New type of convection in a magnetoactive plasma

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It is shown that the corrections of second order in the gradients of the parameters of the expression for the momentum-flux tensor lead to a new solution of the equations of two-velocity hydrodynamics of a magnetized plasma.

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In this paper we present an example of plasma motion in the magnetic field that is determined by the higher-order terms (those following the Navier–Stokes terms) in the equations of motion of two-velocity hydrodynamics. Interest in such "Burnett" terms has greatly increased recently. In particular, in the review of Kogan *et al.*[1] are given examples in which these terms play an important role in ordinary gasdynamics. It has turned out that the Burnett approximation[2] (second-order approximation in the Knudsen number) leads not merely to a refinement of the Navier–Stokes approximation (first-order approximation, see Ref. 3), but also to new solutions. In other words, gas can flow as a continuous medium in such a way that, no matter how small the Knudsen number, the flow cannot be described by Navier–Stokes gasdynamics.

We shall show that exactly this situation arises in natural fashion in a magnetoactive plasma, and we shall consider the case that distinguishes more strongly this medium from an ordinary gas: we shall assume the ionization to be complete and the magnetic field strong, so that

$$\omega_\alpha = e_\alpha H/m_\alpha c \gg \nu_\alpha$$

($e_\alpha, m_\alpha, \nu_\alpha$ are the charge, mass, and effective frequency of the collisions of the particles of species $\alpha, \alpha = e, i$). Under these conditions, the small parameter of the hydrodynamic approximation is the ratio of the Larmor radius $\rho_\alpha = v_{T\alpha}/\omega_\alpha (v_{T\alpha}^2 \sim T_\alpha/m_\alpha)$ to the characteristic inhomogeneity dimension a.

It is known that in two-velocity plasma hydrodynamics the Navier–Stokes approximation corresponds to the system of equations obtained by Braginskiĭ[4,5] by the Chapman–Enskog method.[6] The Burnett approximation, in turn, corresponds to additional terms in these equations; these terms were calculated by a number of workers. The complete system of equations, which takes into account all the terms of second order in the gradients of the parameters, is given in Mikhaĭlovskiĭ's book[7] (see Appendix I of Chap. 16). It will be shown below that allowance for these terms in the equation of motion of the ions leads to a new type of laminar convection in a plasma.

1. EQUATION OF MOTION. POSSIBILITY OF ONSET OF CONVECTION

We consider the stationary equations of two-velocity hydrodynamics, which describe the equilibrium of a plasma in a magnetic field:

$$(\mathbf{V}_\alpha \nabla) V_{\alpha i} = -\frac{\partial P_\alpha}{\partial x_i} - \frac{\partial \Pi_{\alpha ij}}{\partial x_j} + n\{e_\alpha E_i + m_\alpha [\mathbf{V}_\alpha \times \boldsymbol{\omega}_\alpha]_i\} + R_{\alpha i}; \quad i, j = x, y, z. \tag{1.1}$$

Here $P_\alpha = nT_\alpha$ is the partial pressure, $\Pi_{\alpha ij}$ and $R_{\alpha i}$ are the momentum-flux tensor and the friction force, which are defined in the usual manner in terms of the particle distribution function (see Ref. 4). In the zeroth approximation in the small parameter ν_α/ω_α and in first-order approximation in ρ_α/a, the hydrodynamic velocity takes the form

$$\mathbf{V}_\alpha = (m\omega_\alpha^2 n)^{-1}\, \boldsymbol{\omega}_\alpha \times [\nabla P_\alpha - n e_\alpha \mathbf{E}]. \tag{1.2}$$

We assume that the gradients of all the parameters in the field $\mathbf{E}_\perp = \mathbf{E} \times \mathbf{H}/H^2$ are directed along a specified x axis in space, perpendicular to the magnetic field ($\mathbf{H} \parallel \mathbf{z}$). By the same token we neglect the bending of the force lines of the magnetic field. We shall also assume that the plasma is bounded along the x axis, so that under stationary conditions we should have[1)] $V_{\alpha x} \equiv 0$. Then, adding together the y components of Eqs. (1.1) and recognizing that $\mathbf{R}_i + \mathbf{R}_e = 0$, we arrive at the equality

$$\frac{\partial}{\partial x}(\Pi_{exy} + \Pi_{ixy}) = 0,$$

in which we can, however, neglect the electronic part of the viscosity tensor.

