Optical analogy in the problem of galvanomagnetic properties of a quasi-two-dimensional conductor

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(Submitted 5 June 1979)
Zh. Eksp. Teor. Fiz. **78**, 1020–1029 (March 1980)

A theory of the transverse magnetoresistance of a quasi-two-dimensional electron gas in a strong magnetic field is proposed for the case when the electron-phonon interaction is not weak. It is shown that under these conditions the picture of the carrier mobility can be conveniently described in terms of multiphonon transitions of localized electrons. The described approach yields equations in closed form for the transverse-conductivity tensor and makes it possible to investigate the character of its dependence on the temperature, on the magnetic field intensity, and on the electric-field frequency.

PACS numbers: 72.15.Gd, 71.38. + i, 63.20.Kr

The present paper is motivated in part by the recent interest in the electronic properties of quasi-two-dimensional systems connected with the study of inversion layers in metal-insulator-semiconductor junctions, of electrons over the surfaces of liquid helium, of thin conducting films, and of a few other physical objects.[1-4] To be sure, a considerable part of the cited articles is devoted to effects for which the interaction between electrons is responsible (in the presence or absence of a magnetic field).[1-3] The realization of these effects, however, imposes rather stringent requirements on the electron density, temperature, magnetic field, etc., so that for most conditions (which we henceforth assume to be satisfied) the interaction between electrons can be ignored. In addition, the considered transverse conductivity of the electron gas in the case when both the magnetic field and the coupling of the carriers with the phonons are strong, is of independent interest, since the available calculations take the electron–phonon interaction into account only in the lowest (second) order of perturbation theory (see, e.g., the review of Kubo *et al.*[5]). Corresponding to this approximation is a known mechanism, according to which the conductivity is determined by the diffusion of the cyclotron-orbit centers of the carriers on account of their collisions with thermal phonons.

If the conductivity is governed significantly by multiphonon processes, then there is no simple diffusion picture, and the calculation of terms of order higher than the second is made much more complicated by the presence of degenerate states that correspond to different positions of the orbit centers and are labeled by a quantum number that depends on the gauge of the vector potential.

One of the possible methods of eliminating gauge-invariant quantities from the theory of electron–phonon systems in a magnetic field was proposed in Ref. 6 and used there to obtain the ground-state energy and the longitudinal effective mass of formations of the polaron type—magnetic condensons. It will be shown below, in the calculation of the transverse conductivity in the quasi-two-dimensional situation, that this method is also convenient and leads to a lucid and physically verified picture.

I. HAMILTONIAN OF SYSTEM. UNITARY-TRANSFORMATION METHOD

We assume that the states of the conduction electrons transverse to the (x, y) plane in which the electrons move are frozen at the lowest of the discrete levels due to spatial quantization. The single-electron Hamiltonian of the system then takes in the Landau gauge the form

$$H=\frac{(p_x-\rho_0^{-2}y)^2}{2m}+\frac{p_y^2}{2m}+\sum\omega_{\mathbf{k}}n_{\mathbf{k}}+\sum(V_{\mathbf{k}}a_{\mathbf{k}}\exp\{i(k_xx+k_yy)\}+\text{H.c.}),\tag{1}$$

where $\rho_0=(eH)^{-1/2}$ is the magnetic length ($\hbar=c=1$); $n_{\mathbf{k}}=a_{\mathbf{k}}^*a_{\mathbf{k}}$ is the operator of the number of phonons of frequency $\omega_{\mathbf{k}}$; the effective mass of the electron m is assumed for simplicity to be isotropic. The quantities $V_{\mathbf{k}}$ differ from the corresponding matrix elements of the electron-phonon interaction of a three-dimensional system by factors that depend on the form of the wave function of the film ground state. The chosen approximation requires that the distance between the ground and first-excited film levels be much larger than the characteristic energies of the problem (temperature, cyclotron frequency $\Omega_c=(m\rho_0^2)^{-1}$, frequency of the phonons most actively interacting with the electrons). If this condition is violated and several film levels participate in the conduction process, then the problem ceases to be purely two-dimensional (the possibility of this situation justifies the term "quasi-two-dimensional" used in the article), but the corresponding generalizations of the calculations do not lead to fundamental difficulties.

