and a determination of the probability of intervalley scattering and of the presence of a correlation between the number of the electrons before and after reflection with an intervalley transition. The sensitivity of the EF to the crystallographic structure of the surface uncovers new methodological possibilities of the EF. It is of interest in this connection to use EF for the study of "brittle" two-dimensional structures on metallic surfaces. It is known that at helium temperatures and at very small degrees of coating (less than one monolayer) two-dimensional crystalline structures of impurity atoms are produced on the surfaces of metals. In a number of cases they have small defect-formation energies and are destroyed even when bombarded with slow electrons. From the Bragg condition $n\lambda = d\cos\theta$ it follows that to observe the diffraction of the electrons incident on the surface from the outside, the electron wavelength λ should be less than d, and consequently the electron energy $\gtrsim 10$ eV. The use of conduction electrons as a probe in these cases has obvious advantages, inasmuch as in EF the excitation energy of the focused electrons can be of the order of several degrees ($\sim 10^{-4}$ eV).

Interest attaches also to the possibility of determining with the aid of EF the crystallographic structure of the surface under a layer of adsorbed impurity atoms and molecules, whose presence does not interfere with the measurements, since the "irradiation" of the surface takes place "from the inside." Similarly, the EF can be used to determine the crystallographic structure of the interphase boundary.

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Resonant changes of optical orientation of electrons in anti-intersection of the spin levels of the nuclei of a semiconductor lattice

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We investigate the circular polarization of the luminescence of GaAlAs crystals under conditions of optical orientation of the electrons and mixing of the spin states of the lattice nuclei. This mixing is the result of local violation of the crystal cubic symmetry, and depends on the crystal orientation in the external magnetic field. Optical detection of NMR transitions at the triple Larmor frequency without change of the magnetic quantum number ($\Delta m = 0$) demonstrates the mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ of the As^{75} nuclei in a narrow angle range corresponding to anti-intersection of the spin levels. The change of the mixing with change of the crystal orientation in a magnetic field is resonant and leads to the onset of a region with two stable polarization states of the spin system near the anti-intersection point. Optical NMR detection in the anti-intersection region has made it possible to determine the electric-field-gradient asymmetry parameter ($\eta \leq 0.06$).

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Optical orientation of electrons in semiconductors in interband absorption of circularly polarized light is accompanied by dynamic polarization of the nuclei.[1] In turn, the polarized nuclei can exert a strong influence on the behavior of oriented nuclei on account of the hyperfine interaction.[2,3] The effective field H_N of the nuclei, which acts on the average electron spin $\langle \mathbf{S} \rangle$, is determined by various factors. By studying the

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recombination transitions of oriented electrons we can obtain information on the structure of the optically induced hyperfine field in a semiconductor.

In III-V semiconductors, the degree ρ of the circular polarization is equal to the projection $\langle S_z \rangle$ of the average electron spin on the propagation direction of the exciting light (the z axis). The field H_N when added or subtracted from the external field, changes the rate of the magnetic depolarization of the radiation (the Hanle effect). The $\rho(H_x)$ dependence turned out to be a convenient indicator of the optical cooling of the nuclear spin system.[4-6] With decreasing spin temperature of the nuclei, their thermodynamic-equilibrium polarization along the external field is increased.

Interesting features of the hyperfine-field structure manifest themselves in the case of local violation of the cubic symmetry of the crystal.[7,8] This situation arises, for example, in solid solutions based on gallium arsenide, which are in fact our topic here. Thus, when some of the Ga atoms are replaced by Al the cubic symmetry is violated and electric field gradients (EFG) appear. If only one atom adjacent to an As atom is replaced Fig. 1a, the EFG has axial symmetry (if the more remote neighbors are not taken into account). The spin of As^{75} is $I = 3/2$. The quadrupole interaction splits the $\frac{1}{2}$ and $\frac{3}{2}$ levels by an amount $\Delta = \frac{1}{2}e^2 q_{z'z'} Q$, where e is the electron charge $eq_{z'z'}$ is the component of the EFG tensor with principal axis z' along the As–Al bond, and Q is the quadrupole moment of the As^{75} nucleus.

