) and retaining only terms that are significant for the development of the parametric instability in m_z, we arrive at the following system of equations

$$\partial m_z/dt-\gamma\chi_s(\omega_l^2(t)-c_\parallel^2 a_{ij}\nabla_i\nabla_j)\delta d_x=0,$$
$$\frac{\partial\delta d_x}{\partial t}+\frac{1}{\gamma\chi_s}m_z=0, \quad (2.7)$$

where

$$\omega_l^2(t)=(1+a(\omega)\operatorname{tg}\alpha\cos(\omega t+\delta))\Omega_A^2\cos^2\alpha.$$

Considering oscillations of the type $m_z(\mathbf{r},t)=m_z(\mathbf{q},t)$ $\exp(i\mathbf{q}\cdot\mathbf{r})$ and $\delta d_x(\mathbf{r},t)=\delta d_x(\mathbf{q},t)\exp(i\mathbf{q}\cdot\mathbf{r})$, we verify that $\delta d_x(\mathbf{q},t)$ satisfies the Mathieu equation

$$\{\partial^2/\partial t^2+(1+b(\mathbf{q}\omega)\cos(\omega t+\delta))\omega_l^2(\mathbf{q})\}\delta d_x(\mathbf{q},t)=0, \quad (2.8)$$

while the depth of modulation is given by

$$b(\mathbf{q}\omega)={}^1/_2 a(\omega)[\Omega_A/\omega_l(\mathbf{q})]^2\sin2\alpha. \quad (2.9)$$

The threshold value $b=b_c$, above which parametric instability sets in, can be estimated by a known method from the formula

$$b_c\approx 2\Gamma_l/\omega_l, \quad (2.10)$$

where Γ_l determines the damping of the longitudinal spin waves of the corresponding frequency. Under conditions of transverse resonant pumping we have $\omega=\omega_{tr}\approx\omega_0$ and parametric excitation will take place of the high-frequency longitudinal spin waves with $\omega_l(\mathbf{q})=\omega_{tr}/2$ and $q\approx\omega_0/2c_\parallel\approx(\omega_0/\Omega_A)\xi_D^{-1}\gg\xi_D^{-1}(\xi_D=c_\parallel/\Omega_A)$ is the characteristic dipole length). In the case under consideration we obtain for the threshold value h_c of the transverse pump field amplitude the estimate

$$h_c\approx 2H_0(\Gamma_{tr}\Gamma_l\Omega_A^{-2})/\sin2\alpha. \quad (2.11)$$

Noting that under strong-field condition ($\omega_0\gg\Omega_A$) the width of the transverse resonance is $\Gamma_{tr}\approx(\Omega_A/\omega_0)^2\Gamma_l$, with $\Gamma_l/\Omega_A\approx 0.1$ (see, e. g., Ref. 7), we arrive at the estimate

$$h_c\approx\frac{0.02}{\sin2\alpha}(\Omega_A/\omega_0)^2H_0. \quad (2.12)$$

Thus, even at small values of the angle α the threshold amplitude of the transverse pump field is $h_c\ll H_0$ (conditions with $h_c\approx 0.1$ G are fully realistic).

On the other hand, for parametric excitation of longitudinal spin waves with $q<\xi_D^{-1}$ we must use nonresonant frequencies $\omega\gtrsim 2\omega_l(0)\ll\omega_{tr}$, and then

$$h_c\approx 2H_0(\Gamma_l\Omega_A^{-1})/\sin\alpha\approx 0.2H_0/\sin\alpha. \quad (2.13)$$

We see that in this case the conditions for the development of parametric instability are much more stringent.

3. SPIN DYNAMICS OF ^{3}He-A AT $\alpha\neq 0$ UNDER PULSED CONDITIONS

In this section we continue the investigation of certain characteristic features of the spin dynamics of the superfluid phase A of liquid ^{3}He in the case when $\mathbf{l}\neq\pm\mathbf{d}$ at equilibrium, i. e., at $\alpha\neq 0$. It will be convenient hereafter to use the system of Hamilton's equations considered in Refs. 8 and 9 and modified to separate effectively the "slow" motions of the spin degrees of freedom (due to the action of the dipole-dipole forces) from their "fast" motions.

