Dynamics of the resistive state of a superconductor

B. I. Ivlev, N. B. Kopnin, and L. A. Maslova

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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The dynamics of the resistive state of a superconductor is initially investigated with relatively simple model equations as an example. It is shown that at almost the entire distance between the phase-slip centers (PSC) the picture is stationary, and in a narrow region near the PSC the order parameter experiences strong oscillations ranging from zero to approximately the equilibrium value. It turns out that the current-voltage characteristics (CVC) can be obtained by using the static equations, so that an exact analytic expression for the CVC can be obtained. The developed approach is used next for a real superconductor. One of the features of the obtained CVC is the existence, on top of the normal current, an extra current that does not depend on the voltage at large values of the latter. With decreasing temperature, the extra current begins to depend on the voltage.

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1. INTRODUCTION

A superconductor state that has a finite resistance to direct current is called resistive. It is known that a constant and homogeneous electric field cannot exist in an infinite superconductor with a homogeneous and stationary order parameter. Otherwise, the electromagnetic potentials that increase in time or in space would destroy the superconductivity. Therefore, if a constant electric field is present inside the superconductor, this means that the macroscopic phase coherence must be lost at some point of the sample and at some instants of time.

In a type-II superconductor in a magnetic field, the resistive state is produced by transport of the magnetic flux by the moving vortices, and an electric field is induced as a result. In sufficiently narrow (quasi-one-dimensional) superconducting samples, however, the current flowing through them produces practically no magnetic field, and the resistive-state mechanism is different. This is precisely the situation considered in the present paper.

The superconductor is characterized by the gauge-invariant potentials

$$\mathbf{Q}=\mathbf{A}-\frac{c}{2e}\nabla\chi,\quad \Phi=\varphi+\frac{1}{2e}\frac{\partial\chi}{\partial t},$$

the order parameter being $\Delta=|\Delta|e^{i\chi}$. If the potential φ increases with increasing coordinate x along the sample, then in order for the invariant quantity Φ, on which the behavior of the superconductor depends, to remain on the average finite in space, a definite adjustment of the phase of the order parameter is required. For a long narrow superconducting channel, the character of this adjustment should be such that at individual points of the superconductor the phase changes jumpwise at certain instants of time by a multiple of 2π. Naturally, at these points of the sample, the macroscopic phase coherence of the superconductor is disturbed at the instant of the phase jump, and the order parameter vanishes. These points are called phase-slip centers (PSC), and were first introduced in superconductivity theory by Langer and Amboegaokar.[1]

Two different ideas concerning the structure of the PSC were advanced subsequently. Some workers (for example, Rieger, Scalapino, and Mercereau[2]; Skocpol, Beasley, and Tinkham,[3]) have assumed as before that the PSC is produced at definite instants of time at certain points of the sample. Others (for example, Fink and Poulsen[4,5]; Galaĭko et al.[6,7]), on the contrary, have assumed that the PSC are stationary formations characterized by a jump of the derivative $\partial\chi/\partial t$ at definite points of the sample. This approach was argued most consistently in the papers of Galaĭko *et al.*.[6,7]

It is shown in the present paper that the static structure of the PSC is unstable, i.e., that the jump of the chemical potential of the pairs is smeared in space, and that the order parameter at the PSC has two principal length scales expressed in terms of the coherence length $\xi(T)$ and the depth of penetration of the longitudinal electric field l_E. If, as is usually the custom, $l_E \gg \xi(T)$, then the order parameter changes periodically with time in a rather narrow region $x\sim(\xi l_E)^{1/2}$ near the PSC, where $|\Delta|$ fluctuates between zero and values on the order of unity. At the same time, at distances $x\sim l_E$ from the PSC, the order parameter and the gauge-invariant potentials change little in time (see also Ref. 8). The distance between the PSC is determined principally by the relaxation length of the chemical potential Φ, and is therefore of the order of l_E. Thus, the static approximation is valid in almost all the space and is violated only near the PSC, which is essentially a dynamic object. At the same time, to obtain the current-voltage characteristics (CVC) it becomes possible to employ the static equations, which admit of an exact solution and make it possible to understand sufficiently well the entire picture.

It is also shown in this paper that the qualitative picture of the resistive state remains the same for a real superconductor with a gap. Here, too, an exact expression is obtained for the CVC in the temperature region near T_c.

2. MODEL-DEPENDENT EQUATIONS OF SUPERCONDUCTIVITY

The exact dynamic equations that describe a superconductor with a gap in the spectrum are generally

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speaking quite complicated. In some cases the writing down of the equations themselves is a problem, let alone their solution. It would be therefore difficult, naturally, to track such a complicated dynamic phenomenon as the formation of the PSC by starting directly from the general equations for a real superconductor. It is logical to use as the first step an investigation of the simplest dynamic equations of superconductivity. Despite the impossibility of obtaining a mathematically exact solution even for the model equations that will be discussed below, it does become possible to draw a number of important conclusions.

In this section we consider the dynamics of the PSC on the basis of equations of a special type, which differ from the Ginzburg-Landau equations in that dissipative terms are added to the derivatives with respect to time:

$$u\left(\frac{\partial\psi}{\partial t}+i\varphi\psi\right)-\frac{\partial^2\psi}{\partial x^2}+\psi(|\psi|^2-1)=0, \tag{1a}$$

$$j=\mathrm{Im}\left(\psi\times\frac{\partial\psi}{\partial x}\right)-\frac{\partial\varphi}{\partial x}. \tag{1b}$$

The complex order parameter is here $\psi=|\psi|e^{i\chi}$.