In the case of a magnetic field with arbitrary orientation, for which $\mu_I H \ll \Delta$ (μ_I is the magnetic moment of the As^{75} nucleus), the optical-pumping scheme corresponds to Fig. 1c. The eigenstates

$$|+\rangle = |+^1/_2\rangle \cos\delta + |-^1/_2\rangle \sin\delta, \quad |-\rangle = -|+^1/_2\rangle \sin\delta + |-^1/_2\rangle \cos\delta$$

are superpositions of the states $|+\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$. Mixing is produced under the influence of the external-field component perpendicular to the z' axis of the quadrupole interaction.[8] The quantity δ is determined by the angle θ between the external field and the z' axis; if $\mathbf{H} \| z'$, then $\delta = 0$ and the pure states $|+\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$ are realized.

Electron-hole flipflop transitions in optical orientation of electrons are accompanied by an increase of the

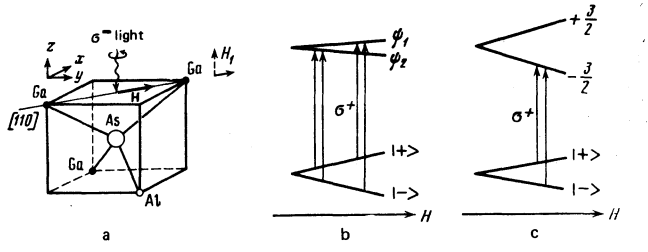

FIG. 1. Experimental setup. a) Dynamic polarization of As^{75} nuclei under conditions of anti-intersection of the spin levels $\mathbf{H} \| [110]$, the As—Al bond is in the (110) plane). b) Transition scheme of the As^{75} nuclei pumped by σ^+ light under anti-intersection conditions (Fig. 1a). c) Transition scheme of As^{75} nuclei pumped by σ^+ light far from the anti-intersection point $\mathbf{H} \perp [110]$.

populations of the $-\frac{3}{2}$ levels in the case when right-circular polarized light (σ^+) is absorbed. Similarly, when left-circular polarized light (σ^-) is absorbed the population of the $+\frac{3}{2}$ levels is increased. Optical pumping of the $\pm\frac{3}{2}$ levels leads to the appearance of a component $\mathbf{H}_{Qz_i'}$ of the hyperfine field $\mathbf{H}_N$, directed along the z_i' axis.[8] Since the substituting Al atom can occupy one of four equivalent positions in the unit cell, there are four preferred quadrupole-interaction axes that coincide with the threefold axes.

Dynamic polarization of the nuclei in the considered crystals is produced mainly because of their contact interaction with electrons localized on shallow donor centers. The field $\mathbf{H}_{Qz_i'}$ is then determined by the combined action of all those As^{75} nuclei within the limits of the Bohr radius of the donor, for which the As–Al bond lies along the z_i' axis.

Experiment and calculations show that the change of the populations of the + and – levels under optical pumping plays a relatively minor role. The resultant nuclear field $\mathbf{H}_Q$ produced by quadrupole interaction at the electrons can then be represented in the form

$$\mathbf{H}_Q \approx \sum_{i=1}^{4} \mathbf{H}_{Qz_i'} = a \sum_{i=1} (\langle \mathbf{S} \rangle \mathbf{n}_i) \mathbf{n}_i = a \langle \mathbf{S} \rangle, \tag{1}$$

where $\mathbf{n}_i$ is a unit vector along the z_i' axis, and a is a coefficient that depends on the external magnetic field.

The magnetic depolarization of the electrons under conditions of optical orientation is described by the Bloch equation

$$d\langle \mathbf{S} \rangle / dt = \langle \mathbf{S} \rangle - \mathbf{S}_0 T_s / \tau - \gamma_e T_s [\langle \mathbf{S} \rangle, (\mathbf{H} + \mathcal{H}_N)], \tag{2}$$

where $\mathbf{S}_0$ is the average spin of the electrons at the instant of the excitations, and T_s, τ, and γ_e are the lifetime of the orientation and the lifetime and gyromagnetic ratio of the electrons; $\mathcal{H}_N$ is the instantaneous value of the field of the nuclei.

