Quantum gravitational effects in an isotropic universe

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Effects of vacuum polarization by an external gravitational field in a system of spinor, scalar, and massive vector particles are considered in the model of conformally flat space-time. Expressions are obtained for the radiative corrections, and the part they play in gravitational theory is analyzed in the limits of weak ($|R_i^k| \ll m^2$) and strong ($|R_i^k| \gg m^2$) gravitational fields. The Appendix gives expressions for the rate of spontaneous production of real particles by a strong gravitational field and the energy-momentum tensor of real particles in the ultrarelativistic limit; a law of increase of the entropy of an ultrarelativistic medium is also formulated.

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INTRODUCTION

The theory of quantum gravitational phenomena has by now been developed in some detail. The theory is based on a classification of quantum effects, in which one distinguishes vacuum polarization, production of real particles, and their interaction with a self-consistent gravitational field[1-8]; the treatment is based on Einstein's equations, which are assumed to hold right down to curvature values $R \sim l_{pl}^{-2} = 10^{66}$ cm^{-2} (Ref. 9). However, it has been conjectured[10] that the quantum behavior of matter will lead to a significant modification of the classical theory of gravitation already at Compton curvatures. In the present paper, this conjecture is discussed on the basis of expressions for the radiative corrections.

In §1, we derive the basic equations describing the isotropic model of the universe, the matter behaving as a mixture of ideal gases, which corresponds to modern ideas about the interactions of elementary particles.[11] In §2, we derive and analyze the radiative corrections of second order to Einstein's equations. The radiative corrections of third order are obtained in §3. Here, after studying the general structure of the perturbation theory series, we consider the conjecture that the theory of gravitation is nonlocal in nature. §4 is devoted to the alternative (local) approach to the theory of strong fields, and we discuss the gravitational Lagrangian of the classical theory and establish an expression for the quantum corrections to it. In the Appendix, we consider the spontaneous production of real particles by a strong gravitational field, formulate a law of increase of the entropy of an ultrarelativistic medium, and obtain expressions for the energy–momentum tensor of real particles in the ultrarelativistic limit.

§ 1. BASIC EQUATIONS

We restrict ourselves to a model which describes a mixture of ideal gases of scalar, spinor, and vector particles in a homogeneous isotropic space with self-consistent conformally flat metric of the form

$$g_{ik} = a^2(t) g_{ik}^{(0)}, \qquad g_{ik}^{(0)} = \mathrm{diag}(1, -1, -1, -1). \tag{1.1}$$

The total Lagrangian of the system is

$$\mathscr{L} = -\frac{1}{2\varkappa_0} R + \mathscr{L}_{(0)} + \mathscr{L}_{(1/2)} + \mathscr{L}_{(1)}, \tag{1.2}$$

where

$$\mathscr{L}_{(0)} = \varphi_{;l}^{+}\varphi^{;l} + (R/6 - m_{(0)}^2)\varphi^{+}\varphi,$$
$$\mathscr{L}_{(1/2)} = {}^1/_2 i(\bar\psi\gamma^l\psi_{;l} - \bar\psi_{;l}\gamma^l\psi) - m_{(1/2)}\bar\psi\psi,$$
$$\mathscr{L}_{(1)} = {}^1/_2\psi_{ik}^{+}\psi^{ik} - {}^1/_2\psi^{+ik}(\psi_{k;i} - \psi_{i;k}) - {}^1/_2\psi^{ik}(\psi_{k;i}^{+} - \psi_{i;k}^{+}) + m_{(1)}^2\psi_k\psi^k,$$

where $\varkappa_0$ is the unrenormalized gravitational constant, and the subscript in brackets indicates the spin of the field. We shall not go through the well-known procedure for obtaining from (1.2) the field equations and the energy–momentum tensor and the transition to the momentum representation of the field operators,[3] but rather write down directly the complete system of equations of the model with allowance for the choice of the metric (1.1) (where no confusion is possible, the spin index j will be omitted for brevity):

$$D_{\mathbf{p}\lambda} = \langle a_{\mathbf{p}\lambda}^{+} a_{\mathbf{p}\lambda} + b_{-\mathbf{p}-\lambda}^{+} b_{-\mathbf{p}-\lambda} \rangle,$$
$$D_{\mathbf{p}\lambda}(t) = D_{\mathbf{p}\lambda}(-\infty) + 4\int_{-\infty}^{t} dt'\, W_{\mathbf{p}\lambda}(t') \int_{-\infty}^{t'} dt''\, W_{\mathbf{p}\lambda}(t'') \times (1 \pm D_{\mathbf{p}\lambda}(t'')) \cos\left(2\int_{t''}^{t'} \omega_{\mathbf{p}}(\xi)\, d\xi\right), \tag{1.3}$$

