Properties of "high-temperature" oscillations of the magnetoresistance of bismuth

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The temperature dependence of the amplitude of the "high-temperature" oscillations (HTO) of the magnetoresistance (MR) of bismuth is investigated in the range 7–60 K. At $T \approx 10$ K the amplitude reaches a maximum value, after which it decreases. The rate of attenuation of the amplitude, determined by the derivative $\partial \ln \tilde{\rho}ik/\partial T$, does not depend on the cyclotron frequency, and at a fixed temperature (20 K) the amplitude of the oscillations of the diagonal component of the MR tensor does not depend on the direction of the magnetic field **H**. It is established that for **H** directions in the C_1C_3 plane (binary plane) the HTO are predominantly superpositions of two frequencies that differ by a factor ≈ 1.22. It is shown that for any direction of the vector **H** in the binary plane the period of the HTO varies as a function of the temperature in accordance with the same law. In the range $3° < \theta < 8° [\theta = \measuredangle(\mathbf{H}, C_3)]$ it became possible to identify, for the first time ever, the HTO connected with the electron Fermi surface. The experimental results are discussed within the framework of resonances of the magnetophonon type. The dependence of the plasma frequencies on the magnetic field, with account taken of the real structure of the Fermi surface of bismuth, is calculated as part of an evaluation of the resonant energy. The influence of the nonparabolicity of the electron spectrum on the resonances is considered.

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In 1973, two of us[1] observed oscillations of the transverse magnetoresistance (MR) of bismuth with the following characteristic features: 1) periodicity in the reciprocal magnetic field, 2) weak temperature damping of the amplitude. The observed oscillations cannot be identified with the Shubnikov-de Haas (SdH) effect: Their period does not fit the accepted ideas concerning the Fermi surface of bismuth, and the damping of the amplitude at a relatively small period is such that the oscillations can be observed up to nitrogen temperatures. (In the interval 20–65 K the period decreases by 20–30%.[2])

A connection was subsequently[3,4] established between the "high-temperature" oscillations (HTO) and the energy spectrum of the carriers. It turned out that the dependence of the HTO frequency on the direction of **H** duplicates the angular dependence of the cross sections S and of the cyclotron masses m^* connected with the hole Fermi surface ($S \approx 2\pi m^* \varepsilon_F$ for holes). The position of the extrema of the HTO in fields up to 56 kOe satisfy[5] the resonance condition

$$\varepsilon_F(H, T) + \varepsilon_{01} = (N + 1/2)\hbar\Omega_c, \qquad (1)$$

which correspond to transitions from the Fermi level ε_F into states determined by the quantum numbers N, k_y, and $k_z = 0$. The magnetic fields at resonance are determined by the relation

$$(1/H)_{ext} = (N + 1/2) e\hbar/m^* c\{\varepsilon_F(H, T) + \varepsilon_{01}\}, \qquad (2)$$

and at a constant Fermi energy their difference is

$$\Delta(1/H)_{ext} = P = e\hbar/m^* c(\varepsilon_F + \varepsilon_{01}). \qquad (3)$$

Here $\Omega_c = eH/m^*c$ is the cyclotron frequency, N is the number of the Landau level, and $\varepsilon_{01} = \hbar\omega_{01} \approx 20$ meV is the characteristic energy calculated with the aid of formula (3) at $H < 15$ kOe, when $\varepsilon_F \approx \text{const} \approx 11.8$ meV. The values of the Fermi energies of the holes as functions of the magnetic field were taken in the preceding paper[4] from Édel'man's paper.[5]

The possibility of describing the evolution of the HTO in a magnetic field with the aid of the Fermi energy points to a relation between the period of the oscillations and the carrier density n. This is attested by the correlation between $(H^{-1})_{ext}$ and n when the temperature is varied: according to Ref. 4, raising the temperature from 18 to 70 K produces in $(H^{-1})_{ext}$ a change equivalent to an increase of the carrier density by 75%, in agreement with the known published data.[6]

The condition (1) was obtained in Ref. 7, where multiphonon resonance (MPR) was predicted for the longitudinal magnetic conductivity of a degenerate electron gas. In contrast to transverse MPR, at $\mathbf{j} \parallel \mathbf{H}$ a contribution is made to the current by electrons near the Fermi surface (**j** is the current-density vector). For anisotropic crystals, the analysis of Ref. 7 can be formally generalized to the case of transverse MR, inasmuch as each element of the matrix $|\rho_{ik}|$ contains generally speaking all the elements of the matrix $|\sigma_{ik}|$.

The end-point energy of the optical phonons in bismuth is[8] 9.2 and 12.4 meV, i.e., much less than ε_{01}. Therefore the HTO oscillations cannot be treated, at least, as MPR with participation of one optical phonon. It must be added that the resonance condition (1) is not limited to the electron-phonon interaction alone, and is applicable also to interactions of electrons with quasiparticles of other types.

Since the resonance (1) is due to the scattering of carriers located on the Fermi surface, it is clear that the amplitude of the oscillations with period (3) contains the same temperature factor as the amplitude of the SdH effect:

$$kT\,\mathrm{sh}^{-1}(2\pi^2 kT/\hbar\Omega_c).$$

As already mentioned, the HTO are connected with "heavy" carriers ($m^* \sim 0.1 m_0$ for holes), and consequently from the point of view of the considered resonance (1) the damping of the amplitude should be quite

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intense. Nevertheless the HTO are observed up to 65 K, in contrast to the SdH oscillations which are observed at $H\parallel C_1, H\approx 20$ kOe only at $T\leq 4$ K. Therefore the characteristic energy ε_{01} can be naturally connected with excitations of the Bose type, the number of which at $kT \ll \varepsilon_{01}$ increases like $\exp(-\varepsilon_{01}/kT)$. The foregoing premise is the basis of our problem, that of investigating in detail the dependence of the HTO amplitude ($\tilde{\rho}$) on the temperature and of comparing the results with the law

$$\tilde{\rho}\sim kT\left[\operatorname{sh}\left(\frac{2\pi^2kT}{\hbar\Omega_c}\right)\left(\exp\left\{\frac{\hbar\omega_{01}}{kT}\right\}-1\right)\right]^{-1}. \quad (4)$$

