Resonant cooperative scattering of light in field-induced splitting of atomic levels

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The theory of cooperative Raman scattering of light is considered under conditions when the exciting field splits the energy levels of the atoms. An approximation is used in which the scattering is resolved into two spectral components with Raman and steplike transition frequencies. It is shown that in the case of exact resonance of the exciting-field frequency with the atomic-transition frequency, the characteristic times and the intensity of the light scattering are equal in both components. In the case of deviation from resonance, the delay time and the intensity of the Raman component are larger than the corresponding characteristics of the steplike-transition component.

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1. INTRODUCTION

A number of papers were published recently on the experimental investigation of cooperative emission (CE) in atomic gases.[1-4] The CE phenomenon has a threshold and is observed at sufficiently high population densities of the excited levels produced within a relatively short time (shorter than the population relaxation time). This requirement is satisfied by the optical method of excitation. In all the cited papers[1-4] the excitation source was a dye laser, whose emission had a one-photon or two-photon resonance with some atomic transition. The CE was observed in this case in an adjacent transition. If the duration of the exciting pulse exceeds the characteristic time of the CE, the process evolves in the presence of a high-power exciting field and one can speak of resonant cooperative Raman scattering (RCRS). A similar situation took place, for example, in Ref. 1.

The earlier papers on the theory of cooperative Raman scattering either disregard a significant feature of resonant scattering-population of an intermediate state[5-7]—or treat the RCRS in a linear approximation, assuming the populations of the initial and final states to be constant in time.[8] In the present paper we consider the RCRS in an essentially nonlinear case, when all the atoms go over in the course of scattering from the initial to the final state. It is assumed that the amplitude of the exciting field has a steplike form in time and that the characteristic duration 2τ of the scattering pulse is much longer than the period of the oscillations of the populations of the initial and intermediate state under the influence of the exciting field.

As shown by Bowden and Sung,[8] the oscillations of the populations modulate the scattering field in time. In spectral language, the modulation corresponds to a multicomponent equi-distant structure of the scattering spectrum with frequencies $\omega = \omega_e - \omega_{21} + \frac{1}{2}\Omega \pm (\frac{1}{2} + n)\Omega_g$, where $n = 0, 1, 2, \ldots$; ω_e is the frequency of the exciting field, ω_{21} is the frequency of the forbidden transition between the initial and final states, $\Omega = \omega_e - \omega_{31}$, and ω_{31} is the frequency of the transition between intial and intermediate states (see Fig. 1). When the condition $\tau\Omega_g \gg 1$ is satisfied, the amplitude of the n-th spectral component is small as the n-th power of the small $(\tau\Omega_g)^{-1}$. We consider in this paper therefore scattering only into two spectral components $\omega = \omega_e - \omega_{21} + \Omega/2 \pm \Omega_g/2$. If we interpret the oscillations of the populations of the initial and intermediate states as the splitting of the energy levels of the atom in the exciting field into two pairs of energy quasilevels, then the employed approximation corresponds to allowance for only the spectral components of the scattering that are at resonance with the transitions between the energy quasilevels of state 3 and the final state 2 (Fig. 1).

2. DERIVATION OF BASIC EQUATIONS

We consider the RCRS process within the framework of a three-level scheme (Fig. 1). The exciting field interacts with the transition 1–3, and the scattering fields interact with transition 2–3. The volume occupied by the atoms is elongated along the z axis. The exciting field is incident at a right angle to the z axis, and the cooperative scattering takes place in the positive and negative z directions. If the characteristic scattering time t_0 is much less than the time required by the light to pass through the medium, then in the case of transverse excitation we can neglect propagation effects and consider the scattering in the spatially homogeneous approximation.[9]