$$\frac{1}{\varkappa_0}\left(R_0^0 - \frac{1}{2}R\right) = \frac{3}{\varkappa_0}\frac{\dot a^2}{a^4} = \frac{1}{a^4}\sum_j \hat T_0^0 = \frac{1}{a^4}\mathscr{H} = \sum_j \sum_{\mathbf{p}\lambda} \omega_{\mathbf{p}}(D_{\mathbf{p}\lambda} \pm 1),$$
$$-\frac{1}{\varkappa_0}R = \frac{6}{\varkappa_0}\frac{\ddot a}{a^3} = \frac{1}{a^4}\sum_j \hat T = \frac{1}{a^4}\sum_j\sum_{\mathbf{p}\lambda}\left[\frac{\mu^2}{\omega_{\mathbf{p}}^2}(D_{\mathbf{p}\lambda} \pm 1) - \frac{\omega_{\mathbf{p}}}{\dot a/a}\dot D_{\mathbf{p}\lambda}\right]. \tag{1.4}$$

Here, $a_{\mathbf{p}\lambda}$ and $b_{\mathbf{p}\lambda}$ are, respectively, the operators of annihilation of particles and antiparticles, the brackets $\langle \ldots \rangle$ denote averaging with respect to the density matrix, the dot denotes differentiation with respect to the time t, and the upper and lower signs correspond to bosons and fermions, respectively; finally, by $\hat T$ and $\hat T_0^0$ we denote the components of the energy–momentum tensor after separation of the conformal factor a^{-4}. Information about the individual properties of the particles is contained in the "masses" μ, "frequencies" $\omega_{\mathbf{p}}$, and the kernels $W_{\mathbf{p}\lambda}$:

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$$j=0,\ {}^1/_2,\ 1;\quad \mu_{(j)}=m_{(j)}a,\quad \omega^2_{p(j)}=p^2+\mu^2_{(j)},$$

$$W_{p\lambda(j)}=\frac{1}{2}\frac{\dot a}{a}V_{p\lambda(j)};$$

$$j=0:\quad V_p=\mu^2/\omega_p^2;$$

$$j={}^1/_2:\quad V_{p(\pm^1/_2)}=p\mu/\omega_p^2;$$

$$j=1:\quad V_{p(0)}=-(p^2+\omega_p^2)/\omega_p^2,\quad V_{p(\pm1)}=\mu^2/\omega_p^2.$$

Equations (1.3)–(1.4) form a complete system that describes quantum gravitational phenomena in an isotropic universe.

Assuming that the metric is weakly nonstationary, i.e., that

$$\frac{W_{p\lambda}^2}{\omega_p^2}\ll1,\quad \frac{1}{\omega_p^n}\left|\frac{1}{W_{p\lambda}}\frac{d^nW_{p\lambda}}{dt^n}\right|\ll1,\quad \frac{1}{\omega_p^n}\left|\frac{1}{\omega_p}\frac{d^n\omega_p}{dt^n}\right|\ll1,\quad n=1,2,\ldots,\tag{1.5}$$

we can construct a solution to Eq. (1.3) in the form of a perturbation series. The propagator can be represented in the form

$$D_{p\lambda}(t)=D_{p\lambda}^{(pol)}(t)+D_{p\lambda}^{(real)}(t).\tag{1.6}$$

In (1.6), $D_{p\lambda}^{(pol)}(t)$ is the local part of the propagator, and it vanishes in flat space-time and, therefore, describes polarization of the physical vacuum by the gravitational field; $D_{p\lambda}^{(real)}(t)$ is the distribution function of the real particles with allowance for the effects of their interaction with the self-consistent field and pair production.

The calculation of $D_{p\lambda}^{(pol)}$ up to terms of third order inclusively in perturbation theory gives

