Optico-magnetic effects in nondestructive quantum counting

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(Submitted 16 October 1979)
Zh. Eksp. Teor. Fiz. **78**, 1712–1717 (May 1980)

A measurement procedure that permits the counting of optical quanta without their absorption is considered. The inverse Faraday effect leads under specific resonant conditions to an effective generation of microwave quanta from optical ones. The number of optical quanta remains unchanged. The existence of two effects, optico-magnetic transition radiation and optico-magnetic Cerenkov radiation, is predicted.

PACS numbers: 42.50. + q, 78.20.Ls

The possibility of measuring very small values of magnetic microwave fields with the aid of the magneto-optical Faraday effects was noted in Ref. 1. In fact, the angle $\Delta\varphi_{opt}$ of rotation of the plane of polarization of a light wave propagating in an optical fiber with a Verdet constant V^*,

$$\Delta\varphi_{opt}=V^*H_eL,$$

reaches the easily measured value $\approx 10^{-7}$ rad if $V^*=10^{-5}$ Oe^{-1} cm^{-1}, the fiber length is $L=10^6$, and the microwave magnetic field strength is $H_e=10^{-8}$ Oe. The value $H_e=10^{-8}$ Oe corresponds to an energy of the order of one microwave quantum $\hbar\omega_e\approx 2\times 10^{-17}$ erg, if the volume of the inductive part of the resonator is ≈ 1 cm^3. In order for a linear connection to exist between $\Delta\varphi_{opt}$ and L, it is necessary that each turn of the optical fiber be in an integer ratio with the microwave wavelength (a unique geometric resonance). The linear connection between $\Delta\varphi_{opt}$ and L means the possibility, in principle, of an arbitrarily accurate measurement of only one of the quadrature components of the magnetic microwave field.[2,3] The second quadrature component, in accordance with the Heisenberg uncertainty relations, should then be strongly disturbed. It was indicated in Ref. 1 that inasmuch as the interaction of the light wave with the microwave field of the resonator is via the magnetic component of the oscillations, it follows that the source of the perturbation should also be a certain fluctuating microwave field induced by the light.

The purpose of the present paper is to consider this fluctuation influence of light on a microwave resonator, and to discuss a new possibility of nondemolition quantum counting in two optical effects that have apparently not been considered earlier.

In 1965, van der Ziel, Pershan, and Malmstrem[4] have discovered (see also Refs. 5 and 6) the inverse Faraday effect, namely that a circularly polarized light wave produces constant magnetization of a substance with a nonzero Faraday constant. The magnetization vector **I** is connected with the Poynting vector **Π** of the light wave by the simple relation

$$\mathbf{I}=\frac{V^*}{\omega_{opt}}\mathbf{\Pi}, \tag{1}$$

where ω_{opt} is the frequency of the optical radiation. We note that this magnetization effect is similar to the ponderomotive effect and to the effect of optical detection (the effect is quadratic). If the power of the circularly polarized optical wave is modulated, then the alternating magnetization of the medium can serve as a source of electromagnetic radiation at the modulation frequency. It is precisely this effect which is responsible for the fluctuation reaction on the investigated microwave resonator in the measurement system used in Ref. 1. It is due to the fluctuating circularly polarized component in the probing light flux (which is linearly polarized on the average). This component is always present, if for no other reason, by virtue of the Heisenberg uncertainty relations for electromagnetic-field oscillators.[7] Straightforward but rather cumbersome calculations, which will not be presented here, yield the following smallest measurement errors $(\Delta H_e)_{\mathrm{I}}$ and $(\Delta H_e)_{\mathrm{II}}$ of the two quadrature components of the magnetic microwave field in the resonator. If the probing light flux is constant, then the two quadrature components are simultaneously measured with equal accuracy

$$(\Delta H_e)_{\mathrm{I}}=(\Delta H_e)_{\mathrm{II}}=(2\pi\hbar\omega_e/V_e)^{1/2}, \tag{2}$$

where V_e is the volume of the inductive part of the microwave resonator. Their perturbations in the course of the measurements are also equal and given by (2). If the light beam is intensity modulated in time with a period equal exactly to $2\pi/\omega_e$, then only one quadrature component is measured, and the accuracy of its measurement can in principle be arbitrarily high. The second component is perturbed in this case, and the following condition is satisfied

$$(\Delta H_e)_{\mathrm{I}}(\Delta H_e)_{\mathrm{II}}=2\pi\hbar\omega_e/V_e. \tag{3}$$

We consider now an experimental setup that is the inverse of the one considered above. In this setup the energy of low-power optical radiation is measured, without absorption, by the inverse Faraday effect—the source of information on the optical energy is a constant or alternating (relatively low frequency) magnetic field.

