Higher harmonics of spontaneous radiation of ultrarelativistic channeled particles

M. A. Kumakhov and Kh. G. Trikalinos

Nuclear Physics Institute, Moscow State University
(Submitted 27 November 1979)
Zh. Eksp. Teor. Fiz. 78, 1623–1635 (April 1980)

Spontaneous radiation of channeled ultrarelativistic particles is discussed in the case in which the dipole condition is not satisfied. Account is taken of the change of longitudinal velocity of the particle, which affects the maximum frequency of the radiation. The frequency and angular characteristics of the radiation are investigated for ultrahigh energies. The effect of slowing down on the trajectory of the particle is estimated. A detailed comparison is made between the characteristics of spontaneous radiation of channeled particles and bremsstrahlung. It is shown that there is an optimal energy at which the density of the radiation is maximal. The effect of the angle of entry of the electrons into the crystal and of the divergence of the beam on the radiation is analyzed.

PACS numbers: 61.80.Mk

The channeling of charged particles in crystals has been studied intensively in recent years (see for example the review by Gemmel[1]). The main theoretical ideas regarding channeling were formulated by Lindhard.[2]

One of the present authors[3] has indicated the possibility of existence of a new effect—the spontaneous radiation of relativistic channeled particles in the x-ray and γ-ray regions. The theory of this effect has been discussed by a number of authors.[4-11] The existence of radiation of this type[1)] presents a significant interest as a result of the fact that the spectral density of the radiation in channeling is several orders of magnitude higher than the spectral density of bremsstrahlung.[4] In Refs. 3—7 the radiation was discussed in the dipole approximation. This condition in channeling can be described approximately as follows: $v_\perp/c\ll 1/\gamma$, where $v_\perp$ is the transverse component of the particle velocity in the channel, $\gamma=(1-\beta^2)^{-1/2}$, and $\beta=v/c$. The dipole condition is satisfied for channeled-particle energies up to several GeV in planar channeling of positrons and approximately up to 1 GeV in axial channeling of electrons. At higher energies it is necessary to take into account the nondipole nature of the radiation and to consider the radiation in higher harmonics. The purpose of the present work is the solution of this problem.

In Section 1 we discuss the motion of the particle with inclusion of the conservation of relativistic momentum and find the change of the longitudinal velocity of the particle, which when the nondipole nature is taken into account affects the maximum frequency of the radiation (Section 3). In Section 2 we consider the principal characteristics of the radiation in higher harmonics. In Section 4 we evaluate the influence of radiation forces on the trajectory of the particle. In Section 5 we consider the question of capture of electrons in the channeling regime, and in Section 6 we discuss the effect of the electron energy, initial angle of incidence, and beam divergence on the basic characteristics of the spontaneous radiation.

1. EQUATION OF MOTION OF A CHANNELED PARTICLE

At high velocities of channeled electrons and positrons the concepts of classical mechanics are applicable.[1] We shall obtain the equation of motion of a particle in a channel in the general case. Here we shall designate the potential acting on the particle in the channel by U. In channeling

$$U\ll\mathscr{E}_0, \tag{1.1}$$

where $\mathscr{E}_0$ is the initial kinetic energy of the particle. The field in the channel can be represented in the form

$$\mathbf{E}=\mathbf{E}_\parallel+\mathbf{E}_\perp,$$

where $\mathbf{E}_\parallel$ is the field component parallel to the channel axis and $\mathbf{E}_\parallel$ is the component perpendicular to the channel axis. Here

$$e\mathbf{E}_\parallel=-\nabla_\parallel U=0,\quad e\mathbf{E}_\perp=-\nabla_\perp U, \tag{1.2}$$

where e is the charge of the particle.

With inclusion of (1.2) the equation of motion has the form

$$d\mathscr{E}/dt=e\mathbf{v}_\perp\mathbf{E}_\perp, \tag{1.3}$$

where $\mathscr{E}=\mathscr{E}_0-U$ is the kinetic energy of the particle and $\mathbf{v}=\mathbf{v}_\perp+\mathbf{v}_\parallel$ is its velocity. Using Eq. (1.3) and also the equation for the relativistic momentum[14]

$$d\mathbf{p}/dt=e\mathbf{E} \tag{1.4}$$

with allowance for the inequality $v_\perp{}^2/c^2\ll 1$, we obtain

$$m_0\gamma\frac{d\mathbf{v}_\perp}{dt}=e\mathbf{E}_\perp=-\nabla U, \tag{1.5}$$

$$m_0\gamma\frac{d\mathbf{v}_\parallel}{dt}=-\frac{e}{c^2}\mathbf{v}_\parallel(\mathbf{v}_\perp\mathbf{E}_\perp). \tag{1.6}$$

 0038-5646/80/040815-07$02.40

It follows from Eq. (1.6) that the conservation of relativistic momentum (1.4) has the result that in the general case the velocity along the channel axis is not constant, but changes with time, although no forces are acting on the particle in this direction.

Let us consider now the equations of motion of a particle in the channel for various specific cases.