The amplitude of the exciting field is specified in the form of a "step"

$$E_e = {}^1/_2\mathscr{E}_e u(t)\, e^{i\omega_e t} + \text{c.c.}, \quad u(t) = \begin{cases} 1, & t \geqslant 0 \\ 0, & t < 0 \end{cases},$$

and that of the scattering field in the form of two quasiharmonics

$$E_s = \frac{1}{2}\sum_{j=1}^{2} (\mathscr{E}_{sj}^{+} e^{-ik_j z} + \mathscr{E}_{sj}^{-} e^{ik_j z})\, e^{i\omega_j t} + \text{c.c.},$$

$$\omega_j = \omega_e - \omega_{21} + \varepsilon_j \quad j = 1, 2.$$

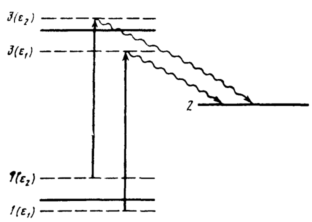

FIG. 1. Transition scheme in a three-level system.

The quantities ε_j describe the shift of the energy quasilevels of the states 1 and 3 in the exciting field, relative to the position of the unperturbed levels, and are given by

$$\varepsilon_{1,2} = -(\Omega/2 \pm \Omega_g/2), \quad (1)$$

where

$$\Omega = \omega_e - \omega_{31}, \quad \Omega_g = [\Omega^2 + 4|G_e|^2]^{1/2}, \quad G_e = d_{13}\mathscr{E}_e/2\hbar.$$

We represent the amplitudes of the states in the form

$$a_1^{(n)} = \sum_j [b_{1j}^{(n)+}(t)\exp(ik_jz_n) + b_{1j}^{(n)-}(t)\exp(-ik_jz_n)]\exp(i\varepsilon_jt),$$

$$a_2^{(n)} = b_2^{(n)}(t), \quad (2)$$

$$a_3^{(n)} = \sum_j [b_{3j}^{(n)+}(t)\exp(ik_jz_n) + b_{3j}^{(n)-}(t)\exp(-ik_jz_n)]\exp(i\varepsilon_jt),$$

where n is the number of the atom.

In the representation (2), the quantities

$$b_{mj}^{(n)+}\exp(ik_jz_n) + b_{mj}^{(n)-}\exp(-ik_jz_n)$$

have the meaning of amplitudes of states with quasienergy $\hbar\omega_j$. We assume that the characteristic time of variation of the amplitudes $b_{mj}^{(n)\pm}$ (the scattering time τ) is much less than the reciprocal $1/\Omega_g$ of the field splitting of the levels. Under this condition, the processes of scattering with participation of different quasilevels of a given state can be regarded as independent, and the equations for the amplitudes of states of different quasilevels can be uncoupled.

Scattering in the positive and negative z direction will be assumed equal, i.e., $\mathscr{E}_{sj}^+ = \mathscr{E}_{sj}^-$. We assume that all the atoms are in identical states, i.e., the amplitudes $b_{mj}^{(n)\pm}$ are the same for all atoms. The polarization density is written in the form

$$P_j^\pm(t) = Nd_{32}b_2^*b_{3j}, \quad (3)$$

where N is the density of the total number of the atoms.

We require also that the level shifts under the influence of the scattering fields be negligibly small compared with the shifts due to the interactions with the exciting field. In the case of small detunings of the exciting-field frequency from resonance, $|\Omega| \lesssim |G_e|$, the following estimates are valid

$$\Omega_g \sim 2|G_e|, \quad |G_{s\,1,2}| \sim \tau^{-1}$$

and the requirement that the field shifts be small coincides with the condition $\Omega_g \gg \tau^{-1}$. At large detunings, $|\Omega| \gg |G_e|$, the following estimate is valid for the matrix elements of the interaction with the scattering fields [see expressions (18a, b)]

$$|G_eG_{s1}/\Omega| \sim |G_{s2}| \sim \tau^{-1}$$

and the condition that the field shifts be small corresponds to the inequality $|G_e|^2\tau/|\Omega| \gg 1$.

