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# Combined phonon resonance in semiconductors

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The absorption of electromagnetic radiation by band carriers whose motion is quantized by a magnetic field is considered. It is shown that the electronic transitions with spin flip that take place on photon absorption and *LO*-phonon emission result in resonance of the absorption coefficient (combined phonon resonance—CPR). The line shape of the CPR is investigated and the parameters of the resonance peaks are determined. The results are compared with the experimental data.

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## 1. INTRODUCTION

Absorption of electromagnetic radiation by band carriers under conditions in which their motion is quantized by a magnetic field leads to various resonance effects. Part of such effects is connected with the scattering of the carriers by optical phonons. A comparison of the existing experimental data with the results of theoretical researches in this region gives essentially good agreement; however, as is noted in the review of Ref. 1, a number of experimental results has not yet found theoretical explanation. In particular, the nature of the resonance found experimentally in $n$-InSb at the frequency $\omega_r = \omega_H + g\beta_0 H + \omega_{LO}$ for longitudinal polarization of the electromagnetic field[2] remains unclear.

As follows from the frequency condition of this resonance, it is obviously determined by the electronic transitions with spin flip upon absorption of a photon and emission of a longitudinal optical phonon. The spin flip in the electronic transition can be connected in this case either with the interaction of the band electron with the high-frequency electromagnetic field or with the spin-phonon interaction. We shall consider both cases. Resonance in the absorption of electromagnetic radiation on account of spin-phonon interaction was studied theoretically in Ref. 3, where it was shown that the resonance in the case of an isotropic energy spec-

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trum of electrons arises in the presence of transverse polarization of the wave and is due to the interaction with the transverse optical mode; the interaction with $LO$ phonons does not lead to spin-cyclotron-phonon resonance.

The excitation of spin transitions by an electromagnetic field has been studied both theoretically and experimentally.[4] These transitions can be excited by both the magnetic and the electric component of the field. However, in the case of strong spin-orbit interaction the intensity of the absorption line under the action of the electric component is several orders above the intensity due to the magnetic excitation. The circumstances set forth above indicate that the resonance observed in Ref. 2 is connected with combined phonon transitions (CPT)–spin transitions excited by the electric component of the field and accompanied by the emission of $LO$ phonons.

We consider the electromagnetic-radiation absorption due to CPT in semiconductors with the symmetry of zincblende (for example, $n$-InSb). Here two types of combined transitions are possible: transitions connected with the presence in the electron dispersion law of terms $\sim K^3$ due to spin-orbit coupling, and also transitions connected with the dependence of the $g$ factor of the electrons on the quasimomentum $K$.[4] The first type of transition depends on the orientation of the magnetic field relative to the crystallographic axes and determines the anistropic part, while the second determines the isotropic part of the absorption coefficient due to CPT.

## 2. ANISOTROPIC PART OF THE ABSORPTION COEFFICIENT

The CPT matrix elements in the lowest approximation in electron-phonon and spin-flipping electromagnetic interaction can be determined with the help of the general expression

$$\langle -, i|\hat{H}|f, +\rangle = \sum_v \frac{\langle -, i|H_L|v, -\rangle\langle -, v|H_R|f, +\rangle}{E_i^{(-)} - E_v^{(+)}} + \sum_v \frac{\langle -, i|H_R|v, +\rangle\langle +, v|H_L|f, +\rangle}{E_i^{(-)} - E_v^{(+)}}. \quad (2.1)$$

Here $E$ is the energy of the system, $|f,\pm\rangle \equiv |\nu, \pm\mathbf{q}, \varkappa\rangle$ is the wave function of the non-interacting electrons, phonons, and photons; the signs $\pm$ indicate the direction of the spin; $\nu\pm, \mathbf{q}, \varkappa$ determine the state of the electrons, phonons, and photons, respectively. Interaction with optical longitudinal phonons is described by the Hamiltonian $H_L$ with matrix elements