At $u=12$, this system of equations, as shown by Gor'kov and Éliashberg,[9] corresponds to a gapless superconductor with large concentration of paramagnetic impurities. Introducing $\Delta=|\psi|$ and the gauge-invariant potentials $Q=A-\partial_\chi/\partial x$ and $\Phi=\varphi+\partial_\chi/\partial t$ ($A=0$ in our one-dimensional case), we can rewrite the system (1) in the form

$$u\frac{\partial\Delta}{\partial t}-\frac{\partial^2\Delta}{\partial x^2}+\Delta(\Delta^2+Q^2-1)=0, \tag{2a}$$

$$j=-\Delta^2Q+E,\qquad E=-\frac{\partial\Phi}{\partial x}-\frac{\partial Q}{\partial t}, \tag{2b}$$

$$u\Delta^2\Phi+\frac{\partial}{\partial x}(\Delta^2Q)=0. \tag{2c}$$

In contrast to the system (1), the system (2) contains only the gauge-invariant quantities Δ, Q, and Φ.

From (2b) and (2c) we easily obtain the equation

$$-\frac{\partial^2\Phi}{\partial x^2}+u\Delta^2\Phi=\frac{\partial^2Q}{\partial x\,\partial t}, \tag{3}$$

which describes in the static case the penetration of a longitudinal electric field into a superconductor. As seen from (2), the characteristic length l_E over which the penetration decreases is proportional to $u^{-1/2}$. Since the length scale, as seen from (2a), was chosen to be the coherence length $\xi(T)$, the free parameter u in (1) and (2) is connected with the ratio of the characteristic lengths

$$u\sim\xi^2/l_E^2.$$

In this section we investigate the case of small $u\ll1$; this is useful, because in a real superconductor with a gap we have $\xi(T)\ll l_E$ in the entire temperature range of practical interest.

If the density of the current through the sample exceeds a certain value j_c, called the Ginzburg-Landau critical current, then a homogeneous and stationary state of the superconductor, as is well known, becomes impossible. For the system (2) the critical current is $j_c=2/3^{3/2}$. Let us dwell on the possible types of solution that the system (2), or its equivalent system (1) can have at $j>j_c$.

We investigate for stability the normal state of relatively infinitesimally small fluctuations of the order parameter. We linearize for this purpose the system (1):

$$u\frac{\partial\psi}{\partial t}-ijux\psi-\frac{\partial^2\psi}{\partial x^2}-\psi=0. \tag{4}$$

The solution of this equation can be written in the form

$$\psi(x,t)=\left(\frac{u}{4\pi t}\right)^{1/2}\exp\left(\frac{t}{u}-\frac{jt^3}{12u}\right)\int_{-\infty}^{\infty}dy\,\psi(y,0)\times\exp\left[\frac{ji}{2}t(x+y)-u\frac{(x-y)^2}{4t}\right]. \tag{5}$$

The quantity t^3 in the argument of the exponential is connected with the dispersal of the Cooper pairs by the electric field and leads to suppression of the order parameter. From this, in accordance with the papers of Gor'kov[10] and Kulik,[11] we can conclude that the normal state is stable in the small.

This conclusion, however, does not extend to fluctuations of finite magnitude. If the one-dimensional sample is broken up into infinitesimally small segments along the coordinate, $x_1, x_2, \ldots$, and the behavior of the system in the phase space $\Delta(x_1), \Delta(x_2), \ldots$, is considered, then the origin corresponding to the normal state is a stable singular point. One can imagine that at a finite distance from the origin there is located a separatrix that serves as the borderline between the region of attraction of the trajectories to the normal state and the region of attraction to a certain limit cycle, which corresponds to a solution periodic in time and gives rise to the resistive state of the superconductor. We shall present below results of numerical calculations that confirm the existence of a limit cycle of the system (1) at $u\ll1$.

Using (5), we can estimate for a certain class of functions the value of the stationary threshold solution that is unstable to infinitesimally small fluctuations. If the modulus of the order parameter was constant in space at the initial instant of time, it will vary subsequently like

$$\Delta(x,t)=\Delta_0\exp(t/u-j^2t^3/3u), \tag{6}$$

and go through a maximum. If we assume that at $u\ll1$ the trajectories that lie in the region $\Delta\lesssim1$ can be attracted to the normal state, then we obtain from (6) an estimate for the critical fluctuations at $uj\ll1$:

$$\Delta_c\sim\exp(-2/3uj). \tag{7}$$

The threshold solution that separates the region of stability of the normal state was investigated numerically for the system (1) at $u>1$ by Kramer and Baratoff.[12]

Thus, both the normal and the dynamic resistive states are stable at $j>j_c$, and a transition from one of them to the other at large currents should take the form of a finite jump. It should be noted here that experimental observation of this jump can be strongly hindered by Joule heating, which becomes stronger at larger currents. In the normal state, some decrease of the resistance can result from quantum fluctuations, as shown in Refs. 10 and 11.[1]

As mentioned above, the physical realization of the resistive states can in principle be visualized as follows. For example, it was proposed in Refs. 4-7 that the order parameter at the PSC point is zero at all times. In this case the PSC is reminiscent of a Josephson transition with constant difference of the chemical potentials of the Cooper pairs, and it can be shown that the alternating Josephson current exerts a weak influence on the entire static picture.

We examine now what happens to such a static PSC if the special condition $\psi(0,t)=0$ is not imposed. Near the point $x=0$, at the initial instant of time $\Delta = 2^{-1/2}|x| \ll 1$ [in accordance with the solution $\Delta = \tanh(2^{-1/2}|x|)$], and we can therefore use the linearized equations (5). From (2c) we find that $Q = 1/3u\Phi(0)|x|$. The order parameter near the static PSC should then be of the form

$$\psi_{st}(x,t)=2^{-1/2}|x| \exp[i\Phi(0)(t-{}^1/_6 x^2 u)\,\mathrm{sign}\,x]. \quad (8)$$

It is easily seen from (5) that actually ψ_{st} is not a solution of Eq. (4), if for no other reason that at $t \ll u$ the solution (5) with initial condition $\Delta = 2^{-1/2}|x|$ yields $\psi(0,t)=(t/2u)^{1/2}$.

Thus, the PSC cannot exist as a static formation, inasmuch as there are no physical reasons that dictate the condition $\psi(0,t)=0$. In other words, ψ_{st} in (8) is an unstable solution and diffuses in time.