Indeed, estimating the viscosity tensor with aid of expression

$$\Pi_{\alpha xy} = \eta_\alpha dV_{\alpha y}/dx \quad (\eta_\alpha = {}^3/_{10} \nu_\alpha n T_\alpha/\omega_\alpha^2), \tag{1.3}$$

obtained by Braginskiĭ, and substituting here (1.2), we find that $\Pi_{exy} \ll \Pi_{ixy}$. Physically this means that the force of the friction of the ions against the light particles (electrons) has a negligible effect on the motion of the former. Thus,

$$d\Pi_{ixy}/dx = 0. \tag{1.4}$$

Substituting here (1.3) we obtain an equation for the velocity $\mathbf{V}_i$; this equation, in conjunction with the frequently encountered boundary conditions

$$V_{iy}(0) = 0, \quad V_{iy}(x_0) = 0, \tag{1.5}$$

has the solution $V_{iy}(x) \equiv 0$. In the second condition of (1.5), the coordinate x_0 marks the boundary of the fully ionized plasma. The wall or neutral gas that bounds the plasma is assumed immobile.

The just-obtained solution of (1.4), which expresses the absence of convection, is in fact in error. The point is that in the Chapman–Enskog method the velocity $\mathbf{V}_\alpha$ is assumed to be a quantity of zeroth order in the pa-

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rameter ρ_α/a, so that expression (1.3) is valid only in first order in ρ_i/a. This approximation is analogous to Navier–Stokes gasdynamics. If, however, we now substitute (1.2) in (1.3), then we obtain $\Pi_{ixy}\propto(\rho_i/a)^2$, which in the given approximation is an exaggeration of the accuracy. Thus, expression (1.3) is not valid in the case of drift flow, and it is necessary to use in (1.4) the viscosity tensor calculated accurate to terms of order ρ_i^2/a^2 [see Eq. (2.1) below]; in ordinary gasdynamic this would correspond to the Burnett approximation. The plasma motion considered by us is thus analogous to slow flow of gas.[1]

We note that the plasma flow has a velocity of the order of the ion drift velocity $v_i^*\sim v_{iT}\rho_i/a$, i.e., the convective flux turns out to be quite appreciable and of the same order as the flux due to boned diffusion in a turbulent plasma (diffusion along x is forbidden in this case because of the stationarity condition). Our flux has, however, a laminar thermal-stress character.

2. THE COMPONENT Π_{ixy} ACCURATE TO TERMS OF ORDER ρ_i^2/a^2

We shall consider hereafter mainly ions, and use the subscript $\alpha=e$ only for quantities pertaining to electrons. If the velocity V is of zeroth order in ρ/a, then the sought expression should coincide, accurate to ρ/a, with (1.3) and therefore the velocity dependence always takes the "Navier–Stokes" form (1.3). The remaining terms of second order in the gradients should be proportional either to d^2T/dx^2, or to $(dT/dx)\,(dA/dx)$, where A stands for T, n, or H. In fact, the electric field should not enter explicitly in the expression for Π_{xy}, since it can be expressed in terms of V_y by means of (1.2). On the other hand, there should be no temperature-gradient terms in Π_{xy}, since the gradients ∇n and ∇H by themselves do not take the plasma out of the equilibrium state. As a result, Π_{xy} should vanish at constant $\mathbf{V}(x)$ and constant $T(x)$. This conclusion is obviously valid for all components of the momentum-flux tensor.[2)]