$$\begin{aligned}D_{p\lambda}^{(pol)}(t)=&\frac{1}{2}\frac{W_{p\lambda}^2}{\omega_p^2}+\frac{1}{4\omega_p^4}\Big[\pm\frac{3}{2}W_{p\lambda}^4+\frac{1}{2}\dot W_{p\lambda}^2\\&+W_{p\lambda}^2\Big(\frac{\ddot\omega_p}{\omega_p}-\frac{5}{2}\frac{\dot\omega_p^2}{\omega_p^2}\Big)+2W_{p\lambda}\dot W_{p\lambda}\frac{\dot\omega_p}{\omega_p}-\ddot W_{p\lambda}W_{p\lambda}\Big]\\&+\frac{1}{16\omega_p^6}\Big[W_{p\lambda}^{(4)}W_{p\lambda}-9\frac{\dot\omega_p}{\omega_p}\dddot W_{p\lambda}W_{p\lambda}+\Big(42\frac{\dot\omega_p^2}{\omega_p^2}-11\frac{\ddot\omega_p}{\omega_p}\Big)\ddot W_{p\lambda}W_{p\lambda}\\&+\Big(-4\frac{\dddot\omega_p}{\omega_p}+49\frac{\ddot\omega_p\dot\omega_p}{\omega_p^2}-84\frac{\dot\omega_p^3}{\omega_p^3}\Big)\dot W_{p\lambda}W_{p\lambda}\\&+\Big(14\frac{\dddot\omega_p\dot\omega_p}{\omega_p^2}-\frac{\omega_p^{(4)}}{\omega_p}+\frac{21}{2}\frac{\ddot\omega_p^2}{\omega_p^2}-98\frac{\ddot\omega_p\dot\omega_p^2}{\omega_p^3}+\frac{189}{2}\frac{\dot\omega_p^4}{\omega_p^4}\Big)W_{p\lambda}^2\\&-\dddot W_{p\lambda}\dot W_{p\lambda}+\frac{1}{2}\ddot W_{p\lambda}^2+3\ddot W_{p\lambda}\dot W_{p\lambda}\frac{\dot\omega_p}{\omega_p}+\Big(4\frac{\ddot\omega_p}{\omega_p}-\frac{21}{2}\frac{\dot\omega_p^2}{\omega_p^2}\Big)\dot W_{p\lambda}^2\\&\pm\Big(-10W_{p\lambda}^3\ddot W_{p\lambda}-5W_{p\lambda}^2\dot W_{p\lambda}^2+40\frac{\dot\omega_p}{\omega_p}\dot W_{p\lambda}W_{p\lambda}^3\Big)\\&\pm\Big(10\frac{\ddot\omega_p}{\omega_p}-35\frac{\dot\omega_p^2}{\omega_p^2}\Big)W_{p\lambda}^4+5W_{p\lambda}^6\Big].\end{aligned}\tag{1.7}$$

The result (1.7) will be used in §2 and §3 to calculate the radiative corrections to Einstein's equations. The effect of particle production will be discussed in the Appendix.

From the representation of the propagator $D_{p\lambda}$ in the form (1.6) there follows an analogous representation for the energy–momentum tensor:

$$T_i^k=T_i^{k(pol)}+T_i^{k(real)},$$

the polarization part $T_i^{k(pol)}$ depending only on the metric. It is therefore natural to write Einstein's equation in the form

$$\frac{1}{\varkappa_0}\Big(R_i^k-\frac{1}{2}\delta_i^kR\Big)+\Pi_i^k=T_i^{k(real)},\quad \Pi_i^k\equiv-T_i^{k(pol)}\tag{1.8}$$

and interpret the tensor Π_i^k as the quantum radiative corrections to the equations of the classical theory of gravitation.

§2. RADIATIVE CORRECTIONS OF SECOND ORDER AND THE PRINCIPLE OF RENORMALIZABILITY

An asymptotic expansion of $T_i^{k(pol)}$ in powers of the curvature can be obtained by assuming that the inequalities (1.5) hold for all p, including p=0. It is readily seen that the condition of applicability of the asymptotic expansion is

$$|R_i^k|/m^2\ll1.$$

Substituting the expansion (1.7) in the expressions for $\hat{\mathcal{H}}$ and $\hat T$ (1.4) and restricting ourselves to the second order of perturbation theory, we obtain

$$\mathscr{H}^{(0)}_{(pol)}=\mathscr{H}_{(vac)}=\pm\sum_{p\lambda}\omega_p,\quad T^{(0)}_{(pol)}=T_{(vac)}=\pm\sum_{p\lambda}\frac{\mu^2}{\omega_p},\tag{2.1}$$

$$\mathscr{H}^{(1)}_{(pol)}=\frac{1}{2}\sum_{p\lambda}\frac{W_{p\lambda}^2}{\omega_p},$$

$$T^{(1)}_{(pol)}=\frac{1}{2}\sum_{p\lambda}\frac{1}{\omega_p}\Big(\frac{\mu^2}{\omega_p}W_{p\lambda}^2+\dot W_{p\lambda}V_{p\lambda}-\frac{\dot\omega_p}{\omega_p}W_{p\lambda}V_{p\lambda}\Big),\tag{2.2}$$

