Excitation of spin waves by parallel pumping in ferromagnets with magnetic imhomogeneities

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An investigation is made of the influence of randomly distributed magnetic inhomogeneities on the interaction of an alternating magnetic field with the spin system of a ferromagnet. A new mechanism is proposed for excitation of spin waves; it consists in the possibility of stimulated decay of the external wave into two spin waves with randomization of the magnon subsystem at impurities or dislocations. Conditions are determined under which this mechanism leads to larger effects than does the usual parametric resonance. Also investigated is the effect of randomization of magnons at inhomogeneities on the value of the nonequilibrium magnetization of the ferromagnet. It is shown that in contrast to the case $\Omega > 2\varepsilon_0$ (Ω is the frequency of the field, ε_0 the energy of a magnon at $\mathbf{k} = 0$), when suppression of the magnetization occurs, when $\Omega < 2\varepsilon_0$ the magnetization increases (the magnetization stimulation effect).

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As is well known, one of the basic problems of the generation of spin waves is parallel pumping, that is, a situation in which the external alternating magnetic field is applied parallel to the magnetization **M**. Then a process is possible in which an external spatially homogeneous wave of frequency Ω decays into two spin waves:

$$\Omega=\varepsilon_{\mathbf{k}}+\varepsilon_{-\mathbf{k}} \tag{1}$$

($\varepsilon_{\mathbf{k}}$ is the energy of a magnon with wave vector **k**), when $\Omega > 2\varepsilon_0$. A theory of parametric resonance excitation of spin waves in ideal ferromagnets was constructed by Zakharov, L'vov, and Starobinets[1] (see also the review[2] by the same authors).

Real ferromagnets usually contain various inhomogeneities (impurities, dislocations, etc.), which, as has been established,[3] substantially affect the threshold for parametric excitation. A systematic theory of this phenomenon was developed in Ref. 4. In addition, as was explained in Ref. 5, for parallel orientation of the external alternating magnetic field and of the magnetization of the ferromagnet an important role is played by effects of modulation of the phase of the magnons by the external field; these lead to the possibility of absorption of the field even in a nonresonance situation ($\Omega < 2\varepsilon_0$) because of "interference" between the interaction of the magnons with the field and magnon-magnon processes. In the presence of impurities, because of the relatively large amplitude of the interaction of magnons with impurities,[6] these effects lead to significant absorption of the field energy when $\Omega < 2\varepsilon_0$.[7] When $\Omega > 2\varepsilon_0$, modulation of the magnon-impurity interactions by the external field leads, along with (1), to still another channel for generation of spin waves (Fig. 1; the dotted line denotes impurities, the wavy line the alternating magnetic field), by virtue of the process

$$\Omega=\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{k}'}. \tag{2}$$

We shall make a preliminary estimate of the effectiveness of processes of generation of spin waves because of impurities as compared with the effectiveness of direct processes of formation of parametric waves, (1). Noting that for $\mu h_0 \ll \Omega$ (h_0 is the amplitude of the external field) the probability of the processes (2) (Fig. 1) is proportional to

$$(\mu h_0/\Omega)^2 U^2 N_i,$$

where N_i is the number of impurities per elementary cell and where U is the matrix element for interaction of a magnon with an impurity, whereas the probability of the processes (1) (Fig. 2) is simply proportional to $(\mu h_0)^2$, we see that the effectiveness of magnon pumping by processes (2) is higher when the inequality

$$N_i > (\Omega/U)^2.$$

is satisfied. For example, when $\Omega \sim 10^{11}$ sec^{-1} and $U \sim 10^{13}$ sec^{-1},[6] $N_i > 10^{-4}$. Below, we shall derive a more exact inequality. But these preliminary remarks already indicate the importance, in principle, of the effects of magnetic inhomogeneities on the kinetics of magnons in parallel pumping, and also the fact that these effects by no means reduce simply to a renormalization of the excitation threshold.

Still another illustration of the importance of allowing for the scattering of spin waves on inhomogeneities that is induced by an alternating field is the redistribution of spin waves in **k** space, with conservation of their number. Such processes (see Fig. 3) are interesting not only in the case of parametric resonance ($\Omega > 2\varepsilon_0$), since they may be an effective method of limitation of the number of parametric magnons, but also in the parametrically nonresonant case ($\Omega < 2\varepsilon_0$), when the processes (1) and (2) are impossible because of the law of conservation of energy. In the last-mentioned case, as will be

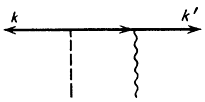

FIG. 1.

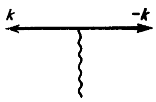

FIG. 2.

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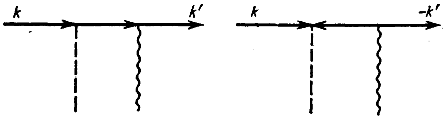

FIG. 3.

shown in the present paper, redistribution of the magnons with respect to energy, toward higher energies, leads to a very interesting effect: an increase of the magnetization of the ferromagnet as compared with the equilibrium magnetization corresponding to the given temperature T.