The starting point is the Leggett adiabatic Hamiltonian

$$\mathscr{H}=S^2/2\chi_s-\gamma SH-{}^1/_2\chi_s\Omega_A^2(dl)^2, \tag{3.1}$$

where S is the summary spin of the system of the ^{3}He nuclei. We assume that initially the spin system was in equilibrium in an external magnetic field $H_0 \parallel z$ and measure the magnetization in units of $\gamma^2\chi_s H_0$, i. e., we put $M=S/\chi_s\omega_0$. Measuring next the energy in units $\chi_s\omega_0^2$, we have

$$\mathscr{H}={}^1/_2M^2-MH/H_0+U(d), \tag{3.2}$$

where the dimensionless dipole-dipole potential is $U(d)=-\frac{1}{2}(\Omega_A/\omega_0)^2(d\cdot l)^2$.

We consider first the situation wherein the action of the transverse radiofrequency pulse causes the orientation of M to deviate from z by a certain angle ϑ, and the principal field remains unchanged ($H=H_0$). This case with $\alpha=0$ was already investigated in experiment[5] and considered theoretically.[10] The evolution of the spin system is described by the Hamiltonian

$$\mathscr{H}={}^1/_2M^2-M_z+U(d), \tag{3.3}$$

where the instantaneous orientation of the vector d is conveniently specified by the Euler angles φ, ϑ, and ψ:

$$d(t)=\hat{R}(\varphi,\vartheta,\psi)d(0),$$

where $\hat{R}$ is the matrix of three-dimensional rotations (φ and ϑ can be regarded as the aximuthal and polar angles of the vector M). Following Ref. 9, we start with the system of canonical equations (the time is measured in units of ω_0^{-1}):

$$\dot{\varphi}=-1+\partial U/\partial M_z,\quad \dot{\psi}=M+\partial U/\partial M, \qquad \dot{M}_z=-\partial U/\partial\varphi,\quad \dot{M}=-\partial U/\partial\psi, \tag{3.4}$$

with

$$U=-{}^1/_2(\Omega_A/\omega_0)^2\{\cos^2\alpha d_y^2+\sin^2\alpha d_z^2+\sin 2\alpha d_y d_z\},$$

where

$$d_y=-\sin\varphi\sin\psi+\cos\varphi\cos\vartheta\cos\psi,\quad d_z=-\cos\varphi\sin\vartheta.$$

In the case $|M-1|\ll 1$, the dipole-dipole potential averaged over the fast variables φ and ψ is

$$\bar{U}=-{}^1/_8(\Omega_A/\omega_0)^2\{(1-3\sin^2\alpha)\cos^2\vartheta+{}^1/_2\cos^2\alpha(1+\cos\vartheta)^2\cos 2\Phi\}, \tag{3.5}$$

where the slow variable is $\Phi=\varphi+\psi$. By introducing the adiabatic invariant $P=M_z-M(-2\leqslant P\leqslant 0)$ and by virtue of the fact that $\cos\vartheta=M_z/M=1+P/M$, the effective Hamiltonian of our problem acquires the form

$$\mathscr{H}={}^1/_2(M-1)^2-P+\bar{U}(\Phi,P/M), \tag{3.6}$$

where

$$\bar{U}(\Phi,P/M)=-{}^1/_8(\Omega_A/\omega_0)^2\{(1-3\sin^2\alpha)(1+P/M)^2 +{}^1/_2\cos^2\alpha(2+P/M)^2\cos 2\Phi\}. \tag{3.7}$$