$$\begin{aligned}\mathscr{H}^{(2)}_{(pol)}=&\frac{1}{4}\sum_{p\lambda}\frac{1}{\omega_p^3}\Big[\frac{1}{2}\dot W_{p\lambda}^2+W_{p\lambda}^2\Big(\frac{\ddot\omega_p}{\omega_p}-\frac{5}{2}\frac{\dot\omega_p^2}{\omega_p^2}\Big)\\&+2\frac{\dot\omega_p}{\omega_p}W_{p\lambda}\dot W_{p\lambda}-\ddot W_{p\lambda}W_{p\lambda}\pm\frac{3}{2}W_{p\lambda}^4\Big],\\T^{(2)}_{(pol)}=&\frac{1}{4}\sum_{p\lambda}\Big\{\frac{\mu^2}{\omega_p^5}\Big[\frac{1}{2}\dot W_{p\lambda}^2+W_{p\lambda}^2\Big(\frac{\ddot\omega_p}{\omega_p}-\frac{5}{2}\frac{\dot\omega_p^2}{\omega_p^2}\Big)\\&+2\frac{\dot\omega_p}{\omega_p}W_{p\lambda}\dot W_{p\lambda}-\ddot W_{p\lambda}W_{p\lambda}\pm\frac{3}{2}W_{p\lambda}^4\Big]\\&\pm\frac{1}{\omega_p^3}W_{p\lambda}^2V_{p\lambda}\Big(\dot W_{p\lambda}-\frac{\dot\omega_p}{\omega_p}W_{p\lambda}\Big)\Big\}.\end{aligned}\tag{2.3}$$

The zeroth term of the expansion (2.1) is the energy–momentum tensor of the undeformed vacuum. The expressions (2.2) and (2.3) also contain divergent integrals and require renormalization. A natural procedure is to subtract from $T^k_{i(pol)}$ the energy–momentum tensor of the undeformed vacuum, this last including additional counterterms whose form is readily established in each concrete case. Such a renormalization amounts to a shift in the origin for the energy–momentum tensor. However, only power divergences can be eliminated from $T^k_{i(pol)}$ by such a method. No shift of the origin can eliminate from (2.2) and (2.3) the logarithmic terms, but there are no physical grounds for other renormalization methods. Cutting off the logarithmically diverging integrals at some limiting momentum p_0, we obtain in the framework of the adopted model the following results.

$j=0$:

$$\mathscr{H}^{(reg)}_{(pol)}=\frac{m^2}{8\pi^2}\dot a^2+\frac{1}{240\pi^2}\Big(-\frac{\dddot a\dot a}{a^2}+\frac{1}{2}\Big(\frac{\ddot a}{a}\Big)^2+2\frac{\ddot a\dot a^2}{a^3}\Big)-\frac{1}{480\pi^2}\frac{\dot a^4}{a^4},$$

$$\begin{aligned}\bar T^{(reg)}_{(pol)}=&\frac{m^2}{4\pi^2}a\ddot a+\frac{1}{240\pi^2}\Big(-\frac{a^{(4)}}{a}+4\frac{\dddot a\dot a}{a^2}+3\Big(\frac{\ddot a}{a}\Big)^2-6\frac{\ddot a\dot a^2}{a^3}\Big)\\&-\frac{1}{120\pi^2}\frac{\dot a^2}{a^2}\Big(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\Big).\end{aligned}\tag{2.4}$$

$j=\frac{1}{2}$:

$$\mathscr{H}^{(reg)}_{(pol)}=\frac{m^2}{8\pi^2}\dot{a}^2\left(\ln\frac{2p_0}{ma}-\frac{4}{3}\right)$$

$$+\frac{1}{80\pi^2}\left(-\frac{\dddot{a}\dot{a}}{a^2}+\frac{1}{2}\left(\frac{\ddot{a}}{a}\right)^2+2\frac{\ddot{a}\dot{a}^2}{a^3}\right)-\frac{11}{960\pi^2}\frac{\dot{a}^4}{a^4},$$

$$\hat{T}^{(reg)}_{(pol)}=\frac{m^2}{4\pi^2}a\ddot{a}\left(\ln\frac{2p_0}{ma}-\frac{4}{3}\right)-\dot{a}^2\frac{m^2}{8\pi^2}+\frac{1}{80\pi^2}\left(-\frac{a^{(4)}}{a}+4\frac{\dddot{a}\dot{a}}{a^2}\right.$$

$$\left.+3\left(\frac{\ddot{a}}{a}\right)^2-6\frac{\ddot{a}\dot{a}^2}{a^3}\right)-\frac{11}{240\pi^2}\frac{\dot{a}^2}{a^2}\left(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\right); \qquad (2.5)$$