1. THE KINETIC EQUATION FOR MAGNONS

We consider a single-domain specimen of a uniaxial ferromagnet, placed in a constant magnetic field H parallel to the anisotropy axis. The Hamiltonian of the magnon system of such a ferromagnet, in an alternating magnetic field polarized parallel to the constant field, has the form

$$\mathscr{H}=\sum_{\mathbf{p}}(\varepsilon_{\mathbf{p}}+h_{\mathbf{p}}^0\cos\Omega t)c_{\mathbf{p}}^+c_{\mathbf{p}}+{}^1/_2\sum_{\mathbf{p}}(H_{\mathbf{p}}e^{-i\Omega t}c_{\mathbf{p}}^+c_{-\mathbf{p}}^++\text{H.c.})$$
$$+\sum_{\mathbf{p}\mathbf{p}'}V_{\mathbf{p},\mathbf{p}'}c_{\mathbf{p}}^+c_{\mathbf{p}'}+\sum_{\mathbf{p}\mathbf{p}'}(W_{\mathbf{p},\mathbf{p}'}c_{\mathbf{p}}^+c_{\mathbf{p}'}^++\text{H.c.})+\mathscr{H}_{ss},\qquad(3)$$

where $h_{\mathbf{p}}^0=\mu h_0(|u_{\mathbf{p}}|^2+|v_{\mathbf{p}}|^2)$ and $H_{\mathbf{p}}=\mu h_0 u_{\mathbf{p}}^* v_{\mathbf{p}}^*$. The values of $\varepsilon_{\mathbf{p}}$, $u_{\mathbf{p}}$, and $v_{\mathbf{p}}$ and the magnon-magnon interaction Hamiltonian $\mathscr{H}_{ss}$ were determined in Ref. 8; the matrix elements of the interaction of magnons with magnetic inhomogeneities are

$$V_{\mathbf{p},\mathbf{p}'}=U_{\mathbf{p},\mathbf{p}'}(u_{\mathbf{p}}^*u_{\mathbf{p}'}+v_{\mathbf{p}}^*v_{\mathbf{p}'}),\quad W_{\mathbf{p}\mathbf{p}'}=U_{\mathbf{p},-\mathbf{p}'}u_{\mathbf{p}}^*v_{\mathbf{p}'}^*.$$

In the case of impurities (the exchange mechanism of interaction), we have

$$U_{\mathbf{p},\mathbf{p}'}=\frac{U}{2N}\sum_j\exp\{-i(\mathbf{p}-\mathbf{p}',\mathbf{R}_j)\},\qquad(4)$$

where N is the number of magnetic-atom lattice sites, U is the exchange integral between an impurity atom and an atom of the matrix ($U\sim\Theta_c$, where Θ_c is the Curie temperature), and $\mathbf{R}_j$ is the coordinate of the j-th impurity. In the case of dislocations (ring-shaped, randomly distributed),

$$U_{\mathbf{p},\mathbf{p}'}=\frac{i}{qV}(s^2pp'\varphi_{jp}+B\Phi_{jp})\sum_l b_j^{(l)}T_p^{(l)}\exp\{-i\mathbf{q}\mathbf{r}^{(l)}\},\qquad(5)$$

where $\mathbf{q}=\mathbf{p}-\mathbf{p}'$, V is the volume of the system, $s^2=\Theta_c a^2$, $B=\gamma_0\mu M_0$, γ_0 is the magnetostriction constant, and $b^{(l)}$ is the Burgers vector of the lth dislocation; expressions for the functions φ_{jp}, Φ_{jp}, and $T^{(l)}$ were given in Ref. 9.

In the derivation of the kinetic equation for the magnon distribution function $f_{\mathbf{p}}=\langle c_{\mathbf{p}}^+c_{\mathbf{p}}\rangle$ (the angular brackets denote an average over a nonequilibrium Gibbs distribution), we shall take into account that, as a rule, the amplitude of the external alternating field is small ($\mu h_0\ll\varepsilon_0$), and we may therefore restrict ourselves to second-order perturbation theory with respect to the field. This means that in the derivation of the term in the kinetic equation that describes ordinary resonance excitation of magnons [coming from the second term in (3)], we shall neglect the higher resonances ($n\Omega=2\varepsilon_{\mathbf{k}}$, $n>1$). At the same time, we shall initially take exact account of the first term in (3), which describes the interaction of magnons with the field, in order subsequently to write the kinetic equation correctly in the quadratic approximation with respect to the field.

As was shown in Ref. 10 (see also Ref. 7), the kinetic equation for normal representative $f_{\mathbf{p}}$, for systems described by a Hamiltonian of the type (3), in the second order of perturbation theory with respect to the interaction between the particles, can be obtained as follows:

$$\frac{\partial f_{\mathbf{p}}}{\partial t}=-\int_{-\infty}^{0}d\tau e^{\eta\tau}\,\mathrm{Sp}\,\rho^{(0)}\{f(t)\}[\mathscr{H}_{int}^{\tau}(t+\tau),[\mathscr{H}_{int}(t),c_{\mathbf{p}}^+c_{\mathbf{p}}]],\quad\eta\to+0,\qquad(6)$$

where

$$\mathscr{H}_{int}^{\tau}=S^+(t+\tau,t)\mathscr{H}_{int}S(t+\tau,t),$$
$$S(t,t')=\exp\left\{-i\sum_{\mathbf{p}}\left[\varepsilon_{\mathbf{p}}(t-t')+\frac{h_{\mathbf{p}}^0}{\Omega}(\sin\Omega t-\sin\Omega t')\right]c_{\mathbf{p}}^+c_{\mathbf{p}}\right\},$$
$$\rho^{(0)}\{f\}=\exp\left\{-\sum_{\mathbf{p}}\left[\ln(1+f_{\mathbf{p}})-c_{\mathbf{p}}^+c_{\mathbf{p}}\ln\left(\frac{1+f_{\mathbf{p}}}{f_{\mathbf{p}}}\right)\right]\right\},$$

and $\mathscr{H}_{int}$ includes all the terms of (3), beginning with the "resonance" generation of magnons by the external field.