As already discussed, the sought terms of second order in the gradients were already calculated in various papers (see, e.g., Ref. 8). These calculations, however, were not aimed at revealing new effects, and this is apparently why no correct expression for Π_{ij} in the approximation needed by us is cited explicitly anywhere. Nonetheless, there is no need to perform all the calculations anew. Using expressions 26, 21, and 22 in Appendix 16 of the book by Mikhaĭlovskiĭ,[7] we obtain accurate to terms of first order in ν/ω and of second order in ρ/a the component Π_{xy} in the drift stationary approximation[3)]:

$$\Pi_{xy}=-\eta\left\{\frac{dV_y}{dx}+\frac{T}{m\omega}\left[\frac{19}{12}\frac{d^2\ln T}{dx^2}+\frac{5}{12}\left(\frac{d\ln T}{dx}\right)^2+\frac{11}{3}\frac{d\ln T}{dx}\frac{d\ln n}{dx}-\frac{35}{12}\frac{d\ln T}{dx}\frac{d\ln\omega}{dx}\right]\right\}, \qquad (2.1)$$

$$\eta=\frac{2}{5}\frac{\pi^{1/2}\lambda e^4n^2}{m^{1/2}\omega^2T^{1/2}};$$

here $V_y=V_{iy}$ is given by (1.2), $\eta=\eta_i$ [see (1.3)], λ is the Coulomb logarithm, and the ions remain singly charged.

For a magnetized plasma, such as in our case, we can calculate the viscous-stresses tensor by a more direct method, the one used by Braginskiĭ[9] to find the transverse particle and heat fluxes. It is necessary to consider here in the expansion of the distribution function of the ions in spherical functions, besides the harmonics of zeroth and first order, which were taken into account by Braginskiĭ, also the second-order harmonic. This harmonic makes it possible to find the component Π_{xy}. As a result, after rather cumbersome manipulations, we obtain again Eq. (2.1).

The viscosity tensor was calculated accurate to terms of order ρ^2/a^2 by Fradkin.[10] His result, however, differs from (2.1) and is in our opinion in error: it contains the "forbidden" terms discussed at the beginning of this section. It is impossible to indicate the concrete cause of the error, since almost all the intermediate steps have been left out of Ref. 10.

We note that expression (2.1) is valid if the ion velocity distribution function differs little from Maxwellian. Summarizing all the assumptions used in the derivation of (2.1), we find that the plasma parameters should satisfy the conditions (see Ref. 9)

$$\partial/\partial t\ll v_T/a\ll(\omega\nu)^{1/2}\ll\omega\leqslant(4\pi ne^2/m)^{1/2}, \qquad (2.2)$$

where

$$a=\min[\,|\nabla\ln n|^{-1},\quad|\nabla\ln T|^{-1},\quad T/|eE_x|,\quad|\nabla\ln H|^{-1}].$$

3. THERMAL-STRESS CONVECTION

To determine $V_y(x)$ from (1.4) we must know the functions $T(x)$, and $n(x)$, and $H(x)$, which can be obtained only by solving completely the problem of the equilibrium of the plasma in a magnetic field. To this end, at any rate, it is necessary to use the heat-transfer equation (see Ref. 4)

$$\frac{dq_x}{dx}=Q=3\frac{m_e}{m}\nu_e(T_e-T)n. \qquad (3.1)$$

Here

$$q_x=\frac{2nT\nu}{m\omega^2}=\frac{20}{3m}\eta\frac{dT}{dx} \qquad (3.1')$$

is the heat flux carried by the ions along x. In (3.1) account is taken of the ion-electron collisions, which we have neglected up to now.

In the equation of motion for the electrons

$$d\Pi_{exy}/dx=R_{ey} \qquad (3.2)$$

the component Π_{exy} is obviously of the same order as $\eta_e dV_{ey}/dx$, where V_{ey} is determined by relation (1.2). At the same time R_{ey} is generally speaking of the order of

$$\frac{n\nu_e}{\omega_e}\left(\left|\frac{dT_e}{dx}\right|+\frac{T_e}{a}\right),$$

which is much more than the left-hand side of (3.2). From this we obtain the condition $R_{ey}=0$, which is equivalent, if we recall the expression for R_{ey} (see Ref. 4) and take (1.2) into account, to the equation