$j=1$:

$$\mathscr{H}^{(reg)}_{(pol)}=-\frac{3m^2}{8\pi^4}\dot{a}^2\left(\ln\frac{2p_0}{ma}-1\right)$$

$$+\frac{1}{8\pi^2}\left[\left(\ln\frac{2p_0}{ma}-\frac{37}{30}\right)\left(-\frac{\dddot{a}\dot{a}}{a^2}+\frac{1}{2}\left(\frac{\ddot{a}}{a}\right)^2+2\frac{\ddot{a}\dot{a}^2}{a^3}\right)+\frac{\ddot{a}\dot{a}^2}{a^3}-\frac{21}{20}\frac{\dot{a}^4}{a^4}\right]$$

$$\hat{T}^{(reg)}_{(pol)}=-\frac{3m^2}{4\pi^2}a\ddot{a}\left(\ln\frac{2p_0}{ma}-1\right)+\dot{a}^2\frac{3m^2}{8\pi^2}$$

$$+\frac{1}{8\pi^2}\left[\left(\ln\frac{2p_0}{ma}-\frac{37}{30}\right)\left(-\frac{a^{(4)}}{a}+4\frac{\dddot{a}\dot{a}}{a^2}+3\left(\frac{\ddot{a}}{a}\right)^2-6\frac{\ddot{a}\dot{a}^2}{a^3}\right)\right.$$

$$\left.+2\frac{\dddot{a}\dot{a}}{a^2}+\frac{3}{2}\left(\frac{\ddot{a}}{a}\right)^2-5\frac{\ddot{a}\dot{a}^2}{a^3}-\frac{21}{5}\frac{\dot{a}^2}{a^2}\left(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\right)\right]. \qquad (2.6)$$

The conservation conditions $T^k_{i(pol);k}=0$ are satisfied identically for (2.4)–(2.6) in each perturbation order.

The limiting conformal momentum p_0 is related to the physical momentum by $P_0=p_0/a$, and it is therefore not a renormalization constant; if we set $P_0=\text{const}$, we come into conflict with the Bianchi identities. We can extricate ourselves from the resulting dilemma by calculating and transforming the Lagrangians corresponding to (2.4)–(2.6). Using the formula

$$\delta\hat{\mathscr{L}}_{(j)}=-\hat{T}_{(j)}\frac{\delta a}{a}, \qquad (2.7)$$

and making some simple transformations, we obtain for the considered system of fields

$$\hat{\mathscr{L}}^{(reg)}_{(pol)}=\frac{1}{2\pi^2}\left[-6a\ddot{a}\sum_j\sum_l^{N_j}m_{jl}^2\left(q_j^{(1)}\ln\frac{2p_0}{m_{jl}a}+\alpha_j^{(1)}\right)\right.$$

$$\left.+36\left(\frac{\ddot{a}}{a}\right)^2\sum_j\sum_l^{N_j}\left(q_j^{(2)}\ln\frac{2p_0}{m_{jl}a}+\alpha_j^{(2)}\right)-\frac{\dot{a}^4}{a^4}\sum_j N_jZ_j^{(2)}\right], \qquad (2.8)$$

where

$$j=0:\quad q^{(1)}=q^{(2)}=0,\ \alpha^{(1)}=\frac{1}{24},\ \alpha^{(2)}=\frac{1}{8640},\ Z^{(2)}=\frac{1}{2880};$$

$$j=1/2:\quad q^{(1)}=\frac{1}{24},\ q^{(2)}=0,\ \alpha^{(1)}=-\frac{1}{18},\ \alpha^{(2)}=\frac{1}{2880},\ Z^{(2)}=\frac{11}{1440}; \qquad (2.9)$$

$$j=1:\quad q^{(1)}=-\frac{1}{8},\ q^{(2)}=\frac{1}{288},\ \alpha^{(1)}=\frac{1}{8},\ \alpha^{(2)}=-\frac{37}{8640},\ Z^{(2)}=\frac{7}{80};$$

here, N_j is the number of species of particle with spin j in the system, and m_{jl} is the rest mass of the particle with spin j of species l.