On calculating in the usual manner the averages in (6) and on carrying out an average over a random distribution of inhomogeneities, with allowance for the remarks made above, we derive the equation in the form

$$\frac{\partial f_{\mathbf{p}}}{\partial t}=-\frac{2n}{(2\pi)^3}\int d\mathbf{p}'|V_{\mathbf{p},\mathbf{p}'}|^2K_{\mathbf{p},\mathbf{p}'}^{(1)}(h,\Omega,t)(f_{\mathbf{p}}-f_{\mathbf{p}'})$$
$$+\frac{2n}{(2\pi)^3}\int d\mathbf{p}'|W_{\mathbf{p},\mathbf{p}'}|^2K_{\mathbf{p},\mathbf{p}'}^{(2)}(h,\Omega,t)(1+f_{\mathbf{p}}+f_{\mathbf{p}'})$$
$$+2|H_{\mathbf{p}}|^2(1+f_{\mathbf{p}}+f_{-\mathbf{p}})\frac{2\gamma_{\mathbf{p}}}{(2\varepsilon_{\mathbf{p}}-\Omega)^2+4\gamma_{\mathbf{p}}^2}+\mathscr{L}_{\mathbf{p}}^{ss}\{f\},\qquad(7)$$

where n is the concentration of defects (n_i of impurities, n_d of dislocations[1]),

$$K_{\mathbf{p},\mathbf{p}'}^{(1,2)}(h,\Omega,t)=\int_{-\infty}^{0}d\tau e^{\eta\tau}\cos\left\{\tau(\varepsilon_{\mathbf{p}}\mp\varepsilon_{\mathbf{p}'})\right.$$
$$\left.+\frac{h_{\mathbf{p}}^0\mp h_{\mathbf{p}'}^0}{\Omega}[\sin\Omega(t+\tau)-\sin\Omega t]\right\},$$

$\mathscr{L}_{\mathbf{p}}^{(ss)}$ are the collision integrals that describe the magnon-magnon processes, in general renormalized by the external field,[7] and

$$\gamma_{\mathbf{p}}=-\frac{1}{2}\frac{\delta\mathscr{L}_{\mathbf{p}}^{ss}\{f\}}{\delta f_{\mathbf{p}}}.$$

In the present paper we shall be interested in a high-frequency field $\Omega\tau\gg1$ (where τ is the relaxation time of the magnons), when the magnetic system can be described by a distribution function "smoothed out" over intervals Δt that satisfy the inequality

$$\tau\gg\Delta t\gg\Omega^{-1}.$$

On passing to the limit $\mu h_0\ll\Omega$ and averaging over the oscillations of the external field (for details, see Ref. 7), we get the kinetic equation in the form

$$\frac{\partial f_p}{\partial t}=-\frac{n}{(2\pi)^2}\int dp'|V_{p,p'}|^2(f_p-f_{p'})\left[1-\frac{1}{2}\left(\frac{h_p{}^0-h_{p'}{}^0}{\Omega}\right)^2\right]\delta(\varepsilon_p-\varepsilon_{p'})$$
$$-\frac{n}{(2\pi)^2}\int dp'|V_{p,p'}|^2(f_p-f_{p'})$$
$$\times\frac{1}{4}\left(\frac{h_p{}^0-h_{p'}{}^0}{\Omega}\right)^2[\delta(\varepsilon_p-\varepsilon_{p'}-\Omega)+\delta(\varepsilon_p-\varepsilon_{p'}+\Omega)]$$
$$+\frac{n}{(2\pi)^2}\int dp'|W_{pp'}|^2(1+f_p+f_{p'})\left(\frac{h_p{}^0+h_{p'}{}^0}{\Omega}\right)^2\delta(\varepsilon_p+\varepsilon_{p'}-\Omega)$$
$$+2|H_p|^2(1+f_p+f_{-p})\frac{2\gamma_p}{(2\varepsilon_p-\Omega)^2+4\gamma_p{}^2}+\mathscr{L}_p{}^{ss}\{f\}. \qquad (8)$$

The first term in (8) describes the ordinary mechanism of scattering of spin waves by defects, which leads to "isotropization" of the magnon distribution function. The second term corresponds to processes of redistribution of magnons with respect to energy (with change of the magnitude and direction of the vectors **k**), with conservation of their number. The third and fourth terms are nonzero only under the condition of parametric resonance ($\Omega > 2\varepsilon_0$) and describe, respectively, the processes of stimulated and of parametric resonance generation of two spin waves, as represented in Figs. 1 and 2. The term $\mathscr{L}_p^{(ss)}\{f\}$ describes relaxation of magnons to a nonequilibrium state.