A. Planar channeling of positrons

In this case the potential of the plane in the first approximation can be represented in the form[4]

$$U=V_0x^2, \tag{1.7}$$

where $V_0=0.35(4\pi NZ_1Z_2e^2 l\, b)e^{-bl}$, $l=d_p/2$, d_p is the channel width, N is the density of atoms, and Z_1e and Z_2e are respectively the charges of the particle and the atom of the target; $b=0.3/a$; a is the screening parameter in the Thomas-Fermi model of the atom.[2)]

Solving Eqs. (1.5) and (1.6) with inclusion of (1.7), we obtain

$$x=x_m \sin \bar{\omega} t, \tag{1.8}$$

where x_m is the initial amplitude,

$$\bar{\omega}=(2V_0/m_0\gamma)^{1/2}, \tag{1.9}$$

$$v_z=v_0 \exp\left(-\frac{x_m^2\bar{\omega}^2}{2c^2}\sin^2 \bar{\omega} t\right). \tag{1.10}$$

The quantity $x_m^2\bar{\omega}^2/c^2$ is small, and therefore

$$v_z=v_0[1-{}^1/_2(x_m\bar{\omega}/c)^2 \sin^2 \bar{\omega} t] \tag{1.11}$$

or on the average

$$\bar{v}_z=v_0[1-{}^1/_4(x_m\bar{\omega}/c)^2]. \tag{1.12}$$

Then the trajectory of the particle in the channel can be described by the equation

$$\mathbf{r}(t)=\mathbf{i}x_m \sin \bar{\omega} t+\mathbf{k}\bar{v}_z t. \tag{1.13}$$

In the case in which $x_m\bar{\omega}/c \ll 1/\gamma$ (this is the dipole condition, since $x_m\bar{\omega}\approx v_\perp$) we can assume that $\bar{v}_z\approx v_0$, i.e., it is not necessary to take into account the change of longitudinal velocity. At the same time on violation of the dipole condition, as we shall see below, inclusion of the change of longitudinal velocity leads to a shift of the maximum frequency of radiation.

B. Axial channeling of electrons

In this case the potential of the string can be represented in the form[15]

$$U=-Ze^2/r+C, \tag{1.14}$$

where $r=(x^2+y^2)^{1/2}$; Z and C are parameters chosen so that the potential U coincides with the Lindhard potential.[2]

We shall assume that an electron with energy $\mathscr{E}$ enters a channel at an angle Ψ_i to the axis of the string. Then the equation of motion has the form[16]

$$t=\int dr\left\{\frac{2}{m_0\gamma}[\mathscr{E}_\perp-U(r)]-\frac{L^2}{m_0^2\gamma^2r^2}\right\}^{-1/2}, \tag{1.15}$$

where $\mathscr{E}_\perp=U(r_i)+\mathscr{E}^*\Psi_i^2$ is the transverse energy; $L=pr_i\Psi_i \sin\chi_i$ is the projection of the angluar momentum on the axis of the string; $\mathscr{E}^*=0.5pv$; r_i and χ_i are the initial polar coordinates of the particle.

Solving Eq. (1.15) with inclusion of the potential (1.14), we find that the particle trajectory is an ellipse which is defined by the parametric equations[14]

$$x=a(\cos\delta-\varepsilon),\quad y=a(1-\varepsilon^2)^{1/2}\sin\delta, \qquad t=(\delta-\varepsilon\sin\delta)/\bar{\omega}, \tag{1.16}$$

where

$$a=Ze^2/2|\mathscr{E}_\perp|,\quad \varepsilon=(1-|\mathscr{E}_\perp|L^2/m_0\gamma(Ze^2)^2)^{1/2}, \tag{1.17}$$

$$\bar{\omega}=(2|\mathscr{E}_\perp|)^{3/2}/Ze^2m_0^{1/2}\gamma^{1/2}, \tag{1.18}$$

$$\bar{v}_z=v_0(1-\bar{\omega}^2a^2/2c^2). \tag{1.19}$$

2. SPECTRAL PROPERTIES OF THE RADIATION

Let the trajectory of the particle in the general case be given by the equation

$$\mathbf{r}(t)=\mathbf{i}x(t)+\mathbf{j}y(t)+\mathbf{k}\bar{\beta}_z ct. \tag{2.1}$$

We shall assume that

$$l_1\gg 2\pi\bar{\beta}_z c/\bar{\omega}, \tag{2.2}$$

where l_1 is the length of the crystal (i.e., that in the entire length of the crystal the particle can complete a large number of oscillations).

Then in analogy to wiggler radiation[17] we fine that the spectral and angular distributions of the intensity of the k-th harmonic of the radiation are

$$\frac{dI_k}{d\omega}=\frac{e^2}{2\pi\bar{\beta}_z\omega}\int_0^{2\pi}|\mathbf{a}_k(\vartheta_k,\varphi)|^2\,d\varphi, \qquad \frac{dI_k}{d\Omega}=\frac{e^2\omega_k}{2\pi c\bar{\omega}_k}|\mathbf{a}_k(\omega_k,\vartheta,\varphi)|^2, \tag{2.3}$$

where

$$\omega_k=\frac{k\bar{\omega}}{1-\bar{\beta}_z\cos\vartheta},\qquad \vartheta_k=\arccos\frac{1-k\bar{\omega}/\omega}{\bar{\beta}_z}, \qquad |\mathbf{a}_k|^2=|\mathbf{b}_k|^2-|\mathbf{n}\mathbf{b}_k|^2; \tag{2.4}$$

$$\mathbf{b}_k(\omega,\vartheta,\varphi)=-i\frac{\bar{\omega}\omega}{2\pi}\int_0^{2\pi/\bar{\omega}}\boldsymbol{\beta}\exp\left\{i\left[k\bar{\omega}t-\frac{\omega}{c}(n_xx+n_yy)\right]\right\}dt. \tag{2.5}$$

We shall discuss the spectral and angular characteristics of the radiation of the particle in various cases of channeling.