In (2.8), we go over to the physical momentum. In addition, we use the arbitrariness in the choice of the cosmological time and make the transformation

$$t'=t+\xi\frac{\dot{a}}{a^3} \qquad (2.10)$$

where ξ is a small parameter. Making the change of variable (2.10) in (2.8) to first order in ξ, omitting in the Lagrangian the total derivatives with respect to the time, and choosing ξ in the form

$$\xi=-\frac{1}{12}\left(\sum_j N_jZ_j^{(2)}\right)\left[\sum_j\sum_l^{N_j}m_{jl}^2\left(q_j^{(1)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(1)}\right)\right]^{-1},$$

we find the final physical Lagrangian of the radiative corrections:

$$\hat{\mathscr{L}}^{(phys)}_{(pol)}=\frac{1}{2\pi^2}\sum_j\sum_l^{N_j}\left[-6a\ddot{a}m_{jl}^2\left(q_j^{(1)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(1)}\right)\right.$$

$$\left.+36\left(\frac{\ddot{a}}{a}\right)^2\left(q_j^{(2)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(2)}\right)\right]. \qquad (2.11)$$

The result (2.11), and also the tensor Π_i^k, can now be represented in a covariant four-dimensional form:

$$\mathscr{L}^{(phys)}_{(pol)}=\mathscr{L}^{(1)(phys)}_{(pol)}+\mathscr{L}^{(2)(phys)}_{(pol)}$$

$$=\frac{1}{2\pi^2}\left[R\sum_j\sum_l^{N_j}m_{jl}^2\left(q_j^{(1)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(1)}\right)\right.$$

$$\left.+R^2\sum_j\sum_l^{N_j}\left(q_j^{(2)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(2)}\right)\right], \qquad (2.12)$$

$$\Pi^{(1)k}_i+\Pi^{(2)k}_i=-\left(R_i^k-\frac{1}{2}\delta_i^kR\right)\sum_{j,l}\frac{m_{jl}^2}{\pi^2}\left(q_j^{(1)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(1)}\right)$$

$$+\left(R^{;k}_{;i}-\delta_iR^{;l}_{;l}-RR_i^k+\frac{1}{4}\delta_iR^2\right)\sum_{j,l}\frac{2}{\pi^2}\left(q_j^{(2)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(2)}\right). \qquad (2.13)$$

Note that the expressions (2.12) and (2.13) are not the most general; namely, (2.12) does not contain the quadratic invariant of the Weyl tensor, which vanishes in a conformally flat world.

Let us now discuss the results. From (2.12), we readily conclude that the effect of the first-order radiative corrections reduces to a renormalization of the bare gravitational constant $\varkappa_0$, and the observed value $\varkappa$ of the gravitational constant is related to $\varkappa_0$ by

$$\varkappa^{-1}=\varkappa_0^{-1}-\sum_{j,l}\frac{m_{jl}^2}{\pi^2}\left(q_j^{(1)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(1)}\right). \qquad (2.14)$$

The quantity $\Delta(1/\varkappa)=1/\varkappa-1/\varkappa_0$ is the correction to the linear elasticity of the vacuum due to the polarization effects.[12] Thus, allowance for the first-order radiative corrections does not give significantly new physical information.

Allowance for the second-order corrections renders the theory nonrenormalizable in the sense that there does not exist a finite limit $\lim\mathcal{L}^{(2)(phys)}_{(pol)}$ as $P_0\to\infty$. The formal similarity of the divergences in $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$ suggests that the effect of $\mathcal{L}^{(2)(phys)}_{(pol)}$ also consists of renormalization of some new physical constant.[13,14] Introducing in the unrenormalized gravitational Lagrangian a term quadratic in the curvature,

$$\mathscr{L}_g^{(0)}=-\frac{1}{2\varkappa_0}R-\frac{1}{2\nu_0}R^2, \qquad (2.15)$$

we obtain after renormalization

$$\nu^{-1}=\nu_0^{-1}-\sum_{j,l}\frac{1}{\pi^2}\left(q_j^{(2)}\ln\frac{P_0}{m_{jl}}+\alpha_j^{(2)}\right). \qquad (2.16)$$

Thus, when we treat the divergent radiative corrections of second order from the point of view of the renormalizability principle the order of the equations of the theory of gravitation is raised from the second to the fourth. This principle evidently also justifies the appearance of two new non-Einstein solutions in the theory of gravitation with quadratic invariants.