It is necessary, however, to make the following remark. For a systematic description of the process of parametric resonance within the framework of the kinetic approach, it is necessary to consider simultaneously the equations for f_p and for $\sigma_p=\langle c_p c_{-p}\rangle$. The necessity of introducing σ_p in order to describe a system in parametric resonance was first pointed out in Ref. 2, where the importance was noted of considering the equation for σ_p in deriving the phase mechanism of limitation of the amplitude of spin waves. In solution of the equations for f_p and σ_p by perturbation theory, by restricting ourselves both to the second order in the interaction of the system with the external field ($\frac{1}{2}\sum_p[H_p e^{-i\Omega t}c_p^+c_{-p}^+ + \text{H. c.}]$), and to the second order in the magnon-magnon interactions ($\mathscr{H}_{ss}$), we can obtain an expression for σ_p in the self-consistent approximation.

If we then substitute the expression obtained for σ_p in the equation for f_p, the kinetic equation for f_p will have the same form (8), but ε_p and H_p must now be interpreted as the renormalized magnon energy and pumping field.[2] These renormalizations of the magnon energy and of the pumping field are obtained in the same way as in the s-theory of Zakharov, L'vov, and Starobinets,[2] who used the dynamic-equation approach to describe the process of parametric excitation of magnons. Hereafter we shall suppose that the principal mechanism of randomization of magnons is scattering by magnetic inhomogeneities, and therefore we shall neglect "interference" magnon-magnon interactions and interactions of magnons with the alternating field.[7]

2. MECHANISMS OF SPIN-WAVE EXCITATION. THE CASE OF MAGNON-IMPURITY SCATTERING

For analysis of the role of the various terms in (8) in the kinetics of magnons, it is convenient to go over to the approximation of isotropy in the wave vectors p, using the fact that because of the large concentration of randomly located magnetic inhomogeneities, there occurs an effective "isotropization" of the distribution functions f_p. Omitting quite simple calculations, in the case $M_0 \ll H$ for the isotropic part of the distribution function we get a kinetic equation in the form

$$\frac{\partial f}{\partial t}=\zeta_1(n_i v_0)\frac{U^2m^2}{\Theta_c}\left(\frac{\mu h_0}{\Omega}\right)^2\frac{(\Omega-\varepsilon_0-\varepsilon)^{1/2}}{(\Omega-\varepsilon)^2}(1+f_\varepsilon+f_{\Omega-\varepsilon})$$
$$\times\theta(\Omega-\varepsilon_0-\varepsilon)-\zeta_2(n_i v_0)\frac{U^2m}{\Theta c^{3/2}}\left(\frac{\mu h_0}{\Omega}\right)^2\Big\{(\varepsilon+\Omega-\varepsilon_0)^{1/2}$$
$$\times\left[\frac{3}{2}\left(\frac{1}{\varepsilon}+\frac{1}{\varepsilon+\Omega}\right)-\frac{2}{(\varepsilon(\varepsilon+\Omega))^{1/2}}\right](f_\varepsilon-f_{\varepsilon+\Omega})$$
$$\times\theta(\varepsilon+\Omega-\varepsilon_0)+(\varepsilon-\Omega-\varepsilon_0)^{1/2}\left[\frac{3}{2}\left(\frac{1}{\varepsilon}+\frac{1}{\varepsilon-\Omega}\right)\right.$$
$$\left.-\frac{2}{(\varepsilon(\varepsilon-\Omega))^{1/2}}\right](f_\varepsilon-f_{\varepsilon-\Omega})\theta(\tau-\Omega-\varepsilon_0)\Big\}$$
$$+\zeta_3(\mu h_0)^2\frac{m^2}{\varepsilon^2}(1+2f_\varepsilon)\frac{2\gamma_\varepsilon}{(2\varepsilon-\Omega)^2+4\gamma_\varepsilon{}^2}+\mathscr{L}_\varepsilon^{(ss)}\{f\}, \qquad (9)$$

where $\zeta_1 \sim 10^{-1}$, $\zeta_2 \sim 2.10^{-2}$, $\zeta_3 \sim 1$, $m=4\pi\mu M_0$, v_0 is the volume of the elementary cell, and $\theta(x)=1$ for $x \geqslant 0$ and $=0$ for $x<0$. In the present paper we are primarily interested in the mechanism of stimulated nonresonance pumping of spin waves, described by the first term in (9); therefore we shall not give an analysis of the effect of impurities on the threshold of resonance excitation of magnons (with energy $\Omega/2$), since this has already been done in Ref. 4. We remark only that the effect of threshold rise as a result of magnon-impurity interactions follows directly from the equation for the isotropic part of the distribution function.

Having derived Eq. (9), we can now estimate (at least in the τ-approximation for $\mathscr{L}_\varepsilon^{(ss)}\{f\}$) the effectiveness of the sources of generation of nonequilibrium magnons by the mechanism of magnon-impurity interactions [the mechanism of stimulated nonresonance pumping of spin waves (nr)] and by the usual parametric resonance excitation of magnons of the system (r), described by the term in (9) containing the Lorentz function. The ratio of the excess number of magnons, as compared with the equilibrium number, generated by the nr-mechanism to the number of nonequilibrium magnons originating in the resonance r-situation is, from (9),

$$N_{nr}^{(1)}/N_r \approx \zeta_0(n_i v_0)(\Theta_C/\varepsilon_0)^{1/2}\alpha^{3/2}, \qquad (10)$$

where $\zeta_0 \sim 5\times10^{-2}$ and $\alpha \equiv (\Omega-2\varepsilon_0)/2\varepsilon_0$.

