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Velocity dependence of the density of the normal component, and hydrodynamics of superfluid flow at high velocities

L. V. Kiknadze and Yu. G. Mamaladze

Physics Institute, Georgian Academy of Sciences
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It is shown that the dependence of the densities of the normal and superfluid components of helium II on their relative flow velocity gives rise to a velocity dependence of the type of the equation to which the hydrodynamics of the superfluid flow reduces. As a result, hyperbolicity regions enclosed in the ellipticity region of the equation for the velocity potential (similar to "supersonic zones in subsonic flow") can arise in nonuniform flow around corners and roughnesses. The critical velocity of the transition from the elliptic type to the hyperbolic type is determined and its temperature dependence is calculated.

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We discuss in this article the circumstances that reveal certain singularities of supersonic-flow hydrodynamics, which escape attention when an analogy is drawn with the flow of an ideal classical incompressible liquid. For example, it must be assumed that when a superfluid flows around sharp corners or convexities on a solid surface there exist in the vicinities of the latter regions where the continuity equation changes from elliptic to hyperbolic. These regions are similar to the supersonic inclusions in superfluid flow, which are know from gasdynamics (see, e.g., Refs. 1 and 2), and whose presence is accompanied usually by formation of discontinuity surfaces (shock waves[3]). It seems to us that these circumstances are of importance for the hydrodynamics of helium II in general, and when vortex formation is considered in particular. Their physical basis is the dependence of the density of the normal component on the relative velocity of the components, as determined by Khalatnikov.[4]

1. To demonstrate the hydrodynamically important consequences of the velocity dependence of the components of helium II, we confine ourselves to the relativly simple case of stationary isothermal flow of the superfluid component while the normal component is at rest ($v_n = 0$) and in the absence of an external-force field. The system of equations of two-velocity hydrodynamics[4,5] reduces in this case to the equations that specify the potential character and the continuity of the flow:

$$\operatorname{rot} \mathbf{v}_s = 0, \tag{1}$$

$$\operatorname{div} \mathbf{j}_s = 0, \tag{2}$$

where $\mathbf{j}_s = \rho_s \mathbf{v}_s$. The continuity equation for the entropy is satisfied identically. The continuity equation for the momentum and the equation of motion of the superfluid component yield one and the same equation, which determines the pressure gradient[1].

$$\operatorname{grad} P = -\rho_s \operatorname{grad} ({}^1/_2 v_s^2). \tag{3}$$

The system of hydrodynamic equations is supplemented by the dependence of ρ_s on v_s, the concrete form of which is immaterial for the time being (see Sec. 4 below).

2. Equation (2) can be transformed in the following manner:

$$\rho_s \operatorname{div} \mathbf{v}_s + \mathbf{v}_s \operatorname{grad} \rho_s = 0, \tag{2a}$$

or

$$\rho_s \operatorname{div} \mathbf{v}_s + \frac{d\rho_s}{dv_s} \mathbf{v}_s \operatorname{grad} v_s = 0, \tag{2b}$$

where $v_s \equiv |\mathbf{v}_s|$ and $d\rho_s/dv_s = \partial\rho_s/\partial v_s + (\partial\rho_s/\partial P)(dP/dv_s)$, and according to (3) we have $dP/dv_s = -\rho_s v_s$. It follows from the last two equations that the solutions of the Laplace equation that holds for an incompressible fluid

$$\Delta\varphi = 0, \tag{4}$$

where φ is the velocity potential ($\mathbf{v}_s = \text{grad}\,\varphi$, $\Delta\varphi = \text{div}\,\mathbf{v}_s$), are applicable only in those cases when the stream lines coincide with the lines $\rho_s = \text{const}$ or, equivalently,[2)] with the lines $v_s = \text{const}$. We note that the latter coincide with the lines $P = \text{const}$.

In this connection, the presence of a dependence of ρ_s on v_s does not contradict the fact that the only realized flow regimes are described by the incompressible-liquid equations that satisfy the just-obtained condition $\mathbf{v}_s\,\text{grad}\rho_s = 0$ or $\mathbf{v}_s\,\text{grad}v_s = 0$. These regimes are: 1) potential rotation around one vortex filament $\varphi = (\Gamma/2\pi)\alpha$ [Γ is the circulation and $\alpha = \tan^{-1}(y/x)$], 2) uniform flow, $\varphi = v_s x (v_s = \text{const})$.