A. Planar channeling of positrons

1. *The dipole approximation* ($x_m\bar{\omega}/c\ll 1/\gamma$, $\gamma\gg 1$). In this case the motion of the particle is given by Eq. (1.13), where $\bar{v}_z\approx v$. Then from Eqs. (2.3)—(2.5) we obtain

$$\frac{dI}{d\omega}=\frac{3\bar{I}}{\omega_m}\frac{\omega}{\omega_m}\left[1-2\frac{\omega}{\omega_m}+2\left(\frac{\omega}{\omega_m}\right)^2\right], \qquad (2.6)$$

where

$$\bar{I}=x_m^2e^2\bar{\omega}^4\gamma^4/3c^3, \qquad (2.7)$$

$\bar{\omega}$ is given by Eq. (1.9) and $\omega_m=2\bar{\omega}\gamma^2$ is the maximum frequency of the radiation.

Equations (2.6) and (2.7) were obtained previously in Refs. 3 and 4.

2. *The dipole condition is not satisfied* ($x_m\bar{\omega}/\gamma c \ll 1$, $\gamma \gg 1$). The motion of the particle occurs in accordance with the law (1.13). After using Eq. (2.5) we obtain

$$b_{kx}=-i\frac{k\bar{\omega}}{\sin\vartheta\cos\varphi}J_k(k\varkappa). \qquad (2.8)$$

$$b_{kz}=-i\frac{k\bar{\omega}\beta_z}{1-\beta_z\cos\vartheta}J_k(k\varkappa), \qquad (2.9)$$

where

$$\varkappa=\frac{x_m\bar{\omega}\sin\vartheta\cos\varphi}{c(1-\beta_z\cos\vartheta)}, \qquad (2.10)$$

and $J_k(k\varkappa)$ is the Bessel function. Consequently

$$|a_k|^2=\bar{\omega}^2k^2\left[\frac{1}{\sin^2\vartheta\cos^2\varphi}-\frac{1-\bar{\beta}^2}{(1-\beta_z\cos\vartheta)^2}\right]J_k^2(k\varkappa). \qquad (2.11)$$

In the relativistic case for $\vartheta \ll 1$, i.e., in the interval of angles where the principal part of the radiated energy is concentrated, we obtain

$$\frac{dI_k}{d\Omega}=\frac{e^2\bar{\omega}^2\gamma^4}{c}\frac{\xi^2}{\pi k}G,\quad \frac{dI_k}{d\omega}=\frac{e^2\bar{\omega}}{2c}\frac{1}{2\pi}\int_0^{2\pi}G\,d\varphi, \qquad (2.12)$$

where

$$G=\left[\frac{k^2}{(k-\xi\alpha)\cos^2\varphi}-4\alpha\xi\right]J_k^2(k\varkappa),$$
$$\varkappa=\frac{2}{k}\frac{x_m\bar{\omega}\gamma}{c}(\xi(k-\xi\alpha))^{1/2}\cos\varphi,$$
$$\alpha=1+\frac{1}{2}\left(\frac{x_m\bar{\omega}\gamma}{c}\right)^2, \qquad (2.13)$$
$$\xi=\frac{\omega}{2\bar{\omega}\gamma^2}=\frac{k}{1+(\vartheta\gamma)^2+{}^1/_2(x_m\bar{\omega}\gamma/c)^2}.$$

It is easy to find also that for $\vartheta=0$ only the first harmonic is radiated, with an intensity

$$\left.\frac{dI_1}{d\Omega}\right|_{\vartheta=0}=\frac{e^2\bar{\omega}^4\gamma^6x_m^2}{\pi c^3}\xi^3. \qquad (2.14)$$

B. Axial channeling of electrons

1. *The dipole approximation*: Motion of the particle in this case is described by Eqs. (1.16) and (1.19), where $\bar{v}_z \approx v \approx c$. The spectral distribution of the radiated intensity is[4]

$$\frac{dI_k}{d\omega}=\frac{3\bar{I}_k}{\omega_{km}}\frac{\omega}{\omega_{km}}\left[1-2\frac{\omega}{\omega_{km}}+2\left(\frac{\omega}{\omega_{km}}\right)^2\right], \qquad (2.15)$$

where

$$\bar{I}_k=\frac{8e^2k^2a^2\bar{\omega}^4\gamma^4}{3c^3}\left[J_k'^2(k\varepsilon)-\frac{1-\varepsilon^2}{\varepsilon^2}J_k^2(k\varepsilon)\right],$$
$$\omega_{km}=2k\bar{\omega}\gamma^2.$$

2. *General case*. Using Eq. (2.5) and also the properties of Bessel functions, we obtain

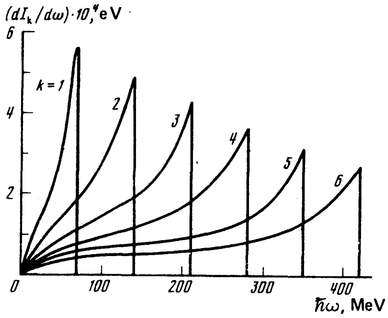

FIG. 1. Spectral distribution of radiation from electrons with $\mathscr{E}=5$ GeV in channeling through diamond in the ⟨110⟩ direction.