Thus, for example, when $(n_i v_0) \sim 10^{-1}$, $\Theta_c \sim 500$ K, $\varepsilon_0 \sim 0.2$ K, and $\alpha > 2$, the excess number of nonequilibrium magnons is provided chiefly by the nr-mechanism suggested here. As is evident, the effectiveness of this mechanism is not solely determined by the concentration of the impurities but also depends substantially on the width of the interval $(\varepsilon_0, \Omega-\varepsilon_0)$ within which generation of nonequilibrium magnons occurs. In contrast to the usual resonance mechanism of excitation of spin waves, which provides generation of spin waves in the vicinity of the point $\varepsilon_k=\Omega/2$ and is of threshold character, the mechanism that we are considering has no threshold and thus is the one that produces prethreshold absorption of the energy of an alternating magnetic field in ferromagnets containing impurities. In the case

$$\alpha > \zeta_0^{-2/3}(n_i v_0)^{-2/3}(\varepsilon_0/\Theta_C)^{1/3} \qquad (11)$$

the mechanism of stimulated nonresonance pumping of spin waves is responsible also for beyond-threshold absorption.

To elucidate the role of this mechanism in the absorption of the energy of the field, we shall calculate the imaginary part χ_i'' of the high-frequency magnetic susceptibility due to nonresonance decay processes. On determining in the usual way the energy absorbed by the system in unit time and normalizing it in relation to the flow of energy from the source of the external alternating field, we get for the value of χ_i'' an expression of the form

$$\chi_i'' = \frac{2\zeta_1}{\pi}(n_i v_0)\left(\frac{\mu^2}{v_0\Omega}\right)\frac{U^2 m^2}{\Theta_c^3\Omega} I_1(x_0, \omega), \tag{12}$$

where $x_0 \equiv \varepsilon_0/T$, $\omega \equiv \Omega/T$, and

$$I_1(x_0, \omega) = \int_{x_0}^{\omega - x_0} dx \frac{x[(x-x_0)(\omega-x_0-x)]^{1/2}}{(\omega-x)^2(e^x-1)(e^{\omega-x}-1)}.$$

In the usual experimental situation, $2\varepsilon_0 < \Omega \ll T$, the expression (12) simplifies:

$$\chi_i'' = {}^1/_4\zeta_1(n_i v_0)(\mu^2/v_0\Omega)(U/\Theta_c)^2(m^2/\Omega\Theta_c)(T/\varepsilon_0) \times (\Omega-2\varepsilon_0)^2/\varepsilon_0^{1/2}(\Omega-\varepsilon_0)^{3/2}. \tag{13}$$

We shall not consider here the case $\Omega < 2\varepsilon_0$, since it was analyzed in detail in Ref. 7. We shall give numerical estimates of the value of χ'' determined by (13). For a constant-field value $H = 1.5$ $k0e$ and for $(n_i v_0) \sim 0.1$, $\Omega \sim 10^{11}$ sec^{-1}, $T \sim 300$ K, and $\Theta_c \sim 500$ K, $\chi'' \sim 10^{-2}$ to 10^{-3}.

3. INTERACTIONS OF MAGNONS WITH DISLOCATIONS IN AN EXTERNAL ALTERNATING MAGNETIC FIELD

Randomization of magnons by randomly located dislocations is described by Eq. (7) with the substitution (5).

As in the case of impurities, we shall compare the effectiveness of the sources that provide pumping of nonequilibrium magnons in the system by nonresonance decay processes at dislocations and by the usual resonance mechanism of excitation of spin waves. The ratio of the corresponding excess numbers of magnons is ($\Omega \gtrsim 2\varepsilon_0$)

$$\frac{N_{nr}^{(d)}}{N_r} \approx 10^{-2}(n_d b^2)\frac{R^3}{v_0}\frac{\Omega^2}{\Theta_c^{3/2}\varepsilon_0^{1/2}}\alpha^{7/2}, \tag{14}$$

where R is the mean radius of the dislocation loops and where b is the mean value of the Burgers vector. In contrast to impurities, the present ratio, for a real experimental situation, may exceed unity even in the case of narrow sources ($\alpha < 1$). In fact, when $n_d \sim 10^8$ cm^{-2}, $R \sim 10^{-3}$ cm, $b \sim 10^2 a$, and $\Theta_c \sim 500$ K, the chief mechanism of magnon pumping is the nonresonant for $\alpha > 0.1$. Thus interaction of magnons with dislocations produces the most favorable situation for realization of the mechanism of parametric excitation of magnons suggested in the present paper.

In analogy to the preceding case (impurities), calculations of the imaginary part of the high-frequency magnetic susceptibility in the case of dislocations (when $\Omega > 2\varepsilon_0$) lead to the following result:

$$\chi_d'' = \frac{16\zeta_1}{35} n_d b^2 \frac{R^3}{v_0}\left(\frac{\mu^2}{v_0\Omega}\right)\frac{m^2 T^2}{\Omega\Theta_c^3} I_2(x_0, \omega),$$

$$I_2(x_0, \omega) = \int_{x_0}^{\omega - x_0} dx \frac{x(x-x_0)^{3/2}(\omega-x_0-x)^{3/2}}{(\omega-x)^2(e^x-1)(e^{\omega-x}-1)}. \tag{15}$$

This expression is valid when $T \gg \Omega > 2\varepsilon_0$.