Opposite examples are: flow of an incompressible liquid containing a pair of vortices, $\varphi = (\Gamma/2\pi)(\alpha_1 - \alpha_2)(\alpha_{1,2} = \tan^{-1}y/(x \mp x_0)$, $\pm x_0$ are the coordinates of the vortex and of the antivortex), flow around a right corner, $\varphi = A r^{\frac{2}{3}} \cos(2\alpha/3)$ ($r^2 = x^2 + y^2$, $A = \text{const}$). These solutions of Eq. (4), which correspond to stream lines that are not constant-velocity lines, do not satisfy Eq. (2), and can not describe the flow of a superfluid liquid at velocities such that the $\rho_s(v_s)$ dependence is significant. Included among these examples are solutions of Eq. (4) that describe flow around convexities on a flat surface.

The circumstances noted above are worthy of attention also for the following reason. It is known that very plausible general considerations and convincing experiments (see, e.g., Refs. 9–11) confirm the assumption that vortex formation in helium II is localized on inhomogeneities of the boundary surfaces, such as corners and roughnesses. A hydrodynamic analysis, however, based as usual on Eq. (4), shows that a vortex produced in the vicinity of a solid surface can be carried away by the flow into the interior of the liquid (and not towards the wall, where it would be absorbed) only from a macroscopic distance (even if a large inhomogeneity is present on the surface; see, e.g., Ref. 12). The problem raised by this contradiction must apparently be solved outside the framework of the hydrodynamics of incompressible liquids.

We advanced similar assumptions in Ref. 2, where the treatment was within the framework of the phenomenological theory of superfluidity.[6] It will be shown below that its conclusions are not limited to the region of applicability of this theory[3)] and are confirmed by the Landau theory.[4,5]

3. We replace Eq. (4) by a more general equation for the velocity potential φ, which follows from (2b):

$$\sum_{i=1}^{3}\sum_{k=1}^{3} a_{ik}\frac{\partial^2\varphi}{\partial x_i \partial x_k} = 0, \tag{5}$$

where

$$a_{ik} = \rho_s\delta_{ik} + \frac{1}{v_s}\frac{d\rho_s}{dv_s}\frac{\partial\varphi}{\partial x_i}\frac{\partial\varphi}{\partial x_k}.$$

The discriminant of the corresponding equation of characteristics can be readily shown, after elementary but somewhat unwieldy transformations, is equal to[4)]

$$|a_{ik}| = \rho_s^2\frac{dj_s}{dv_s} \quad (j_s \equiv |\rho_s \mathbf{v}_s|). \tag{6}$$

Consequently, the type of equation (5) depends on the sign of the derivative dj_s/dv_s: it is elliptic at $dj_s/dv_s > 0$, parabolic at $dj_s/dv_s = 0$, and hyperbolic at $dj_s/dv_s < 0$. We shall return to this question in Sec. 5.

The critical velocity v_s at which the transition from the elliptic to the parabolic type takes place will be designated v_t. The surface (line) on which $v_s = v_t$ is called the transition surface.

4. Thus, the type of Eq. (5), and consequently also the properties of the solutions of the system (1) and (2) (the character of their dependence on the boundary conditions), are determined by the concrete form of the function $j_s(v_s)$, which are specified in turn by the function $\rho_s(v_s)$. We examine now the latter without the simplifying restriction used in Secs. 1-3.

The dependence of the density of the normal component ρ_s on the relative velocity $u = |\mathbf{v}_n - \mathbf{v}_s|$ is determined by the already mentioned Khalatnikov formula[4]

$$\rho_n(u) = \rho_r f_1(u) + \rho_{ph} f_2(u), \tag{7}$$

where

$$f_1(u) = \frac{3}{2}\frac{e^x(x-1) + e^{-x}(x+1)}{x^3}, \quad x = \frac{p_0 u}{kT},$$
$$f_2(u) = (1 - u^2/u_1^2)^{-3},$$

ρ_r and ρ_{ph} are the known contributions of the rotons and phonons to the normal density at $u = 0$, p_0 is the momentum of the immobile roton, and u_1 is the velocity of the first sound.