$$b_{kx}=-\frac{\omega\bar{\omega}a}{c}\left[J_k'(k\varkappa)\cos\psi-\frac{i}{\varkappa}J_k(k\varkappa)\sin\psi\right],$$
$$b_{ky}=-\frac{\omega\bar{\omega}a(1-\varepsilon^2)^{1/2}}{c}\left[J_k'(k\varkappa)\sin\psi+\frac{i}{k}J_k(k\varkappa)\cos\psi\right], \qquad (2.16)$$
$$b_{kz}=\bar{\omega}\beta_z\left[\varepsilon J_k'(k\varkappa)\sin\psi-\left(1-\frac{\varepsilon}{\varkappa}\cos\psi\right)J_k(k\varkappa)\right],$$

where

$$\varkappa=(\varkappa_x^2+\varkappa_y^2)^{1/2},\quad \psi=0{,}5\,\mathrm{arc\,cos}\left(\frac{\varkappa_y^2-\varkappa_x^2}{\varkappa_x^2+\varkappa_y^2}\right),$$
$$\varkappa_x=\frac{a\bar{\omega}\sin\vartheta}{c(1-\beta_z\cos\vartheta)}\cos\varphi,\quad \varkappa_y=\frac{a(1-\varepsilon^2)^{1/2}\bar{\omega}\sin\vartheta}{c(1-\beta_z\cos\vartheta)}\sin\varphi+\varepsilon, \qquad (2.17)$$

where $J_k'(k\varkappa)$ is the derivative of the Bessel function.

From Eqs. (2.3) and (2.4) we can find the spectral and angular distributions of the radiation.

In this case the radiation does not have axial symmetry. However, as was shown by Kumm *et al.*,[18] in the axial channeling of electrons there is a motion of the rosette type which can be described approximately as the rotation of an ellipse in a plane around the axis of the string. Consequently, from a certain depth the radiation can be considered symmetric. We shall estimate this depth.

We shall use the potential

$$U(r)=-Ze^2/r^{1+\lambda}, \qquad (2.18)$$

where $\lambda \ll 1$. Then for the depth we obtain

$$l=\pi\cdot 4Lc/\lambda|\mathscr{E}_\perp|. \qquad (2.19)$$

For $\mathscr{E}=1$ GeV and $\lambda \sim 0.05$ we find $l \sim 50$ μm, which is much less than the dechanneling depth.

In Fig. 1 we have shown the spectral distribution of intensity in axial channeling of an electron with energy $\mathscr{E}=5$ GeV through a diamond crystal in the ⟨110⟩ direction.

3. EFFECT OF NONDIPOLE CONDITION ON THE MAXIMUM FREQUENCY OF RADIATION

The Doppler formula for the frequency of radiation of the first harmonic has the form

$$\omega=\bar{\omega}/(1-\beta_z\cos\vartheta). \qquad (3.1)$$

Consequently the maximum frequency of radiation is

$$\omega_m=\bar{\omega}/(1-\beta_z). \quad (3.2)$$

As we have already shown in the general case, $\bar{\beta}_z \neq \beta$. For example, for planar channeling of positrons $\bar{\beta}_z$ is determined by Eq. (1.12). Consequently

$$\omega_m=\frac{\bar{\omega}}{1-\beta+{}^1/_4\beta(x_m\bar{\omega}/c)^2}. \quad (3.3)$$

Taking into account that $\beta \approx 1, 1-\beta \approx 1/2\gamma^2$, we obtain

$$\omega_m=\frac{2\bar{\omega}\gamma^2}{1+{}^1/_2(x_m\bar{\omega}\gamma/c)^2}\sim\gamma^{1/2}. \quad (3.4)$$

In the dipole approximation, where the condition $x_m\bar{\omega}/c \ll 1/\gamma$ is satisfied, we obtain

$$\omega_m\approx 2\bar{\omega}\gamma^2\sim\gamma^{3/2}. \quad (3.5)$$

Similarly, for axial channeling of electrons the maximum frequency of the harmonic is

$$\omega_m=\frac{2\bar{\omega}\gamma^2}{1+\bar{\omega}^2a^2\gamma^2/2c^2}. \quad (3.6)$$

Equations (3.4)—(3.6) have another important consequence. It is well known that $\bar{\omega}\sim\gamma^{-1/2}$. Thus, at low energies where the dipole condition is satisfied the maximum frequency of the first harmonic rises rapidly ($\omega_m\sim\gamma^{3/2}$). However, on increase of the energy the rise occurs more slowly ($\omega_m\sim\gamma^{1/2}$).

We find from Eqs. (2.6), (3.4), and (3.5) that at low energies (when the dipole condition is satisfied and the maximum frequency of radiation is proportional to $\gamma^{3/2}$) the spectral intensity of the radiation at $\omega=\omega_m$ is proportional to $\gamma^{1/2}$. At high energies (when the maximum frequency of radiation is proportional to $\gamma^{1/2}$) the spectral intensity of the radiation at $\omega=\omega_m$ is proportional to $\gamma^{-1/2}$. This important result agrees with the result obtained by Zhevago[11] in a quantum-mechanical treatment of the problem.

Let us consider the physical meaning of this result. We shall define the transverse velocity of the particle (for example, a positron in planar channeling) in the system of coordinates in which on the average it is at rest. This system moves along the z axis with a velocity v_z. Then from Lorentz transformations we find that in the new system the transverse velocity is

$$v_x'=\frac{v_x(1-v_z^2/c^2)^{1/2}}{1-\bar{v}_z^2/c^2}=\frac{v_x}{(1-\bar{v}_z^2/c^2)^{1/2}}\approx v_x\gamma. \quad (3.7)$$

We know that $v_x\approx x_m\bar{\omega}$, and therefore

$$v_x'^2/c^2\sim(x_m\bar{\omega}\gamma/c)^2. \quad (3.8)$$

Thus, when the dipole condition is not satisfied the transverse velocity of the particle in the new system becomes relativistic.