Assume that a circularly polarized wave circulates in a closed light-conducting ring (see the figure). The produced magnetization is proportional to the energy of the wave and does not contain information on its phase. This, in accordance with Ref. 8, is a sufficient condition for an approximately nondemolishing measurement of the energy. The magnetic flux can be measured, for example, with a quantum magnetometer. Estimates show, however, that by using light-conducting materials

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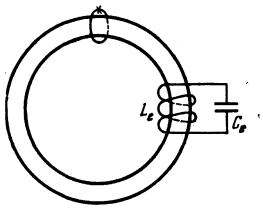

FIG. 1.

with the presently known values of the Faraday constant it is impossible to attain a sensitivity level of one quantum of light energy. In fact, the quantum-magnetometer sensitivity attained to date is such that within a time τ they can sense a magnetic flux of the order of[9,10]

$$\Phi \approx 10^{-7}\Phi_0\tau^{-1/2}\ \text{G}\cdot\text{cm}^2,$$

where Φ_0 is the magnetic-flux quantum. On the other hand, the magnetic flux induced in a light-conducting ring is equal, in accordance with (1), to

$$\Phi = \frac{2V^*\lambda_{opt}}{l}\mathscr{E}_{opt}, \tag{4}$$

($\mathscr{E}_{opt}$ is the energy of the wave circulating in the optical waveguide, and l is the length of this waveguide). In a measurement time $\tau \approx 100$ μsec (which corresponds to an attenuation of 0.2 dB/km for the best optical waveguides known at present) $l = 1$ cm, $\lambda_{opt} = 0.5$ μm, and $V^* = 1\times 10^{-4}$ Oe^{-1} cm^{-1} (the highest value for substances weakly absorbing in the optical band[11]) the energy measurement error is $\Delta\mathscr{E}_{opt} 1\times 10^{-4}$ erg, which corresponds to approximately 2×10^7 optical quanta.

We consider another possible method of measuring the induced magnetization. Assume that the light pulse has a length shorter than the length of the optical waveguide ring, and that part of the optical waveguide is inside the inductive part of an electromagnetic resonator (see the figure). Then, for each passage of the light pulse to this part of the optical waveguide there will be induced in the resonator a current pulse proportional to the energy of the light. By choosing the period of the natural oscillations of the resonator equal to the time of circulation of the light in the ring, we can attain resonant accumulation of the effect.

The equation of motion of the field in the microwave resonator is of the form

$$\ddot{H}_e + \omega_e^2 H_e = \frac{4\pi c\omega_e^2 V^*}{V_e\omega_{opt}}\mathscr{E}_{opt} f(t), \tag{5}$$

where H_e is the magnetic field of the microwave resonator, chosen to be its generalized coordinate, ω_e is its natural frequency, V_e is the volume occupied by the inductive part, $\mathscr{E}_{opt}$ is the energy of the light pulse, and $\mathscr{E}_{opt} f(t)$ is the fraction of energy contained at a given instant of time inside the inductant [$f(t)$ is a periodic function of the time with a period $2\pi/\omega_e$]. Equation (5) is similar to the equation of motion for a ponderomotive energy meter.[8] In both cases, the force acting on the probing oscillator (in this case, the microwave resonator) is proportional to the energy of the investigated system. The fluctuating reaction in Ref. 8 reduces to a random shift of the frequency of the investigated oscillator. In our case the situation is somewhat more complicated, since a light pulse bounded in space constitutes an excited state of a large number of radiation oscillators. It can be shown that in the course of the measurement the light pulse becomes "smeared out" over the entire optical waveguide ring under the influence of the direct Faraday effect that is due to the fluctuating component of the magnetic field of the microwave resonator. The energy exchange between the light pulse and the microwave resonator takes place in this case at the expense of the change of the frequency of the light quanta. In particular, when the microwave resonator is in the ground state prior to the start of the measurement, an overall downward shift of the light frequency takes place. The number of the light quanta remains unchanged in this case.