With allowance for the assumptions made, the amplitudes of the states satisfy the following system of equations:

$$\frac{db_{3j}}{dt} = \frac{i|\varepsilon_j|}{\Omega_g}G_{sj}b_2, \quad (4a)$$

$$\frac{db_2}{dt} = 2i\sum_j G_{sj}^*b_{3j}, \quad (4b)$$

$$\left(\frac{d}{dt} + \Gamma\right)G_{sj} = \frac{i}{\tau_0^2}b_{3j}b_2^*, \quad (4c)$$

where Γ is a constant that describes the linear losses-absorption, diffraction, etc., and $\tau_0^{-2} = 2\pi\omega_j|d_{32}|^2\hbar^{-1}N$.

The system of equation (4a–c) has an integral of motion

$$|b_2|^2 + \sum_j \frac{2\Omega_g}{|\varepsilon_j|}|b_{3j}|^2 = n_0, \quad (5)$$

where n_0 is a certain constant determined by the initial conditions.

We assume that at the initial instant of time there was no correlation between the atoms, and the scattering fields were equal to zero: $G_{sj}(t=0) = 0$. The RCRS process begins with incoherent spontaneous scattering. Since it is assumed that the characteristic duration of the scattering process greatly exceeds the period of the oscillations of the populations of states 1 and 3, we assume for the initial amplitudes b_{mj} the values established when the atom interacts with the exciting field in the absence of a scattering field. If there was no atom in state 3 at the instant when the exciting field was turned on, then the amplitudes b_{mj} are given by the expressions

$$b_{1j} = -\frac{1}{\sqrt{2}}\frac{\Omega + \varepsilon_j}{\Omega_g}n_{10}^{1/2}, \quad b_{3j} = (-1)^j\frac{|G_e|}{\Omega_g}n_{10}^{1/2}, \quad (6)$$

where n_{10} is the initial population of the state 1.

The initial amplitude of the state 2 will be chosen to correspond to spontaneous incoherent scattering during the initial stage. The estimate for $b_2(t=0)$ can be obtained in analogy with the procedure used by one of us in Ref. 9, and takes the form

$$b_2(t=0) \approx (N\lambda^2 l)^{-1/2}, \quad (7)$$

where l is the characteristic range of the photon in the medium. The quantity $N\lambda^2 l$ is equal to the number of atoms correlated in the scattering process, and is assumed to be much larger than unity. In the case $\tau \gg \Gamma^{-1}$ we must choose l to be $c\Gamma^{-1}$, and in the case $\tau \ll \Gamma^{-1}$ we have $l = c\tau$.

3. SCATTERING AT EXACT RESONANCE ($\Omega = 0$)

When the frequency of the exciting field is equal to the frequency of the 1–3 transition, the shifts of the quasilevels relative to the unperturbed levels are symmetrical:

$$\varepsilon_1 = -\varepsilon_2 = -|G_e|$$

and the intensities of scattering into both components are equal: $|G_{s1}| = |G_{s2}|$. The substitution

$$b_2 = n_0^{1/2}\sin(\theta/2), \quad b_3 = n_0^{1/2}\cos(\theta/2),$$

where

$$\theta = \sqrt{8}\int_0^t dt'|G_s(t')| + \theta_0, \quad \theta_0 = 2\arcsin[b_2(t=0)/n_0^{1/2}],$$

reduces the system of equation (4a–c) to the equation of the oscillations of a mathematical pendulum with damping

$$\frac{d^2\theta}{dt^2}+\Gamma\frac{d\theta}{dt}=\frac{n_0}{2\tau_0^2}\sin\theta. \tag{8}$$

It is known[10] that Eq. (8) describes the CE by a medium of two-level atoms. In the limiting cases $\tau_0 \gg \Gamma^{-1}$ and $\tau_0 \ll \Gamma^{-1}$, analytic solutions are known.[10,11] At $\tau_0 \gg \Gamma^{-1}$ the solution is of the form

$$\sin\theta=\text{sech}[(t-t_0)/\tau], \tag{9a}$$

$$I_j(t)=\frac{c\hbar\omega_j N n_0}{16\Gamma\tau}\,\text{sech}^2\frac{t-t_0}{\tau}, \tag{9b}$$

$$\tau=\frac{2\tau_0^2\Gamma}{n_0},\quad t_0=\frac{\tau}{2}\ln\frac{1+\cos\theta_0}{1-\cos\theta_0},$$

where $I_j(t)$ is the scattering intensity. As follows from (9a) the scattering pulse has a duration 2τ, and the intensity maximum is reached at the instant t_0.