$$\langle \pm, \nu, 0|H_L|\nu', \pm, \pm\mathbf{q}\rangle = \pm ic_q[N(q)+{}^1/_2 \pm {}^1/_2]^{1/2}\delta(p_x, p_x' \pm \hbar q_x) \times \delta(p_z, p_z' \pm \hbar q_z)\exp\{\pm 2i\hbar q_y p_x/p_H^2\} M_{ll'}(\pm\mathbf{q}_\perp), \quad (2.2)$$

where $|\nu\pm\rangle = |p_x, p_z, l\pm\rangle$, $c_q$ is the Fourier transform of the electron-phonon interaction potential, $N(q)$ is the number of phonons,

$$M_{ll'}(q_\perp) = \exp\{-i\hbar^2 q_x q_y/p_H^2\}\exp\{i(l'-l)\varphi\} Q_{ll'}(\hbar q_\perp/p_H), \quad (2.3)$$

and the function $Q_{ll'}(x)$ is defined by the expression

$$Q_{ll'}(x) = (-1)^{l-l'}(l'!/l!)^{1/2} x^{l-l'} L_{l'}^{l-l'}(x^2) e^{-x^2/2}. \quad (2.4)$$

Here $L_l^{l-l'}(x)$ is a generalized Laguerre polynomial, $p_H = (2m^*\hbar\omega_H)^{1/2}$, $\omega_H$ is the cyclotron frequency, and $q_\perp$ and $\varphi$ are the polar coordinates in $q$ space.

The matrix elements of the spin-flipping interaction $H_R$ are easily determined by using the results of Ref. 5. In the dipole approximation, we obtain

$$\langle +, \nu, 0|H_R|\nu', -, -\varkappa\rangle = \frac{e}{m^* c} N^{1/2}(\omega)\left(\frac{2\pi c^2\hbar}{\omega\varepsilon V}\right)^{1/2}\frac{2L\hbar\omega_H}{c} \times \delta(p_x, p_x')\delta(p_z, p_z')\beta^*\left(\frac{l-j+\beta^*}{l-j+S_\alpha+\beta^*}\right)\sum_{\beta,\gamma} B_{\alpha\beta\gamma}\langle j|a_\beta a_\gamma|l\rangle, \quad (2.5)$$

where $V$ is the normalization volume, $\omega$ is the frequency of the electromagnetic wave, $N(\omega)$ is the number of photons, $L = 8(\delta_0 m^* m_s c/\hbar^3)^2/\varepsilon'$, $\varepsilon'$ is the dielectric constant, $\delta_0$ is the parameter of band divergence due to the spin-orbit interaction and to the absence of a center of inversion; $l$ and $j$ are the numbers of the Landau levels, $\alpha, \beta, \gamma$ run through the values 1, 2, and 3, with $\alpha = 1$ for the left, $\alpha = 2$ for the right, and $\alpha = 3$ for the longitudinal polarization; $S_1 = -1, S_2 = 1, S_3 = 0$; $\beta^* = m^*/m_s, m_s = 2m_0/g$, $m_0$ is the mass of the free electron, $m^*$ is the effective mass; the operators $a_\beta$ and the coefficient $B_{\alpha\beta\gamma}$ are defined in Ref. 5. The angular dependence of the matrix elements (2.5) is determined by the coefficients $B_{\alpha\beta\gamma}$.

Using the expression (2.1)–(2.5), after cumbersome transformations which involve the recurrence relations for the Laguerre polynomials, we obtain an expression for the CPT matrix elements in the form

$$\langle -, \nu, 0, 0|\hat{H}|+, \nu', \pm\mathbf{q}, -\varkappa\rangle = \pm ic_q[N(q)+{}^1/_2 \pm {}^1/_2]^{1/2} N^{1/2}(\omega) \times \frac{2e\hbar\omega_H}{m^* c^2}\left(\frac{2\pi\hbar c^2}{\omega\varepsilon V}\right)^{1/2} L^{1/2}\delta(p_x, p_x' \pm q_x)\delta(p_z, p_z' \pm \hbar q_z) \times \exp\left\{\pm\frac{2ip_x\hbar q_y}{p_H^2}\right\} D_{ll'}^{\alpha}(\omega), \quad (2.6)$$