We apply to the Hamiltonian (1) the unitary transformation proposed in Ref. 6. For the sake of clarity it is convenient to divide it into two successive stages: the first is realized by the operator

$$U_1=\exp\{i\rho_0^2p_xp_y\}\tag{2}$$

and leads to the transfer of the centers of all the Landau oscillators to the point $y=0$. The second is represented with the aid of

$$U_2=\exp\Big\{i\sum(k_xx+\rho_0^2k_yp_x)n_{\mathbf{k}}\Big\}\tag{3}$$

and excludes entirely the operators p_x and x from the

 0038-5646/80/030515-05$02.40

Hamiltonian. As a result of these transformations we have

$$H^{(2)} \equiv U_2 U_1 H U_1^{-1} U_2^{-1} = \Omega_c (c^* c + 1/2) + \sum \omega_k n_k + \sum (V_k S_k a_k \Gamma_k + \text{H.c.}), \tag{4}$$

where we have introduced the oscillator creation and annihilation operators

$$c^* = 2^{-1/2}(\rho_0 p_y + i\rho_0^{-1} y), \quad c = 2^{-1/2}(\rho_0 p_y - i\rho_0^{-1} y) \tag{5}$$

and the unitary operators

$$S_{\mathbf{k}} = \exp\left\{ \frac{i\rho_0^2}{2} \sum_{\mathbf{k}'} [\mathbf{k}\times\mathbf{k}']_z n_{\mathbf{k}'} \right\}. \tag{6}$$

We note here that the use of rather general approximations (certain variants of the self-consistent-field method, allowance for diagrams without crossings of the phonon lines, etc., dealt with in more detail in Ref. 6) makes it possible to replace the operators $S_{\mathbf{k}}$ by unit operators, but in the present article this replacement is used only in Sec. III in the analysis of a simple illustration of the proposed method.

Most important in everything that follows is the operator

$$\Gamma_{\mathbf{k}} = \exp(-1/2|\alpha_{\mathbf{k}}|^2) \exp(i\alpha_{\mathbf{k}} c^*) \exp(i\alpha_{\mathbf{k}} c), \tag{7}$$

where $\alpha_{\mathbf{k}} = 2^{-1/2}\rho_0(k_x + ik_y)$. It is convenient to express $\Gamma_{\mathbf{k}}$ in the form of a normal product for strong magnetic fields, when states of one or several Landau bands near the Fermi level take part in the electronic processes (we note that the transformations (2) and (3) do not intermix states corresponding to different Landau levels). In this situation, the intensity of the electron–phonon coupling increases effectively with increasing field intensity on account of the first factor in (7). It is easy to show that for an arbitrary function $\psi(y)$

$$\Gamma_{\mathbf{k}} \psi(y) = \exp(1/2 i\rho_0^2 k_x k_y) \exp(ik_y y) \psi(y + \rho_0^2 k_x), \tag{8}$$

i.e., the interaction with the phonons causes random wandering of the electron: a step $\rho_0^2 k_x$ is connected with a phonon whose wave vector is equal to k. Thus, we can reduce our problem with the aid of (2) and (3) to the problem of the motion of a single electron oscillator in a phonon field, but the coupling between them is essentially nonlinear, in contrast with systems of similar type with linear interaction, which have been investigated in detail.[7,8]

Before we proceed to a study of the kinetic properties, we point out some "exact" relations for the electron velocity $\mathbf{v} = i[\mathbf{r}, H]$, which follows essentially only from the form of $H^{(2)}$. Since the Hamiltonian (1) commutes with the projection of the total angular momentum

$$P_x = p_x + \sum k_x n_{\mathbf{k}}$$

and $U_2 U_1 P_x U_1^{-1} U_2^{-1} = p_x$, it follows that the absence of p_x from $H^{(2)}$ means that the vector of the initial Hamiltonian $E(P_x)$ does not depend on P_x and the mean values of v_x in the states ψ_{P_x} are

$$\bar{v}_x = (v_x)_{P_x P_x} = \frac{\partial E(P_x)}{\partial P_x} = 0. \tag{9}$$

Since the system is isotropic in the (x, y) plane, it is also obvious that $\bar{v}_y = 0$. If an external electric field of intensity E is applied along the y axis, we have as before $\bar{v}_y^E = 0$, while $\bar{v}_x^E = v_H^0$, where $v_H^0 = E/H$ is the Hall velocity in the absence of interaction with the phonons. In fact, for this system the dissipation mechanisms that determine its kinetic behavior are usually connected either with violation of spatial homogeneity or with limitation of the phonon dragging by the electrons; both refute the choice of P_x as the quantum number, meaning also deductions of the type (9). Nonetheless, it is useful to bear these deductions in mind.