If the damping time is long compared with the oscillation time, $\tau_0 \ll \Gamma^{-1}$, the damping in (8) can be neglected. The solution can then be expressed in terms of Jacobi elliptic functions[11]:

$$\cos(\theta/2)=\cos(\theta_0/2)\,\text{sn}[K(k)-t/\tau,\ k], \tag{10a}$$

$$I_j(t)={}^1/_4 Nc\hbar\omega_j n_0\cos^2(\theta_0/2)\,\text{cn}^2[K(k)-t/\tau,k], \tag{10b}$$

where $K(k)$ is a complete elliptic integral of the first kind

$$k=\cos(\theta_0/2),\quad \tau=(2/n_0)^{1/2}\tau_0.$$

The scattering intensity, defined by (10b), is a periodic function of the time with a period $T=2\tau K(k)$. At $\theta_0 \ll 1$ the following estimate is valid[12]

$$T\approx 2\tau\ln(4/\theta_0). \tag{11}$$

The scattering intensity at $\theta_0 \ll 1$ is a sequence of pulses of duration 2τ, separated by a time interval T much longer than the pulse duration. Inasmuch as at $\theta_0 \ll 1$ the function $\text{cn}^2[K(k)-t/\tau, k]$ is approximated on the interval $2nT < t < (2n+1)T$ by the function $\text{sech}^2[(t-t_0)/\tau]$, where $t_0=\tau K(k)$, the form of the individual pulse is close to the form of the pulse in the monopulse regime. Under conditions when the damping time is long, an inverse process evolves, namely scattering with absorption of a photon at the frequency of the scattering field and emission at the frequency of the exciting field; it is this which makes the scattering regime periodic.

In the intermediate case, obviously, the regime is that of damped oscillations of the scattering intensity.

4. NONRESONANT SCATTERING ($|\Omega| \gg |G_e|$)

At large deviation of the exciting-field frequency from resonance with the frequency of the transition 1–3($|\Omega| \gg |G_e|$) the shifts of the quasilevels differ substantially in magnitude and are given by the following approximate expressions:

$$\varepsilon_1\approx -|G_e|^2/|\Omega|,\quad \varepsilon_2\approx -\Omega+|G_e|^2/|\Omega|.$$

We shall henceforth call the component of scattering with frequency $\omega_1=\omega_e-\omega_{21}+\varepsilon_2$ the combination line, and the component with frequency $\omega_2=\omega_e-\omega_{21}+\varepsilon_2$ the steplike transition line.

In analogy with the preceding section, we now analyze two limiting cases: $\tau \gg \Gamma^{-1}$ and $\tau \ll \Gamma^{-1}$. In the case $\tau \gg \Gamma^{-1}$, neglecting the derivative with respect to time in (4c), we have

$$G_{sj}=\frac{i}{\tau_0^2\Gamma}b_{3j}b_2^*. \tag{12}$$

Substituting expression (12) in Eqs. (4a, b), we obtain the following system of equations

$$\frac{db_{3j}}{dt}=-\frac{|\varepsilon_j|}{\tau_0^2\Gamma\Omega_g}b_{3j}|b_2|^2, \tag{13a}$$

$$\frac{db_2}{dt}=\frac{2}{\tau_0^2\Gamma}b_2\sum_j|b_{3j}|^2. \tag{13b}$$

From (13a), the amplitudes b_{3j} can be expressed in terms of the function

$$\varphi=\frac{1}{\tau_0^2\Gamma}\int_0^t dt'|b_2(t')|^2,$$

$$b_{3j}=b_{3j}(t=0)\exp(-|\varepsilon_j|\varphi/\Omega_g). \tag{14}$$

Using the integral of motion (5), we can write for φ the equation

$$\frac{d\varphi}{dt}=\frac{1}{\tau_0^2\Gamma}\left[n_0-\sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\exp\left(-\frac{2|\varepsilon_j|\varphi}{\Omega_g}\right)\right], \tag{15}$$

where $n_{30}=|b_{3j}(t=0)|^2$.