We proceed now to an examination of the dynamic situation. As already mentioned, the dynamic PSC presupposes a vanishing of $\psi(x=0,t)$ only at certain instants of time. This is precisely the situation considered in Ref. 1 and investigated also in Ref. 2. It was shown in an earlier paper[14] that the PSC can be visualized as topological singularities of the type of vortices in the space $\{x,t\}$, which form there a periodic structure similar to the Abrikosov lattice in type-II superconductors.[15] (A similar situation in the case of phase slip in superfluid ^{3}He was considered by Volovik.[16] The electric field satisfies in this case the relation

$$\int E\,dx\,c\,dt = nF_0,$$

where n is an integer, $F_0 = \pi\hbar c/e$ is the "flux quantum," and the integral is taken over the area of the unit cell in $\{x,t\}$ space. In the units chosen above, this relation takes the form

$$\int E\,dx\,dt = 2\pi n. \quad (9)$$

An individual PSC is simply an isolated point in this space. Just as for an ordinary vortex, the integral along an infinitesimally small contour around this point is

$$\int \mathbf{q}\,d\mathbf{l} = 2\pi n, \quad (10)$$

where the vector $\mathbf{q} = \{Q, -\Phi\}$. This relation turns out to be useful also in the consideration of the PSC dynamics.

If the current is not much higher than j_c, we have $j \sim 1$ in our units and, as follows from (2b), $E \sim j$. Relation (9) connects the characteristic spatial and temporal scales x_0 and t_0 of the variation of the electric field:

$$j x_0 t_0 \sim 1. \quad (11)$$

The characteristic spatial scale of the change of the electric field is, just as in the static case, the quantity $x_0 \sim u^{-1/2}$. The characteristic time scale is then $t_0 \sim u^{1/2}$, and, by making the change of variables

$$y = x u^{1/2}, \quad \tau = t u^{-1/2}, \quad (12)$$

we rewrite (2) in the form

$$u^{1/2}\frac{\partial\Delta}{\partial\tau} - u\frac{\partial^2\Delta}{\partial y^2} + \Delta(\Delta^2+Q^2-1) = 0, \quad (13a)$$

$$j = -\Delta^2 Q - u^{-1/2}\frac{\partial Q}{\partial\tau} + \frac{\partial}{\partial y}\left[\frac{1}{\Delta^2}\frac{\partial}{\partial y}(\Delta^2 Q)\right]. \quad (13b)$$

In the current equation, the term with the derivative with respect to time is anomalously large, and the solution must be found by equating this term to zero. We find thus that over scales $x \sim u^{-1/2} (y \sim 1)$ the regime should be static. From an estimate of the values of the derivatives in (13b) it is seen that the derivative with respect to the coordinate becomes just as large as the derivative with respect to time over scales $x \sim u^{-1/4} \times (y \sim u^{1/4})$. The static approximation does not hold in this region. Since the PSC form in the $\{x,t\}$ space a regular lattice, the time derivatives drop out when (2) or (3) is averaged over the time, and the resultant equations determine the static solution over the characteristic lengths $x \gg u^{-1/4}$.

These equations can be written in the form

$$-\partial^2\Phi/\partial x^2 + u\Delta^2\Phi = 0, \quad (14a)$$

$$j = -\Delta^2 Q - \partial\Phi/\partial x, \quad (14b)$$

$$\Delta^2 + Q^2 = 1. \quad (14c)$$

We recall that $\xi(T)=1$ and $l_E = u^{-1/2}$ in terms of the units assumed in (2). It can therefore be stated that the static regime exists over distances of the order of l_E from the PSC and is violated over distances $(\xi l_E)^{1/2}$ from it.

Integration of the system (14) is simple:

$$u\Phi^2 = f(\Delta), \quad f(\Delta) = 2\int_{\Delta_0}^{\Delta} \frac{(3p^2-2)(j-p^2(1-p^2)^{1/2})}{p(1-p^2)^{1/2}}\,dp, \quad (15a)$$

$$\left(\frac{l}{2} - x\right)u^{1/2} = \int_{\Delta_0}^{\Delta} \frac{(3p^2-2)}{p(1-p^2)^{1/2}[f(\Delta)]^{1/2}}\,dp, \quad (15b)$$

where l is the distance between the PSC. The quantity Δ_0 is the integration constant. The sign of j was reversed in (15a) for convenience. With respect to the choice of the integration limits Δ and Δ_0, the following remark must be made. What flows through the sample at large distances from the PSC is principally the superconducting current whose value is expressed in terms of the local value of the order parameter $j_s = \Delta^2(1-\Delta^2)^{1/2}$. The thermodynamically stable branch is then $2/3 \leqslant \Delta^2 \leqslant 1$. Thus, the values of Δ_0 and Δ must be chosen from the indicated interval. As the PSC is approached, the superconducting current decreases, and therefore $\Delta > \Delta_0$. At distances $x \ll u^{-1/2}$ from the PSC, the superconducting current is small, and the main contribution to the total current is made by its normal component. For this reason, at distances $u^{-1/4}$

$\ll x \ll u^{-1/2}$ we must assume that $j_s = 0$ and $\Delta \approx 1$. An estimate with the aid of Eq. (13a) shows that $1-\Delta^2 \sim u^{-1/2}$ in this region.

The coordinate dependence of the modulus of the order parameter is shown in Fig. 1, where the static solution (15) is almost the entire curve, with the exception of the vicinities of the points $x = 0$ and $x = l$, in which phase slip takes place periodically in time, and $x_1 \sim (\xi l_E)^{1/2} \sim u^{-1/4}$. The constant Δ_0 is the value of Δ halfway between two PSC. Curves 1, 2, and 3 correspond to different instants of time. Thus, curve 1, for example, corresponds to the instant of formation of the PSC, and recalls the static solution of Fink and Poulsen,[5] except that in their case $x_1 \sim \xi$.