II. GENERAL EXPRESSIONS FOR THE CONDUCTIVITY. OPTICAL ANALOGY

We calculate the conductivity by using a variant, proposed by Lax,[9] of the Kubo formula. If the electron subsystem is nondegenerate (let, for example, the number of carriers in the conducting layer be much less than the degeneracy multiplicity of the Landau levels, i.e., when $S/2\pi\rho_0^2$, where S is the area of the layer), then the diagonal component of the conductivity tensor is

$$\sigma_{yy}(\nu, T) = -i\frac{ne^2}{m} \int_0^\infty dt \exp(-i\nu_\varepsilon t) \operatorname{Sp}\{p_y(t)[\rho_\beta, y]\}, \tag{10}$$

and the Hall conductivity is

$$\sigma_{xy}(\nu, T) = -i\frac{ne^2}{m} \int_0^\infty dt \exp(-i\nu_\varepsilon t) \operatorname{Sp}\{(p_x(t) - \rho_0^{-2} y(t))[\rho_\beta, y]\}, \tag{11}$$

where $\nu_\varepsilon = \nu - i\varepsilon$ (ν is the frequency of the external electromagnetic field, $\varepsilon \to +0$), n is the carrier density in the layer, $\mathbf{p}(t)$ and $y(t)$ are the operators in the Heisenberg representation with the Hamiltonian H, $\rho_\beta = Z^{-1} e^{-\beta H}$, $\beta = T^{-1}$, and Z is the partition function. We note that the conductivity of a degenerate electron gas is also described by Eqs. (10) and (11), but in this case ρ_β must be replaced by the operator ρ_F—the Fermi function of the Hamiltonian H, divided by the number of carriers in the layer.

We use the known identity

$$[\rho_\beta, y] = \frac{i}{m} \int_0^\beta \rho_\beta p_y(-i\lambda)\, d\lambda \tag{12}$$

and transform the operators in (10) and (11) under the trace sign with the aid of (2) and (3); as a result we obtain

$$\sigma_{yy}(\nu, T) = \frac{\Omega_c}{2}\frac{ne^2}{m} \int_0^\infty dt \exp(-i\nu_\varepsilon t) \int_0^\beta d\lambda \operatorname{Sp}\{\rho_\beta^{(2)} c_+^{(2)}(-i\lambda) c_+^{(2)}(t)\}, \tag{13}$$

$$\sigma_{xy}(\nu, T) = \frac{i\Omega_c}{2}\frac{ne^2}{m} \int_0^\infty dt \exp(-i\nu_\varepsilon t) \int_0^\beta d\lambda \operatorname{Sp}\{\rho_\beta^{(2)} c_+^{(2)}(-i\lambda) c_-^{(2)}(t)\}, \tag{14}$$

where $c_\pm = c^* \pm c$, and the superscript (2) denotes that ρ_β and $c_\pm(\tau)$ are determined by the transformed Hamiltonian (4).

Expressions (13) and (14) are quite convenient for calculations if the electron and vibrational subsystem in $H^{(2)}$ are separated with good accuracy, for example with the aid of the adiabatic approximation. Inasmuch as in a strong magnetic field the conduction electrons of most crystals interact most actively with the acoustic lattice vibrations,[6] the electron subsystem is fast compared with the phonon subsystem if the in-

equality $s/\rho_0 < \Omega_c$ is satisfied, where s is the speed of sound (as a rule this inequality is well satisfied in fields 10^4–10^5 Oe). In addition, just as in the absence of a magnetic field, the adiabatic approximation is more reliable the stronger the electron–phonon coupling. A suitable object for the adiabatic situation may also be electrons over liquid He^4, which interact with the surface oscillations (ripplons); the characteristic frequency of the ripplons is of the order of 10^{-4} eV, and the electron–ripplon interaction can be varied in a rather wide range by means of a clamping electric field.