The solution of (15) can be written in quadratures:

$$\int_0^\varphi d\varphi'\left[n_0-\sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\exp\left(-\frac{2\varphi'|\varepsilon_j|}{\Omega_g}\right)\right]^{-1}=\frac{t}{\tau_0^2\Gamma}. \tag{16}$$

The scattering intensity is, according to (12),

$$I_j(t)=\frac{c\hbar\omega_j N n_{30}}{\tau_0^2\Gamma^2}\exp\left(-\frac{2|\varepsilon_j|\varphi}{\Omega_g}\right)\left[n_0-\sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\exp\left(-\frac{2|\varepsilon_j|\varphi}{\Omega_g}\right)\right]. \tag{17}$$

From the condition $dI_j/dt=0$ we estimate the values of G_{sj}, I_j, and φ at which the intensities reach maxima. Recognizing that $n_0 \sim 1$, $|\varepsilon_2| \gg |\varepsilon_1|$, we obtain the estimates

$$\varphi_{1max}\approx\frac{\Omega_g\ln 2}{2|\varepsilon_1|},\quad G_{s1max}\approx\frac{n_{30}^{1/2}}{2\tau_0^2\Gamma},\quad I_{1max}=\frac{c\hbar\omega_1 N n_{30}}{4\Gamma^2\tau_0^2}; \tag{18a}$$

$$\varphi_{2max}\approx\frac{\Omega_g}{|\varepsilon_2|},\quad G_{s2}\approx\frac{2n_{30}^{1/2}e^{-1}}{\tau_0^2\Gamma}\left|\frac{G_e}{\Omega}\right|,\quad I_{2max}\approx\frac{c\hbar\omega_2 N}{\Gamma^2\tau_0^2}\left|\frac{G_e}{\Omega}\right|^2 n_{30}4e^{-2}. \tag{18b}$$

According to the estimates (18a, b) the intensity of the Raman line at the maximum is much larger than the intensity of the steplike-transition line

$$I_{2\,max}/I_{1\,max}\approx 2|G_e/\Omega|^2. \tag{19}$$

The results of the numerical integration of (16) are shown in Fig. 2, from which it follows that the intensity of the Raman line is always larger than the intensity of the steplike transition line, and the maximum values are reached at different instants of time.

Using a linear expansion of the exponentials in the integral (16), we can obtain an analytic estimate of the duration Δt_2 of the pulse and of the delay time t_{02} of the steplike transition. Taking the estimate (7) into account, we have

$$t_{02}={}^1/_4\tau\ln(8n_{30}N\lambda_2^2 c\Gamma^{-1}),\quad \Delta t_2=\tau, \tag{20}$$

where $\tau=\tau_0^2\Gamma/2n_{30}$.

In the limiting case of large values of φ we can also obtain an analytic dependence of the intensity of the combination line on the time. To this end we use the circumstance that the maxima of the intensities of the lines are reached in substantially different instants

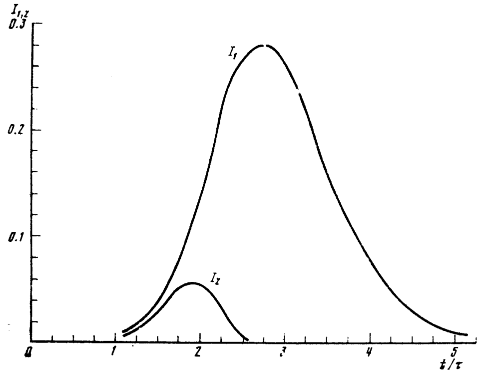

FIG. 2. Dependence of the scattering intensity on the time. Case of strong damping of the field, $\tau \gg \Gamma^{-1}$. $I_{1,2}$—intensities of scattering at the frequencies of the Raman and steplike transitions in units of $I_0^{(1,2)} = c\hbar\omega_{1,2}N/4\tau\Gamma$, $\tau = \tau_0^2\Gamma/2n_{30}$ at $|G_e/\Omega|^2 = 0.1$, $n_0 = 1.0001$, $n_{10} = 1$.