The last expression is a generalization of the result of Ref. 9 to the case $\alpha\neq 0$. From the pair of the Hamiltonian equations

$$\dot{\Phi}=M-1,$$
$$\dot{M}=-\partial\bar{U}/\partial\Phi=-{}^1/_8(\Omega_A/\omega_0)^2\cos^2\alpha(2+P/M)^2\sin 2\Phi, \tag{3.8}$$

it follows that the phase Φ satisfies the equation

$$\ddot{\Phi}+{}^1/_2(\omega_l/\omega_0)^2\sin 2\Phi=0, \tag{3.9}$$

where the square of the frequency of the longitudinal resonance is $\omega_l^2=\frac{1}{4}\Omega_A^2\cos^2\alpha(1+\cos\vartheta)^2$. On the other hand, from the equation

$$\dot{\varphi}=-1+\partial\bar{U}/\partial P \tag{3.10}$$

it follows that the dipole shift of the magnetization-precession frequency is

$$\delta\omega_{tr}=\omega_{tr}-\omega_0={}^1/_4(\Omega_A^2/\omega_0)\{(1-3\sin^2\alpha)\cos\vartheta+{}^1/_2\cos^2\alpha(1+\cos\vartheta)\}. \tag{3.11}$$

It is known[11] that in the absence of a strong magnetic field and sufficiently close to the temperature of the transition to the normal phase it is necessary to take into account the fact that the amplitudes of the quasiparticle pairings in the spin states (↑↑) and (↓↓) are not equal to each other ($\Delta_\uparrow\neq\Delta_\downarrow$). Taking this circumstance into account, we can easily show that

$$\delta\omega_{tr}={}^1/_4(\Omega_A^2/\omega_0)\{(1-3\sin^2\alpha)\cos\vartheta+{}^1/_2\beta\cos^2\alpha(1+\cos\vartheta)\}, \tag{3.12}$$

where $\beta=\Delta_\uparrow\Delta_\downarrow/\Delta^2$, with $\Delta^2=\frac{1}{2}(\Delta_\uparrow^2+\Delta_\downarrow^2)$. It is curious that that at $\alpha=\pi/2$, i. e., in the case when $l\parallel H_0$, the dipole frequency shift $\delta\omega_{tr}=-(\Omega_A^2/\omega_0)\cos\vartheta$ and does not depend on β (a weak dependence on β appears only in the higher order in the small parameter $(\Omega_A/\omega_0)^2$; (see Ref. 12).

We turn now to a more interesting situation, which also pertains to the case $\alpha\neq 0$. We assume that following the radio-frequency pulse, which takes M out of the equilibrium orientation, the principal magnetic field decreased jumpwise to a value $H=H_0/2$. It will become obvious subsequently that the actual requirement is $\gamma|H_0-2H|<\Omega_A$. It is clear that the evolution of the spin system is now described by the effective Hamiltonian

$$\mathscr{H}={}^1/_2M^2-{}^1/_2M_z+U(d), \tag{3.13}$$

and the canonical equations for the angles φ and ψ take the form

$$\dot{\varphi}=-\frac{1}{2}+\partial U/\partial M_z,\quad \dot{\psi}=M+\partial U/\partial M. \tag{3.14}$$

In the situation considered and at $|M-1|\ll 1$ the variable $\Phi=2\varphi+\psi$ is slow and therefore

$$\overline{d_y^2}={}^1/_4(1+\cos^2\vartheta),\quad \overline{d_z^2}={}^1/_2\sin^2\vartheta,$$
$$\overline{d_y d_z}=-{}^1/_4\sin\vartheta(1+\cos\vartheta)\cos\Phi. \tag{3.15}$$

Recognizing furthermore that the adiabatic invariant is now the quantity $P=\frac{1}{2}M_z-M(-\frac{3}{2}\leqslant P\leqslant\frac{1}{2})$, we obtain the following dipole-dipole potential averaged over the "fast" motions $[\cos\vartheta=2(1+P/M)]$:

$$\bar{U}(\Phi,P/M)=-{}^1/_2(\Omega_A/\omega_0)^2\{(1-3\sin^2\alpha)(1+P/M)^2 -{}^1/_4[1-4(1+P/M)^2]^{1/2}(3+2P/M)\sin 2\alpha\cos\Phi\}. \tag{3.16}$$