It is important to emphasize that the adiabatic potential surfaces corresponding to $H^{(2)}$ are not degenerate, inasmuch as in the given configuration of the lattice $Q_{\mathbf{k}}$ the electron spectrum is discrete and nondegenerate. We denote the spectrum of the system in the adiabatic approximation by $E_{N,\{n_{\mathbf{k}}\}}$, where N is the number of the electronic state and $\{n_{\mathbf{k}}\}$ is the set of the occupation numbers of the phonons corresponding to this state, and choosing the functions

$$\psi_{N,\{n_{\mathbf{k}}\}}(y,\{Q_{\mathbf{k}}\}) = \varphi_N(y,\{Q_{\mathbf{k}}\}) \prod_{\mathbf{k}} \Phi_{n_{\mathbf{k}}}(Q_{\mathbf{k}} - Q_{\mathbf{k}0}^N),$$

we obtain after simple transformations

$$\operatorname{Re}\sigma_{yy}(\nu,T) = \pi e^{-\beta\nu/2} \operatorname{sh}\left(\frac{\beta\nu}{2}\right) \frac{\Omega_c}{\nu} \frac{ne^2}{m} F_{yy}(\nu,T); \tag{15}$$

$$F_{yy}(\nu,T) = \sum_{\alpha,\alpha'} \mu_\alpha |(c_+)_{\alpha\alpha'}|^2 \delta(\omega_{\alpha'\alpha} - \nu), \tag{16}$$

$$\alpha \equiv (N,\{n_{\mathbf{k}}\}), \quad \mu_\alpha = Z^{-1}\exp(-\beta E_\alpha), \quad \omega_{\alpha'\alpha} = E_{\alpha'} - E_\alpha,$$

where $\operatorname{Re}\sigma_{xy}(\nu, T)$ is also determined by (15), but in place of $|(c_+)_{\alpha\alpha'}|^2$ it contains the products $(c_+)_{\alpha\alpha'}(c_-)_{\alpha'\alpha}$. The quantities $\operatorname{Im}\sigma_{\mu\lambda}$, as is well known, are connected with the dielectric properties of the system and will henceforth be disregarded; for the static conductivity we have $\operatorname{Im}\sigma_{yy}(0,T) = 0$ and $\operatorname{Im}(\sigma_{xy}(0,T) + \sigma_{yx}(0,T) = 0$.

Sums of the form $F_{\mu\lambda}(\nu, T)$ determine also the optical and nonradiative processes in local centers, and their investigation is the subject of an extensive literature.[10,11] The main purpose of the present article is to indicate this optical analogy and to use the optical methods developed in those studies to investigate $\sigma_{\mu\lambda}(\nu, T)$.

We note first, however, a number of characteristic properties that distinguish our problem from the optical problem. First, the formulas for the absorption and luminescence spectra contain in place of $|(c_+)_{\alpha\alpha'}|^2$ and $(c_+)_{\alpha\alpha'}(c_-)_{\alpha'\alpha}$ the squares of the moduli of the matrix element of the dipole-moment operator of the optical electrons, and the formulas for the probability of nonradiative transitions contain the non-adiabaticity operator elements. Second, the optical and nonradiative transitions are usually assumed to take place between pairs of states with definite values of N and N', with $N \neq N'$, whereas in (16) it is necessary to take into account, generally speaking, also terms with $N = N'$, and the summation is limited only by the temperature. Finally, the most important difference is connected with the form of H_{int}. In the conductivity problem $H_{\text{int}}^{(2)}$ depends according to (7) not only on y but also on p_y. Certain consequences of this unusual fact will be discussed in Sec. IV. In all other respects the analogy is complete, so that to calculate (16) we can use a picture extensively used in the optics of local centers, which leads to results that describe the experiment splendidly. According to this picture, the multiphonon processes are due to three causes: the change of the positions of the equilibrium of the lattice operators in the electronic transition $N \to N'$, the change of the phonon frequencies in the transition, and finally, the Q_k dependence of the matrix elements, which take in the conduction problem the form

$$M_{NN'}^{\pm}(\{Q_{\mathbf{k}}\}) = \int \varphi_N^*(y,\{Q_{\mathbf{k}}\}) c_\pm \varphi_{N'}(y,\{Q_{\mathbf{k}}\})\, dy. \tag{17}$$

The last cause constitutes violation of the Franck–Condon principle, a formulation of which can be the equality

$$M_{NN'}^{\pm}(\{Q_{\mathbf{k}}\}) = \text{const}. \tag{18}$$

In optics, the most frequently used is the Δ approximation—the case in which only the equilibrium positions change in the electronic transition. It is assumed that the phonon frequencies remain unchanged and the Franck–Condon principle is valid. It is precisely for this case (and only for it) that the calculation leads to the "mirror symmetry" of the light absorption and emission spectra, which is usually observed in experiment. The conductivity in the Δ approximation is the subject of the next section.