The system (1) was numerically integrated at small u and the result confirmed fully the foregoing arguments. The boundary conditions $\partial\Delta/\partial x = 0$ and $\Phi = 0$ at the PSC point were used in the calculations. The result was a time-periodic regime at currents both larger and smaller than j_c. Figure 1 corresponds to the calculations for $u = 0.01$ and $j = 0.4$ (we recall that $j_c = 2/3^{3/2} \approx 0.3849$), and to a distance $l = 40$ between the PSC. If the instant of the PSC formation (curve 1) corresponds to $t_1 = 0$, then at the instant when the maximum of $\Delta(x = 0, l; t)$ is reached (curve 2) we have $t_2 = 0.053$, while curve 3 represents some intermediate value $t_3 = 2.188$. The time period for this case is $T = 2.52$, which contradicts at first glance the estimate $T \sim u^{1/2} = 0.1$. However, as follows from (9), $T = \pi\Phi_0^{-1}$ and calculation by formula (15a) yields for $u^{1/2}$ a numerical coefficient of the order of 10, thus resolving this contradiction.

In the numerical integration it was noted that with decreasing u the amplitude of the time oscillations decreased in the central part of the curve of Fig. 1.

The coordinate dependence of the potential Φ is shown schematically in Fig. 2 up to the region of the strong nonstationarity. This region, from zero to values of the order of x_1, makes a small contribution to the relation (9). In fact, the contribution from the region $0 < x \lesssim x_1 \sim u^{-1/4}$ is of the order of

$$\int_0^{x_1} \bar{E}\, dx = -\frac{\partial \bar{\varphi}}{\partial x} x_1,$$

where the bar denotes averaging with respect to time. It follows from (3) that

$$\partial^2\varphi/\partial x^2 = u\Delta^2\Phi.$$

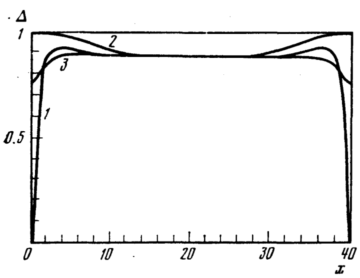

FIG. 1. Coordinate dependence of the modulus of the order parameter for three instants of time: $t_1 = 0$, $t_2 = 0.053$, and $t_3 = 2.188$ for curves 1, 2, and 3, respectively.

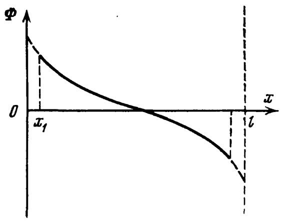

FIG. 2. Schematic coordinate dependence of the scalar gauge-invariant potential in the static region. The static approximation does not hold at distances on the order of x_1 from the PSC.

Therefore the contribution of the considered region is of the order of $u\Phi(x_1)x_1^2 \sim u^{1/2}\Phi(x_1)$ and is small compared with $\Phi(x_1) = \Phi_0$, which is half the voltage on the PSC.

In the static region we have

$$E = -\partial\Phi/\partial x, \tag{16}$$

and therefore, introducing the oscillation frequency $\omega = 2\pi/T$ and $\Phi_0 \approx \Phi(x = 0)$ we obtain the relation

$$2\Phi_0 = n\omega. \tag{17}$$

The quantity $2\Phi_0$ plays the role of the voltage V on a Josephson junction. Relation (17) differs by a coefficient from the usual expression $2eV = n\omega$ by virtue of the specific form in which the initial equations (1) are written, where $i\varphi\psi$ is added to the derivative instead of $2i\varphi\psi$.

It is now easy to write down an expression for the CVC. As shown earlier[14] the electric field averaged over time and space, which we shall also designate by E, is expressed in terms of the periods in the coordinate l and the time T:

$$E = 2\pi n/lT. \tag{18}$$

Using (17) we find that

$$E = 2\Phi_0/l \tag{19}$$

which is the result that can be obtained by using the static approximation(16). By the same token, to obtain the CVC we can use the static equations (14), which were already used earlier to describe the resistive state at $l_E \gg \xi$ in Refs. 4-7. The form of the CVC is obtained from (19) and (15):

$$[f(1)]^{1/2} = E\int_{\Delta_0}^{1} \frac{(3\Delta^2-2)\,d\Delta}{\Delta(1-\Delta^2)^{1/2}[f(\Delta)]^{1/2}}. \tag{20}$$

The period of the structure with respect to the coordinate, i.e., the distance between neighboring PSC, is

$$l = \frac{2}{u^{1/2}}\int_{\Delta_0}^{1} \frac{(3\Delta^2-2)\,d\Delta}{\Delta(1-\Delta^2)^{1/2}[f(\Delta)]^{1/2}}. \tag{21}$$

The free parameter Δ_0, which varies in the range $2/3 \leqslant \Delta_0^2 \leqslant 1$, defines a single-parameter family of CVC.

The CVC defined by Eq. (20) have a zero slope at small values of E and begin with a certain values of the current j_0, which is determined by the parameter Δ_0, namely $j_0 = \Delta_0^2(1-\Delta_0^2)^{1/2}$. We consider now the initial section of the CVC. We put $j = j_0 + j_1, j_1 \ll j$. At large distances from the PSC we then have $\Delta = \Delta_0 + \Delta_1$ and the function $f(\Delta)$ takes the form

$$f(\Delta) = \frac{2(3\Delta_0^2-2)}{j_0}\left[j_1\Delta_1 + \frac{3\Delta_0^2-2}{2(1-\Delta_0^2)^{1/2}}\Delta_1^2\right]\Delta_0.$$

The period of the structure is therefore, with logarithmic accuracy,

$$l=\frac{2}{\Delta_0 u^{1/2}}\ln\left(\frac{Cj_0}{j-j_0}\right), \tag{22}$$

where $C\sim 1$.

Thus, the initial section of the CVC is of the form

$$(j-j_0)/j_0=C\exp\{-[f(1)]^{1/2}\Delta_0/E\}. \tag{23}$$

It is clear therefore that the role of the critical current, i.e., the current at which the electric field appears for the first time, is played by the quantity j_0, which lies in the range between zero and j_c, corresponding to the Δ_0 interval from 1 to $(2/3)^{1/2}$. The concrete choice of the parameter Δ_0 (i.e., of the form of the CVC) should be determined from additional physical considerations.