4. BREMSSTRAHLUNG

Let us consider the change in energy of a particle due to its radiation in channeling. It is well known that the retarding force is[12]

$$\mathbf{f}=-\frac{2e^4E^2\gamma^2}{3m_0^2c^5}\mathbf{v}, \quad (4.1)$$

where E is the electric field strength in the channel. We shall assume that the particular energy is $\mathscr{E}=m_0\gamma c^2$. Then the change in energy in time dt is

$$-\frac{d\mathscr{E}}{dt}=\frac{2e^4\overline{E^2}\gamma}{3m_0^2c^5}v^2. \quad (4.2)$$

Taking into account that $v\approx c$, we obtain

$$-\frac{d\mathscr{E}}{dt}=\frac{2e^4\overline{E^2}}{3m_0^4c^7}\mathscr{E}^2, \quad (4.3)$$

and after integration

$$\frac{1}{\mathscr{E}}=\frac{1}{\mathscr{E}_0}+\frac{2e^4\overline{E^2}}{3m_0^4c^7}t. \quad (4.4)$$

Thus, we have found the change in energy of a spontaneously radiating channeled particle as a function of time. In order that the energy loss be small, it is necessary that the following condition be satisfied:

$$\mathscr{E}_0\ll 3m_0^4c^7/2e^4\overline{E^2}t. \quad (4.5)$$

Let us evaluate this condition. For axial channeling of electrons in silicon in the ⟨110⟩ direction we have

$$e^2\overline{E^2}\approx 1.2\cdot 10^3\ \mathrm{eV^2/Å^2} \text{ and } t=\Delta x/v\approx\Delta x/c,$$

where Δx is the dechanneling depth at the given energy. In silicon at energies of several tens of GeV we can assume $\Delta x\sim 10^8$ Å. Then we obtain $\mathscr{E}_0\ll 100$ GeV. Consequently, up to energies of several tens of GeV the energy loss due to the radiation can be neglected in thin single crystals.

Similarly, for planar channeling of positrons in silicon through the (100) channel for $\Delta x\sim 10^7$ Å we obtain $\mathscr{E}_0\ll 10^3$ GeV.

We shall consider now the question of the change in the amplitude of transverse motion of a particle in channel in the case of planar channeling. The equation of motion (1.5) takes the form

$$m_0\gamma\frac{dv_x}{dt}=-2V_0x-\frac{8e^2x^2V_0^2\gamma^2}{3m_0^2c^5}\frac{dx}{dt}. \quad (4.6)$$

We shall assume that the condition (4.5) is satisfied. Then we can assume that γ does not depend on time. From Eq. (4.6) we obtain

$$\frac{d^2x}{dt^2}+\bar{\omega}^2x=\mu\left(-\frac{1}{m_0\gamma}x^2\frac{dx}{dt}\right), \quad (4.7)$$

where $\mu=8e^2\gamma^2V_0^2/3m_0^2c^5$ is a small parameter. The solution of Eq. (4.7) is obtained in the form

$$x=x_1\sin\bar{\omega}t, \quad (4.8)$$

where

$$x_1=2x_m[m_0\gamma/(4m_0\gamma+x_m^2\mu t)]^{1/2}. \quad (4.9)$$

It is easy to see that if the condition (4.5) is satisfied, we can set $x_1 \approx x_m$.

5. CAPTURE OF ELECTRONS INTO THE CHANNELING REGIME

It is known that in channeling of positively charged particles practically all of the particles are captured into the channeling regime. A different situation exists in channeling of electrons. It has been shown[18] that in axial channeling of electrons the entire beam is not captured into the channeling regime, but only a certain part determined from the conditions

$$r_{min} \geqslant r_c, \tag{5.1}$$

$$r_{max} \leqslant D/2. \tag{5.2}$$

$$\mathscr{E}_\perp \leqslant 0, \tag{5.3}$$

where r_{min} and r_{max} are the minimum and maximum distances of the particle from the string in channeling, D is the distance between strings, and r_c is the critical radius. The condition (5.1) means that the particle cannot approach a string of atoms closer than a distance r_c. The condition (5.3) is necessary in order that the particle be captured into the potential well.

Using the potential (1.14), we obtain

$$r_{min} = a(1-\varepsilon) \geqslant r_c, \tag{5.4}$$

$$r_{max} = a(1+\varepsilon) \leqslant D/2, \tag{5.5}$$

where

$$\varepsilon = (a^2 - b^2)^{1/2}/a,$$

and a and b are determined from Eq. (1.17).

From Eqs. (5.4) and (5.5) we determine r_{min} and r_{max} and consequently the fraction of particles n which are captured into the channeling regime. As can be seen from Eqs. (5.3)–(5.5) and (1.17), n depends on the initial angle of incidence Ψ_i and on the critical radius r_c. For $E \gg m_0 c^2$ the value of n does not depend on the electron energy.