III. THE Δ APPROXIMATION

We discuss now the approximations that will be used henceforth. First, the operators $S_{\mathbf{k}}$ in $H^{(2)}$ will be replaced by unit operators. The conditions for the replacements were indicated in Sec. I; we add only that this approximation, while retaining the spatial homogeneity, does not lead to violation of relations of the type (9), and can therefore not be an independent cause of dissipative processes. Next, since the magnetic field is assumed strong enough and the adiabatic approximation is applicable, the operators $\Gamma_{\mathbf{k}}$ can be expanded in powers of c and c^* in the following manner:

$$\Gamma_{\mathbf{k}} = f_{\mathbf{k}}^0(N) + i\alpha_{\mathbf{k}} f_{\mathbf{k}}^1(N) c + i\alpha_{\mathbf{k}}^* c^* f_{\mathbf{k}}^1(N) + \ldots, \tag{19}$$

where $f_{\mathbf{k}}^{(0)}$ and $f_{\mathbf{k}}^{(1)}$ are functions of the operator $N = c^*c$; their matrix elements are expressed in terms of Laguerre polynomials $L_n^\gamma(x)$:

$$\begin{aligned} (f_{\mathbf{k}}^0)_{NN} &= \exp(-{}^1\!/_2|\alpha_{\mathbf{k}}|^2) L_N(|\alpha_{\mathbf{k}}|^2), \\ (f_{\mathbf{k}}^1)_{NN} &= (N+1)^{-1/2} \exp(-{}^1\!/_2|\alpha_{\mathbf{k}}|^2) L_N^1(|\alpha_{\mathbf{k}}|^2) \end{aligned} \tag{20}$$

(a distinction must be made between the operator N and the level number designated by the same letter). For the problem considered, the terms of the form $f_{\mathbf{k}}^l(N) c^l c$ $l > 1$, with $l > 1$, contained in the expansion of $\Gamma_{\mathbf{k}}$, are usually inessential. The approximation (19) results in the general case in a change of the phonon equilibrium positions and the frequencies, and also of a dependence of $M_{NN'}^{\pm}$ on $Q_{\mathbf{k}}$. The Δ approximation corresponds to allowance for only the first term in the expansion (19). Indeed, if $\Gamma_{\mathbf{k}} = f_{\mathbf{k}}^0(N)$, i.e.,

$$H^{(2)} = \Omega_c N + \sum \omega_{\mathbf{k}} n_{\mathbf{k}} + \sum f_{\mathbf{k}}^0(N)(V_{\mathbf{k}} a_{\mathbf{k}} + V_{\mathbf{k}}^* a_{\mathbf{k}}^*), \tag{21}$$

then in a state with a definite value of N the phonon eq-

uilibrium positions are shifted, with the magnitude of the shift depending on N; the frequencies of the phonons do not change as a result of the transition, and $M^{\pm}_{NN'} = \text{const}$.

In the Δ approximation, the conductivity tensor can be calculated exactly. For this purpose we express (16) in the form

$$F_{\nu\nu}(\nu, T)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\nu t}\,\mathrm{Sp}\{\rho_\beta^{(2)} c_+ c_+^{(2)}(t)\}\,dt, \tag{22}$$

where, in accord with the assumed approximation, $H^{(2)}$ is given by (21). We carry out next, under the trace sign in (22), a unitary transformation with the aid of the operator

$$U_3=\exp\Big\{-\sum \frac{f_{\mathbf{k}}^0(N)}{\omega_{\mathbf{k}}}(V_{\mathbf{k}}a_{\mathbf{k}}-V_{\mathbf{k}}^* a_{\mathbf{k}}^*)\Big\}. \tag{23}$$