$$\frac{d}{dx}(nT_e+nT)-\frac{3}{2}n\frac{dT_e}{dx}=0. \qquad (3.3)$$

Finally, we must take into account Maxwell's equations, from which it follows that the pressure in the plasma, including the magnetic pressure, must be constant:

$$\frac{d}{dx}\left(\frac{H^2}{8\pi}+nT_e+nT\right)=0. \tag{3.4}$$

Thus, the functions $n(x)$, $T(x)$, and $H(x)$ can be obtained from equations (3.1), (3.3), and (3.4) only if one knows the function $T_e(x)$, which determines completely the state of the plasma. The $T_e(x)$ distribution, however, is not universal, is determined by the energy balance for the electrons, and depends on the concrete manner in which the plasma discharge is maintained.

We assume that the conditions

$$T_e\gg T,\quad H^2/8\pi\gg nT_e \tag{3.5}$$

are satisfied, and obtain then from (3.3) and (3.4)

$$\frac{d\ln n}{dx}=\frac{1}{2}\frac{d\ln T_e}{dx}-\frac{T}{T_e}\frac{d\ln T}{dx},\quad \frac{d\ln H}{dx}=0; \tag{3.6}$$

while Eq. (3.1) takes the form

$$-\frac{d}{dx}\left(T_e\frac{dT^{1/2}}{dx}\right)=3\omega^2\left(\frac{m_e m}{8}\right)^{1/2}T_e^{1/2}. \tag{3.7}$$

If we assume that the plasma layer is symmetrical about the (y,z) plane, then the boundary conditions for the temperature at $x=0$ are

$$T(0)=T_0,\quad \left.\frac{dT}{dx}\right|_{x=0}=0$$

and the solution of (3.7) can be easily found:

$$T^{1/2}=T_0^{1/2}-2\frac{(T_0T_e(0))^{1/2}}{x_0^2}\int_0^x\frac{dx'}{T_e(x')}\int_0^{x'}T_e^{1/2}(x'')\,dx'', \tag{3.8}$$

where

$$x_0=\frac{2}{\sqrt{3}\omega}\left(\frac{2T_0T_e(0)}{m_em}\right)^{1/4}\sim a$$

is the characteristic dimension of the inhomogeneity.

Substituting (3.6) in (1.4), we reduce the latter to the form

$$-\eta\left\{\frac{dV_y}{dx}+\frac{T}{m\omega}\left[\frac{19}{12}\frac{d^2\ln T}{dx^2}+\frac{5}{12}\left(\frac{d\ln T}{dx}\right)^2+\frac{11}{6}\frac{d\ln T}{dx}\frac{d\ln T_e}{dx}\right]\right\}=\Pi_{xy}^0, \tag{3.9}$$

where $T(x)$ is expressed in terms of $T_e(x)$ by means of the relation (3.8). The quantity Π_{xy}^0 in (3.9) is an integration constant that must be chosen on the basis of the boundary conditions of the problem.

We note that if the variation of the electron temperature in space is not faster than that of the ion temperature,

$$|\nabla\ln T_e|\lesssim|\nabla\ln T|, \tag{3.10}$$

then the concrete form of the function $T_e(x)$ does not influence significantly the final answer, since the solution $V_y(x)$ is determined by the largest of the terms in the square brackets of (3.9). For this region the characteristic value of the conduction velocity is $T/m\omega x_0$. As to the boundary conditions, one of them, obviously, is $V_y(0)=0$, and the second must be specified at the point where the ion temperature vanishes. According to (3.8), this temperature is a monotonically decreasing function and must vanish at the end point $x=x_0^*$ provided that $T_e(x)$ does not increase more rapidly than x^4 towards the plasma periphery.