It follows from (2) that the field $\mathbf{H}_Q = a\langle \mathbf{S} \rangle$ that enters in $\mathcal{H}_N$ should not manifest itself in experiments on magnetic depolarization of nuclei. However, if the equivalence of the optical polarization of the nuclei along the different threefold axes is violated, then the vector $\mathbf{H}_Q$ does not coincide with $\langle \mathbf{S} \rangle$ in direction and alters the electron-spin precession rate in accordance with (2).

The fact that the dynamic polarization of the nuclei as a result of quadrupole effect is not the same along the different axes z_i' is the result of the magnetic anisotropic mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$. This effect is particularly strongly pronounced when the direction of the external magnetic field coincides with the normal to one or two axes of the quadrupole interaction ($\theta = \pi/2$).

In the zeroth approximation in the parameter $\mu_I H/\Delta$ the levels $\pm\frac{3}{2}$ are randomly degenerate. This degeneracy is lifted if account is taken of the interaction due to the admixture to the states $\chi_{\pm 3/2}$ of the states $|\pm\frac{1}{2}\rangle$. The presence of this admixture is unequivocally confirmed by NMR observations at the triple Larmor

frequency of the As^{75} nuclei[7,10,11]

$$f_i = 3\gamma_I H \cos\theta_i.$$

The alternating magnetic field H_1 causes transitions between the $\pm\frac{3}{2}$ levels and equalizes the population difference produced by the light. The change of the corresponding component of the hyperfine field $H_{Qz_i'}$ leads to deviation of the vector H_Q from the direction $\langle S\rangle$, a fact easily detected optically by the change of the luminescence polarization.

The admixture of the $|\pm\frac{1}{2}\rangle$ states to the $\chi_{\pm 3/2}$ states can be due either to the H component normal to the z' axis, in the higher-order approximations in the parameter $\mu_I H/\Delta$, or to the EFG non-axiality due to the substitution of Ge by Al in the third and more remote coordination spheres. In the former case the states $\chi_{+3/2}$ and $\chi_{-3/2}$ are mixed respectively with the states $|+\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$, and in the latter with $|-\frac{1}{2}\rangle$ and $|+\frac{1}{2}\rangle$.

Our optical detection of NMR in a longitudinal field (3 to 18 Oe) has shown that the mixing of states in weak fields is due to nonaxiality of the EFG. The experiments were performed with n-type $Al_{0.24}Ga_{0.76}As$ crystals at 4.2 K.

The presence of $\mp\frac{1}{2}$ admixtures to the $\pm\frac{3}{2}$ levels gives rise to an interaction between levels, which lifts the random degeneracy at the point $\theta=\pi/2$ in an external magnetic field, and anti-crossing of the levels sets in.[8] When account is taken of the anti-crossing, the Zeeman energies of the $\pm\frac{3}{2}$ levels are determined for an arbitrary angle θ by the expression

$$\lambda_{\pm 3/2} = \pm[(\tfrac{3}{2}\mu_I H\cos\theta)^2 + |V|^2]^{1/2}, \qquad (3)$$

where $|V|$ is the matrix element of the interaction with the magnetic field between the states $\pm\frac{3}{2}$. At $\theta\approx\pi/2$ we have $2|V| = 3\mu_I H\eta\sin\theta$, where η is the EFG asymmetry parameter.

The wave functions ψ_1 and ψ_2 of the stationary states $|+\frac{3}{2}\rangle$ and $|-\frac{3}{2}\rangle$ are of the form

$$\psi_1 = \chi_{+1/2}\cos\alpha + \chi_{-1/2}\sin\alpha, \quad \psi_2 = -\chi_{+1/2}\sin\alpha + \chi_{-1/2}\cos\alpha. \qquad (4)$$

Here $\chi_{\pm 3/2}$ are the wave functions of the pure states and

$$\cos 2\alpha = \tfrac{3}{2}\mu_I H\cos\theta_i/|\lambda_{3/2}|. \qquad (5)$$

The relative contribution of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ is determined by the equation

$$\operatorname{tg}\alpha = -(\tfrac{3}{2}\mu_I H\cos\theta_i - [(\tfrac{3}{2}\mu_I H\cos\theta_i)^2 + |V|^2]^{1/2})/|V|. \qquad (6)$$

Expression (6) has a resonant character. At $\theta_i \to \pi/2$, $\tan\alpha \to 1$, and in the range of angles θ_i for which $|V| \ll \frac{3}{2}\mu_I H\cos\theta_i$, the states ψ_1 and ψ_2 turn out to be practically pure $|\pm\frac{3}{2}\rangle$.