An experimental confirmation of the relation (4) with the proof of the fact that the HTO is a manifestation of the resonance (1). The obtained data, however turned out to be quite unexpected: whereas the properties of the HTO period (its dependence on the carrier density with change both of temperature and of the quantizing magnetic field) can be described by the resonance conditon (1), the properties of the amplitude pertain more readily to a resonance of the type

$$M\hbar\Omega_c=\varepsilon_0, \quad (5)$$

where $\varepsilon_0\approx 32$ meV at $T\lesssim 20$ K, $H\sim 10$ kOe, $M=1,2,\ldots$. This circumstance raised a large number of questions. These include the dependence of the dielectric anomaly of bismuth on the magnetic field (we have in mind the plasma frequency ω_p at zero plasmon wave vector), the influence of the nonparabolicity of the band on the period of the HTO, and the temperature dependence of the electric conductivity in the 20–70 K range.

We investigated here in greater detail than before[3] the angular dependence of the period of the HTO, namely, a considerable part of the range of angles between $H\parallel C_3$ (the trigonal axis) and $H\parallel C_1$ (the bisector axis) was covered in steps of 1°. As a result we were able to identify the oscillations connected with the electrons, as well as to observe a nonmonotonic dependence of the period of the HTO on the direction of the magnetic-field vector. (In the preceding paper[3] we reported observation, besides the fundamental frequency, also of doubled and tripled HTO frequencies. In the present paper we deal throughout with the fundamental frequency.)

It must be noted that despite the detailed investigation reported below, the question of the origin of the resonance energy still remains open. The study of the kinetic properties of bismuth alloys has so far not yielded any positive results. Investigations of the optical properties of bismuth in the IR band may be promising. Diverse arguments on this question have been advanced in the discussions of the results.

EXPERIMENTAL RESULTS

1. Temperature dependence of the HTO amplitude

The main results were obtained for a bulky (cross section 5×5 mm) single-crystal (Bi-X) of high purity ($\Delta\approx 700$, where Δ is the ratio of the resistances at room and helium temperatures in a zero magnetic field), whose longitudinal axis was parallel to the binary axis C_2. Data are also presented for a sample with 2×2 mm cross section and $\Delta\approx 126$ (Bi-XI), with the longitudinal axis along C_3. We measured both the diagonal and the off-diagonal components of the MR tensor, expressed in terms of the axes $X\parallel C_2, Y\parallel C_1, Z\parallel C_3$. The experiments were performed at temperatures from 4.2 to 70 K. The temperature pickup was a KGG thermometer produced at the Semiconductor Institute of the Ukrainian Academy of Sciences.[1)]

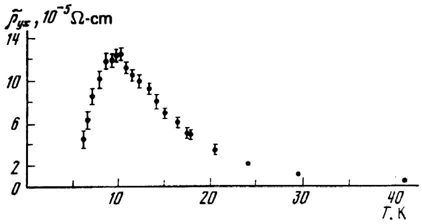

FIG. 1. Temperature dependence of the HTO amplitude in the binary plane at $\theta=60°$ and $H=15.1$ kOe.

A study of the HTO at $T<15$ K entails considerable difficulties because of the superpositon of SdH oscillations, whose amplitude increases rapidly with decreasing temperature. The HTO can be separated from the background of the much larger-amplitude SdH oscillations only if the periods differ substantially, when it is possible to compensate for the change of the SdH signal over a relatively large section of one half-period. From this point of view the favorable angles between the magnetic field and the C_3 axis were $\theta=60°$ (in the measurement of ρ_{yx}) and 90° (in the measurement of ρ_{xx}). The temperature dependence of the HTO amplitude at $\theta=60°$ is shown in Fig. 1. It is seen that the amplitude reaches the maximum value in the region of 10K. We note that the relative value of $\tilde{\rho}/\rho$ for HTO at $T\sim 20$ K and $H\sim 10$ kOe does not exceed several tenths of 1% (ρ is the monotonic part of the MR).

Detailed measurements of the HTO amplitude at $T>15$ K were made for the orientations $\theta=5$, 17, and 60° in different magnetic fields. The results are shown in Fig. 2 together with the data for $H\parallel C_1$, obtained for the Bi-XI sample in a superconducting solenoid. It is seen that, at least in the investigated temperature range, the HTO-amplitude damping, which is determined by the derivative $\partial \ln\tilde{\rho}/\partial T$ does not depend on the direction or magnitude of the magnetic field, even though the cyclotron mass changes by more than 3 times when the vector H is rotated from C_3 to C_1. The inset of Fig. 2 shows for comparison with experiment the function

$$f=\frac{2\pi^2kT}{\hbar\Omega_c}\left[\operatorname{sh}\left(\frac{2\pi^2kT}{\hbar\Omega_c}\right)\left(\exp\left\{\frac{\hbar\omega_{01}}{kT}\right\}-1\right)\right]^{-1},$$

which describes the damping of the amplitude at the resonance (1), and "follows" the values of the cyclotron mass and of the magnetic field. Thus, starting with 20 K, the plots of $\ln f(T)$ are straight lines whose slopes are proportional to m^*/H.