of time, so that for certain values of the time, corresponding to the inequality $\varphi > \varphi_{2\max}$, scattering at the frequency of the steplike transition can be neglected. This approximation was used by us earlier in Ref. 6. As noted there, the problem of cooperative Raman scattering is in this case mathematically equivalent to the CE problem, so that the waveform of the scattering pulse coincides with that of the CE pulse:

$$I_1(t) = \frac{c\hbar\omega_1 N n_{30}}{4\tau_0^2\Gamma}\operatorname{sech}^2\frac{t-t_{01}}{\tau}; \tag{21}$$

Here

$$t_{01} = {}^1/_2\tau\ln(2N\lambda_1^2 c\Gamma^{-1}) - {}^1/_4\tau\ln(n_{30}N\lambda_1^2\varphi_2 c\Gamma^{-1}). \tag{22}$$

We turn now to the analysis of the limiting case $\tau \ll \Gamma^{-1}$. We leave out the term that describes the damping in (4c). The system of equations (4a–c) takes the form

$$\frac{db_{3j}}{dt} = \frac{i|\varepsilon_j|}{\Omega_g}G_{ej}b_2, \tag{23a}$$

$$\frac{db_2}{dt} = 2i\sum_j G_{ej}^* b_{3j}, \tag{23b}$$

$$\frac{dG_{ej}}{dt} = \frac{i}{\tau_0^2}b_{3j}b_2^*. \tag{23c}$$

In addition to the integral of motion (5), the system (23a–c) is satisfied by one more integral of motion:

$$\frac{\Omega_g}{|\varepsilon_j|\tau_0^2}|b_{3j}|^2 + |G_{ej}|^2 = \frac{\Omega_g n_{30}}{|\varepsilon_j|\tau_0^2}. \tag{24}$$

Taking (5) and (24) into account, the sought functions in (23a–c) can be obtained in the form

$$b_{3j} = n_{30}^{1/2}\cos\left(\left(\frac{|\varepsilon_j|}{\Omega_g}\right)^{1/2}\psi\right),$$

$$b_2 = \left[n_0 - \sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\cos^2\left(\left(\frac{|\varepsilon_j|}{\Omega_g}\right)^{1/2}\psi\right)\right]^{1/2}; \tag{25a}$$

$$G_{ej} = \left[\frac{\Omega_g n_{30}}{|\varepsilon_j|\tau_0^2}\right]^{1/2}\sin\left(\left(\frac{|\varepsilon_j|}{\Omega_g}\right)^{1/2}\psi\right), \tag{25b}$$

where

$$\psi(t) = \frac{1}{\tau_0}\int_0^t dt' b_2(t'). \tag{26}$$

The function $\psi(t)$ satisfies the integral equation

$$\int_0^\psi d\psi'\left[n_0 - \sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\cos^2\left(\left(\frac{|\varepsilon_j|}{\Omega_g}\right)^{1/2}\psi'\right)\right]^{-1/2} = \frac{t}{\tau_0}. \tag{27}$$

A qualitative idea of the character of the time dependence of the function $\psi(t)$ can be obtained by using a mechanical analogy. Equation (27) is the solution of the equation of motion of the particle in a biharmonic potential

$$U = -\frac{1}{2\tau_0^2}\left[n_0 - \sum_j\frac{2\Omega_g n_{30}}{|\varepsilon_j|}\cos^2\left(\left(\frac{|\varepsilon_j|}{\Omega_g}\right)^{1/2}\psi\right)\right]. \tag{28}$$

The particle has zero energy and an initial velocity $d\psi/dt = n_{20}/\tau_0$, so that in a negative potential the motion is infinite. Since $|\varepsilon_1| \ll |\varepsilon_2|$, the contribution made to the potential by the harmonic with the high frequency (the steplike-transition) has a lower amplitude. The particle has a low velocity $\sim n_{20}/\tau_0$ in the regions of the maxima of the potential energy $\psi \sim (\Omega_g/|\varepsilon_1|)^{1/2}\pi n$ and acclerates to the velocity $\sim(n_0/\tau_0)^{1/2}$ in the regions of the minima of the potential $\psi \sim (\Omega_g/|\varepsilon_1|)^{1/2}\pi(2n+1)/2$.