Thus, complete mixing of the $\pm\frac{3}{2}$ states takes place in the anti-intersection point at $\theta_i = \pi/2$. The wave functions ψ_1 and ψ_2 are the half-sum and half-difference of the pure states $|\pm\frac{3}{2}\rangle$. It is shown on the level scheme of Fig. 1b that near the quasi-intersection point, owing to the mixing of the $\pm\frac{3}{2}$ levels under the influence of σ^+ light, transition take place with population of levels corresponding both to the ψ_1 states and to the ψ_2 states, with the field $H_{Qz_i'} \to 0$.

Figure 1a correspond to an experimental geometry in which half of the As^{75} nuclei with one substituted neighboring atom are under conditions of anti-intersection of the spin levels. This takes place in the case when the field H is directed along the [110] axis and is perpendicular to the plane that includes two quadrupole-interaction axes.

Under anti-intersection conditions, characteristic features are observed in the behavior of polarized luminescence, namely, a strong anisotropy appears and regions appear with two stable states or undamped polarization oscillations.[7]

Figures 2b and 2c show the changes in the width of the hysteresis loop on the magnetic-depolarization curve as functions of the angle φ between the vector H and the [110] axis in the (001) plane. The measurements were made with an $Al_{0.24}Ga_{0.76}As$ crystal excited along a fourfold axis at 77 K. The degree ρ of the circular polarization of the luminescence was determined by the method of two-channel counting of the "left-hand" and "right-hand" photons.[12] As seen from Fig. 2c, which shows the angular dependence of the width ΔH of the region with two stable states, a non-single-valued dependence of ρ on H is observed only in a relatively narrow range of angles near the anti-intersection point ($\varphi=0$, $\theta_{1,2}=\pi/2$).

The shift of the maximum of the plot of ΔH relative to the point $\varphi=0$ is attributed to the influence of the field H_e of the optically oriented electrons on the nuclei. Since the vector $H_e \sim \langle S\rangle$ is inclined to the (001) plane, a true anti-intersection should be observed in a certain inclined external field H at which the vector $H + H_e$ is perpendicular to the z_i' axis. Nonetheless, the data in Fig. 2c illustrates the resonant character of the manifestation of the hysteresis phenomena when

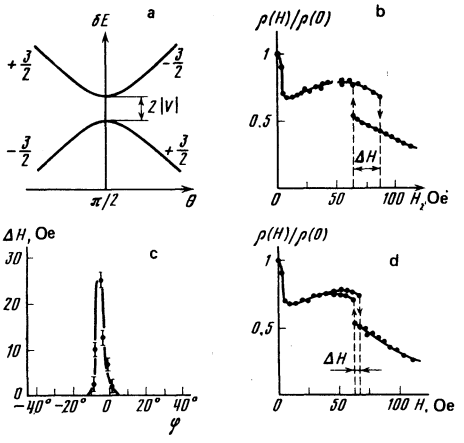

FIG. 2. Magnetic depolarization of luminescence in the region of anti-intersection of the spin levels. a) Angular dependence of the distribution of the $\pm\frac{3}{2}$ levels in a magnetic field. b, d) Depolarization of the luminescence in a transverse magnetic field at $\varphi=3°$ (b) and $\varphi=5.5°$ (d). c) Dependence of the width of the region with two stable states on φ.

the states $\chi_{+3/2}$ and $\chi_{-3/2}$ in the scheme of the quadrupole-split spin levels of Fig. 2a are mixed.

Thus, by varying the orientation of the crystal in the magnetic field, we can equalize the populations of the $\pm\frac{3}{2}$ levels and effect a situation similar to the saturation of an NMR line at triple the Larmor frequency. This allows us to call the observed effect an orientational resonance or resonance at zero frequency.

The strong anisotropy of the optical orientation of the electrons under conditions of anti-intersection of nuclear spin levels makes possible sensitive registration of NMR at low frequency. Thus, partial mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ near the anti-intersection decreases the corresponding components H_{Qzi} of the nuclear field.