On carrying out the integration in (15), we obtain the final result in the form

$$\chi_d'' = \frac{24\pi^3}{35}\zeta_1(n_d b^2)\left(\frac{R^3}{v_0}\right)\left(\frac{\mu^2}{v_0\Omega}\right)\left(\frac{\mu M_0}{\Theta_c}\right)^2\left(\frac{T}{\Omega}\right)\frac{1}{\Theta_c}\left\{\frac{\Omega^2+4\Omega\varepsilon_0-4\varepsilon_0^2}{4(\varepsilon_0(\Omega-\varepsilon_0))^{1/2}} - \Omega\right\}. \tag{16}$$

For the system parameters already used above, we get for the magnitude of χ_d'' the value $10^{-1} - 10^{-2}$.

4. THE EFFECT OF PARALLEL PUMPING ON THE MAGNETIZATION OF FERROMAGNETS

Production of nonequilibrium stationary states of magnons by an external field leads to a change of the kinetic and thermodynamic properties of a ferromagnet. In this section we shall consider the effects of the action of a high-frequency alternating magnetic field on the magnetization $M(h, T)$ of a ferromagnet with magnetic inhomogeneities. Such effects have been treated earlier[7] for the case of an ideal ferromagnet when $\Omega < 2\varepsilon_0$.

We shall begin with the case $\Omega > 2\varepsilon_0$ with allowance for magnon-impurity interactions. On solving Eq. (9) in the τ-approximation and determining only the contribution of parametrically nonresonant processes, and after substitution of the answer for f_ε in the magnetization expression

$$M(h, T) = M_0 - \mu\sum_{\mathbf{p}}(|u_{\mathbf{p}}|^2 + |v_{\mathbf{p}}|^2) f_{\mathbf{p}} \equiv M(T) - \delta M, \tag{17}$$

we get (Ω, $\varepsilon_0 \ll T$)

$$\frac{\delta M}{M_0} = \frac{\zeta_1}{4\pi}(n_i v_0)\frac{U^2 m^2}{\tau^{-1}\Theta_c^3}\left(\frac{\mu h_0}{\Omega}\right)^2\left(\frac{T}{\Omega}\right)\frac{(\Omega-2\varepsilon_0)^2}{\Omega[\varepsilon_0(\Omega-\varepsilon_0)]^{1/2}}\left\{\frac{\Omega^2}{4\varepsilon_0(\Omega-\varepsilon_0)} + 1\right\}. \tag{18}$$

When the inequality (11) is satisfied, as has already been mentioned above, the suppression of the magnetization of the ferromagnet is determined by the nr-mechanism and consequently by formula (18).

A more interesting situation is the case $\Omega < 2\varepsilon_0$, when, at least in the quadratic approximation with respect to the field, processes of generation of an excess number of nonequilibrium magnons are forbidden by the law of conservation of energy. In this case, as is seen from (9), the external field leads only to a redistribution of magnons with respect to energy; the energy of the field goes to increase of the mean energy of the magnons. This redistribution of magnons with respect to energy, toward higher energies, leads, because of the dependence on $\mathbf{p}$ of the coefficient of $f_{\mathbf{p}}$ in formula (17), to a change of the magnetization in a nonequilibrium state, despite the fact that the total number of magnons is conserved when $\Omega < 2\varepsilon_0$. Since the structure of the coefficient $(|u_{\mathbf{p}}|^2 + |v_{\mathbf{p}}|^2)$ is such that it decreases with increase of the wave vector $\mathbf{p}$, a redistribution of the magnons with respect to energy, toward higher energies, leads to a smaller negative contribution to the magnetization (17) as compared with the equilibrium state, when $f_{\mathbf{p}}$ is a Bose function. Thus in the case $\Omega < 2\varepsilon_0$, we may expect the phenomenon of an increase of the magnetization of the specimen (needless to say, when $T \neq 0$) as compared with the equilibrium case (stimulated magnetization). We emphasize that here we have, in analogy with the case of a superconductor placed in an alternating electromagnetic field of frequency $\Omega < 2\Delta$ (where Δ is the superconducting energy gap[11]), the effect of stimulation of an order parameter of the system by a high-frequency field.

By treating the kinetic Eq. (9) in the τ-approximation[12] when $\Omega < 2\varepsilon_0$, we get for the case $\Omega \ll 2\varepsilon_0 \ll T$ the following result for the relative increase of magnetization of the ferromagnet:

$$\frac{(\delta M)_s^{(i)}}{M_0} \sim 10^{-1}(n_i v_0)\frac{(\mu h_0)^2}{\tau^{-1}\Theta_C}\left(\frac{U}{\Theta_C}\right)^2\left(\frac{\mu M_0}{\varepsilon_0}\right)^3\left(\frac{T}{\varepsilon_0}\right). \qquad (19)$$

Here we have considered only the influence of the alternating magnetic field on the magnon-impurity interactions and have disregarded the magnon-magnon interactions induced by the external field.[7] As was shown earlier,[7] triple magnon-magnon interactions, accompanied by absorption and radiation of quanta Ω of the external field, lead effectively to increase of the total number of magnons and consequently to suppression of the magnetization in the nonequilibrium state. Thus the magnetization-stimulation effect can occur only at sufficiently large concentrations of impurities, when the inequality

$$\zeta_4 (n_i v_0)(\mu M_0/T)(U/\Theta_C)^2(\Theta_C/\varepsilon_0)^2 > 1, \qquad (20)$$

where $\zeta_4 \sim 10^2$, is satisfied.