We calculate now the energy measurement error in the described setup. In the usual methods used to pick off information from a microwave resonator (for example, the use of a linear low-noise amplifier), action on the resonator is sensed if it corresponds to a change of the resonator energy by at least one microwave quantum at zero initial conditions. With this taken into account, when the conditions

$$\tau < \tau_e^*, \quad \tau < \tau_{opt}^*$$

are satisfied, where τ_e^* and τ_{opt}^* are the relaxation times of the resonator and of the light in the waveguide, respectively, the energy measurement error is

$$\Delta\mathscr{E}_{opt} = 2(\pi\hbar\omega_e V_e)^{1/2}/\lambda_{opt}V^*\omega_e\tau. \tag{6}$$

At $\lambda_{opt} = 0.5$ μm, $V^* = 10^{-4}$ Oe^{-1} cm^{-1}, $\omega_e = 2\times 10^{10}$ sec^{-1}, $\tau = 100$ μsec, and $V_e = 1$ cm^3 we have $\Delta\mathscr{E}_{opt} \approx 1.5\times 10^{-6}$ erg, i.e., approximately 4×10^5 optical quanta.

This numerical estimate was obtained for the experimental-technique level reached at the present time, and does not describe the fundamental limit of the method. It is possible to increase radically the sensitivity of the method and to be able to count single light quanta without absorbing them if substances are discovered having a Verdet constant of the order of 10 to 1 Oe^{-1} cm^{-1} at the attenuation typical of optical glasses. From among the presently known substances, certain garnets come closest to satisfying this requirement. They have Verdet constants up to 0.1 Oe^{-1} cm^{-1} (TbAlG at 4 K, Ref. 11).

In conclusion, we note the presence of two phenomena whose cause is the inverse Faraday effect.

1. OPTICO-MAGNETIC TRANSITION RADIATION

When a spatially bounded circularly polarized light pulse passes through the boundary between two media with different Verdet constants, the abrupt change of the induced magnetic moment gives rise to radiation that can be called, in analogy with the usual transition radiation,[12] optico-magnetic transition radiation. Its spectrum obviously lies in the range from zero to τ_{fr}^{-1} (τ_{fr} is the duration of the front of the pulse), and the total energy $\mathscr{E}_e$ can be estimated by assuming that the radiation is mainly of magneto-dipole origin[13]:

$$\mathscr{E}_e \approx \frac{(\lambda_{opt}\Delta V^*)^2}{6\pi^2}\int\left(\frac{d\rho_z}{dz}\right)^2 dz. \tag{7}$$

The z axis is chosen along the propagation direction of the light pulse,

$$\rho_z = \int \rho dx\, dy,$$

ρ is the density of the optical energy in the pulse, and ΔV^* is the jump of the Verdet constant.

The value of $\mathscr{E}_e$ depends on the shape of the pulse, particularly on the slopes of its edges. Choosing for the sake of argument a bell-shaped form:

$$\rho_z = \mathscr{E}_{opt} \frac{\exp\{-2z^2/(c\tau_0)^2\}}{(\pi/2)^{1/2} c\tau_0}$$

($\mathscr{E}_{opt}$ is the total energy in the pulse and c is the speed of light) we obtain

$$\mathscr{E}_e = \frac{(\lambda_{opt}\Delta V^* \mathscr{E}_{opt})^2}{3\pi^{1/2} c^3 \tau_0^3}. \tag{8}$$

If $\mathscr{E}_{opt} = 10^3$ erg, $\tau_0 = 1\times 10^{-10}$ sec, $\lambda_{opt} = 0.5$ μm, and $\Delta V^* = 1\times 10^{-4}$ Oe^{-1} cm^{-1}, then $\mathscr{E}_e \approx 1.5\times 10^{-14}$ erg. We emphasize that this optico-magnetic radiation is accompanied by a general "reddening" of all the optical quanta by a relative amount $\mathscr{E}_e/\mathscr{E}_{opt}$, without a change in the number of optical quanta.