We can use here, just as in Ref. 7, the principle of the minimum entropy production; this corresponds to the minimum of the expression

$$l^{-1}\int_0^l jE\,dx. \tag{24}$$

The latter is attained at the values $\Delta_0=(2/3)^{1/2}$ and $j_0=j_c$. In this case

$$(j-j_c)/j_c=C_1\exp(-0.32/E). \tag{25}$$

We present by way of example also the expression for the CVC in the case $\Delta_0\to 1$. It takes the form

$$(j-j_0)/j_0=C_2\exp(-j_0/E);\quad j_0=\Delta_0^2(1-\Delta_0^2)^{1/2}\ll j_c. \tag{26}$$

We turn now to the CVC section corresponding to high voltages. For this approach to be valid, the upper current limit can be established from the following considerations. With increasing current, the length l of the structure decreases in proportion to $(uj)^{-1/2}$. It follows from (11) that the characteristic time scale decreases like $(u/j)^{1/2}$. If we now compare in (2b) the terms with the time and coordinate derivatives, in the same manner as in (13b), then it turns out that the region of static behavior corresponds to $x\gg(uj)^{-1/4}$. Therefore the static approximation is violated at $(uj)^{-1/4}\lesssim(uj)^{-1/2}$, i.e., at $j\ll u^{-1}$. At these values of the current, all the quantities oscillate strongly in time over the entire length of the sample. In this regime one can expect the resistive state to go over into the pure normal state. The described theory is therefore valid at $j\ll u^{-1}$ (at $j/j_c\ll l_E^2/\xi^2$ in the customary units).

From Eqs. (15), (20), and (21) it is easy to obtain the CVC in the case of large currents $j_c\ll j\ll u^{-1}$. It takes the form

$$j=E+j_{exc} \tag{27}$$

and differs from Ohm's law in the presence of an excess current

$$j_{exc}=\frac{1}{\nu(1)}\int_{\Delta_0^2}^{1}\frac{(3p-2)}{2\nu(p)}dp,\qquad \nu^2(p)=\int_{\Delta_0^2}^{p}\frac{3x-2}{x(1-x)^{1/2}}dx, \tag{28}$$

which does not depend on the electric field.

The CVC for the three values of Δ_0 are shown in Fig. 3.

Thus, we can draw the following conclusions.

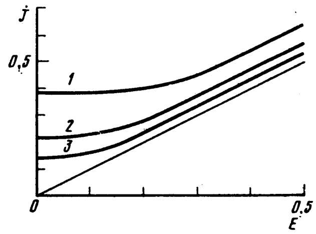

FIG. 3. CVC calculated for a superconductor from Eqs. (15) and (20) at different values of the constant $\Delta_0^2=0.68$, 0.95, 0.98 respectively for curves 1, 2, and 3.

1) The closer Δ_0^2 to 2/3, the higher the corresponding curve; at Δ_0 close to unity, the curves start out from very small currents (this agrees with the already mentioned results of the numerical calculations). The Ginzburg-Landau thermodynamic critical current j_c is the critical current for only one value $\Delta_0^2=2/3$, and at larger values of this parameter the critical current is less than j_c.

2) The initial section of the CVC has a zero slope (this is a consequence of the infinite length of the sample).

3) At currents greatly exceeding the critical value, the CVC proceeds in parallel with the normal law, with an excess current independent of E. The reason for the latter is that in this model there is no direct influence of the scalar potential Φ on the order parameter, and the superconducting current remains finite with increasing Φ.

At very large currents, the static approximation no longer holds and it appears that the CVC is no longer parallel to Ohm's law, so that the changeover to the normal state becomes possible.

3. CURRENT-VOLTAGE CHARACTERISTICS OF A SUPERCONDUCTOR WITH A GAP

In this section we shall dwell on the case of a superconductor with a gap in the energy spectrum. The dynamics of such superconductors is much more complicated that in the gapless situation, and the equations are not as simple in form as (1). In the properties of the superconductor with a gap, however, there is an essential singularity that makes the dynamics of the PSC of a real superconductor highly similar to the dynamics of the model considered in the preceding section. This singularity is the large ratio of the depth of penetration of the electric field to the coherence length l_E/ξ. It is precisely because of this hierarchy of the characteristic lengths, in accordance with the ideas developed by Galaĭko,[6] that the order parameter adjusts itself rapidly to the static variation of the electric field, as shown in the preceding section. It can be concluded on this basis that the dynamics of the PSC in a real superconductor is similar to that considered above, i.e., it is characterized by the presence of a broad region in which the picture is stationary, and a narrow region in which Δ varies with time from zero to a certain maximum value.

The dynamics of a superconductor with a gap depends essentially on the ratio of the characteristic frequencies ω and of the wave vectors k, and on the reciprocal energy relaxation time $\gamma \sim T^3/\omega_D^2$. If ω and Dk^2 (D is the diffusion coefficient of the normal electrons) are much less than γ, then it is easy to write down the dynamic equations near the transition temperature. Under these conditions, as shown by Artemenko and Volkov,[17] we can use the approximation of the effective chemical potential Φ. Putting $\tau = (T_c - T)/T$, we write down the equations for the modulus of the order parameter, for the current density, and for the scalar potential:

$$\tau\Delta - \frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2} - \frac{\pi D}{2T}\left(\frac{e}{c}Q\right)^2\Delta - \frac{\pi}{4\gamma T}\Delta\frac{\partial\Delta}{\partial t} = 0, \tag{29a}$$

$$j = -\frac{\pi\sigma\Delta^2}{2cT}Q + \sigma E, \qquad E = -\frac{1}{c}\frac{\partial Q}{\partial t} - \frac{\partial\Phi}{\partial x}, \tag{29b}$$

$$-D\frac{\partial^2\Phi}{\partial x^2} + \frac{\pi\Delta}{4T}\gamma\Phi = \frac{D}{c}\frac{\partial^2 Q}{\partial x\,\partial t}. \tag{29c}$$

Equation (29a) is similar to the equation of Ref. 18. We have here the invariant combinations

$$Q = A - \frac{c}{2e}\frac{\partial\chi}{\partial x}, \qquad \Phi = \varphi + \frac{1}{2e}\frac{\partial\chi}{\partial t},$$

where χ is the phase of the order parameter.