The temperature dependence of the experimentally measured amplitude $\tilde{\rho}$ of the oscillations of the MR are influenced by the monotonic part of the magnetoconductivity (MC) σ. In fact, assuming the MC to be the sum of a monotonic part and an oscillating part ($\sigma_{ik}^{\Sigma}=\sigma_{ik}+\tilde{\sigma}_{ik}$), inverting the matrix $|\sigma_{ik}^{\Sigma}|$, and retaining the

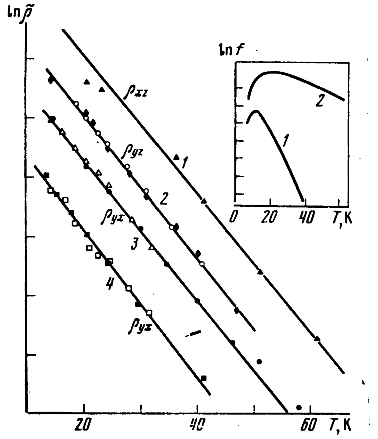

FIG. 2. Temperature dependence of the HTO amplitude (arbitrary scale). The plots of the functions that differ only in the value of the magnetic field are superimposed: 1—$H \parallel C_1$, $H = 29$ kOe; 2—$\theta = 5°$, ♦ and ○ correspond to $H = 15.6$ and 10.8 kOe respectively; 3—$\theta = 17°$, ● and △ correspond to $H = 16.7$ and 6.9 kOe; 4—$\theta = 60°$, ■ and □ correspond to $H = 15.13$ and 6.0 kOe. Inset—the function f at $\omega_{01} = 20$ meV: 1—$H = 8$ kOe, $m^* = 0.12\, m_0$; 2—$H = 16$ kOe, $m^* = 0.06\, m_0$. One division on the ordinate axis of Fig. 2 or 3 corresponds to unity.

terms of the first order in $\tilde\sigma_{ik}$, the summary magnetoresistance can in the general case by expressed in the form

$$\rho_{ik}^{\Sigma}=\rho_{ik}+\tilde\rho_{ik}\approx f_\theta^{(1)}(T)+f_\theta^{(2)}(T)\tilde\sigma_{ik}, \quad (6)$$

where $\tilde\rho_{ik}\approx f_\theta^{(2)}(T)\tilde\sigma_{ik}$, while $f_\theta^{(1,2)}(T)$ are determined generally speaking by the orientation of the magnetic field.

To determine the influence of the possible anisotropy of the temperature dependence of the monotonic part of the MC on the anisotropy of the temperature dependence of $\tilde\rho$, we have performed two experiments. In the first we determined the cyclotron mass by measuring the amplitude of the SdH oscillations in a range 4–8 K, when the dependence of the monotonic part of the MC on the temperature is substantial. In the calculation we substituted the values of the relative amplitudes of MR oscillations in the formulas for the determination of m^* with the aid of the relative amplitude of the MC oscillations.[9] The obtained values of m^* are close to those obtained by cyclotron resonance.[10] We therefore assume (see Ref. 6)

$$\tilde\rho/\rho\approx\tilde\sigma/\sigma, \quad \rho^{\Sigma}=\rho(1+\tilde\sigma/\sigma). \quad (7)$$

We measured next the dependence of the monotonic part of the MR on the temperature for $\theta = 5$, 17, and 60° in different magnetic fields. The results are shown in Fig. 3, from which it follows that the temperature factor of the function $\rho(T)$ [or $f_\theta^{(1)}(T)$] remains constant when both the orientation and the value of the magnetic field change. It is logical to assume that $f_\theta^{(2)}(T)$ has the same property, more so because it follows directly from (7) if we assume $\rho\sim\sigma^{-1}$. Thus, the anisotropy of the temperature dependence of $\tilde\rho$ determines completely the anisotropy of the temperature dependence of $\tilde\sigma$.

It should be noted that the presence in $\tilde\rho$ of the factor

$$\mathrm{sh}^{-1}\left(\frac{2\pi^2kT}{\hbar\Omega}\right)\sim\exp\left\{-\frac{2\pi^2kT}{\hbar\Omega}\right\}$$

should influence also the dependence of the amplitude

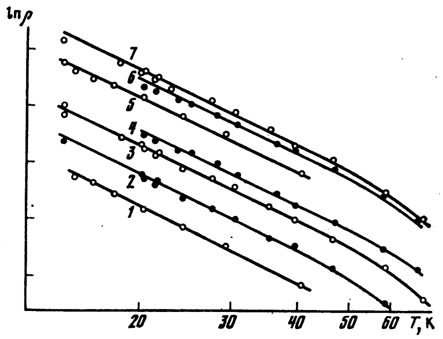

FIG. 3. Temperature dependence of the monotonic component of ρ_{yx} (arbitrary scale): curves 7, 3, and 2 correspond to $H = 18.2$, 9, and 6.9 kOe and $\theta = 17°$; curves 6 and 4 correspond to $H = 17.5$ and 10.8 kOe and $\theta = 5°$; curves 5 and 1 correspond to $H = 15.3$ and 6.9 kOe and $\theta = 60°$. The temperature on the abscissa axis is in the natural logarithm scale.

$\tilde\rho_{xx}$ on the direction of H at fixed temperature. It follows from the experiment, however, that $\tilde\rho_{xx}(\theta)$ is practically constant at 20 K. At $T > 15$ K the damping of the HTO amplitude is better described by an exponential function ($e^{-\gamma T}, \gamma = 0.12$ deg^{-1}) than by a power-law function—the function plotted as a straight line is $\ln\tilde\rho(T)$ and not $\ln\tilde\rho(\ln T)$.

2. Angular dependence of the period of the HTO

In the range $\theta = 0$–40°, the HTO are predominantly superpositions of 2 modes with a ratio of the periods ≈1.22 (Figs. 4c and 4d). It can be stated with full assurance that the observed beats are not due to superposition of electron and hole modes, otherwise the parameters of the electron spectrum would adjust themselves at each value of θ to fit the hole spectrum. At $\theta > 40°$ the influence of the SdH effect on the HTO becomes noticeable and leads to additional modulation of the HTO amplitude. The gist of the phenomenon reduces to the