It is well known[19] that at high incident-particle energies r_c decreases and can reach a value of the order of the amplitude of the thermal vibrations of the atoms of the string. In Fig. 2 we have shown the dependence of

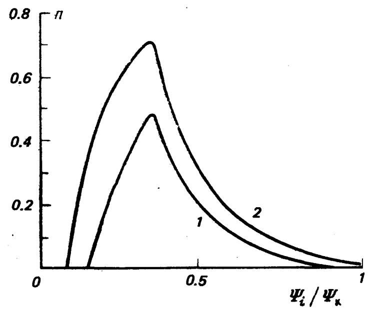

FIG. 2. Fraction of particles captured into the channeling regime, as a function of the initial angle of incidence for the ⟨110⟩ channel of diamond: 1—$r_c = 0.1$ Å, 2—$r_c = 0.05$ Å.

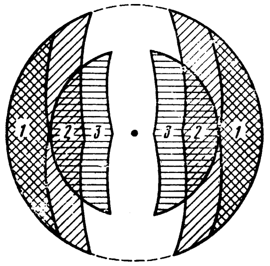

FIG. 3. Initial distribution of particles captured into the ⟨110⟩ channeling regime of a diamond crystal: 1—$\Psi_i = 0.3\Psi_c$; 2—$\Psi_i = 0.5\Psi_c$; 3—$\Psi_i = 0.7\Psi_c$.

the fraction n of electrons captured into the channeling regime on Ψ_i [in units of the Lindhard critical angle Ψ_c (Ref. 2)] for two values of r_c for the ⟨110⟩ direction of a diamond crystal. In Fig. 3 we have shown the initial distribution of particles captured into the channeling regime for various fixed Ψ_i.

Under real experimental conditions the incident beam has some angular divergence $\bar{\vartheta}$. When the divergence is taken into account the number of captured particles changes substantially. For example, for the ⟨110⟩ channel of diamond at an electron energy $\mathscr{E} = 5$ GeV with $\Psi_i = 0$ and a divergence $\bar{\vartheta} = 2 \times 10^{-4}$ rad for $r_c = 0.5$ Å, we have $n \approx 15\%$.

We note that the features of capture of electrons into the axial channeling regime and also the angular dependence of a real beam will lead to the result that the spectrum of the spontaneous radiation will have a fine structure.

6. EFFECT OF ELECTRON ENERGY, INITIAL ANGLE OF INCIDENCE, AND BEAM DIVERGENCE ON THE PRINCIPAL CHARACTERISTICS OF THE SPONTANEOUS RADIATION

The theoretical studies[3–11] carried out up to the present time have for the most part considered the radiation parameters of an individual particle. For comparison of theory with experiment it is necessary to take into account the initial angle of entry of the beam into the channel, the divergence, and so forth. In this sec-

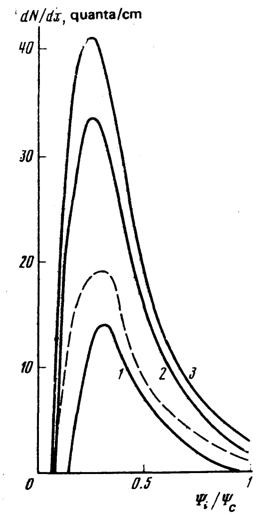

FIG. 4. Number of radiated γ rays as a function of the initial angle of incidence at $\mathscr{E} = 5$ GeV for diamond in the ⟨110⟩ direction: 1—$r_c = 0.1$ Å, 2—$r_c = 0.05$ Å, 3—$r_c = 0.04$ Å; the dashed curve is for $\mathscr{E} = 10$ GeV, $r_c = 0.05$ Å.

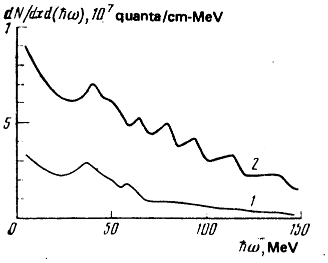

FIG. 5. Spectral distribution of intensity for the ⟨110⟩ channel of diamond at $\mathscr{E}=5$ GeV, $\Psi_i=0.25\Psi_c$: 1—$r_c=0.1$ Å, 2—$r_c=0.05$ Å.

tion we present the results of such a calculation.

We calculated the number of γ rays emitted in 1 cm of path with energy from 15 to 70 MeV in channeling of 5-GeV electrons through a diamond crystal in the ⟨110⟩ direction.

In Fig. 4 we have shown the dependence of the number of radiated quanta on the initial angle of incidence for three values of r_c. For comparison we have also shown a curve for $\mathscr{E}=10$ GeV.

In Fig. 5 we have shown the spectral distribution of intensity for a fixed angle of incidence with inclusion of six harmonics.

It is known that the number of bremsstrahlung photons in an unoriented target is

$$N_b=\frac{4}{3}\frac{\Delta x}{R}\ln\frac{\omega_2}{\omega_1}, \quad (6.1)$$

where Δx is the thickness of the crystal, R is a radiation length, and ω_2 and ω_1 are the maximum and minimum frequencies of the detected photons.

If the angular divergence of the beam is known, we can determine the ratio of effect to background

$$\eta=N_\gamma/N_b. \quad (6.2)$$

For $\Psi_i=0$, $\vartheta=2\times10^{-4}$ rad, and $r_c=0.05$ Å we obtain $\eta\approx50$. For $\Psi_i=0.25\Psi_c$, $\vartheta=0.1\Psi_c$, and $r_c=0.05$ Å we have $\eta=200$; for $r_c\approx0.4$ Å, $\eta\approx300$. In Fig. 6 we have shown dN_γ/dx as a function of the incident beam energy for $\Psi_i=0$, $\vartheta=2\times10^{-4}$ rad, $r_c=0.05$ Å, and for the specified range of detected γ-ray energies (15—70 MeV). As can be seen from this figure, under the conditions specified it is most effecient to use an electron beam with energy $\mathscr{E}\approx2$ GeV. In that case $\eta\approx250$.