The length over which the longitudinal electric field falls off, as seen from (29c), and the coherence length are given by $l_E^2 \sim DT/\gamma\Delta$ and $\xi^2 \sim D/T\tau$, respectively, and their ratio is

$$l_E^2/\xi^2 = (T^2\tau/\gamma^2)^{1/2} \gg 1. \tag{30}$$

For this ratio to be large, it is necessary to satisfy the inequality $2^{-3/2}\pi[8\pi^2/7\zeta(3)]^{5/4} = a$.

We shall measure next all the quantities in units of

$$\Delta_0^2 = \frac{8\pi^2 T^2\tau}{7\zeta(3)}, \qquad \frac{1}{x_0^2} = \frac{\pi\gamma}{4D}\left(\frac{8\pi^2\tau}{7\zeta(3)}\right)^{1/2},$$

$$\left(\frac{e}{c}Q_0\right)^2 = \frac{2T\tau}{\pi D}, \tag{31}$$

$$j_0 = \frac{\pi\sigma\Delta_0^2 Q_0}{2cT}, \qquad (e\Phi_0)^2 = 2\left(\frac{8\pi^2}{7\zeta(3)}\right)^{3/2}\left(\frac{T^2\tau}{\gamma^2}\right)^{1/2}\tau^2 T^2, \qquad t_0 = \frac{1}{e\Phi_0}.$$

The system (29) can then be written in the form

$$1 - \Delta_1^2 - Q_1^2 = a\left(\frac{l_E}{\xi}\right)^3\frac{\partial\Delta_1}{\partial t_1}, \tag{32a}$$

$$j_1 = -\Delta_1^2 Q_1 - \frac{\partial\Phi_1}{\partial x_1} - b\frac{l_E}{\xi}\frac{\partial Q_1}{\partial t_1}, \tag{32b}$$

$$-\frac{\partial^2\Phi_1}{\partial x_1^2} + \Delta_1\Phi_1 = b\frac{l_E}{\xi}\frac{\partial^2 Q_1}{\partial x_1\,\partial t_1}, \tag{32c}$$

where $2^{-3/2}\pi[8\pi^2/7\zeta(3)]^{5/4} = a$ and $b = 2\pi^{-1}[7\zeta(3)/2\pi^2]^{1/4}$. By virtue of the Josephson relation for the PSC, the reciprocal characteristic time of the problem coincides with $e\Phi_0$, and the necessary condition $\omega \ll \gamma$ for the applicability of (29) is equivalent by virtue of (31) to the inequality $\tau \ll (\gamma/T)^{6/5}$. Therefore, taking (30) into account, we shall consider Eqs. (29) and (32) under the condition

$$(\gamma/T)^2 \ll \tau \ll (\gamma/T)^{6/5} \ll 1. \tag{33}$$

The derivatives with respect to time in (32), just as in (13b), are preceded by anomalously large coefficients. Repeating arguments of the type used in the preceding section, we arrive at an analogous conclusion, that the picture over a distance of the order of l_E from the PSC is in the main static, and over a distance $(\xi l_E)^{1/2}$ this approximation is violated. [At still shorter distances, an essential role is assumed by the derivative with respect to the coordinate in (32a).]

Equations (29) are similar to those investigated numerically by Kramer and Watts-Tobin.[19] They also obtained for these equations oscillating solutions corresponding to PSC.

Having determined the region of applicability of the static approximation, we now write down static equations of somewhat more general form, which are valid in a larger temperature interval that will be established below:

$$\tau - \frac{7\zeta(3)}{8\pi^2 T^2}(\Delta^2 + 2\Phi^2) - \frac{\pi D}{2T}\left(\frac{e}{c}Q\right)^2 = 0, \tag{34a}$$

$$j = -\frac{\pi\sigma\Delta^2 Q}{2cT} + \sigma E, \qquad E = -\frac{\partial\Phi}{\partial x}, \tag{34b}$$

$$-D\frac{\partial^2\Phi}{\partial x^2} + \frac{\pi\Delta}{4T}\gamma\Phi q\left(\frac{4D}{\gamma}\frac{e^2}{c^2}Q^2\right) = 0. \tag{34c}$$

The difference between (34) and (29) is that (34a) contains the term Φ_2, which describes the direct influence of the scalar potential on the order parameter.[6] In addition, account is taken in (34a) of the influence of the superconducting current on the rate of the relaxation of the unbalance of the electron-hole branches[20]:

$$q(z) = 1 + \frac{2z}{\pi}\int_1^\infty \frac{(x^2-1)^{1/2}dx}{x[x(x^2-1)^{1/2}+z]} = \begin{cases} 1+2z/\pi, & z \ll 1 \\ z^{1/2}, & 1 \ll z \ll T^2/\Delta^2 \end{cases}. \tag{35}$$

Changing to new units, which differ from (31) only in the definitions of x_0 and Φ_0,

$$\frac{1}{x_0^2} = \frac{\tau}{D}\left(\frac{4\pi^3\gamma T}{7\zeta(3)}\right)^{1/2}, \quad (e\Phi_0)^2 = \left(\frac{4\pi^2}{7\zeta(3)}\right)^2\left(\frac{7\zeta(3)T\tau^2}{\pi\gamma}\right)^{1/3}T^2\tau, \tag{36}$$

we write down the system (34), retaining the previous notation for all the quantities

$$\Delta^2 + Q^2 + \Phi^2/\Phi_1^2 = 1, \tag{37a}$$

$$j = -\Delta^2 Q + E, \quad E = -\partial\Phi/\partial x, \tag{37b}$$

$$-\partial^2\Phi/\partial x^2 + \Delta\Phi B(Q) = 0, \tag{37c}$$

where

$$B(Q) = \left(\frac{\pi\gamma}{8T\tau}\right)^{1/2} q\left(\frac{8T\tau}{\pi\gamma}Q^2\right), \qquad \Phi_1^2 = \frac{1}{4\pi}\left(\frac{7\zeta(3)\gamma}{\pi T\tau^2}\right)^{1/3}.$$

The second derivative of Δ with respect to the coordinate, which is present in the Ginzburg-Landau equation, turns out to be insignificant within the scale of the problem.