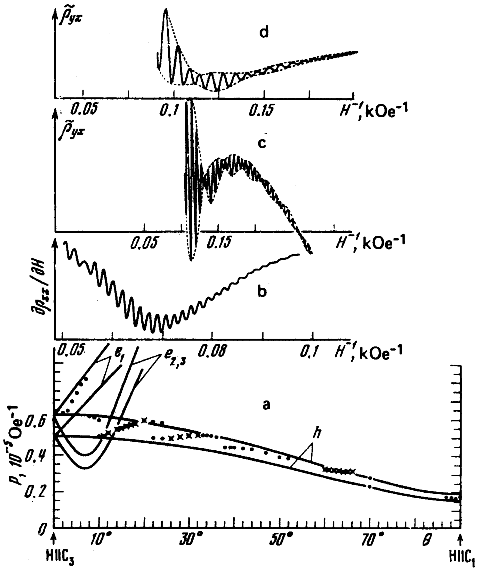

FIG. 4. a) Angular dependence of the period of the HTO: ●—average. ×—period obtained from the angle shift of the extrema; b), c), d) HTO for $\theta = 90°$, 17°, and 2°, respectively; $T = 20$ K.

following. Assuming [see (7)], for example, $\tilde{\rho}_{xx}=\tilde{\sigma}_{xx}/\sigma_{xx}^2$ and representing σ_{xx} as a sum of the monotonic part of the MR and of the oscillating term $\tilde{\sigma}^{(1)}$ due to the SdH effect, we obtain

$$\tilde{\rho}_{xx}\approx\tilde{\sigma}_{xx}/\sigma_{xx}^2-2\tilde{\sigma}_{xx}\tilde{\sigma}_{xx}^{(1)}/\sigma_{xx}^3, \tag{8}$$

where the term $\sim\tilde{\sigma}_{xx}\tilde{\sigma}_{xx}^{(1)}$ represents the HTO modulated by the low-frequency mode of the SdH.

The contribution of the considered effect should be larger the closer the vector of the magnetic field to the bisector axis in the vicinity of which the relative amplitude of the SdH oscillations is $\gtrsim 10\%$ at $H\sim 10$ kOe and $T\sim 20$ K. Assuming next that, for example, $\rho_{zx}^{\Sigma}\approx\sigma_{zx}^{\Sigma}/(\sigma_{xx}^{\Sigma})^2$, i.e., $\sigma_{zx}^{\Sigma}<\sigma_{xx}^{\Sigma}$, and

$$\rho_{zx}^{\Sigma}\approx\frac{\sigma_{zx}+\tilde{\sigma}_{zx}}{\sigma_{xx}^2(1+\tilde{\sigma}_{xx}/\sigma_{xx})^2},$$

and separating in σ_{ik} the terms connected with the SdH effect, we have at $\tilde{\sigma}_{zx}/\tilde{\sigma}_{xx}<\tilde{\sigma}_{zx}^{(1)}/\tilde{\sigma}_{xx}^{(1)}$

$$\tilde{\rho}_{zx}\approx\frac{\tilde{\sigma}_{zx}}{\sigma_{xx}^2}-\frac{2\tilde{\sigma}_{xx}\tilde{\sigma}_{zx}^{(1)}}{\sigma_{xx}^3}-\frac{2\tilde{\sigma}_{xx}}{\sigma_{xx}^2}\frac{\sigma_{zx}}{\sigma_{xx}}. \tag{9}$$

By comparing expressions (8) and (9) we can see that the phase shift of $\tilde{\sigma}_{xx}^{(1)}$ and $\tilde{\sigma}_{zx}^{(1)}$ should lead to a relative shift with respect to the magnetic field of the extrema of the additional beats of $\tilde{\rho}_{xx}$ and $\tilde{\rho}_{zx}$, which are due to the modulation of the amplitude of the HTO by the SdH effect. Furthermore, if in (9)

$$\frac{\tilde{\sigma}_{zx}}{\sigma_{xx}^2}\gtrsim\frac{\tilde{\sigma}_{xx}}{\sigma_{xx}^2}\frac{\sigma_{zx}}{\sigma_{xx}}$$

and $\tilde{\sigma}_{xx}$ and σ_{zx} are shifted in phase, a phase shift not due to the modulation should be observed also for the $\tilde{\rho}_{xx}$ and $\tilde{\rho}_{zx}$ carrier frequencies.

The predicted singularities of the HTO are seen in Fig. 5, which shows the plots of $\partial\rho_{xx}/\partial H$ and $\partial\rho_{zx}/\partial H$ for the direction $H\parallel C_1$ ($\rho_{yz}\to 0$ for the given orientation of the magnetic field).

Figure 4a shows the dependence of the average period of the HTO on the direction of the vector **H** in the binary plane. The determination of the period as a function of θ was carried out by two methods—by constructing for each θ the dependence of the number of the extremum on the reciprocal magnetic field, and also on the basis of

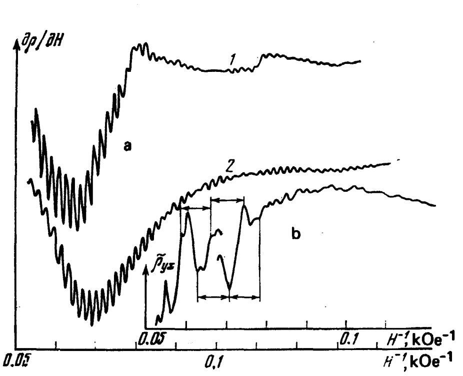

FIG. 5. a) Plots of HTO at $T=20$ K, $H\parallel C_1$: 1—$\partial_{zx}/\partial H$, 2—$\partial\rho_{xx}/\partial H$. It is seen that the carrier frequencies on curves 1 and 2 are shifted in phase. The relative shift of the extrema of the beats due to modulation of the amplitude by SdH are well noticeable near $H^{-1}=0.06$ and 0.1 kOe^{-1}. b) $\tilde{\rho}_{yx}$ at $\theta=7°$, $T=20$ K.

the phase shift (inasmuch as at sufficiently large values of the number of the extremum the phase shift can be large also for a small change of the period). In the latter case we used the relation $P=P_0H_0^{ext}$, where H_0^{ext} is the value of the magnetic field at the extremum point corresponding to the period P_0 for a certain direction of **H**. Both procedures of determining the period lead to identical results (see the crosses and points in Fig. 4a).[2)] It is seen that the period of the oscillations is a nonmonotonic function of θ.