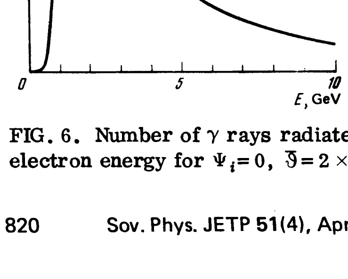

FIG. 6. Number of γ rays radiated in 1 cm as a function of electron energy for $\Psi_i=0$, $\vartheta=2\times10^{-4}$ rad, and $r_c=0.05$ Å.

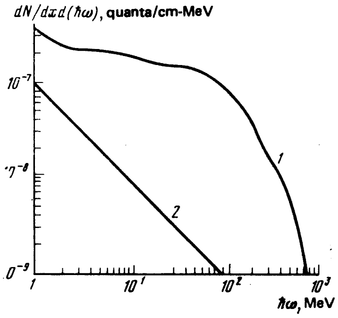

FIG. 7. Spectral distribution of spontaneous radiation (1) and bremsstrahlung (2) for the ⟨110⟩ channel of diamond at $\mathscr{E}=5$ GeV, $\Psi_i=0$, $\vartheta=2\times10^{-4}$ rad, and $r_c=0.05$ Å.

In Fig. 7 we have shown the spectral distributions of the spontaneous radiation and of bremsstrahlung for $\mathscr{E}=5$ GeV, $\Psi_i=0$, $\vartheta=2\times10^{-4}$ rad, and $r_c=0.05$ Å. It is evident that by choosing $\hbar\omega_1$ and $\hbar\omega_2$ one can obtain an increase of η. For example, for $\hbar\omega_1=100$ MeV and $\hbar\omega_2=150$ MeV we obtain $\eta=150$. From these calculations it can be seen that under certain conditions there is an optimal energy and an optimal detection interval for which the spectral density of the radiation will have a maximum. There is also an optimal angle of entry of the electrons into the crystal. The beam divergence also greatly affects the number of radiated γ rays.

We shall consider also the effect of the rms scattering angle on the ratio of the numbers of γ rays radiated in the spontaneous radiation and in bremsstrahlung. In a length $\Delta x=10^{-2}$ cm for $\mathscr{E}=1$ GeV in the frequency interval $0.1\omega_m\leqslant\Delta\omega\leqslant\omega_m$ in the ⟨111⟩ direction of a tungsten crystal we obtain $N_\gamma\approx12$, $N_b\approx0.06$. The number of photons emitted in a given direction in bremsstrahlung is determined by the multiple scattering of the electrons. The mean square scattering angle is

$$\overline{\Delta\theta^2}=440\frac{\Delta x}{R}\frac{4}{\gamma^2}, \quad (6.3)$$

where R is a radiation length. In regard to the spontaneous radiation, here the crystal itself governs the motion of the particle and therefore the half-width of the angular distribution is not due to multiple scattering, but is determined by the radiation, i.e., it is equal to $\sim1/\gamma$. Therefore the ratio of the number of γ rays in the frequency interval $0.1\omega_m\leqslant\Delta\omega\leqslant\omega_m$ emitted in a given direction is approximately

$$\eta\approx1760\alpha\frac{\Delta x}{R}\frac{N_\gamma}{N_b}, \quad (6.4)$$

where α is the fraction of particles which enter the channeling regime. We shall assume that $\alpha\approx0.4$.

From Eq. (6.4) we find that $\eta=2.5\times10^3$, while $N_\gamma/N_b\sim2\times10^2$. Thus, the flux of γ rays produced in the frequency interval $0.1\omega_m\leqslant\Delta\omega\leqslant\omega_m$ and a given range of angles in channeling can in the case of heavy elements exceed the corresponding flux from bremsstrahlung by three to four orders of magnitude.

It should be noted the results obtained by us in this work and also in Refs. 3–6 on all of the main parameters of the radiation (intensity, spectral and angular density, polarization, and so forth) differ fundamentally from the results obtained by Baryshevskiĭ *et al.*[20] This is due to the erroneous nature of their work[20] (a detailed critical analysis of Ref. 20 is given in Refs. 21 and 22).

A substantial effect on the radiation of channeled particles is exerted by secondary processes which occur in the crystal (for example, multiple scattering, etc.). Here dechanneling of the particles and some broadening of the spectrum occur. Therefore a separate article will be devoted to the discussion of this question.

In conclusion the authors express their gratitude to Yu. V. Kononets for a helpful discussion and for a number of remarks which made possible improvement of this article.

[1)]The first indications of the existence of this effect for electrons were obtained by Agan'yants *et al.*[12] In the current Soviet-American experiment at the Stanford Linear Accelerator Center (SLAC) spontaneous γ radiation was observed for positrons with energy 1–14 GeV in planar channeling of positrons through a diamond crystal.[13] For electrons and positrons of low energies (E= 28–56 MeV) the effect was measured by Datz *et al.*[23]

[2)]The real potential differs from harmonic. For more accurate calculation of the radiation it is necessary to take into account the anharmonic part of the potential. Such a calculation was carried out for the first time in Refs. 4 and 6 (see for example Fig. 1 in Ref. 6). In Refs. 4 and 6 at the same time the authors carried out an averaging over the amplitudes of oscillation of the particles in evaluation of the intensity of radiation [see for example Eqs. (5.4)–(5.6) in Ref. 6].