The density ρ_s, equal to $\rho - \rho_n$, depends correspondingly on the relative velocity u. The function $\rho_s(u)$ determines two critical velocities: the maximum velocity, which is compatible with the superfluidity v_m

$$\rho_s(v_m) = 0 \tag{8}$$

and the velocity v_t, at which

$$\left[\frac{d}{du}\rho_s(u)u\right]_{u=v_t} = 0. \tag{9}$$

If $v_n = 0$, then the last formula coincides with the equation $dj_s/dv_s = 0$ that determines the transition of Eq. (5) from elliptic to hyperbolic. The quantity $j_s \equiv \rho_s(u)$ $u = \rho_s|\mathbf{v}_s - \mathbf{v}_n|$ is the modulus of the flux density of the superfluid component or its momentum (per unit volume) in a reference frame tied to the normal component.

The velocities v_m and v_t at different temperatures can be determined by numerically solving Eqs. (8) and (9). The velocity v_m was already calculated in similar fashion in Refs. 13 and 14 (see also Refs. 16 and 17). Simultaneously with the calculation of v_t we repeated also the calculation of v_m at various temperatures in the interval 0.2–2.1 K. We used Eq. (7) and the experimental data, gathered in Wilks's book,[15] on the parameters u_1, p_0, μ, and Δ of the energy spectrum of helium III, and also on the total density ρ, which are needed to tabulate ρ_r and ρ_{ph} and to convert from ρ_n to ρ_s.

In view of the lack of sufficiently accurate and detailed information on the temperature dependence of the derivative $\partial(\rho_n/\rho)/\partial P$ we have neglected in the numerical calculations (see below) the u-dependence of the total density ρ (as is well known, $\partial\rho/\partial u^2 = 0.5\rho^2\partial(\rho_n/\rho)/\partial P$) and the pressure dependence of the parameters of the energy spectrum of helium II [this is equivalent to replacing the total derivative $d\rho_s/dv_s$ by the partial derivative $\partial\rho_s/\partial v_s$, which differs from the former by the amount given following Eq. (26) in Sec. 2]. It can be shown that (in the temperature and velocity interval of interest to us) the errors due to this neglect does not exceed several percent.

In addition to the temperature dependences of v_m and v_t (Fig. 1), we plotted the functions $\rho_s(u)$, (Fig. 2), $j_s(u)$ (Fig. 3), and $\rho_s(j_s)$ (Fig. 4) at various temperatures.

It must be emphasized that in the units in which the curves of Figs. 2, 3, and 4 were plotted, the dimensionless quantities ρ_s/ρ_{s0} and $j_s/\rho_{s0}v_m$, as well as the ratio v_t/v_m, ($\rho_{s0} \equiv \rho_s(0)$) practically coincide with the results of the calculations by the phenomenological superfluidity theory even at $T = 2.1$ K (for details see Sec. 6).

5. We turn now to a discussion of Eq. (5) (we recall that it pertains to the case $v_n = 0, u = v_s$). It can be reduced by the coordinate transformation $x, y, z \to \xi, \eta, \zeta$ to the form

$$\lambda_1\frac{\partial^2\varphi}{\partial\xi^2}+\lambda_2\frac{\partial^2\varphi}{\partial\eta^2}+\lambda_3\frac{\partial^2\varphi}{\partial\zeta^2}=0, \tag{10}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the roots of the equation $|a_{ik} - \lambda\delta_{ik}| = 0$, which can also be solved directly, but we shall write out only the connection between the quantities $\lambda_1, \lambda_2, \lambda_3$ and the invariants of the quadratic form

$$\sum a_{ik}x_i x_k \quad (i, k = 1, 2, 3),$$

which are relatively easy to calculate:

$$\lambda_1\lambda_2\lambda_3 = |a_{ik}| = \rho_s^2 dj_s/dv_s, \tag{11}$$

$$\begin{aligned}\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 &= A_{33}+A_{22}+A_{11}\\ &\equiv a = \rho_s(\rho_s+2dj_s/dv_s),\end{aligned} \tag{12}$$

$$\lambda_1+\lambda_2+\lambda_3 = a_{11}+a_{22}+a_{33} \equiv S = 2\rho_s + dj_s/dv_s, \tag{13}$$

where A_{ik} are the minors of a_{ik} in the determinant $|a_{ik}|$.