In fact, because of the large value of the matrix element for magnon-impurity interactions, stimulation of the magnetization sets in even at quite small concentrations n_i. Thus, for example, when $T \sim 300$ K, $\Theta_c \sim 500$ K, $\mu M_0 \sim 0.02$ K, and $\varepsilon_0 \sim 0.3$ K, the stimulation effect is already larger than the magnetization-suppression effect when $(n_i v_0) > 10^{-4}$.

As was shown in section 3, linear defects—dislocations—may be more effective for transfer of the field energy of the magnon system. It should also be emphasized that for experimental observation of the effects considered here, a situation may prove more favorable in which it is possible to produce the magnetic inhomogeneities by plastic deformation of the specimen; this leads, as is well known, to the formation of dislocations. If we consider the case of scattering of magnons on dislocations in Eq. (8), for the case $\Omega \ll 2\varepsilon_0 \ll T$ the relative increase of magnetization is described by the formula

$$\frac{(\delta M)_s^{(d)}}{M_0} = \frac{1}{35}(n_d b^2)\frac{R^3 m^3}{2^4\pi^2\Theta_C^3 v_0}\frac{(\mu h_0)^2}{\tau^{-1}T} I_3(x_0, \omega), \qquad (21)$$

where

$$I_3(x_0, \omega) = \int_{x_0}^{\infty} dx \frac{(x-x_0)^{3/2}(x+\omega-x_0)^{3/2}e^x}{x^4(e^x-1)^2}.$$

After some calculations we get

$$\frac{(\delta M)_s^{(d)}}{M_0} \sim 2\, 10^{-2}(n_d b^2)(R^3/v_0)(\mu M_0/\Theta_C)^3(\mu h_0/\varepsilon_0)^2 T/\tau^{-1}. \qquad (22)$$

We shall give an estimate of the magnitude of the effect. When the amplitude of the alternating magnetic field is $h_0 \sim 1$ Oe and when $n_d \sim 10^8$ cm^{-2}, $b \sim 10^2 a$, $R \sim 10^{-3}$ cm, $4\pi\mu M_0/\Theta_c \sim 10^{-3}$, $\Omega/2\varepsilon_0 \sim 10^{-1}$, $T \sim 300$ K, and $\tau \sim 10^{-7} - 10^{-8}$ sec, the relative increase of magnetization amounts to $10^{-3} - 10^{-2}$.

Finally we consider the magnetization-suppression effect produced by the nonresonant mechanism of generation of spin waves in ferromagnets containing dislocations. We give only the answer for the case $T \gg \Omega > 2\varepsilon_0$:

$$\frac{\delta M}{M_0} = \frac{8\pi^2\zeta_1}{70}(n_d b^2)\left(\frac{R^3}{v_0}\right)\left(\frac{\mu M_0}{\Theta_C}\right)^2\left(\frac{\mu h_0}{\Omega}\right)^2(\Omega\tau)\left(\frac{T}{\varepsilon_0}\right)\frac{(\Omega-2\varepsilon_0)^2}{\varepsilon_0^{1/2}(\Omega-\varepsilon_0)^{3/2}}\frac{E}{\Theta_C}, \qquad (23)$$

where

$$E = \frac{(\Omega-2\varepsilon_0)^2[\Omega^2+2(\Omega-\varepsilon_0)\varepsilon_0]}{\Omega^3} - \frac{[(\Omega-\varepsilon_0)^{1/2}-\varepsilon_0^{1/2}]^2}{[(\Omega-\varepsilon_0)^{1/2}+\varepsilon_0^{1/2}]^2}[\Omega+4(\varepsilon_0(\Omega-\varepsilon_0))^{1/2}].$$

Here we have considered a model of randomly distributed ring-shaped dislocations. It should be mentioned that the results obtained remain valid in order of magnitude also for a model of straight-line, randomly located dislocations. In the latter case, it is necessary in the estimates to interpret R as the mean length of the dislocation lines.

In conclusion, we note that the approach developed in this paper to the description of parametric excitation of spin waves in ferromagnets containing magnetic inhomogeneities may also be useful for treatment of antiferromagnets. Here one of the most interesting problems is the consideration of the influence of nonequilibrium states on phase transitions of the spin-reorientation type.

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[1)] In this paper we shall use the standard definition of the concentration n_d of dislocations, as the ratio of the total length of the dislocations to the volume of the crystal.

[2)] A detailed presentation of a description of parametric excitation of magnons within the framework of the kinetic-equation approach will be the subject of a separate paper.

[1] V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **59**, 1200 (1970) [Sov. Phys. JETP **32**, 656 (1971)].

[2] V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

[3] D. G. Scotter, J. Appl. Phys. **42**, 4088 (1971).

[4] V. E. Zakharov and V. S. L'vov, Fiz. Tverd. Tela **14**, 2913 (1972) [Sov. Phys. Solid State **14**, 2513 (1973)].

[5] V. P. Seminozhenko, V. L. Sobolev, and A. A. Yatsenko, Solid State Commun. **31**, 277 (1979).