§3. RADIATIVE CORRECTIONS OF HIGHER ORDERS

In the higher orders of perturbation theory, none of the terms in $T^k_{i(pol)}$ contains divergences, and their calculation does not lead to any fundamental difficulties. Omitting the simple but fairly lengthy calculations, we give the final expression for the Lagrangian of the radiative corrections of third order directly in covariant four-dimensional form:

$$\mathscr{L}^{(3)}_{(pol)}=\mathscr{L}^{(3)(phys)}_{(pol)}=\frac{1}{2\pi^2m^2}(\nu^{(3)}R^{;l}R_{;l}+\alpha^{(3)}R^3+\beta^{(3)}RR^k_iR^i_k+\gamma^{(3)}R^l_iR^k_lR^i_k), \tag{3.1}$$

where

$$j=0:\ \nu^{(3)}=-\frac{1}{30240},\ \alpha^{(3)}=-\frac{1}{272160},\ \beta^{(3)}=-\frac{1}{30240},\ \gamma^{(3)}=\frac{1}{7560};$$

$$j=1/2:\ \nu^{(3)}=-\frac{1}{161280},\ \alpha^{(3)}=\frac{613813}{1660538880},\ \beta^{(3)}=-\frac{616493}{332107776}.$$

$$\gamma^{(3)}=\frac{12293}{27675648};$$

$$j=1:\ \nu^{(3)}=-\frac{1}{2280},\ \alpha^{(3)}=-\frac{2827}{8640},\ \beta^{(3)}=\frac{121}{216},\ \gamma^{(3)}=-\frac{67}{120}.$$

The tensor Π^k_i of the radiative corrections corresponding to the Lagrangian (3.1) has the form

$$\begin{aligned}\Pi^{(3)k}_i\equiv-T^{(3)k}_{i(pol)}=-\frac{1}{\pi^2m^2}\{&\nu^{(3)}(R_{;i}R^{;k}-{}^1\!/_2\,\delta^k_iR_{;l}R^{;l}-2R^k_iR^{;l}{}_{;l}\\&+2R^{;l}{}_{;l}{}^{;k}{}_{;i}-2\delta^k_iR^{;l}{}_{;l}{}^{;m}{}_{;m})+\alpha^{(3)}[3R^2R^k_i-{}^1\!/_2\,\delta^k_iR^3-3(R^2)^{;k}{}_{;i}\\&+3\delta^k_i(R^2)^{;l}{}_{;l}]+\beta^{(3)}[R^m_lR^l_mR^k_i+2RR^l_iR^k_l-{}^1\!/_2\,\delta^kRR^mR^l_m-(R^l_mR^m_l)^{;k}{}_{;i}\\&-(RR^k_l)_{;i}{}^{;l}-(RR^l_i)^{;k}{}_{;l}+\delta^k_i(R^l_mR^m_l)^{;q}{}_{;q}+(RR^k_i)^{;l}{}_{;l}+\delta^k_i(RR^m_l)^{;l}{}_{;m}]\\&+\gamma^{(3)}[3R^m_lR^k_mR^l_i-{}^1\!/_2\,\delta^k_iR^m_lR^q_mR^l_q-{}^3\!/_2(R^l_mR^m_i)^{;k}{}_{;l}-{}^3\!/_2(R^k_mR^m_l)_{;i}{}^{;l}\\&+{}^3\!/_2(R^m_iR^k_m)^{;l}{}_{;l}+{}^3\!/_2\,\delta^k_i(R^l_mR^m_q)^{;q}{}_{;l}]\}.\end{aligned} \tag{3.2}$$

The expression (3.2) makes it possible to discuss the part played by the radiative corrections of higher order in the theory of gravitation. The terms in (3.1) cubic in the curvature introduce additional nonlinearities in (3.2) and, therefore, in the equations of the theory (1.8). As in the case of the second-order radiative corrections, these terms must be regarded as the effects of the nonlinear reaction of the vacuum to the gravitational field which deforms it. In this sense, analysis of the terms in (3.1) cubic in the curvature cannot yield qualitatively new physical information; allowance for them when $|R^k|\ll m^2$ only leads to a more precise quantitative determination of effects that are already contained in the corrections to the gravitational Lagrangian that are quadratic in the curvature.

A particular part in (3.1) is played by the term with $\nu^{(3)}$, which arises because of the four derivatives of the curvature in (3.2), which corresponds to the presence in Eqs. (2.1) of six derivatives of the metric tensor and therefore, the existence of two further new solutions, whose physical interpretation presents a serious problem. Analayis of the radiative corrections of higher orders shows that the solution to the problem of the new solutions can in principle be found only by studying the structure of the complete perturbation series. In fact, one can show that the Lagrangian of the radiative corrections of n-th order has the form

$$\mathscr{L}^{(n)}=\sum_{i=1}^{n}\xi^{(n)}_i\mathscr{L}^{(n)}_i,$$

with

$$\mathscr{L}^{(n)}_1\sim\frac{R^n}{m^{2n-4}}, \tag{3.3.1}$$

$$\mathscr{L}^{(n)}_2\sim\frac{R^{n-4}}{m^{2n-4}}R^{;l}{}_{;l}, \tag{3.3.2}$$

$$\mathscr{L}^{(n)}_3\sim\frac{R^{n-6}}{m^{2n-4}}R^{;l}R_{;m}R_{;l}{}^{;m}, \tag{3.3.3}$$

$$\mathscr{L}^{(n)}_n\sim\frac{1}{m^{2n-4}}R^{;l_1;l_2\ldots;l_{n-2}}R_{;l_1;l_2\ldots;l_{n-2}}. \tag{3.3n}$$