At $v_s = 0$ Eq. (7) yields $d\rho_s(v_s)/dv_s \propto v_s = 0$ and $dj_s/dv_s = \rho_s$. In this case the fact that all three coefficients $\lambda_1, \lambda_2, \lambda_3$ in (10) are positive follows directly from Eq. (10), which goes over in this limit into the Laplace equation [$\lambda_1 = \lambda_2 = \lambda_3 = \rho_s$ in (10)]. At small v_s expression (11) remains positive. It vanishes before expressions (12) and (13), which remain positive at $dj_s/dv_s = 0$, since $\rho_s(v_t) > 0$. It follows therefore that at $v_s = v_t$ only one of the coefficients $\lambda_1, \lambda_2, \lambda_3$ vanishes, and at $v_s > v_t$ it reverses sign and Eq. (10) [and with it also (5)] becomes hyperbolic. Since the function $j_s(v_s)$ (Fig. 3) has only one extremum, reversal of the sign of another coefficient is already impossible in the entire interval $v_t < v_s < v_m$, where Eqs. (10) and (5) are thus properly hyperbolic.

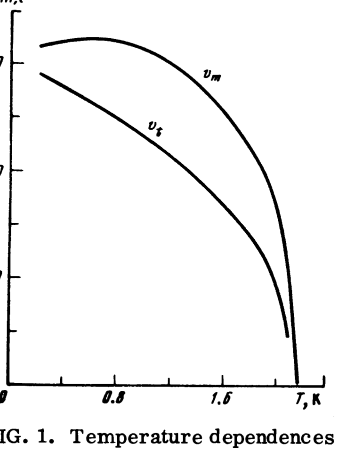

FIG. 1. Temperature dependences of the critical velocities: of the break of the superfluidity (v_m) and of the transition from the elliptic type to hyperbolic (v_t).

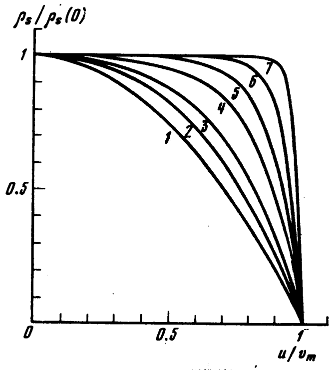

FIG. 2. Dependence of the density of the superfluid component ρ_s on the relative flow velocity u of the components at various temperatures: 1—T = 2.1 K, 2—1.8 K, 3—1.5 K, 4—1.1 K, 5—0.8 K, 6—0.5 K, 7—0.2 K.

6. In the phenomenological theory of superfluidity the function $\rho_s(u)$ is defined by the formula[6]

$$\rho_s = \rho_s(0)\,(1 - u^2/v_m^2) \tag{14}$$

and $\rho_s = 0$ at $v = v_m$, where[6,16,17]

$$v_m = 5.76\cdot 10^3 (T_\lambda - T)^{1/3}\,[\text{cm/sec}]. \tag{15}$$

In this theory the critical velocity v_t turns out to be[5)]

$$v_t = 3^{-1/2} v_m = 3.33\cdot 10^3 (T_\lambda - T)^{1/3}\,[\text{cm/sec}]. \tag{16}$$

It was first introduced in Ref. 18, where it was shown that at a given value of j_s the states with $u > v_t$ (the lower branches of each of the $\rho_s(j_s)$ plots of Fig. 4) are unstable (but they are stable for a given u). At $u = v_t$

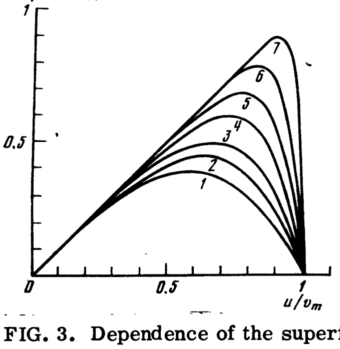

FIG. 3. Dependence of the superfluid flow $j_s = \rho_s u$ on the relative velocity u at the various temperatures listed in the caption of Fig. 2.