It is natural to assume that at $|x|>x_0^*$ the plasma is surrounded by a quiescent neutral gas, and that the transition region in the vicinity of x_0^* (which is small compared with a) is occupied by a partially ionized plasma. The friction between the ions and the neutral atoms (this friction force, in contrast to electron friction, is not small and cannot be neglected in the equation of motion) leads to vanishing of the macroscopic velocity of the neutrals at the point x_0^*. On the other hand, our theory does not take into account this friction force, and according to the definition (1.2) the boundary condition at x_0^* should take the form

$$V_y(x_0^*)=\frac{c}{eH}\left[\frac{1}{n}\frac{d(nT)}{dx}-eE_x\right]\Big|_{x=x_0^*}, \tag{3.11}$$

but is not necessarily equivalent to $V_y(x_0^*)=0$, i.e., in the transition region $V_y(x)$ decreases rapidly (over a length on the order of the Larmor radius) from the value (3.11) to zero.[4)]

It may happen, however, that at $x=x_0^*$ not only the temperature but also its derivative dT/dx vanishes [see formula (3.13) below]. Then, assuming the electric field outside the plasma to be equal to zero [this means that also $E(x_0^*)=0$ by virtue of the condition $D_x=\varepsilon_{xx}E_x(x)=\text{const}$], we find that (3.11) reduces to $V_y(x_0^*)=0$. As a result, the conditions (1.5) take place.

The final solution of (3.9) can be obtained, for example, by assuming

$$|d\ln T_e/dx|\ll|d\ln T/dx|. \tag{3.12}$$

This assumption is not formal. Actually, the electron temperature is determined as a rule not by the laminar thermal conductivity but by the enhanced thermal conductivity due to the turbulent fluctuations. In a plasma with unequal temperatures, at $T_e/T>5.2$, the turbulence due to the drift instability (which is always present in the considered model of the plasma) turns out to be more substantial for electrons than for ions, inasmuch as $\nu_e/\omega_e^*<\nu_i/\omega_i^*$ [$\omega_\alpha^*=\omega_\alpha(\rho_\alpha/a)^2$ is the characteristic drift velocity]. Assuming now in (3.8) that the electron temperature is independent of x, we find

$$T=T_0(1-x^2/x_0^2)^2,\quad |x|<x_0. \tag{3.13}$$

This formula was obtained also by Kapitza.[12]

The point x_0^* where the temperature vanishes coincides thus with x_0. Therefore the boundary conditions at a zero external electric field take the form (1.5). Solving Eq. (3.9) with allowance for (3.12), (3.13) and (1.5), we obtain ultimately

$$V=\frac{3T_0}{m\omega x_0^2}\left(\frac{x^3}{x_0^2}-x\right). \tag{3.14}$$

Accordingly, the mechanical moment per unit area of the layer is

$$L=mn\int_{-x_0}xV_ydx=-\frac{4}{5}\frac{nT_0}{\omega}x_0.$$

We note once more that the order of magnitude of V_y and

L is determined only by the conditions (3.5) and (3.6), and not by the condition (3.12).

We consider now the case of a cylindrically symmetrical plasma, which corresponds to a greater degree to real experiments. All the gradients are assumed directed along the radius. The expression for the component $\Pi_{r\varphi}$ of the momentum-flux tensor are of the form

$$\Pi_{r\varphi}=-\eta\left\{r\frac{d}{dr}\frac{V_\varphi}{r}+\frac{1}{m\omega}\left[\frac{19}{12}rT\frac{d}{dr}\left(\frac{1}{rT}\frac{dT}{dr}\right)+\frac{5}{12T}\left(\frac{dT}{dr}\right)^2+\frac{11}{3}\frac{dT}{dr}\frac{d\ln n}{dr}-\frac{35}{12}\frac{dT}{dr}\frac{d\ln\omega}{dr}\right]\right\},$$

in which V_φ is determined as before by Eq. (1.2).