2. OPTICO-MAGNETIC CERENKOV RADIATION

As a result of the action of the inverse Faraday effect, a light pulse propagating through a medium with a nonzero Verdet constant is in fact a magnetic dipole oriented along the direction of motion and moving with a velocity $v = c/n(\omega_{opt})$ (n is the refractive index of the medium). Cerenkov radiation is then produced at those frequencies for which the relation $n(\omega) > n(\omega_{opt})$ is satisfied. For many substances the refractive index at the optical frequencies is lower than in the microwave band.

The energy lost per unit path length, owing to Cerenkov radiation, by a magnetic moment μ oriented along the direction of motion, is[14]

$$\frac{d\mathscr{E}}{dl} = \int\limits_{\beta n(\omega)>1} \frac{\omega^3 \mu^2 (1-\beta^2)}{c^4\beta^2}\left[n^2(\omega) - \frac{1}{\beta^2}\right] d\omega \tag{9}$$

($\beta = v/c$ and v is the velocity of motion). In our case this expression takes the form

$$\frac{d\mathscr{E}}{dl} = \left(\frac{\lambda_{opt} V^* \mathscr{E}_{opt}}{2\pi c^2}\right)^2 [n^2(\omega_{opt}) - 1] \int\limits_{n(\omega)>n(\omega_{opt})} \omega^3 [n^2(\omega) - n^2(\omega_{opt})]\, d\omega. \tag{10}$$

Equation (10) is valid if the Cerenkov-radiation wavelength exceeds the dimensions of the light pulse, and the latter radiates coherently. In that frequency region where the radiation becomes incoherent, the light pulse intensity decreases sharply. Therefore the region of integration in (10) is limited in fact by the frequency $\omega_{max} = 2\pi c/l$, where l is the dimension of the light pulse. Taking this circumstance into account, we can replace (10) by the following approximate formula

$$\frac{d\mathscr{E}_e}{dl} \simeq \left(\frac{\pi V^* \lambda_{opt} \mathscr{E}_{opt}}{l^2}\right)^2 [n^2(\omega_{opt}) - 1][n^2(\omega_e) - n^2(\omega_{opt})]. \tag{11}$$

Assuming $\lambda_{opt} = 0.5$ μm, $V^* = 10^{-4}$ Oe^{-1} cm^{-1}, $\mathscr{E}_{opt} = 1$ erg, $l = 3$ cm, $n^2(\omega_{opt}) = 2.25$, $n^2(\omega_e) = 3.75$ (fused quartz), we find that an energy of the order of 5×10^{-12} erg is radiated over 10 kilometers of path. This corresponds to a relative decrease in the frequency of the light-pulse photons on the order of 5×10^{-12}.

When the light pulse propagates in an anisotropic medium it will also have an electric dipole moment as a result of optical self-detection (see, e.g., Ref. 15), and the above-described radiation effects will be accompanied by analogous effects due to the radiation of the electric dipole. We examine now the extent of which these effects are appreciable compared with the magnetodipole effects.

The modulus of the induced dipole moment for the case of circular polarization of the light pulse is

$$p = 16\pi \mathscr{E}_{opt} d, \tag{12}$$

where $d = [(d_{11}+d_{12})^2 + (d_{21}+d_{22})^2 + (d_{31}+d_{32})^2]^{1/2}$, and d_{ij} are the components of the second-harmonic generation tensor. Accordingly, when the light pulse crosses the boundary between two media with different anisotropy, the radiated energy is of the order of

$$\mathscr{E}_e = \frac{(32\pi\Delta d)^2}{6} \int \left(\frac{d\rho_z}{dz}\right)^2 dz, \tag{13}$$

and the energy loss to Cerenkov radiation is, in accord with Ref. 14, of the order of

$$d\mathscr{E}/dl = (32\pi^3 \mathscr{E}_{opt} d/l^2)^2. \tag{14}$$

[We have left out from (13) and (14) factors of the order of unity, which depend on the direction of propagation of the light wave and on the refractive indices of the medium.]