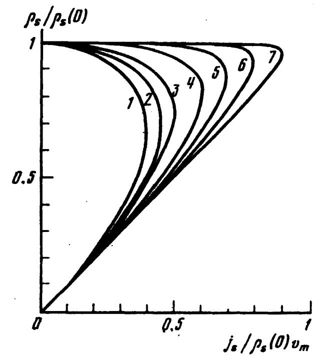

FIG. 4. Dependence of the superfluid component ρ_s on the flux j_s at the various temperatures listed in the caption of Fig. 2.

we have

$$j_s(v_t)=j_{max}=j_t=2\rho_{sb}v_m/3\sqrt{3},\quad \rho_s(v_t)=\rho_{st}=2\rho_{sb}/3.$$

Plots of the dimensionless ratios ρ_s/ρ_{sb} and $j_s/\rho_{sb}v_m$ at $T=2.1$ K, shown in Figs. 2–4, practically coincide with the calculations by Eq. (14), and the ratio v_m/v_t is practically equal to $\sqrt{3}$ at 2.1 K. We note that this almost complete coincidence is observed just for the indicated dimensionless ratios rather than for the calculated quantities themselves, but even the latter are in sufficient agreement. For example, at $T=2.1$ K we have according to (15) $v_m \approx 10.1$ m/sec, whereas Khalatnikov's formula yields $v_m \approx 14.6$ m/sec.

Formulas (14) and (16) are similar to the gasdynamic relations (see, e.g., Ref. 19) between the density ρ and the velocity of an ideal gas, the speed of sound c_* at the point where it is equal to the gas velocity, and the maximum gas velocity v in adiabatic flow. The quantities ρ_s, u, v_t, and v_m correspond to the quantities ρ, v, c_*, and $v_{\max}$, and it is necessary to put $c_P/c_V \equiv \gamma = 2$. It must be emphasized that v_t is not a complete analog of the speed of sound. In particular, there is no dependence of v_t on u similar to the dependence of the speed of sound on the flow velocity in gasdynamics. The velocity v_t is the analog of c_* only in the sense that it is the boundary between the ellipticity and hyperbolicity regions of Eq. (5), in analogy with the subsonic and supersonic regions in gasdynamics. The incomplete analogy between a gas and the superfluid component of helium II (even as $T \to T_\lambda$) manifests itself also in the absence of an inflection point on the $j_s(u)$ curve (Fig. 3), unlike the $j(v)$ plot (cf. Fig. 42 of Ref. 19).

7. Using two-velocity hydrodynamics, we regard the function $\rho_s(u)$ and those ensuing from it (Figs. 2–4) as locally valid at any point of the inhomogeneous flow, whose inhomogeneity is thus connected only with the inhomogeneity of the velocity distribution.

In the phenomenological theory of superfluidity, as already noted in Sec. 2 (see footnote 2), ρ_s depends not only on u but also directly on the spatial coordinates. Equation (14), which is analogous to (7), is valid only in the case of uniform flow. In non-uniform flow Eq. (14) is replaced by a differential equation that takes at $v_n = 0$ the form

$$\Delta f + f - f^3 = (\nabla\varphi)^2 f, \tag{17}$$

where $f^2 = \rho_s/\rho_{sb}$, $v_s = (\hbar/m)\text{grad}\,\varphi$, $\Delta \equiv \nabla^2$ and ∇ are the operators of differentiation with respect to the coordinates measured in units of the coherence length $\xi = 2.73\cdot 10^{-8}(T_\lambda - T)^{-2/3}$ [cm] (see Ref. 17).

Together with (2), which can be rewritten in the form

$$\nabla(f^2\nabla\varphi)=0, \tag{18}$$

Eq. (17) makes up a system shown in Refs. 7 and 8 to be always elliptical.

It appears that the difference thus observed between the Landau two-velocity hydrodynamics and the hydrodynamics of the phenomenological theory may in practice not be as great as might appear at first glance. The point is that the volume of the liquid in which substantial changes of the modulus of the wave function take place is bounded by layers of thickness of the order of the coherence length ξ (we have in mind the layers at the walls and the layers in which are concentrated the abrupt changes of the velocity and density—the discontinuity "surfaces"). With increasing distance from this layer Δf falls off exponentially and Eq. (7) goes over into (14). Consequently the difference between the situation expected at lower temperature and the situation as $T \to T_\lambda$ reduces to the fact that in the latter case the thickness of the discontinuity "surfaces" is of the order of the interatomic distance a, whereas at $T_\lambda - T \ll T_\lambda$ their thickness ξ is much larger than a.