As a result of the transformation (23) the phonon operators are shifted

$$\tilde{a}_{\mathbf{k}}^* = U_3 a_{\mathbf{k}}^* U_3^{-1} = a_{\mathbf{k}}^* - \frac{V_{\mathbf{k}}}{\omega_{\mathbf{k}}} f_{\mathbf{k}}^0(N), \tag{24}$$

and the Hamiltonian takes the form

$$H^{(3)}=\Omega_c N - \sum \frac{|V_{\mathbf{k}}|^2}{\omega_{\mathbf{k}}}[f_{\mathbf{k}}^0(N)]^2 + \sum \omega_{\mathbf{k}} n_{\mathbf{k}} = H_e^{(3)} + H_{ph}^0. \tag{25}$$

The operator

$$-\sum \frac{|V_{\mathbf{k}}|^2}{\omega_{\mathbf{k}}}[f_{\mathbf{k}}^0(N)]^2$$

determines the polaron shift of the electronic levels. The $H_e^{(3)}$ spectrum is

$$E_N^0=\Omega_c N - \sum \frac{|V_{\mathbf{k}}|^2}{\omega_{\mathbf{k}}}\exp(-|\alpha_{\mathbf{k}}|^2)[L_N(|\alpha_{\mathbf{k}}|^2)]^2. \tag{26}$$

The differences of the levels $\{E_N^0\}$

$$\Omega_c^{N'N} = E_{N'}^0 - E_N^0$$

constitute the frequencies of the purely electronic transitions for the pair (N, N').

The transformation of $c_\pm$ under the action of U_3 is conveniently carried out with the aid of the identities

$$\begin{aligned} e^{g(N)} c e^{-g(N)} &= e^{-[g(N+1)-g(N)]} c, \\ e^{g(N)} c^* e^{-g(N)} &= c^* e^{[g(N+1)-g(N)]}, \end{aligned} \tag{27}$$

where g is an arbitrary function of the operator N and I is the unit operator.

Further calculation of (22) leads to expressions of the form

$$(\mathrm{Sp}_{\mathbf{k}} e^{-\beta n_{\mathbf{k}}})^{-1}\,\mathrm{Sp}_{\mathbf{k}}\{e^{-\beta n_{\mathbf{k}}}\exp[\Delta_{\mathbf{k}}^{N+1,N} a_{\mathbf{k}} - (\Delta_{\mathbf{k}}^{N+1,N})^* a_{\mathbf{k}}^*]\exp[-\Delta_{\mathbf{k}}^{N+1,N} a_{\mathbf{k}} e^{-i\omega_{\mathbf{k}}} + (\Delta_{\mathbf{k}}^{N+1,N})^* a_{\mathbf{k}}^* e^{i\omega_{\mathbf{k}} t}]\},$$

where

$$\Delta_{\mathbf{k}}^{N',N} = \frac{V_{\mathbf{k}}}{\omega_{\mathbf{k}}}[L_{N'}(|\alpha_{\mathbf{k}}|^2) - L_N(|\alpha_{\mathbf{k}}|^2)]\exp\Big(-\frac{|\alpha_{\mathbf{k}}|^2}{2}\Big)$$

is the difference between the matrix elements of the transformed phonon operators (24) of the electronic states N' and N; the trace is taken in the subspace of the eigenfunctions of the k-th oscillator. It can be shown that

$$(\mathrm{Sp}_{\mathbf{k}} e^{-\beta n_{\mathbf{k}}})^{-1}\,\mathrm{Sp}_{\mathbf{k}}\{e^{-\beta n_{\mathbf{k}}}\exp(\gamma_{\mathbf{k}} a_{\mathbf{k}} - \gamma_{\mathbf{k}}^* a_{\mathbf{k}}^*)\} = \exp\{-|\gamma_{\mathbf{k}}|^2(\bar{n}_{\mathbf{k}}+{}^1/_2)\} \tag{28}$$

with the Bose function $\bar{n}_{\mathbf{k}}$. Taking (28) into account, we obtain

$$F_{\nu\nu}(\nu, T) = \Big(\sum_{N=0}^{\infty}\exp(-\beta E_N^0)\Big)^{-1}\sum_{N=0}^{\infty}(N+1)\exp(-\beta E_N^0)\cdot \times\{F_a^{N,N+1}(\nu, T) + \exp(-\beta\Omega_c^{N+1,N})F_e^{N+1,N}(-\nu, T)\}; \tag{29}$$