where

$$D_{ll'}^{\alpha}(\omega) = \sum_{l''\beta\gamma} B_{\alpha\beta\gamma}^{\cdot}\left(\frac{l''-l+\beta^*}{l''-l+S_\alpha+\beta^*}\right)\frac{\langle l|a_\beta a_\gamma|l''\rangle M_{l''l'}}{\varepsilon_{\nu''}^{-} - \varepsilon_{\nu'}^{+} + \hbar\omega} + \frac{l'-l''+\beta^*}{l'-l''+S_\alpha+\beta^*}\frac{\langle l''|a_\beta a_\gamma|l'\rangle M_{ll''}}{\varepsilon_{\nu'}^{-} - \varepsilon_{\nu''}^{+} - \hbar\omega} \quad (2.7)$$

and

$$\varepsilon_\nu^{\pm} = \hbar\omega_H\left(l+\frac{1}{2}\right) \pm \frac{g\beta_0 H}{2} + \frac{\hbar^2 K_z^2}{2m^*}.$$

To obtain the absorption coefficients due to CPT, (2.6), we use the well-known expression[6]

$$K(\omega) = \frac{\varepsilon^{1/2}(\omega)}{c^2}\frac{nV}{N(\omega)}\left[1-\exp\left(-\frac{\hbar\omega}{k_0 T}\right)\right]\mathrm{Avi}\sum_f \frac{2\pi}{\hbar}|\langle i|\hat{H}|f\rangle|^2, \quad (2.8)$$

where $n$ is the concentration of electrons, $N(\omega)$ is the number of photons in the initial state, and the operator Avi indicates thermal averaging over the initial states. Substituting (2.6) and (2.7) in (2.8), after tedious calculations, we obtain for the case of longitudinal polarization ($\alpha = 3$) of interest to us the singular part of the absorption coefficient

$$K_A^{\pm}(\omega) = \sum_{ll'} K_{ll'}^{\pm},$$

in the form

$$K_{ll'}^{\pm} = \frac{8\pi^{1/2} nL\beta^*\hbar(\hbar\omega)^2(\hbar\omega_0)^{1/2}\alpha_R\alpha_L w_{-l}}{m^{*2}\omega c^2(k_0 T)^{1/2}}\left[1-\exp\left(-\frac{\hbar\omega}{k_0 T}\right)\right] \times \left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right) A_\alpha(\omega)\exp\left\{-\frac{\hbar\Delta\Omega^{\pm}}{2k_0 T}\right\} K_0\left(\frac{\hbar|\Delta\Omega|^{\pm}}{2k_0 T}\right), \quad (2.9)$$

where $\alpha_R$ and $\alpha_L$ are dimensionless constants of electron-photon and electron-phonon interactions; the detuning from resonance is

$$\Delta\Omega^{\pm}=\omega_H(l'-l)+\frac{1}{\hbar}(g\beta_0 H)+\omega\pm\omega_0;$$

$N_0$ is the Planck distribution function of the phonons. The angular dependence of the longitudinal resonance is determined by the functions

$$A_3(\omega)=\frac{|B_{311}|^2(l'+l+1)}{(2\hbar\omega_H+g\beta_0 H-\hbar\omega)^2}+\frac{4|B_{313}|^2}{(\hbar\omega_H+g\beta_{0H}-\hbar\omega)^2}+\frac{|B_{322}|^2(l'+l+1)}{(2\hbar\omega_H-g\beta_0 H+\hbar\omega)^2} \tag{2.10}$$

As follows from (2.10), $A_3(\omega)$ is also a slowly changing function of the frequency, except for singularities of no interest to us at the points $\omega=2\omega_H+g\beta_0 H$ and $\omega=\omega_H+g\beta_0 H$. $K_0(\hbar|\Delta\Omega|^{\pm}/2k_0T$ is a Macdonald function that has a logarithmic singularity at the point of resonance $\Delta\Omega=0$.