[1]D. S. Gemmel, Rev. Mod. Phys. **46**, 129 (1974).
[2]J. Lindhard, Danske Vid. Selsk. Mat.-fys. Medd. **34**, 14 (1965).
[3]M. A. Kumakhov, Phys. Lett. **57A**, 17 (1976).
[4]M. A. Kumakhov, Dokl. Akad. Nauk SSSR **230**, 1077 (1976) [Sov. Phys. Doklady **21**, 581 (1976)]; Zh. Eksp. Teor. Fiz. **72**, 1489 (1977) [Sov. Phys. JETP **45**, 781 (1977)].
[5]M. A. Kumakhov, Phys. Stat. Sol. (b), **84**, 41 (1977).
[6]M. A. Kumakhov and R. Wedell, Phys. Stat. Sol. (b), **84**, 581 (1977).
[7]V. V. Beloshitsky, Phys. Lett. **64A**, 95 (1977). R. W. Terhune and R. H. Pantell, Appl. Phys. Letters **30**, 265 (1977).
[8]V. A. Bazylev and N. K. Zhevago, Zh. Eksp. Teor. Fiz. **73**, 1697 (1977) [Sov. Phys. JETP **46**, 891 (1977)].
[9]A. I. Akhiezer, V. F. Boldyshev, and N. F. Shul'ga, Dokl. Akad. Nauk SSSR **236**, 830 (1977) [Sov. Phys. Doklady **22**, 569 (1977)].
[10]M. I. Podgoretskiĭ, Preprint JINR R2-10793, 1977; R2-10986, 1977; R2-11140, 1977.
[11]N. K. Zhevago, Zh. Eksp. Teor. Fiz. **75**, 1389 (1978) [Sov. Phys. JETP **48**, 701 (1978)].
[12]A. O. Agan'yants, N. Z. Akopov, Yu. A. Vortanov, and G. A. Vartapetyan, Preprint EFI-312(37)-78, Erevan Physics Institute. A. O. Agan'yants, Preprint EFI-313(38)-78, Erevan Physics Institute, 1978.
[13]I. I. Miroshnichenko, J. J. Murray, R. O. Avakyan, and T. Kh. Figut, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 786 (1979) [JETP Lett. **29**, 722 (1979)].
[14]L. D. Landau and E. M. Lifshitz, Teoriya polya (Theory of Fields), Nauka, Moscow, 1967.
[15]A. Tamura and T. Kawamura, Phys. Stat. Sol. (b), **73**, 391 (1976).
[16]L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics), Nauka, Moscow, 1965.
[17]D. F. Alferov, Yu. A. Bashmakov, and E. G. Bessonov, Trudy FIAN (Proceedings of the Lebedev Institute) 80, 100 (1975).
[18]H. Kumm, F. Bell, R. Sizman, H. J. Kreiner, and D. Harder, Rad. Eff. **126**, 53 (1972).
[19]S. A.Vorob'ev and V. V. Kaplin, Trudy VI Vsesoyuznogo soveshchaniya po fizike vzaimodeĭstviya zaryazhennykh chastits s monokristallami (Proceedings of the Sixth All-Union Conf. on Physics of Interaction of Charged Particles With Single Crystals), Moscow, 1975, p. 221.
[20]V. G. Baryshevskii and I. Ya. Dubovskaya, Phys. Stat. Sol. (b), **82**, 403 (1977); V. G. Baryshevskii, A. O. Grubich, and I. Ya. Dubovskaya, *ibid.* **88**, 351 (1978).
[21]V. A. Bazylev, V. N. Glebov, and N. K. Zhevago, Zh. Eksp. Teor. Fiz. **78**, 62 (1980) [Sov. Phys. JETP **51**, 31 (1980)].
[22]M. A. Kumakhov and Cr. Trikalinos, Phys. Stat. Sol. (b), (in print).
[23]M. J. Alguard, R. L. Swent, R. H. Pantell, B. L. Berman, S. D. Bloom, and S. Datz, Phys. Rev. Lett. **42**, 1148 (1979); R. L. Swent *et al.*, **43**, 1723 (1979).

Translated by Clark S. Robinson

Stochastic self-oscillations in parametric excitation of spin waves

E. V. Astashkina and A. S. Mikhaĭlov

Moscow State University
(Submitted 17 September 1979)
Zh. Eksp. Teor. Fiz. **78**, 1636–1646 (April 1980)

We consider a situation wherein an external monochromatic pump excites parametrically a pair of primary spin waves, each of which breaks up in turn into two secondary waves. The dynamics of the system is simulated numerically and it is shown that instability of the phase trajectories is observed in it when the initial conditions are perturbed. The values of the Kolmogorov entropy are calculated for different values of the excess over the parametric resonance threshold.

PACS numbers: 75.30.Ds, 05.20. – y

The irreversibility of the behavior of complicated dynamic systems, consisting of a large number of particles, is due to the instability of the phase trajectories of such systems relative to some arbitrarily weak perturbation of the initial conditions. Recent mathematical investigations[1] by Smale, Anosov, Sinaĭ, and others have shown that dynamics randomization due to such an instability is possible also in systems having a small number of degrees of freedom. The stochastic behavior in a real physical model, described by only three

 0038-5646/80/040821-06$02.40