$$F_a^{N,N+1}(\nu, T) = \frac{1}{2\pi}\exp\Big\{-\sum|\Delta_{\mathbf{k}}^{N+1,N}|^2(2\bar{n}_{\mathbf{k}}+1)\Big\} \times\int_{-\infty}^{\infty}\exp[i(\Omega_c^{N+1,N}-\nu)t]\exp\Big\{\sum|\Delta_{\mathbf{k}}^{N+1,N}|^2[(\bar{n}_{\mathbf{k}}+1)e^{i\omega_{\mathbf{k}}t} + \bar{n}_{\mathbf{k}}e^{-i\omega_{\mathbf{k}}t}]\Big\}\,dt; \tag{30}$$

$$F_e^{N+1,N}(\nu, T) = \frac{1}{2\pi}\exp\Big\{-\sum|\Delta_{\mathbf{k}}^{N+1,N}|^2(2\bar{n}_{\mathbf{k}}+1)\Big\} \times\int_{-\infty}^{\infty}\exp[i(\Omega_c^{N+1,N}-\nu)t]\exp\Big\{\sum|\Delta_{\mathbf{k}}^{N+1,N}|^2[\bar{n}_{\mathbf{k}}e^{i\omega_{\mathbf{k}}t} + (\bar{n}_{\mathbf{k}}+1)e^{-i\omega_{\mathbf{k}}t}]\Big\}\,dt, \tag{31}$$

where $F_\alpha^{N,N+1}$ and $F_e^{N+1,N}$ are the spectra, normalized to unity, of the absorption and emission of light by local centers, due transitions between the levels N and $N+1$ and calculated in the Δ approximation. The simple connection between them is an expression of the law of mirror symmetry. Replacing the quantity $\bar{n}_{\mathbf{k}}+1$ in (30) by its equivalent $e^{\beta\omega_{\mathbf{k}}}\bar{n}_{\mathbf{k}}$, introducing $z = t - i\beta$, and transfering the integration contour to the real axis, we obtain

$$F_a^{N,N+1}(\nu, T) = \exp\{-\beta(\Omega_c^{N+1,N}-\nu)\}F_e^{N+1,N}(\nu, T). \tag{32}$$

If the electron density in the layer is less than $1/2\pi\rho_0^2$ and the temperature is low enough, $\beta\Omega_c \gg 1$, then it suffices to retain in (29) only the first term, and the static conductivity, with allowance for (32), takes the form

$$\sigma_{\nu\nu}(0, T) = \beta\Omega_c\frac{ne^2}{2m}F_a^{0,1}(0, T). \tag{33}$$

Expressions of the type (30) were investigated in detail in many papers on optics,[10,11] and simple formulas were obtained for some special models. In the general case, the dependence of $F^{N,N\pm1}$ on the temperature, on the intensity of the magnetic field, and on the frequency of the electric field is determined by the details of the phonon spectrum and by the character and intensity of the electron–phonon interaction.

We indicate, in concluding this section, that the temperature dependence of F_a and F_e can be investigated in the general case with the aid of an equation that they satisfy as functions of ν and T and makes it possible to trace the evolution of the form of $F(\nu, T)$ with increasing temperature.[12]

IV. DISCUSSION OF RESULTS

We compare first the results of the preceding section with the expressions given in the literature for $\sigma_{\mu\lambda}$, calculated in second order of perturbation theory and corresponding to the Titeica mechanism. According to Ref. 5 we have for a nondegenerate two-dimensional electron gas

$$\sigma_{\nu\nu}(0, T) = \beta\frac{ne^2}{2}\int_{-\infty}^{\infty}\mathrm{Sp}\{\rho_\beta \dot{Y}\dot{Y}(t)\}\,dt, \tag{34}$$

where $\dot{Y}(t)$ is the Heisenberg operator of the projection of the velocity of the center of a cyclotron orbit. For collisions with phonons,

$$\dot{Y}=-i\rho_0^2\sum k_x(V_{\mathbf{k}}a_{\mathbf{k}}e^{i(k_x x+k_y y)}-\text{H.c.}), \tag{35}$$

i.e., in second order ρ_β and $\dot{Y}(t)$ are determined by the Hamiltonian without interaction, so that we can obtain

$$\sigma_{yy}^{II}(0,T)=\frac{2\pi\beta}{\Omega_c}\frac{ne^2}{2m}\sum|V_{\mathbf{k}}|^2\exp(-|\alpha_{\mathbf{k}}|^2)|\alpha_{\mathbf{k}}|^2(k_x\rho_0)^2\bar{n}_{\mathbf{k}}\delta(\Omega_c-\omega_{\mathbf{k}}). \tag{36}$$