If we neglect the last term in (37a), this system can be easily integrated in analogy with the preceding section:

$$\Phi^2 = g(\Delta), \qquad \frac{l}{2} - x = \int_{\Delta_0}^{\Delta}\frac{(3\Delta^2-2)\,d\Delta}{(1-\Delta^2)^{1/2}[g(\Delta)]^{1/2}B[(1-\Delta^2)^{1/2}]},$$
$$g(\Delta) = 2\int_{\Delta_0}^{\Delta}\frac{[j - p^2(1-p^2)^{1/2}](3p^2-2)\,dp}{(1-p^2)^{1/2}B}. \tag{38}$$

The electric field is determined in this case just as in (19), and the distance between two neighboring PSC is

$$l = 2\int_{\Delta_0}^{1}\frac{(3\Delta^2-2)\,d\Delta}{(1-\Delta^2)^{1/2}[g(\Delta)]^{1/2}B[(1-\Delta^2)^{1/2}]}, \tag{39}$$

where Δ_0 is also the value of the order parameter halfway between two PSC. We then have for the CVC in the customary units

$$\frac{2}{3^{1/2}}\frac{\sigma E}{j_c}\int_{\Delta_0}^{1}\frac{(3\Delta^2-2)\,d\Delta}{(1-\Delta^2)^{1/2}[g(\Delta)]^{1/2}B[(1-\Delta^2)^{1/2}]}=[g(1)]^{1/2}. \quad (40)$$

In the transition to the customary units it is necessary to make the substitution $j\to 2j/3^{3/2}j_c$ also in the function $g(\Delta)$. The condition for the smallness of the discarded term in (37a) is $\Phi_0\ll\Phi_1$. Therefore, taking into account the logarithmic singularity in (38) and the inequality (30), the region of applicability of (40) is

$$\left(\frac{\gamma}{T}\right)^2\ll\tau\ll\left(\frac{\gamma}{T}\right)^{1/2}\Big/\ln\frac{T\tau}{\gamma}. \quad (41)$$

If we go farther down from T_c, then direct suppression of the order parameter by the scalar potential becomes important. The picture of the distribution of Φ becomes the same qualitatively as in Fig. 3, but the maximum value of the potential is now Φ_1, and the order parameter is suppressed to zero, at the assumed accuracy, at the points of the PSC. In this case the electric field is

$$E=2\Phi_1/l. \quad (42)$$

If we move from the midpoint between the PSC, where the potential Φ is still small, then the vector potential Q manages to drop practically to zero over those distances at which the last term in (37a) is still small.

At large distances, suppression of Δ on account of the term Φ^2 sets is in (37a), as shown in Fig. 4. The region in which $Q\sim 1$ has a characteristic scale of the order of unity. Because of this difference between the scales, the distance between the PSC is

$$l=\frac{2\Phi_1}{j}+\frac{2}{j}\int_{\Delta_0}^{1}\frac{(3\Delta^2-2)\Delta^2\,d\Delta}{(1-\Delta^2)^{1/2}[h(\Delta)]^{1/2}},$$
$$h(\Delta)=2\int_{\Delta_0}^{\Delta}\frac{[j-p^2(1-p^2)^{1/2}](3p^2-2)\,dp}{1-p^2}. \quad (43)$$

The CVC is given by Eq. (42). The region of applicability of (42) and (43) is given by the inequality

$$\left(\frac{\gamma}{T}\right)^{1/2}\Big/\ln\frac{T\tau}{\gamma}\ll\tau\ll\left(\frac{\gamma}{T}\right)^{1/2}. \quad (44)$$

This region, of course, is very narrow and we consider it only in order to trace the transition to the regime when the scalar potential influences directly the order parameter.

For the case (41), the CVC family will take the same form as in Fig. 3, and the limiting value $\Delta_0^2=2/3$ corresponds to the uppermost position of the curve. In the principle, hysteresis connected with transitions from

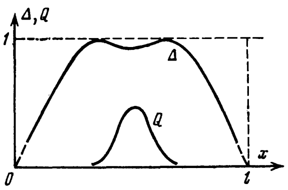

FIG. 4. Coordinate dependence of the modulus of the order parameter and of the gauge-invariant vector potential for the case of direct suppression of the superconductivity by a scalar potential in the temperature region
$$\left(\frac{\gamma}{T}\right)^{1/2}\Big/\ln\frac{T\tau}{\gamma}\ll\tau\ll\left(\frac{\gamma}{T}\right)^{1/2}.$$

one curve to another is possible, but this question calls for a special investigation, at a given current through the sample, the curve with $\Delta_0^2=2/3$ corresponds to the weakest electric field, and consequently, to the smallest entropy production, and we shall therefore consider henceforth only this curve.

We consider now the CVC in different temperature regions:

1. $(\gamma/T)^2\ll\tau\ll\gamma/T$. In this case the influence of the superconducting current on the relaxation of the unbalance of the branches is small.

a) Initial section of the characteristic, $j-j_c\ll j_c$:

$$\frac{j-j_c}{j_c}\sim\exp\left(-0{,}91\frac{j_c}{\sigma E}\right). \quad (45)$$

b) Large currents, $j_c\ll j$. A current in excess of the normal current appears in this case and does not depend on the electric field:

$$j=\sigma E+a_1 j_c,$$
$$a_1=\frac{3^{3/2}}{2\nu_1(1)}\int_{(2/3)^{1/2}}^{1}\frac{\Delta^2(3\Delta^2-2)}{\nu_1(\Delta)}d\Delta\approx 0{,}68,$$
$$\nu_1^2(\Delta)=2\int_{(2/3)^{1/2}}^{\Delta}\frac{(3p^2-2)\,dp}{(1-p^2)^{1/2}}. \quad (46)$$