We note that $K^{+}_{ll'}(\omega)$ for the other initial spin state has a form similar to (2.9), the difference being a different angular dependence of $A_3(\omega)$, determined by the coefficients $B_{3\beta\gamma}$ with indices $\beta\gamma$ other than those in (2.10) and, in addition, $w_-$ in (2.9) is replaced by $w_+$.

The functions $w_l$ and $w_\pm$ are determined for a nondegenerate electron gas by the expressions

$$w_l=2\,\mathrm{sh}\left(\frac{\hbar\omega_H}{k_0T}\right)\exp\left\{-\frac{\hbar\omega_H}{k_0T}\left(l+\frac{1}{2}\right)\right\},$$
$$w_\pm=\frac{1}{2}\,\mathrm{ch}^{-1}\left(\frac{g\beta_0 H}{2k_0T}\right)\exp\left\{\mp\frac{g\beta_0 H}{2k_0T}\right\}. \tag{2.11}$$

In addition to the singular part (2.9), $K^{+}_{ll'}$ has also the following regular part that is regular at the point of resonance:

$$K^{\mathrm{reg}}_{ll'}=A(\omega)(\Delta\Omega)^{\pm}K_0\left(\frac{\hbar|\Delta\Omega|^{\pm}}{2k_0T}\right)+B(\omega)(\Delta\Omega)^{\pm}K_1\left(\frac{\hbar|\Delta\Omega|^{\pm}}{2k_0T}\right), \tag{2.12}$$

where $K_1(\hbar|\Delta\Omega|^{\pm}/2k_0T$ is a Macdonald function of first order. At the resonance point, $K^{\mathrm{reg}}_{ll}$ vanishes.

As follows from (2.9), the singular part of the absorption coefficient has the same singularity both for the longitudinal ($\alpha=3$) and for the other transverse ($\alpha=1,2$) polarizations; however, the angular dependences determined by $A_\alpha(\omega)$ differ for all three values of (right, left, and longitudinal polarization).

## 3. ISOTROPIC PART OF THE ABSORPTION COEFFICIENT

Using the results of Sheka and McCombe[7] it is not difficult to determine the matrix elements of the electron-photon interaction operator which reverses spin because of the dependence of the $g$ factor of the electron on the quasimomentum.

For a longitudinally polarized wave and an initial state $\langle+|$, with account of the inequality $\hbar\omega_H/\varepsilon_g\ll1$, we obtain in the given approximation

$$\langle+,\nu,0|H_{\parallel}|\nu',-\mathbf{K},-\rangle=\frac{e}{m^*c}N^{1/2}(\omega)\left[\frac{2\pi\hbar c^2}{V\varepsilon(\omega)\omega}\right]^{1/2}(l+1)^{1/2}$$
$$\times\frac{\sqrt{2}\,m^{*}p^2K_H}{3}\left(\omega_c-\frac{g\beta_0 H}{\hbar}\right)\left[\frac{1}{\varepsilon_g^2}-\frac{1}{(\varepsilon_g+\Delta)^2}\right]\delta(p_x,p_x')\delta(p_z,p_z')\delta_{l',l+1}, \tag{3.1}$$

where $K_H=(eH/c\hbar)^{1/2}$, $\varepsilon_g$ is the width of the forbidden band, $\Delta$ is the spin-orbit splitting, and

$$p^2=\frac{3g\hbar^2}{4m_0}\left[\frac{1}{\varepsilon_g+\Delta}-\frac{1}{\varepsilon_g}\right]^{-1}.$$

Using the expressions (2.1), (2.2), and (3.1), we obtain the CPT matrix elements for the considered case in the form

$$\langle+,\nu,0,0|H|\nu',\pm\mathbf{q},-\varkappa_z,-\rangle=\pm ic_qN^{1/2}(\omega)[N(q)+{}^1/_2\pm{}^1/_2]^{1/2}$$
$$\times\left(\frac{2\pi\hbar c^2}{\varepsilon(\omega)\omega}\right)^{1/2}\frac{\sqrt{2}\,m^{*}p^2K_H}{3}\left(\omega_c-\frac{g\beta_0 H}{\hbar}\right)\left[\frac{1}{(\varepsilon_g+\Delta)^2}-\frac{1}{\varepsilon_g^2}\right] \tag{3.2}$$
$$\times\frac{\exp\{\pm2\hbar iq_yp_x/p_H^2\}}{\hbar\omega_H-g\beta_0 H-\hbar\omega}[(l+1)^{1/2}M_{l+1,l'}-(l')^{1/2}M_{l,l'-1}].$$