To determine the causes of this phenomenon we started from the assumption that the angular dependences of the period of the HTO and of the reciprocal cyclotron mass are similar at $[m^*(\theta)]^{-1}$.[3] We plotted first the function $P(\theta)\propto[m_h^*(\theta)]^{-1}$ (see the upper curve h, Fig. 4a) from the initial point at $\theta=34°$, where the MR oscillates at practically a single frequency in a relatively wide interval of magnetic fields. (The same curve h is obtained if the starting points for the plotting of $P(\theta)\propto[m_h^*(\theta)]^{-1}$ are the values of P at the angles $\theta=22$ and $60°$ where the HTO are similar to the oscillations at $\theta=34°$.) The points of the upper curve h of Fig. 4a were then divided by the ratio 1.22 of the frequencies that form the beats (Figs. 4c, 4d) and the lower curve h of Fig. 4a was plotted. It turns out that at $\theta>8°$ the experimentally determined average period of the HTO fluctuates between points lying on the h curves of Fig. 4a. It follows therefore that nonmonotonic dependence of the average period of the HTO on the direction of the vector **H** at $\theta>8°$ is the result of the variation, as a function of θ, of the ratio of the amplitudes of the combining modes.

The curves e_1 and $e_{2,3}$ of Fig. 4a are plotted in analogy with the angular dependence of the reciprocal cyclotron masses corresponding to the electron "ellipsoid" e_1, e_2, and e_3 from a starting point at $H\parallel C_3$, where the cyclotron masses of the electrons and holes are identical. The similarity coefficient for like curves, just as for the holes h, was taken to be 1.22. It is seen that in the interval $3°<\theta<8°$ the function $P(\theta)$ is close to the angular dependence of the reciprocal cyclotron masses of the electrons of the "ellipsoid" e_1, and this gives ground for assuming that the corresponding periods of the HTO are connected with the electron Fermi surface. It must be stated that in this angle interval the HTO seem much less effective and more complicated than in the remaining band (see the inset of Fig. 5). We shall discuss this circumstance separately later on.

3. Temperature dependence of the HTO period and singularities of the electric conductivity and of the magnetoresistance

It was established earlier[2] that the period of the HTO at $H\parallel C_3$ depends on the temperature and that its change is connected with the temperature dependence of the concentration of the carriers in bismuth. In the present paper we measured the temperature dependence of the HTO period at different directions of the magnetic-field vector in the plane C_1C_3 ($\theta=5$, 17, 60, 90°) and in the range 20–70 K. We used the method referred to above—we observed the temperature shift of the phase of the oscillations. Within the limits of the errors (~10%) the

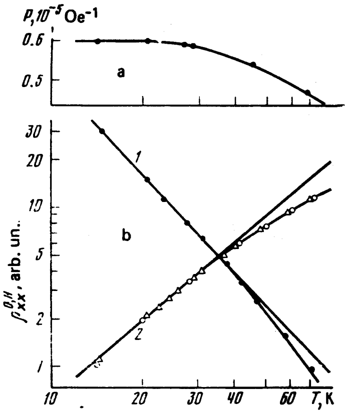

FIG. 6. a) Temperature dependence of the period of the HTO. b) Dependence of the resistivity $\rho_{xx}{}^0$ in a zero magnetic field (curve 2) and of the magnetoresistivity $\rho_{xx}{}^H$ at $H=10$ kOe (curve 1) on the temperature. The points △ and ● correspond to the present results, ○—data of Friedman's paper.[11]

relative shift of the extremum $(H_T-H_{20K})/H_{20K}$ does not depend on the angle θ. In other words, at any direction of the vector **H** the HTO period varies as a function of temperature in accordance with one and the same law.

A study of the HTO makes it possible to understand better the singularities of the temperature dependence of the electric resistance and of the magnetoresistance of bismuth (see Fig. 6, curves 1 and 2; the results obtained in Refs. 12–14 agree fully with those shown in Fig. 6). The maximum effective Debye temperature $\Theta_D^* = 2p_F s/k$ for bismuth, which corresponds to the value of the Fermi momentum along the bisector axis $p^e = 76\cdot 10^{-22}$ cgs esu,[15] is ≈11 K for the transverse sound ($s=1.02\times 10^5$ cm/sec) and ≈28 K for longitudinal sound ($s=2.57\times 10^5$ cm/sec) propagating along C_1.[16] Therefore the function $\rho_{xx}^0 \sim T^{1,2}$ in the interval 50–70 K (Fig. 6) can be naturally connected with scattering by acoustic phonons at $T>\Theta_D^*$, when $\tau\sim T^{-1}$.

On the other hand, since $\rho_{xx}^H\sim\tau$, the deviation of the MR from the low-temperature law T^{-2} (Fig. 6) should be towards T^{-1}, whereas experiment reveals a relation $\rho_{xx}^H \sim T^{-2,7}$ for $T=50$–70 K (see curve 1 of Fig. 6). This contradiction can be reconciled by making use of the data obtained in the study of HTO. Thus, it follows from Fig. 6 that the deviations of ρ_{xx}^0 and ρ_{xx}^H from the low-temperature dependence takes place practically simultaneously with the start of the temperature-induced change of the HTO period, i.e., of the carrier density. Therefore the equations for the electric resistance and the magnotoresistance in the 50–70 K range must be written in the form

$$\rho^0(T)\sim\frac{m(T)}{n(T)\tau(T)}\sim T^{1,2},\quad \rho^H(T)\sim\frac{\tau(T)}{n(T)m(T)}\sim T^{-2,7},$$

and this yields $m\sim T$ and $n\sim T$ at $\tau\sim T^{-1}$.

According to the results of Ref. 17, in the interval 50–70 K the cyclotron mass of the electrons is $m^*\sim T^{0.5}$, which agrees qualitatively with the estimate obtained above for the temperature dependence of the effective mass. Thus, if we take into account the temperature dependence of the concentration and of the effective mass, the electric resistance and the magnetoresistance of bismuth at $T=30$–70 K can be described by taking into account the interaction of the carriers with only the acoustic phonons. This conclusion in turn turns out to be essential in the discussion of the temperature dependence of the HTO amplitude.