By applying an alternating magnetic field of resonant frequency we can additionally decrease these components, equalizing the number of the As^{75} nuclei in the states ψ_1 and ψ_2. It has turned out that this is accompanied by a change in the condition for the stability of the bound electron-nuclear spin system.

Figure 3 illustrates the changes of the states of the polarization of the optically oriented spin system of a semiconductor acted upon by an alternating low-frequency magnetic field. Figure 3a shows schematically the $A \rightarrow B$ transition between two stable polarization states, while Fig. 3b shows automatically recorded plots of such transitions in a constant field $H = 57$ Oe at alternating-field frequencies f equal to 7 kHz (curve 1) and 12 kHz (curve 2). Figure 3d illustrates the dependence of transition times t_{tr} on f. This dependence has a resonant character with a minimum at 7 kHz. The level splitting (see Fig. 2a) due to the non-axiality of the EFG at the anti-intersection point, amounts to $3\mu_I H\eta$. A resonant frequency kHz corresponds to an estimate $\eta \leqslant 2\pi f_i/3\gamma_I H \approx 0.06$.

When the angle φ is increased from 0°, the region

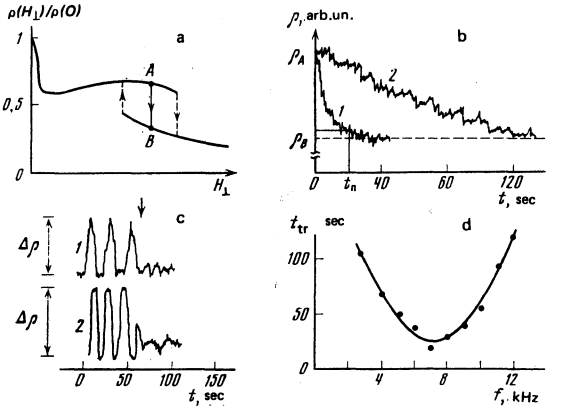

FIG. 3. Optical detection of NMR by means of the change of the states of the spin system near the level anti-intersection region. a) Transition between two stable states under NMR conditions. b) Dependence of the polarization on the time after turning on an alternating field of frequency 7 kHz (1) and 12 kHz (2). c) Termination of the self-oscillations when H_{1h} is turned on at the instant of time marked by the arrow. d) Dependence of the time of the $A \rightarrow B$ transition on the alternating-field frequency.

with two stable states goes over into a region where the spin-system polarization is unstable and self-oscillations of this polarization are observed.[7] Inasmuch as the region of the undamped oscillations corresponds to smaller angles θ, the mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ in this region is smaller than in the hysteresis region, and the field H_Q is larger. By decreasing this field under NMR conditions it is possible to take the spin system out of the self-oscillating regime, as illustrated in Fig. 3c for the two angles $\varphi = 5°$ and $\varphi = 6°$. When an alternating field is turned on at the instants of time marked by the arrows, the undamped polarization oscillations end. Thus, NMR can be registered near the region of anti-intersection of the spin levels of nuclei by observing the qualitative change that takes place in the state of the spin system and manifests itself in large changes in the degree of circular polarization of the luminescence of the semiconductor.

Finally, direct proof of the appearance of the mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ can be obtained with the aid of NMR in a longitudinal alternating field. The transitions that correspond to the selection rule $\Delta m = 0$ are produced between the states ψ_1 and ψ_2 on account of the mixing of $\chi_{+3/2}$ and $\chi_{-3/2}$ in each of these states (the $\pm\frac{3}{2} \rightarrow \pm\frac{3}{2}$). Obviously, the closer the anti-intersection point, the larger the probability of a transition with the selection rule $\Delta m = 0$ compared with the transitions $\Delta m = \pm 1$ in an alternating field perpendicular to the quadrupole interaction axis.

The matrix element M of the transition between the states ψ_1 and ψ_2 under the influence of a field H_{1z}, parallel to the quadrupole axis is of the form

$$M(\theta_i) = -\frac{3}{2}\frac{\operatorname{tg}\alpha}{1+\operatorname{tg}^2\alpha} H_{1z_i}. \qquad (7)$$

The value of $\tan\alpha$ varies at resonance with the angle θ_i in accord with Eq. (6), and consequently the NMR intensity, which is proportional to $M^2(\theta_i)$, increases resonantly as the anti-intersection point is approached. It is convenient to observe this effect by comparing the results of the change of the luminescence polarization under the influence of a longitudinal and a transverse alternating field.