We substitute (3.2) in (2.8) and make use of the relation

$$(l+1)^{1/2}M_{l+1,l'}-(l')^{1/2}M_{l,l'-1}=-\exp\{-i\hbar^2q_xq_y/p_H^2\}$$
$$\times\exp\{i(l'-l-1)\varphi\}(\hbar q_\perp/p_H)Q_{ll'}(\hbar q_\perp/p_H), \tag{3.3}$$

we therefore obtain the following expression for the isotropic part $K^{\pm}_{ll'}$ (the transition from the initial spin state $\langle+|$)

$$K^{\pm}_{ll'}=\frac{n\alpha_R\alpha_L(\hbar\omega_0)^{1/2}\hbar\omega_H\hbar^2m^{*}g^2(\hbar\omega_H-g\beta_0 H)^2}{8\omega(k_0T)^{1/2}m_0(\hbar\omega_H-g\beta_0 H-\hbar\omega)^2}$$
$$\times\left[1-\exp\left(-\frac{\hbar\omega}{k_0T}\right)\right]\left(N_0+\frac{1}{2}\pm\frac{1}{2}\right)w_lw_{(+)}\left[\frac{1}{(\varepsilon_g+\Delta)^2}+\frac{1}{\varepsilon_g^2}\right]$$
$$\times\exp\left(\frac{\hbar\Delta\Omega}{2k_0T}\right)K_0\left(\frac{\hbar|\Delta\Omega|}{2k_0T}\right). \tag{3.4}$$

It can be shown that for a transversely polarized wave, the matrix elements of the effective Hamiltonian vanish. Therefore the isotropic part of the absorption coefficient, which is due to the CPT, appears only in the case of a longitudinally polarized wave.

## 4. DETERMINATION OF THE PARAMETERS OF THE RESONANCE PEAKS AND THEIR COMPARISON WITH EXPERIMENT

As follows from the expressions (2.9) and (3.4), the form of the resonance peaks is determined essentially by the asymptotic forms of the Macdonald function at small and large values of the argument:

$$K^{\pm}_{ll'}\sim\ln(4k_0T/\hbar|\Delta\Omega|) \tag{4.1}$$

at $\hbar|\Delta\Omega|/2k_0T\ll1$;

$$K^{\pm}_{ll'}\sim\begin{cases}(2k_0T/\hbar|\Delta\Omega|)^{1/2}, & \Delta\Omega>0\\ (2k_0T/\hbar|\Delta\Omega|)^{1/2}\exp(-\hbar|\Delta\Omega|/k_0T), & \Delta\Omega<0\end{cases} \tag{4.2}$$

at $\hbar|\Delta\Omega|/2k_0T\gg1$. As follows from (4.1), the absorption coefficient has a logarithmic singularity at the resonance point $\Delta\Omega=0$; the resonance peak is strongly asymmetric; since the behavior of $K^{\pm}_{ll'}$ to the left and right of the resonance point (4.2) is significantly different.

It must be noted that the anisotropic part of the intensity of the peaks depends weakly, (2.9) on $l'$, and the isotropic part does not depend at all, (3.4), on $l'$. The dependence on $l$ is determined by the function $w_l$. The product $w_lw_\pm$ and also $N_0\approx\exp\{-\hbar\omega_0/k_0T\}$ determine the most significant dependence of the intensity of the peaks on the temperature and the intensity of the external magnetic field, as follows from (2.9) and (3.4).