DISCUSSION OF RESULTS

From the general point of view we regard the HTO as the result of a resonant interaction between the carriers and some quasiparticles having energies 20–30 meV. Inasmuch as at $kT\ll 10$ meV the HTO amplitude has a rising section (Fig. 1), i.e., the interaction probability, which is proportional to the number of excitations, increases with increasing temperature, the quasiparticles we deal with should obey Bose-Einstein statistics.

The singularities of the HTO period at $\mathbf{H}\parallel C_1$, and also the location of the maximum of the oscillation amplitude on the temperature scale (Fig. 1, inset in Fig. 2) are quantitatively described by a resonance of type (1) with a characteristic energy ≈20 meV which does not depend on a magnetic field and on the temperature. However, the HTO amplitude, as seen from Fig. 2, does not satisfy the relations corresponding to this type of resonance.

The temperature and angular dependences of the HTO amplitude can be qualitatively described with the aid of the resonance condition (5) with a characteristic energy $\varepsilon_0 = e\hbar/m^*cP\approx 32$ meV, a condition connected with the transitions between the Landau levels at the points $k_z=0$. In fact, using formally the results of Ref. 18, which contains approximate formulas for the amplitude of the oscillations of the transverse MC of a degenerate semiconductor for carrier interaction with optical phonons at a certain end-point frequency ω_0^ℓ, assuming as before that $\tilde\rho\sim\tilde\sigma/\sigma^2$, and assuming further (see above) that the decisive contribution to the monotonic part of the MR is made by scattering by acoustic phonons, i.e.,

$$\sigma\sim ne^2\tau/m(\Omega_c\tau)^2,$$

we obtain[3)]

$$\tilde\rho\sim\frac{1}{2kT}\left(\frac{eH}{c}\right)^3\left(\frac{\omega_0\tau}{n\pi\hbar}\right)^2\frac{\hbar\omega_0}{kT}\exp\left\{-\frac{\hbar\omega_0}{kT}\right\}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)\quad(10)$$

($\varkappa_\infty$ and $\varkappa_0$ are the high-frequency and static dielectric constants of the lattice, and $\hbar\omega_0=\varepsilon_0$).

It follows from (10) that the derivative $\partial\ln\tilde\rho/\partial T$ does not depend on the magnitude or direction of the magnetic-field vector, and the amplitude itself is not connected with the effective mass. This agrees fully with the experimental data. If we put, in accord with the estimates given above, $n\sim T$ and $\tau\sim T^{-1}$ in the range 50–70 K, and $n=$const, $\tau\sim T^{-3}$ at lower temperatures, then Eq. (10) describes a curve with a maximum near 60 K (we know of no data on the temperature dependence of the dielectric constant). Plotting the descending section of the curve in the interval 65–85 K shows that, just as in the experiment, the damping of the MR oscillations can be interpolated by an exponential function, but with an argument 0.02 deg^{-1}.

The theoretical and experimental positions of the maximum on the temperature scale, as well as the rate of damping, differ by a factor 5–6. It must be remembered, however, that the approximate formula used by us to calculate $\tilde{\rho}$ was derived for the simplest model of an electron energy spectrum, and without allowance for the smearing of the resonance due to the temperature excitation of the quasiparticles of the entire branch of the spectrum, and not only with energy ε_0. It should also be remembered that the characteristic energies calculated from the period of the HTO for resonance conditions (1) and (5) are of the same order as the Fermi energy of the carriers in the bismuth. For this reason, when considering the resonance transitions (1) and (5), account must be taken of the Fermi-liquid effects, and in particular of the summary width of the Landau levels, i.e., the smearing of the initial and final states. The MPR theory,[18] however, was developed for $\varepsilon_F \gg \varepsilon_0$.

At a fixed value of θ, the ratio of the amplitudes of the MR oscillations corresponding to the electron and hole Fermi surface of bismuth, for the resonance (1), is approximately equal to

$$\exp\left\{-\frac{2\pi^2 kTc}{\hbar eH}(m_e^*-m_h^*)\right\}, \tag{11}$$

which yields, for example, under the conditions $\theta=50°$ ($m_e^*\approx 0.016m_0, m_h^*\approx 0.085m_0, T\sim 20$ K, $H\sim 10^4$ Oe, the tremendous value ~10^8–10^9. Therefore the registration of HTO connected exclusively with holes at $\theta>10°$ attests one more to the independence of the HTO amplitude of the cyclotron frequency, at any rate in accordance with (11).

This repeatedly confirmed result can serve as a criterion for selecting the type of resonance, that it is necessary to prove first that the Fermi-liquid interaction that broadens the energy levels does not lead to a transformation of Eqs. (4) and (11) into

$$\tilde{\rho}\sim\exp\left\{-\frac{2\pi^2 k[T+T_D^*(T)]}{\hbar\Omega_c}\right\},$$

where $T_D^*(T)\gtrsim T, T_D^*\sim\Omega_c$. It is clear that this transformation weakens substantially the dependence of the oscillation amplitude on the cyclotron frequency, and in our experiments the resonances (1) and (5) may prove to be indistinguishable.