As already noted, when the influence of the scattering at the frequency of the steplike transition is neglected, the equations for the description of the combination lines agree with the formulas that describe the CE:

$$\cos\psi = \operatorname{sn}(K(k) - t/\tau, k), \tag{29a}$$

$$I_1 = c\Omega_g n_{30}\hbar\omega_1 N|\varepsilon_1|^{-1}\operatorname{cn}^2(K(k) - t/\tau, k); \tag{29b}$$

Here

$$k^2 = 2\Omega_g n_{30}/|\varepsilon_1|n_0, \quad \tau = \tau_0(\Omega_g/n_0|\varepsilon_1|)^{1/2}.$$

The period and duration of the oscillations are given by the relations

$$T_1 = 2\tau\ln[4(1-k^2)^{-1/2}], \quad \Delta t_1 \sim 1{,}3\tau. \tag{30}$$

To estimate the duration of the pulses and their repetition interval at the frequency of the steplike transition, we proceed just as in the preceding case of strong damping $\tau \gg \Gamma^{-1}$, namely, we expand the cosines in the integral of (27) in a series. In the vicinity of the maximum of the potential (28) we have

$$T_2 \approx 2^{-1/2}\tau\ln(\pi^2 n_{30}\lambda_2^2 N c\tau), \quad \Delta t_2 \approx \sqrt{2}\ln 2\tau. \tag{31}$$

In the vicinity of the minimum of the potential

$$T_2 \sim \Delta t_2 \sim \tau_0. \tag{32}$$

The results of the numerical calculation are shown in Fig. 3. Comparison with the quantities Δt_1 and T_1,

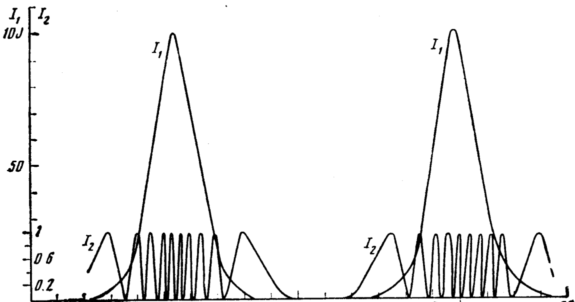

FIG. 3. Dependence of the scattering intensity on the time. Case of weak damping of the field, $\tau \ll \Gamma^{-1}$. $I_{1,2}$—intensities of the scattering of the Raman and steplike transitions in units of $I_0^{(1,2)} = c\hbar\omega_{1,2}n_{30}N$, $\tau = \tau_0(\Omega_g/n_0|\varepsilon_1|)^{1/2}$ at $|G_e/\Omega|^2 = 0.01$, $n_0 = 1.0001$, $n_{10} = 1$.

obtained from the numerical calculation, with the values that follow from Eq. (30) indicates that the influence of the scattering at the frequency of the steplike transition on the temporal properties of the Raman line reduces to a certain increase (30%) of the times Δt_1 and T_1.

The character of the time dependence of the intensity of the scattering at the frequency of the steplike transition corresponds qualitatively to the CE. In the vicinity of the minimum of the intensity of the Raman line there are much fewer atoms in the state 2 than in state 3 (ε_2); this corresponds to an inverted medium of two-level atoms. Scattering in this case is avalanche-like with a delay time longer than the pulse duration. On the other hand if the number of atoms in state 2 is larger than on the sublevel 3 (ε_2) (region of maximum intensity), the scattering process is oscillatory.