If we replace $k_x^{(2)}$ in (36) by $k_\rho^2/2=(k_x^2+k_y^2)/2$ (this is justified because of the isotropy of the system in the (x, y) plane), then the obtained expression coincides exactly with the earlier one (33), expanded up to $|V_{\mathbf{k}}|^2$. In order for the expressions for $\sigma_{\mu\lambda}$ to contain k_x and k_y separately, and not only in a combination of the form k_ρ, it is necessary to go outside the framework of the Λ approximation, and include, for example, terms with $f_{\mathbf{k}}^1$ in the expansion (19); this is particularly important for the calculation of the Hall conductivity, where products k_xk_y are involved and lead to $\sigma_{xy}^{II}(0,T)=0$. Strictly speaking, the comparison presented above is given for mutually exclusive approximations—the adiabatic approximation ($\Omega_c>\bar{\omega}$ or strong coupling) and the Titeica mechanism (weak coupling and $\Omega_c<\bar{\omega}$, since $\sigma_{yy}^{II}(0,T)$ is proportional to the density of the phonon spectrum at the frequency Ω_c). The agreement of the results means that the approach assumed in the article is valid also beyond the boundaries of the formal adiabatic-approximation scheme.

We discuss briefly the contribution made to $\sigma_{\mu\lambda}$ by the processes due to the dependence of $M_{NN'}^{\pm}$ on $Q_{\mathbf{k}}$. This dependence is the only cause of transitions with participation of phonons for $N'=N$. The terms (16) with $N'=N$ may turn out to be significant, for example, in the limit of low temperatures, inasmuch as at $\beta(\Omega_c^{0,1}-\nu)\gg 1(\nu<\Omega_c^{0,1})$ we have $F_\alpha^{0,1}(\nu,T)\sim\exp\{-\beta(\Omega_c^{0,1}-\nu)\}$, and M_{00}^* gives rise to a power-law dependence on the temperature, albeit with a new small parameter as a factor. Allowance for $M_{NN}^{\pm}(\{Q_{\mathbf{k}}\})$ leads also to the appearance of the quantities $\{k_x^2\}$ in σ_{yy} and of the quantities $\{k_xk_y\}$ in σ_{xy}, as should be the case in accordance with a general formula of the type (34). Indeed, $(c_+)_{NN}$ differs from zero only to the extent that $H_{int}^{(2)}$ depends on p_y, since the spectrum of $H^{(2)}$ at fixed values of $Q_{\mathbf{k}}$ is discrete and nondegenerate. On the other hand this dependence is determined by the commutators $[\Gamma_{\mathbf{k}}, y]=k_x\rho_0^2\Gamma_{\mathbf{k}}$. For similar reasons, the matrix element $(c_-)_{NN}$ is proportional to the quantities $\{k_y\}$, since $[\Gamma_{\mathbf{k}}p_y]\sim k_y$. The simplest linear approximation of $M_{NN}^*(\{Q_{\mathbf{k}}\})$ does not contribute to the conductivity, since it gives rise to processes of second order in $\{V_{\mathbf{k}}\}$ that are forbidden by the energy conservation law.

The role of the effect of the change of the phonon frequencies in electronic transitions $N\to N'$ is as a rule inessential for the values of $\sigma_{\mu\lambda}$.

We note in conclusion that the described method is convenient also if it is necessary to take into account several electronic levels corresponding to quantization across the film (quasi-two-dimensionality). In this case the electronic states are numbered additionally by the index of the film levels; in all other respects formulas of the type (15) are suitable, provided only that the transitions between the essential pairs of electronic levels can be described in the adiabatic approximation.

I take the opportunity to express sincere gratitude to Academician I. M. Lifshitz for a discussion of a number of results in the article.

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Translated by J. G. Adashko