2. $\gamma/T\ll\tau\ll(\lambda/T)^{1/2}\ln(T\tau/\gamma)$. In this case the influence of the superconducting current on the relaxation of the unbalance of the branches is appreciable. [The value of z in (35) is large.]

a) The initial section of the characteristic, $j-j_c\ll j_c$:

$$j=\sigma E\left\{1+\left(\frac{2}{3}\right)^{1/4}\ln^{-1/2}\frac{T\tau}{\gamma}\ln\frac{j_c}{j-j_c}\right\}. \quad (47)$$

We can separate here two limiting cases. We introduce the quantity

$$K=\exp\left\{-\left(\frac{3}{2}\right)^{1/4}\ln^{1/2}\frac{T\tau}{\gamma}\right\}.$$

The following cases are possible: $(j-j_c)/j_c\ll K$, and then

$$\frac{j-j_c}{j_c}\sim\exp\left\{-\frac{j_c}{\sigma E}\left(\frac{3}{2}\right)^{1/4}\ln^{1/2}\frac{T\tau}{\gamma}\right\} \quad (48)$$

and $K\ll(j-j_c)/j_c\ll 1$. In the latter case

$$j=\sigma E\left\{1+\left(\frac{2}{3}\right)^{1/4}\ln^{-1/2}\frac{T\tau}{\gamma}\ln\frac{j_c}{\sigma E-j_c}\right\}. \quad (49)$$

b) Large currents. Here, too, we have an excess current

$$j=\sigma E+b_1 j_c\Big/\ln^{1/2}\frac{T\tau}{\gamma}, \quad (50)$$

where

$$\lambda^2(\Delta)=2\int_{(2/3)^{1/2}}^{\Delta}\frac{3p^2-2}{1-p^2}dp,$$
$$b_1=\frac{3^{3/2}}{2}\int_{2/3^{1/2}}^{1}\frac{\Delta^2(3\Delta^2-2)\,d\Delta}{(1-\Delta^2)^{1/2}\lambda(\Delta)}=1{,}22.$$

3. $(\gamma/T)^{1/2}/\ln(T\tau/\gamma)\ll\tau\ll(\gamma/T)^{1/2}$. In this temperature region the direct suppression of the superconductivity by the scalar potential becomes significant, and the superconducting current influences strongly, as before, the relaxation of the branch unbalance.

a) Initial section of the characteristic. Here

$$j=\sigma E\left(1+\frac{2^{1/4}}{3\Phi_1}\ln\frac{j_c}{j-j_c}\right). \quad (51)$$

We introduce the quantity $M = \exp(-3\Phi_1/2^{3/4}) \ll 1$. If $(j - j_c)/j_c \ll M$, then

$$\frac{j-j_c}{j_c} \sim \exp\left(-\frac{j_c}{\sigma E}\frac{3\Phi_1}{2^{3/4}}\right). \tag{52}$$

On the other hand, if $M \ll (j - j_c)/j_c \ll 1$, then we have from (51)

$$j=\sigma E\left(1+\frac{2^{3/4}}{3\Phi_1}\ln\frac{j_c}{\sigma E-j_c}\right). \tag{53}$$

b) Large currents, $j_c \ll j$:

$$j=\sigma E+d_1(\sigma E j_c)^{1/2}/\Phi_1, \quad d_1=3^{3/4}\int_{(2/3)^{1/2}}\frac{\Delta^2(3\Delta^2-2)\,d\Delta}{(1-\Delta^2)^{1/2}[\lambda(\Delta)]^{1/2}}\approx 1.07. \tag{54}$$

This formula is valid at $j/j_c \ll \Phi_1^2 \ll \ln(T\tau/\gamma)$. We see therefore that allowance for the term Φ^2 in the Ginzburg-Landau equation for the order parameter leads to a dependence of the excess current on E at large values of the latter. If we disregard this term, then the increase of Φ with increasing current does not lead to suppression of the order parameter and superconducting current remains constant; this in fact is the cause of the constancy of the excess current with increasing voltage.

Equations (45)-(54) cover the temperature interval

$$(\gamma/T)^2 \ll \tau \ll (\gamma/T)^{1/2} \tag{55}$$

and admit of comparison with experiments on thin long superconducting samples, for which the reciprocal electron-phonon relaxation time γ is known.

4. CONCLUSION

It is shown in the present paper that when the model superconductivity equations are used the nonstationary nature of the PSC manifests itself only in a narrow region on the order of $(l_E\xi)^{1/2}$ near the PSC, while at distances of the order of l_E from it the picture is stationary. By the same token, if $l_E \gg \xi$, then the static equations can be used in the space between the PSC. In addition we have tracked the analogy in the dynamics of the PSC of a real superconductor with a gap in the spectrum, and of PSC described by model equations. On this basis we obtained the CVC of a real superconductor. A feature of the obtained CVC is the existence of an excess current (on top of the normal current), which does not depend on the voltage at large values of the latter. With decreasing temperature, the excess current begins to depend on the voltage because of the stronger influence of the scalar potential on the order parameter. A constant excess current was observed experimentally in Ref. 3.

An excess current was also obtained by Rieger, Scalapino, and Mercereau[3] in their model of the resistive state. Tinkham[21] has recently proposed a simple qualitative picture that describes the interaction of several PSC in a quasi-one-dimensional superconductor, in which the CVC has a similar behavior and also has an excess current. In addition, the excess current appears in superconducting filaments of finite length that join two bulky superconductors between which a potential difference is maintained,[22] and in the CVC of point junctions of two superconductors.[23]

Electromagnetic radiation from a sample in the resistive state was observed in Ref. 24. The high degree of monochromaticity of this radiation makes it possible, in principle, to assume it to be caused by nonstationary processes connected with CVC.

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1) We note that at $u = 0$ the threshold solution (7) vanishes and the normal state becomes unstable at not very large currents. This situation corresponds to a certain degree to the case considered in an investigation[13] of a nonequilibrium superconductor in which the coefficient of the first derivative in the equation for the order parameter was equal to zero because peculiar character of the disequilibrium.

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