5. DISCUSSION

Our results admit of a simple qualitative interpretation. The approximation of scattering resolved into two components is equivalent to the presence of two scattering channels. The first channel (the Raman line) corresponds to a transition that couples the lower energy quasilevels with level 2: $1(\varepsilon_1) \to 3(\varepsilon_1) \to 2$. The second channel (steplike-transition line) corresponds to transition of the atoms through upper energy quasilevels: $1(\varepsilon_2) \to 3(\varepsilon_2) \to 2$. If the frequency of the exciting field coincides with the frequency of the 1–3 transition, the intensities of the scattering into both components are equal and the pulse waveforms and the times τ and t_0 coincide with the corresponding characteristics of CE by a medium of 2-level atoms with initial excited-atom density N.

If the deviation of the exciting-field frequency from the frequency of the 1–3 transition is large enough ($|\Omega| \gg |G_e|$), the initial populations of the quasilevels $1(\varepsilon_1)$ and $1(\varepsilon_2)$ differ substantially in accordance with (6): $n_1(\varepsilon_1) \sim 1, n_1(\varepsilon_2) \sim |G_e/\Omega|^2$. If we neglect the mutual influence of the scattering channels, then the characteristics of the process of scattering at the frequency of the Raman transition correspond to nonresonant cooperative Raman scattering with initial density N of the atoms in the initial states. Scattering in the steplike transition channel at $\tau \gg \Gamma^{-1}$ corresponds to CE by an inverted medium with excited-atom density $|G_e/\Omega|^2 N$. However, the scattering channels are coupled by a common final state, therefore their mutual influence in the course of scattering is significant. The greater part of the atoms go over into the state via the Raman-scattering channel, and its influence on the steplike transition channel is much larger than the inverse action. Thus, expression (22) for the delay time of the Raman-scattering pulse consists of two terms. The first is equal to the delay time of the cooperative Raman scattering pulse, obtained[6] by that allowance for scattering at the frequency of the steplike transition. The second term in (22) describes the influence of the steplike-transition channel, which reduces to a certain shortening of the delay time t_{01}. The duration of the pulse remains in this case practically unchanged.

The delay time and the pulse duration of the steplike-transition line are shortened by one-half in the case $\tau \gg \Gamma^{-1}$ compared with the times obtained without allowance for the interaction of the channels. In the case $\tau \ll \Gamma^{-1}$ the time dependence of the intensity of the line of the steplike transition is due to a greater degree to the presence of the Raman-scattering channel. Comparing expressions (20) and (22) for the scattering-pulse delay times, we see that the delay times can differ substantially because of the difference in the value of the logarithmic factor. The pulse durations differ in this case by a factor of 2.

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[1]M. Gross, C. Fabre, P. Pillet, and S. Haroche, Phys. Rev. Lett. 36, 1035 (1976).

[2]M. Gross, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. 40, 1711 (1978).

[3]A. Flusberg, T. Mosberg, and S. R. Hartmann, Phys. Lett. 58A, 373 (1976).

[4]Q. H. F. Vrehen, H. M. Hikspoores, and H. M. Gibbs, Phys. Rev. Lett. 38, 764 (1977).

[5]T. M. Makhviladze and L. A. Shelepin, Phys. Rev. A 9, 538 (1974).

[6]S. G. Rautian and B. M. Chernobrod, Zh. Eksp. Teor. Fiz. 72, 1342 (1977) [Sov. Phys. JETP 45, 705 (1977)].

[7]V. I. Emel'yanov and V. N. Seminogov, Zh. Eksp. Teor. Fiz. 76, 34 (1979) [Sov. Phys. JETP 49, 17 (1979)].

[8]C. M. Bowden and C. C. Sung, Phys. Rev. A 18, 1558 (1978).

[9]V. M. Chernobrod, Preprint No. 97, Inst. of Automation and Electronics, Siberian Division, USSR Acad. Sci., 1979; B. M. Chernobrod, Opt. Commun. 30, 29 (1979).

[10]R. Bonifacio and L. A. Lugito, Phys. Rev. A 11, 1507 (1975).

[11]M. P. Crisp. Phys. Rev. Lett. 22, 820 (1969).

[12]I. S. Gradshtein and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedenii (Tables of Integrals, Sums, Series, and Products), Nauka, Moscow, 1971 [Academic, 1965].

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