[6] V. G. Bar'yakhtar and G. I. Urushadze, Zh. Eksp. Teor. Fiz. **39**, 355 (1960) [Sov. Phys. JETP **12**, 251 (1961)].

[7] V. P. Seminozhenko, V. L. Sobolev, and A. A. Yatsenko, Zh. Eksp. Teor. Fiz. **77**, 2324 (1979) [Sov. Phys. JETP **50**, 1119 (1979)].

[8] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spinovye volny (Spin Waves), Nauka, 1976 (transl., North-Holland, 1968).

[9] V. G. Bar'yakhtar, M. A. Savchenko, and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **51**, 936 (1966) [Sov. Phys. JETP **24**, 623 (1967)].

[10] V. P. Seminozhenko and A. A. Yatsenko, Preprint ITF-79-51R, Keiv, 1979.
[11] G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. 11, 186 (1970) [JETP Lett. 11, 114 (1970)].
[12] V. P. Seminozhenko and A. A. Yatsenko, Fiz. Nizk. Temp. 6, 399 (1980) [Sov. J. Low Temp. Phys. 6, 199 (1980)].

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Effect of impurities of normal metals on the residual electric resistance of Na, K, Rb, and Cs

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The effect of 12 impurities (Li, K, Rb, Cs, Ca, Sr, Cd, Hg, In, Tl, Sn, Pb) on the residual resistivity ρ_0 of sodium, and of Cs impurities on the ρ_0 of potassium is studied experimentally. It is found that the dependence $\Delta\rho/c = 4.75\,(\Delta Z-1.1)^2 + 5.15$ is valid for sodium with impurities of the V period. The values of $(\Delta\rho/c)_{\rm exp}$ for alloys with Na, K, Rb, and Cs bases are compared with those of $(\Delta\rho/c)_{\rm calc}$ determined by using the phase shifts and making allowance for lattice distortions by means of the Blatt correction. The agreement between experiment and calculation is just barely satisfactory (a discrepancy by a factor of 2–4). It is suggested that the Blatt correction is insufficient for obtaining good agreement, and that the deviation of the real Fermi surface of the matrix from spherical is not the principal cause of the discrepancy.

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In the study of residual electric resistance of dilute alloys based on normal polyvalent metals Sn,[1] Pb,[2] Tl,[3] In, Ga,[4] Zn, Cd, Mg,[5] and Hg,[6] it has been established by the authors that the calculated values of the electric resistance agree with the experimental values by no better than barely satisfactory. In Ref. 7, where we studied the residual resistance of alloys based on lithium, whose Fermi surface is close to spherical (in comparison with polyvalent metals), a rather good agreement between theory and experiment was obtained; therefore, it has been suggested there that an important role in the impurity scattering of conduction electrons is played by the anisotropies of the Fermi surface of the metal matrix. Inasmuch as all our calculations were made within the framework of the free-electron model, and the contribution to the scattering connected with anisotropies of the Fermi surface was nowhere taken into account, a better agreement of the theoretical and experimental results should be expected for metals in which the Fermi surface is closer to spherical. Sodium and potassium belong to such metal matrices. The present research was undertaken with the aim of testing the assumptions stated above.

Prior to the beginning of our investigations, there were a series of papers in the literature on the effect of impurities of Na, Rb, and Cs on the residual resistivity ρ_0 of calcium;[6,8-11] however, the data on the system of alloys K+Cs were unreliable because of a great scatter of the experimental points among the various authors and greater precision is required. For the other alkali metals, the researches of Refs. 12 and 13 are known to us, in which the resistance of dilute alloys of Cs+Rb, Rb+K, and K+Cs were studied (the boldface indicates the metal-matrices).

Researches devoted to the study of ρ_0 of dilute alloys on a sodium base were not known to us at all. Therefore, to perform the undertaken task, it was first necessary to obtain reliable data on the change in the residual resistivity ($\Delta\rho_0$) of sodium or potassium in which one impurity or another is dissolved. The problem was appreciably complicated by the fact that, according to known binary diagrams of state only Cs, Rb and some quantity of Na dissolve in solid potassium, and only Tl and Ba in sodium. With the aim of studying the possibility of formation of binary solid solutions of sodium and potassium with various metals, exploratory research was performed by us,[14] in which a noticeable solubility of ten impurity metals in sodium and practically none for impurity metals (except Rb, Cs, and Na) in potassium were first noted.

METHOD OF EXPERIMENT

All the alloys were prepared on the base of pure (~99.99%) metals with electric resistance ratio $\delta_{4.2} = R_{4.2}/R_{293}$ ($R_{4.2}$ and R_{293}—resistance of the sample at 4.2 and 293 K, respectively) equal to $(1.2-1.6)\times10^{-4}$ for sodium and 3.6×10^{-4} for potassium. The quantity $\delta_{4.2}$ differed from δ_0 (relative residual resistance) by not more than 15% for potassium and not more than 2–3% for sodium. The method of preparation of the alloys and of the wire samples (diameter 3 mm and length ~180 mm), and of also the measurements of their electric resistance at 4.2 and 293 K was reported in detail in Ref. 14.

The error of measurement of R_T did not exceed 0.7%. To increase the reliability of the determination of the quantity $\Delta\rho/c$—the change in ρ_0 of the metal-matrix up-

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