The equations of motion and of the energy balance now take the form

$$r^{-2}\frac{d}{dr}(r^2\Pi_{r\varphi})=0,\quad \frac{1}{r}\frac{d}{dr}(rq_r)=Q,$$

where q_r is given by (3.1′) in which x is replaced by r. By means of a similar substitution we can obtain the two other equations from (3.3) and (3.4). If all the assumptions made above are valid as before, then

$$T=T_0\left(1-\frac{r^2}{r_0^2}\right)^2,\quad r_0=\frac{2}{\sqrt{3}\omega}\left(\frac{8T_0T_e}{m_e m}\right)^{1/4};$$

the expression for V_φ differs from (3.14) in the substitutions $y\to\varphi$; $x, x_0\to r, r_0$, and the mechanical moment per unit length is

$$L=-\frac{\pi}{2}\frac{nT_0}{\omega}r_0^2.$$

4. CONCLUSION

We make a few remarks concerning the applicability of the results. It is well known that an inhomogeneous plasma in a magnetic field is never stable. The presence of gradient instabilities leads to the development in the plasma of a turbulent state. As a result, allowance for dissipative effects with the aid of the pair-collision integral is frequently insufficient, and it is necessary to take into account the nonlinear mechanism of ion scattering by turbulent fluctuations. This scattering mechanism can be formally considered by adding to the friction forces R_α the corresponding friction forces due to the turbulent noise. Thus, the right-hand side of (1.4) is not equal to zero, but contains a force $R_{i\,\text{turb}}$. There are grounds, however, for assuming that under those conditions which we are assuming the level of the turbulent fluctuations is relatively low, so that we can put $R_{i\,\text{turb}}\approx 0$ and use as before Eq. (1.4) for the solution. This conclusion follows from the second inequality of (2.2), which can be transformed into

$$\nu_i\gg\omega_i^*\sim\omega_i(\rho_i/a)^2. \qquad (4.1)$$

The instability growth rate is as a rule of the same order as ω_i^* (at any rate, it does not exceed this value); it follows therefore that the pair collisions will dominate as before, and the solution obtained by us remains in force.

As already noted, the electrons can be substantially influenced by the turbulence. This follows from the fact at $T_e\gg T_i$ it is possible to have the inequality $\nu_e\ll\omega_e^*$, even if the condition (4.1) is satisfied.[5)]

Among the possible consequences of the plasma convection we note, for example, the change of the dielectric tensor, as well as the appearance in the medium of an additional instability channel as a result of the macroscopic motion of the ions.[14]

1) For further conclusions, in fact, the condition $V_{ix}/V_{iy}\ll\nu_i/\omega_i)\times(\rho_i/a)^2$, i.e., $\partial/\partial t\ll\nu_i(\rho_i/a)^4$.

2) The reason why the expression for the tensor π does not contain terms proportional to d^2n/dx^2 and $(dn/dx)^2$ is the neglect of the interaction of the ions with the electrons. In this sense our plasma is a single-component gas. In a mixture of gases, these terms are present in the expression for the momentum flux and can even cause a concentration-stress convection (See Ref. 1).

3) We note that in the same book,[7] in Appendix 5 of Chapter 16, there is given, for π_{ij}, as stated, an expression corresponding to the drift stationary approximation. This expression, however, has not been written down accurately, name namely, the tensor $W_{ij}^{(2)}$, which enters in the expression for π_{ij} (See Eq. 26 of Appendix 16 in Ref. 7), does not contain the terms described by the first and third terms in the correct formula 21 of Appendix 26,

4) This behavior of the velocity is similar to the effect of thermal slippage in ordinary gasdynamics, when the velocity jump is proportional to the tangential component of the temperature gradient on the gas boundary (see Ref. 11). The singularity of the plasma manifests itself in the presence in (3.11) of an electric field, and also in that the tangential component of the gradient is replaced, owing to the strong magnetic field, by a normal component, and the particle mean free path is replaced by the Larmor radius.

5) The fact that the transverse viscous hydrodynamic stresses turn out to be more important than the fluctuations at high collision frequencies, while at small ν_i, on the contrary, the noise is more significant, is due to the presence in the medium of a strong magnetic field, and is valid only so long as $\nu_i<\omega_i$. In an ordinary non-magnetoactive hydrodynamics, the opposite relation holds true (see Ref. 13). The fluctuations predominate at large ν_i (i.e., at small Knudsen numbers), and in liquids the fluctuation thermal correction to the equation of motion turns out to be more important than the Burnett correction, while in gases, when the collisions are small and the Knudsen number is much larger than in liquids, the fluctuations can be neglected.