The results of optical investigations[19,20] point to the possible existence in bismuth of quasiparticles with characteristic energy close to 32 meV, i.e., to the value given by the period of the HTO (see Fig. 7, which gives the results of various studies to the passage of light through single-crystal bismuth plates; minima of light transmission are clearly seen near 34 meV; the transmission flashes at lower energies (19.6 and 23.2 meV) correspond to plasmon excitation). This circumstance, together with the singularities produced in the period by the influence of the temperature and of the quantizing magnetic field on the carrier density n, has prompted us to search for an energy, of corresponding scale, connected with the carrier density. Thus, an attempt was made to describe, with the aid of a resonance of type (5) with a characteristic energy[4)] $\hbar(\omega_p+\omega_p^0)$, the experimentally observed evolution of the curve of the HTO magnetic fields up to 60 kOe.[4] This attempt, however, did not lead to an unambiguous result. On the one

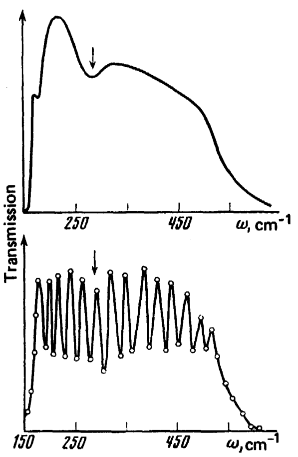

FIG. 7. Transmission spectrum of bismuth according to the data of Refs. 19 and 20. The arrow shows the maximum at $\hbar\omega$ 34 meV.

hand, classical calculation yields too strong an $\omega_p(H)$ dependence (see the Appendix), which does not fit the experimental data for $\mathbf{H}\parallel C_1$. On the other hand, the calculation accuracy may be low because the ultraquantum limit is reached for 3 electron ellipsoids at $\mathbf{H}\parallel C_1$ and for 2 ellipsoids at $\mathbf{H}\parallel C_2$ in magnetic fields larger than 20 kOe. Thus, to solve our problem exactly we need a special theoretical analysis of the tensor of the high-frequency conductivity of bismuth in the ultraquantum limit.

So far we have considered the resonances (1) and (5) within the framework of the ellipsoidal parabolic (EP) model of the electron spectrum. Allowance for nonparabolicity of the hole band (the ENP model) even in the absence of presently established notions concerning the values of the energy gaps in the spectrum of bismuth[5] (the energy gap ε_{gT} at the point T of the Brillouin zone of bismuth, in accord with data by various workers, is 900,[5] 200,[21] or 65 meV[22]) does not exert a substantial influence on the resonance (1). Thus, the ratio (3) becomes

$$P^{ENP}=e\hbar\left(1+\frac{2\varepsilon_F}{\varepsilon_g}\right)\Big/ m^*c(\varepsilon_F+\varepsilon_{01})\left(1+\frac{\varepsilon_F+\varepsilon_{01}}{\varepsilon_g}\right), \tag{12}$$

and this yields at $\varepsilon_F\approx 12$ meV and ε_{gT} from Refs. 5, 21, and 22 the respective values $\varepsilon_{01}=20.2, 19.5, 18.4$ meV.

This situation is entirely different in the case of the resonance (5). In the analysis of the influence of the nonparabolicity of the band on the given type of resonance it is convenient, using fixed parameters of the energy spectrum, to choose the resonance frequency such as to satisfy the experimental results. This approach is necessitated by the non-equidistance of the Landau levels in a magnetic field. The resonance condition (5) for the ENP model is of the form

$$\varepsilon_M+\varepsilon_0=\varepsilon_N, \tag{13}$$

ε_M and ε_N are the "start" and "finish" energy levels and are determined from the equation

$$\varepsilon_N\left(1+\frac{\varepsilon_N}{\varepsilon_{gT}}\right)=\left(N+\frac{1}{2}\right)\frac{e\hbar H}{m^{**}c}, \tag{14}$$

m^{**} is the cyclotron mass at the bottom of the band (see Fig. 8 which shows schematically the shift of the Landau

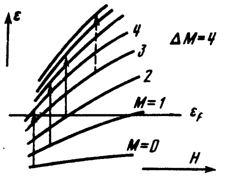

FIG. 8. Schematic diagram of the shift of the Landau levels in a magnetic field in accord with (14).

levels in a magnetic field and the transitions at $\Delta M = N - M = 4$; the transition shown by the dashed arrow is not realized, inasmuch as the Landau levels with the required energy lie above ε_F).

It turns out that the experimental results can be justified by putting $\varepsilon_{gT} = 900$ meV at $\hbar\omega_0 \approx 30$ meV (rounded-off to simplify the calculations). For example, in the case $\mathbf{H} \parallel C_3$, calculation (at $T = 0$ K) yields several series of resonance peaks with distances between the starts of the series—periods—which coincide with the oscillation period $P = e/m^* c\omega_0 = 0.6 \cdot 10^{-5}$ Oe^{-1} calculated in the EP model, and which exceed considerably (by ~15 times) the distance between the peaks in the series (the value of the latter is determined at $\varepsilon_0 \ll \varepsilon_{gT}$ by the formula $P_1 \sim 2e\hbar / m^{**} c\varepsilon_{gT}$). It is clear that under real conditions the series of the peaks become smeared out into equidistant lines (separated by 0.6×10^{-5} Oe^{-1}, the period of the HTO). At values $\varepsilon_{gT} = 200$ and 65 meV, variation of the resonance frequency within reasonable limits (0–50 meV) leads to periods that are far from the experimental value.

The foregoing analysis shows that the absence of established concepts concerning the values of the energy gaps in the spectrum of bismuth is one more obstacle to the interpretation of the HTO. Thus, if we assume that the true value of ε_{gT} is either 200 or 65 meV, then in the interpretation of the HTO we can exclude from consideration the resonance of type (5) since it does not agree with the experimental data. If $\varepsilon_{gT} = 900$ meV, the entire situation is completely different. Moreover, from the point of view of the resonance (5), (13), assuming that $\hbar\omega_0 \approx 30$ meV, we can explain why no HTO connected with the electron Fermi surface are observed in a wide range of angles θ (see Fig. 4a), and also the complicated picture of the oscillation curves in the region $3° < \theta < 8°$ (see the inset of Fig. 5). In fact, calculation of the resonant values of the magnetic field, carried out at $\mathbf{H} \parallel C_3$, $T = 0$ K, $\varepsilon_F^e = 25$ meV, $\varepsilon_{gL} = 50$ meV (the energy gap at the L point of the Brillouin zone of bismuth), and $m^{**} = 0.0144 m_0$, at first glance does not lead at all to a periodic dependence of the resistance on the reciprocal magnetic field. Only by a careful examination of the picture of the resonance peaks does it become possible to separate magnetic-field regions with different densities of the peaks, and the average distance between the centers of the regions with high density is close to 0.6×10^{-5} Oe^{-1}, i.e., to the value of the HTO period at $\mathbf{H} \parallel C_3$. Since the Landau levels have finite width, the resonance peaks become smeared out. The line width can substantially overlap the distance between the peaks in the regions of their largest concentration, and this leads to a nonmonotonic dependence of the resistance on the magnetic field. Thus, the concept of the period of the oscillations connected with the electrons is for the considered model to some degree arbitrary. At the same time, it is extremely important that the numerical value of this arbitrary period agrees with experiment.