The emission peaks on the curve $K(\omega)$ in the actual case $\omega_H>\omega_0$, which corresponds for $n$-InSb to a field

$H \geq 32$ kOe, are located at the points

$$\omega = m\omega_H + \omega_0 \pm g\beta_0 H/\hbar \quad (m=0, 1, 2, \ldots),$$

and the absorption peaks are located at the points

$$\omega = m\omega_H - \omega_0 \pm g\beta_0 H/\hbar \quad (m=1, 2, \ldots).$$

The principal contribution to the intensity is made by transitions from the lowest sublevel ($l=0, l=m$). The intensity of the absorption peaks is smaller by the factor $\exp(-\hbar\omega_0/k_0T)$ than the emission peaks, and at the temperatures of interest ($k_0T \ll \hbar\omega_0$) their contribution is negligible. For larger $m$, the intensity falls off as $m^{-3}$ with increase in $m$.

For an estimate of the amplitude and ahlf-width of the resonance peaks, we introduce the phenomenolgical damping parameter $\gamma$, which is determined by the basic mechanism that limits the height of the peaks. Then the absorption coefficient is determined by the expression

$$\Gamma(\omega) = \operatorname{Re} K(\hbar|\Delta\Omega + i\gamma|). \tag{4.3}$$

We obtain an estimate from this expression for the inesity of the principal line in the form

$$\Gamma_{01}^{+} \approx \alpha_R \alpha_L A_{01}^{+}(\omega_p) \cdot \begin{cases} \ln(\hbar\gamma/4k_0T), & \hbar\gamma \ll k_0T \\ (\pi T/\gamma)^{1/2}, & \hbar\gamma \gg k_0T \end{cases} \tag{4.4}$$

and for the half-width, the estimate $\delta\omega \sim \gamma$.

It is evident that to observe the resonance the ordinary conditions of strong and quantizing magnetic field must be satisfied namely

$$\hbar\omega_H \gg \hbar\gamma, \quad \hbar\omega_H \gg k_0T.$$

We now determine the ratio of the isotropic and anisotropic parts of the intensity of the absorption line. Using (2.9) and (3.4), we get

$$\frac{\Gamma_{01}{}^{i}}{\Gamma_{01}{}^{an}} = \frac{g^2(m^*/m_0)^2(\hbar\omega_H - g\beta_0 H)^2 m^* c^2}{64\pi^{1/2} L\beta^{*2}\hbar\omega_H} \left[\frac{1}{\varepsilon_g + \Delta} + \frac{1}{\varepsilon_g}\right]^2. \tag{4.5}$$

It must be noted that resonance is observed against the background of cyclotron resonance and lattice absorption. However, the latter gives a monotonic background, which can be eliminated by measuring $\Gamma(H) - \Gamma(0)$. Taking into account that in the distant region of the wings of the cyclotron resonance line $\Gamma_{cr} \sim \alpha_R \alpha_L \hbar n/m^* \omega_0$, and using (4.3), we obtain the estimate

$$\frac{\Gamma_{cpr}}{\Gamma_{cr}} \approx \frac{g^2}{8}\left(\frac{m^*}{m_0}\right)^2 \left(\frac{\hbar\omega_H}{\varepsilon_g}\right)^2 \left(\frac{\hbar\omega_0}{k_0T}\right)^{1/2}. \tag{4.6}$$

We now make several numerical estimates for the case of $n$-InSb and upon satisfaction of the conditions of observation of the effect experimentally.[2] At $n \sim 2 \times 10^{16}$, $H \sim 100$ kOe, $T \sim 30$ K, and $\gamma \sim 10^{12}$ sec$^{-1}$, it follows from (4.6) that $\Gamma_{CPR}/\Gamma_{CR} \sim 1$, which means that the line should be rather weak; for the intensity and halfwidth from (4.4), we have $\Gamma_{01}^{+} \approx 0.2$ cm$^{-1}$; $\hbar\delta\omega \approx 1$ meV, which agrees well with experiment.[2]

As follows from (4.5), the ratio $\Gamma^i/\Gamma^{an} \sim 5$; therefore, under the conditions of the discussed experiment, the basic contribution to the absorption is made by the isotropic part, which is due to the dependence of the $g$ factor on the quasimomentum.


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