Figure 4 shows the dependence, on the angle φ, of the ratio H_{1h}/H_{1z} of the amplitudes of the alternating

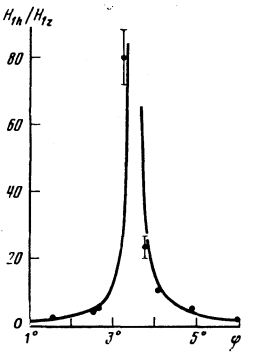

FIG. 4. Relative effectiveness of $\Delta m = 0$ transitions near the anti-intersection point (dependence of the amplitude ratio H_{1h}/H_{1z} on the angle φ).

fields that are applied along the constant field H and along the z axis and lead to the same result when their frequency corresponds to the transition $\psi_1 \rightleftarrows \psi_2$. The constant field H is close to the direction of the [110] axis. The alternating low-frequency equalizes the populations of the spin levels of the As^{75} nuclei, for which the z_i' axes coincide with the axes $[1\bar{1}1]$ and $[\bar{1}11]$ of the crystal. The alternating field H_{1h} gives rise to transitions $\psi_1 \rightleftarrows \psi_2$ corresponding to $\Delta m = \pm 1$, on account of the admixture of the states $|\mp\frac{1}{2}\rangle$ to the states $\chi_{\pm 3/2}$, as a result of non-axiality of the EFG. Those components of the field H_{1z} which are parallel to the z_i' axes give rise to $\Delta m = 0$ transitions if the states $\chi_{+3/2}$ and $\chi_{-3/2}$ are mixed.

As seen from Fig. 4, in a narrow region of angles φ the field H_{1z} is much more effective than the field H_{1h}, this being proof of the mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$. The shift of the maximum on the curve of Fig. 4 relative to the point $\varphi = 0$ is determined by the field of the electrons.

The mixing of the states $\chi_{+3/2}$ and $\chi_{-3/2}$ at an orientation H along the [110] axis turns off part of the field of the As^{75} nuclei with one substituted neighboring atom. The direction of the summary nuclear field due to the quadrupole interaction differs in this case from the direction of the vector $\langle \mathbf{S} \rangle$. A consistent allowance for the effect of mixing of the spin wave function is made in the theory of D'yakonov, Merkulov, and Perel', who explained the sharp magnetic anisotropy and the onset of the ambiguity of the polarization following optical orientation as being due to the influence of the level anti-crossing.

The described experiments confirm the validity of the premises and of the conclusions of this theory.

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The hydrodynamics of two- and one-dimensional liquids

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We evaluate the (first and second) viscosity coefficients and the thermal conductivity coefficient of an arbitrary two-dimensional (nonsuperfluid) liquid. The kinetic coefficients are mainly produced by the contribution of the long-wavelength thermal fluctuations and can be expressed completely in terms of thermodynamic functions. We find a thermodynamic inequality, the violation of which must lead to a singularity in the kinetic coefficients. We evaluate the sound absorption coefficient in a one-dimensional liquid.

PACS numbers: 51.10. + y, 65.50. + m, 66.60. + a

The existence of long-wavelength weakly damped thermal fluctuations (of sound waves, and also of entropy and viscous waves) in liquids causes the occurrence of a number of non-local kinetic effects.[1-9] In particular, power-law "tails" appear in the expressions for the damping in space or time of such excitations which are usually characterized by exponential damping laws.

In the literature, a large of amount of special attention has been paid to a study of the power-law decrease in time of the auto-correlation function of the particle velocity in a liquid, which was detected by Alder and Wainwright[6] as the result of a numerical calculation, and which, as was shown in Refs. 2 to 5, is caused by long-wavelength fluctuations. This law is so slow that there appear (see Refs. 2, 4, 5, 7) formal divergences of the coefficients in the Barnett correction terms to the Navier-Stokes equation (in the three-dimensional case) and of the viscosity coefficients (in the two- and one-dimensional cases).

The present author[1,9] has shown that the system of equations which describes the dynamic properties of a

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