Expressions (13) and (14) differ from the corresponding expressions for the magneto-dipole radiation mainly by the factor

$$k^2 = (32\pi^2 d/\lambda_{opt} V^*)^2.$$

At $d \sim 10^{-8}$ cgs esu, $\lambda_{opt} \sim 1$ μm, $V^* \sim 10^{-4}$ Oe^{-1} cm^{-1} we have $k \approx 10^3$, i.e., the effects connected with optical self-detection are stronger in an anisotropic medium. It must be noted, however, that not all media by far have optical anisotropy, whereas the Faraday effect is observed in practically all substances.

The authors thank V. M. Agranovich, B. Ya. Zel'dovich, G. S. Krinchik, L. V. Keldysh, and I. A. Yakovlev for valuable discussions.

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Transl. by J. G. Adashko

Nonlinear paramagnetic Faraday effect

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(Submitted 21 November 1979)
Zh. Eksp. Teor. Fiz. 78, 1718–1732 (May 1980)

A microscopic theory of the nonlinear Faraday effect is developed for cubic crystals containing paramagnetic impurity ions. The autorotation and deformation of the polarization ellipse are calculated for strong electromagnetic radiation propagated in such crystals. It is shown that a change of polarization of the radiation occurs even for ions whose ground state is a "nonmagnetic" doublet, for which the ordinary paramagnetic Faraday effect is absent. Allowance is made for the effect of the change of polarization of the wave on the magnetization of the crystal, and this permits refinement of Pershan's theory of the inverse Faraday effect. Nonstationary phenomena are considered, and it is shown that a relaxationless change of the crystal magnetization by a short light pulse is possible by virtue of the combinational light scattering that occurs between magnetic sublevels of the ions, split by a constant magnetic field.

PACS numbers: 78.20.Ls

1. INTRODUCTION

In a magnetic field, degenerate states of paramagnetic ions are split and, at not very high temperatures, are occupied with unequal probabilities. Consequently a crystal with paramagnetic impurities possesses circular birefringence, which leads to a Faraday rotation of the plane of polarization of radiation propagated along the magnetic field. The amount of this rotation is proportional to the magnetic field and inversely proportional to the temperature. But if the radiation is sufficiently strong, it also perturbs the spectrum of the ions, and this leads to a dependence of the Faraday rotation on the intensity of the radiation.

Besides the effect on the index of refraction, elliptically polarized radiation leads also to magnetization of the medium. This phenomenon is known as the inverse Faraday effect (IFE)[1] and is similar to the rectification effect known for ferrites.[2] The terminological difference is due to the fact that the IFE is observed in the optical range and is caused by interaction with the electric vector of the wave, whereas the rectification effect observed in the radiofrequency range is caused by magnetic interaction.

The nonlinear corrections to the Faraday rotation and the IFE are closely related to each other. In the present paper, a microscopic treatment of both these phenomena is given for cubic paramagnetic crystals. In Ref. 1, the IFE was treated phenomenologically and without allowance for the change of the polarization characteristics of the wave during is propagation. Also, no analysis was made of the nonstationary phenomena that occur for pulsed electromagnetic radiation, although the existing experimental methods of observation of the IFE are based precisely on the pulsed character of the phenomenon.[3,4] Furthermore, the possibility of the presence of a constant external magnetic field leads to new features of the IFE, connected with a change of the populations of the magnetic sublevels as a result of the process of combinational (Raman) scattering (CS).[5]

In section 2, the effective Hamiltonian is given for interaction of an ion with external fields. In section 3, the kinetic equation for an ion is obtained in the relaxation approximation. In section 4, the permittivity tensor of a paramagnetic crystal is calculated with allowance for nonlinear effects, and a closed system of equations is obtained for the field and the material, with allowance for the change of polarization of the radiation during propagation in the crystal. These equations are solved in the subsequent sections: in section 5 for ions whose ground state is a Kramers doublet (in this case the equations can be solved exactly for an arbitrary variation of the radiation intensity with time), and in section 6 for ions in nonmagnetic doublet, triplet, or quadruplet states (for the case of a slow variation of the intensity with time, in comparison with the paramagnetic relaxation times). The results obtained make it

 0038-5646/80/050862-08$02.40