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Motion of nuclear magnetization under conditions of microscopic inhomogeneity of the hyperfine field

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We investigate the motion of nuclear magnetization under conditions of microscopic inhomogeneity of the hyperfine field (HFF) at low deviations from the equilibrium position. It is shown that if the HFF has a Lorentz distribution function the free-precession damping coefficient depends little on the ratio of the dynamic shift of the NMR frequency to the width of the distribution function. For a quadratic Lorentz function, which falls off more rapidly on the wings, the damping coefficient of the free precession decreases sharply with increasing dynamic frequency shift. Both the frequencies and the damping coefficients of the free precession are independent of the parameters of the pulse-exciting high-frequency field.

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Theoretical and experimental investigations[1-6] carried out during the last ten years have shown that most typical for magnets is a microscopic inhomogeneity of the hyperfine field (HFF): the correlation radius r_0 of the HFF is usually much smaller than the effective exchange-interaction radius r. In this situation, the electronic magnetization M connected with the strong exchange interacts with the resultant field of the nuclear isochromats[5,7] and the density of the macroscopic energy of the hyperfine interaction (HFI) is expressed in the form

$$\mathscr{H}=\mathbf{M}\int A\mu^A\,dA, \qquad (1)$$

where A is the HFI constant, μ^A is the magnetization of the nuclear isochromat, and by isochromat is meant the set of nuclei for which the HFI constant has the same value. Naturally, under conditions of microinhomogeneity of the HFF the motion of the nuclear magnetization μ^A is described by integro-differential equations, since the dynamic HFI $\mathbf{M}_\perp\int A\mu_\perp^A dA$, and consequently also the NMR dynamic frequency shift (DFS) due to this interaction, are "turned on" only at those time intervals when the total transverse nuclear magnetization differs from zero. A mathematical analysis of this situation is in the general case a very complicated problem. Up to recently, the theoretical calculation was carried out either for a model of a macroscopic inhomogeneity of the HFF,[8-11] wherein in the sample is broken up into a quasi-non-interacting sections within which the HFF is homogeneous, or for the case when the inhomogeneity of the HFF is microscopic but the DFS is small.[6,7] In the present paper we consider the simplest phenomena that can be analyzed without restrictions on the value of the HFF—we investigate the motion of nuclear magnetization at small deviations from the equilibrium positions.

We consider for the sake of argument a ferromagnetic sample in the form of a sphere magnetized parallel to the external constant magnetic field **H**. We assume that the Z axis is directed along **H** and that it is possible to apply to the sample a high-frequency (HF) field $h\sim\exp(i\omega t)$ polarized in the XY plane. In place of $\mu^A_{X,Y,Z}$ it is convenient to introduce the relative components

$$u=\mu_X{}^A/\mu^A,\quad v=-\mu_Y{}^A/\mu^A,\quad m=-\mu_Z{}^A/\mu^A, \qquad (2)$$

where μ^A is the static magnetization of the isochromat and is determined at not too low temperatures by the Langevin formula

$$\mu^A=Ng(A)\frac{\gamma_n{}^2\hbar^2I(I+1)}{3kT}(AM-H). \qquad (3)$$

Here N is the concentration of the magnetic nuclei, $g(A)$ is the distribution function of A, γ_n is the nuclear gyromagnetic ratio, and I is the spin of the nucleus. In a coordinate system rotating with frequency ω, the motion of the nuclear magnetization is described by the equations[7,8]

$$\begin{aligned}&\dot u-(\Delta+\delta)v+\Gamma_n u+L_Y=0,\\&\dot v+(\Delta+\delta)u+\Gamma_n v+L_X=-\omega_N m,\\&\dot m+(m-1)/T_1+L_Z=\omega_N v.\end{aligned} \qquad (4)$$

Here Δ is the difference between the frequency of the HF field and the average undisplaced NMR frequency

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