At $\theta \neq 0°$ the picture of the electronic oscillations should become even more complicated for the following reasons: 1) At an arbitrary direction of $\mathbf{H}$, the contribution to the resonance is made generally speaking not by one but by three electron "ellipsoids" (two on Fig. 4a) with different cyclotron masses. 2) The observation of HTO beats connected with holes (Fig. 4) may be the result of the existence of not one but two close resonant frequencies. 3) Finally, account must be taken also of the superposition of the hole periods. An impression is gained that for the considered resonance model the situation in which MR oscillations connected with electrons can be observed is extremely rare and the resultant oscillation picture is extremely complicated.

Thus, when it comes to attributing the HTO to resonant scattering of the carriers, the experimental results admit of an ambiguous treatment, caused to a considerable degree by the absence of the necessary theoretical premises, and also to the opinions held concerning the values of the energy gaps in the electron spectrum of bismuth. The final answer to the crucial questions of the magnitude and origin of the resonant energy has not yet been found, although, on the basis of the HTO singularities connected with the electrons, we give preference to a resonance of type (5). Quite promising for the understanding of the nature of the HTO may be optical investigations of bismuth in the infrared region.

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APPENDIX

The question of the dependence of the plasma frequency on the magnetic field reduces to a solution of the dispersion equation $\mathrm{Det}|\varkappa_{ik}| = 0$ at different directions of the vector $\mathbf{H}$. To determine the components of the high-frequency conductivity tensor σ_{ik}, which enter in the dispersion equation, we have used corresponding expressions for the static conductivity, obtained in the classical approximation,[23] making the substitution $1/\tau \to i\omega$ (the collisionless case). Thus, the problem was solved in the local limit ($k_p l \ll 1$, where k_p is the wave vector of the plasmon and l is the carrier mean-free path), and without allowance for quantization. The numerical values of the static dielectric constant of the lattice ($\varkappa_{0x} = \varkappa_{0y} = 106, \varkappa_{0z} = 69$) were obtained by substituting in the dispersion equation (for the case $H = 0$) the experimentally measured frequencies of the plasma resonance[5] and of the effective masses calculated in the ellipsoidal model from the experimental values of the cyclotron masses.[5] The simplest situation arises at $\mathbf{H} \parallel C_3$. In this case the equation for the determination of the plasma frequencies takes the form

$$1 - \frac{(\omega_p')_x^2}{\omega^2 - \Omega_c^2}\left(\alpha_{xx}^h + \frac{\alpha_{xx}^e + \alpha_{yy}^e}{2}\right) = 0; \quad \mathbf{E} \parallel C_1, C_2,$$
$$1 - \frac{(\omega_p')_z^2}{\omega^2}\left[\alpha_{zz}^e + \alpha_{zz}^h + \frac{\alpha_{xx}^e (\alpha_{yz}^e)^2 \Omega^2}{\alpha_{zz}^e (\omega^2 - \Omega_c^2)}\right] = 0, \quad \mathbf{E} \parallel C_3, \qquad (15)$$

where

$$\Omega=\frac{eH}{m_0c},\quad \Omega_c=(\alpha_{xx}^e\alpha_{yy}^e)^{1/2}\Omega,\quad (\omega_p')_i=\left[\frac{4\pi n(H)e^2}{m_0\varkappa_{0i}}\right]^{1/2},$$

m_0 is the mass of the free electron, $\alpha_{ik}^{e;h}$ are the components of the tensor of the reciprocal effective masses, $n(H)$ is taken in accord with Édel'man's data,[5] and this, in particular, leads to oscillations of $\omega_p(H)$. When $\mathbf{H}\parallel C_3$, the exact solution of the dispersion equation, which we shall not write out here, could be obtained only for $\mathbf{E}\parallel C_2$. At $\mathbf{H}\parallel C_1$ the angle of inclination of the electron ellipsoids in the trigonal-bisector plane was assumed equal to zero and the dispersion equations were solved with a computer.

The minimum value of the plasma frequency in the interval 0–60 kOe for the $\mathbf{H}\parallel C_1$ direction, in contrast to the $\mathbf{H}\parallel C_3$ orientation, when ω_p increases by not more than 20%, is quite large and amounts to ≈100%.

1) The authors thank L. I. Zarubin for kindly placing the resistance thermometer at their disposal.

2) The two values of the period in the region $\theta=10$ and 20° (see Fig. 4a) correspond to different magnetic-field ranges, the smaller period being determined in the larger field. At $\theta=70°$, the two values of P are given by different measurement methods—using the derivative $\partial\rho_{xx}/\partial H$ and ρ_{yx}.

3) In the derivation of (2) it was taken into account that at $\varepsilon_F<\hbar\omega_0$ the contribution to the resonance is made by all the carriers, from the bottom of the band to ε_F. For this reason, the broadening of the Landau levels is determined by the probability of inelastic scattering, i.e., by the number of excitations and by the cyclotron frequency. In fact, the larger Ω_c the smaller the number of levels located below ε_F. Thus, the carrier density at each Landau level increases, and with it the scattering probability.

4) There can be no optical-frequency renormalization of the type $\omega_0^{p*}=(\omega_{p0}^2+(\omega_0^p)^2)^{1/2}$ in bismuth, since the lattice is made up of one species of atoms (ω_{p0} is the end-point plasma frequency at $H=0$).

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