[1)] It is possible to use in place of (3) the Bernoulli integral of the Landau equation $\mu(P, T, v_s) + 0.5v_s^2 = \text{const}$, where μ is the chemical potential per unit mass. Only in the case of small v_s can μ be expressed in the form $\mu_0(P,T) - (\rho_n/2\rho)v_s^2$ and the result $\mu_0(P,T) + (\rho_s/2\rho)v_s^2 = \text{const}$ obtained. At higher velocities the last equation is not exact.

[2)] In the phenomenological theory of superfluidity[6] the lines $\rho_s = \text{const}$ and $v_s = \text{const}$ do not coincide in general, since ρ_s depends on the coordinates not only through its dependence on v_s, so that grad $\rho_s \neq (d\rho_s/dv_s)$ grad v_s. However, as shown in Refs. 7 and 8, even in this case the streamlines must coincide with the line $v_s = \text{const}$ if the solutions of Eq. (4) are to remain applicable.

[3)] We are glad to thank V. L. Ginzburg for a question raised in connection with our paper[8] (which contains preliminary results of Ref. 7), the answer to which is the present article.

[4)] In Refs. 7 and 8 are considered only planar flows. Equation (5) and the conclusions that follow it are free of this limitation. We note that in the case of planar flows the two-dimensional equation (5) has, in contrast to (6), a discriminant $\rho_s dj_s/dv_s$. Thus, in both cases the sign of the discriminant is determined by the derivative dj_s/dv_s, which can be rewritten with the aid of (3) in the form $dj_s/dv_s = \rho_s(1 - v_s^2/v_t^2)$, where $v_t^2 \equiv dP/d\rho_s$.

[5)] We note that the invariant a [Eq. (12)] reverses sign at $v_s(3/7)^{1/2}v_m$, as does the invariant S [Eq. (13)] at $v_s = (3/5)^{1/2}v_m$.

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Dielectric fluctuations in ultrathin metallic filaments

V. N. Prigodin

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
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The Cooper and Peierls instabilities in a metallic filament with a cross section of several atomic units are investigated. It follows in the self-consistent field approximation that although the Peierls transition temperature T_p decreases with the filament diameter exponentially, the transition can be observable because of the large pre-exponential factor. The transition itself is represented as a sequence of transitions of the separate bands. Allowance for the fluctuations in second order of the renormalization-group method, within the framework of the multicomponent Fermi-gas model, shows that below the corresponding T_c there is only short-range order in the system. The superconducting fluctuations suppress the dielectric ones near T_c. It is shown with the aid of the "bosonization" method that this effect is due to the difference between the influence of the long-wave fluctuations of the electron density on these two types of instability. The different susceptibilities and critical exponents are calculated.

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1. INTRODUCTION

Metallic filaments with diameters of several atomic units occupy, with respect to their physical properties, a position intermediate between one-dimensional and three-dimensional systems. The spacing of the discrete energy levels corresponding to different states of the transverse motion turns out in such filaments to be larger than or equal to some energy scale that is a characteristic of some considered phenomenon. This phenomenon has therefore a different behavior in a filament than in the bulk material, and turns out to be close to what should be observed in the one-dimensional case. At the same time, the large number of different transverse-motion states that participate in the phenomenon make thin filaments different from purely one-dimensional systems.

A similar physical situation is realized also in a quasi-one-dimensional system made up of one-dimensional metallic filaments that are weakly coupled.[1] There are known experimental facts that point to a similarlity between the physical properties of the two systems.[2] The three-dimensionality parameter for a quasi-one-dimensional system is the transverse coupling between the filaments. In a filament, this parameter is its diamter d. When d is large we have the well-investigated object—the bulk metal. By decreasing the filament diameter we can track the variation of certain physical properties and see how new ones appear as the filament becomes one-dimensional.

A specific feature of a one-dimensional system is its inherent Peierls instability.[3] It manifests itself in the appearance in the system of a superstructure with a period $\pi/p_0(\hbar=1)$ and is accompanied by a restructuring of the electron spectrum. A gap opens on the Fermi surface and some of the electrons lower the kinetic energy. The corresponding energy gain compensates for the loss of the elastic energy of the lattice and makes such a transition energywise favored.

In the case of a filament, the appearance in the filament of a superstructure with a period corresponding to the Fermi momentum of one of the transverse-motion states produces an energy gain mainly on account of the electrons corresponding to this state of transverse motion, since the Fermi momenta of the different transverse-motion state are not commensurate, whereas the loss in the lattice elastic energy is proportional to the number of atom in the filament