It is important that the dependence of $\bar{U}$ on the slow variable $\Phi=2\varphi+\psi$ appears only at $\alpha\neq(0,\pi/2)$. This constitutes the specific feature of the considered resonant case. The system of canonical equations for Φ and M takes the form

$$\dot{\Phi}=M-1, \tag{3.17}$$

$$\dot{M}=\frac{1}{8}(\Omega_A/\omega_0)^2[1-4(1+P/M)^2]^{1/2}(3+2P/M)\sin 2\alpha\sin\Phi, \tag{3.18}$$

and the stable stationary point is $(M_s=1,\Phi_s=\pi)$. It is easy to verify that the variable $\chi=\pi+\Phi$ satisfies the equation

$$\ddot{\chi}+(\omega_l/\omega_0)^2\sin\chi=0, \tag{3.19}$$

where

$$\omega_t^2 = {}^1/_8\Omega_A^2 \sin 2\alpha \sin\vartheta (1+\cos\vartheta).$$

It follows from (3.18) that the values of the adiabatic invariant P, namely $-1/2$ and $-3/2$, are singular. Using the equation

$$\dot\varphi = -{}^1/_2 + \partial\overline{U}/\partial P, \tag{3.20}$$

we easily verify that far from the singular point $P=-\frac{1}{2}$ the magnetization-precession shift (relative to the Larmor value $\omega_0/2$) is given by

$$\delta\omega_{tr} = {}^1/_2(\Omega_A^2/\omega_0)\{(1-3\sin^2\alpha)\cos\vartheta + {}^1/_2 \sin 2\alpha(\sin\vartheta - (1+\cos\vartheta)\operatorname{ctg}\vartheta)\}. \tag{3.21}$$

4. CONCLUSION

We have discussed in this article some new effects involved in the spin dynamics of the superfluid phase A of liquid ^{3}He and typical of the case when the equilibrium axis **l** of the orbital anisotropy does not coincide in direction with the quantization axis **d** of the Cooper-pair spin. We shall deal briefly below with the possibility of experimentally realizing conditions under which the angle between **l** and $\mathbf{d}_0$ differs from zero in a sufficiently large volume of liquid ^{3}He–A. In the presence of a strong field $\mathbf{H}(\gamma H \gg \Omega_A)$ we can assume that $\mathbf{d}_0 \perp \mathbf{H}$ in the entire volume. On the other hand, near the walls of the container of the liquid helium, the vector **l** prefers to assume an orientation normal to the wall. If the magnetic field is oriented perpendicular to the flat wall of a vessel with ^{3}H–A, then at a distance on the order of the dipole length ξ_D from the wall the orbital vector **l** rotates through an angle $\pi/2$ and settles along $\mathbf{d}_0$ (bearing in mind the fact that the distance to the opposite wall in a plane-parallel geometry greatly exceeds $\xi_D \approx 10^{-3}$ cm). Thus, for this configuration the region where $\alpha \neq 0$ is concentrated near the vessel wall and has a thickness of the order of ξ_D. On the other hand if the magnetic field has an oblique orientation relative to the plane wall, making with the latter an angle $\delta \ll 1$, then the distance over which $\alpha \neq 0$ greatly exceeds ξ_D. This case was recently considered in detail in Ref. 13. The dependence of the angle α on the distance **d** to the container wall can be described with good accuracy by the relation

$$\sin(\delta-\alpha) = \sin\delta \,\operatorname{th}(d/\xi(\delta)),$$

and, as already noted, for small values of the "glancing" angle δ the characteristic length $\xi(\delta) \gg \xi_D$. Consequently, in this geometry, at distances $\mathbf{d} < \xi(\delta)$ from the vessel wall the angle $\alpha \approx \delta$. It must be emphasized that, in the considered example, an almost homogeneous state with $\alpha \neq 0$ can be established in a large volume $[\xi(\delta) \gg \xi_D]$ for small values of α, but this circumstance should not hinder the observation of parametric instability (see Sec. 2). Indeed, as follows from the estimate (2.12), the threshold amplitude is small enough even at a very small value of the angle α.

Note added in proof (mailed to translation editor 23 March 1980). The resonant situation considered in Sec. 3 and realized as a result of the jumplike change of the magnetic field pertains to the case $H = 2H_0$ (and not to the case $H = H_0/2$ as stated in the article).

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