Terms of the type (3.3.1) have the same physical meaning as the terms in (3.1) cubic in the curvature, and therefore allowance for them does not give rise to any objections when $|R^k_i|\ll m^2$ in any perturbation order. Allowance for the terms (3.3) results in an increase in the order and, accordingly, the number of solutions of Eqs. (2.1) to $2n$. The new solutions introduced into the theory of gravitation by the radiative corrections of higher orders are not physical—there is no field-theoretical principle which calls for their inclusion in the region of small curvatures. It is possible that the appearance in Π^k_i of the terms with the higher derivatives is due to an incorrectness of the perturbation theory that is employed, and then the corresponding terms should simply be ignored; but if one adheres to the opposite point of view, an approach to the interpretation of the obtained results can be based on nonlocal quantum field theory.[15]

Indeed, we can show that an inifinite series of terms of the type (3.3n) can in principle be combined into the Lagrangian of a nonlocal field theory. For this, let us consider the action functional

$$S^{(pol)}_2=\int d^4xd^4x'\sqrt{-g(x)}\sqrt{-g(x')}\,K_2(x,x')\mathscr{L}^{(pol)}_2(x,x'), \tag{3.4}$$

where $K_2(x,x')$ is a form factor and $\mathscr{L}^{(pol)}_2(x,x')=\alpha_2R(x)R(x')$. Suppose that the functions $K_2(x,x')$ has a fairly sharp maximum on the surface $x-x'$ and can therefore be expanded in a series in the even derivatives of the covariant delta function $D(x-x')$:

$$K_2(x,x')=\frac{1}{\sqrt{-g(x)}}[D(x-x')+l_0^2D(x-x')^{;l}{}_{;l}+l_0^4D(x-x')^{;l}{}_{;l}{}^{;m}{}_{;m}+\ldots] \tag{3.5}$$

[in (3.5), the covariant derivative is taken with respect to the argument of the D function]. Substituting the series (3.5) in (3.4), we obtain a series for $S^{(pol)}_2$, which for $l_0\sim1/m$ has the same structure as the series

$$S_2=\sum_{l=1}^{\infty}\xi^{(l)}_l\int d^4x\sqrt{-g(x)}\mathscr{L}^{(l)}_l(x),$$

which is obtained by summing expressions of the type (3.3.n). Thus, for a suitable choice of the kernel $K_2(x,x')$, the Lagrangian (3.4) will contain radiative corrections that are quadratic in the curvature but not with derivatives of all orders. Similarly, the radiative corrections of N-th power in the curvature with derivatives of any order can be combined in the nonlocal Lagrangian $\mathcal{L}_N^{(pol)}(x,x')\sim R(x)R^{N-1}(x')/m^{2N-1}$. The action functional for such a Lagrangian has the form

$$S_N^{(pol)}=\int d^4x\,d^4x'\sqrt{-g(x)}\sqrt{-g(x')}K_N(x,x')\mathcal{L}_N^{(pol)}(x,x'),\qquad(3.6)$$

and the total action for the radiative corrections is obtained naturally by summing the expressions (3.6):

$$S^{(pol)}=\sum_{N=2}^{\infty}S_N^{(pol)}.$$

Thus, the results of calculating the radiative corrections can in principle be represented in the form of the expansion of a nonlocal action functional, and one may therefore conjecture that the true theory of gravitation is nonlocal, the nonlocality becoming important at $|R_i^k|\sim m^2=l_c^{-2}$, where l_c is the Compton wavelength of a particle of mass m. It is clear that in this treatment of the radiative corrections with higher derivatives the problem of interpreting the new solutions does not arise. On the other hand, radiative corrections of the type (3.3.1), which do not contain derivatives of the curvature, are evidently the terms of an expansion in an asymptotic series in the parameter $\eta=|R_i^k|/m^2$ of some, in general, nonpolynomial local Lagrangian that takes into account exactly the nonlinear reaction of the physical vacuum to the gravitational field which deforms it. When $|R_i^k|\sim l_c^{-2}$, we have $\eta\sim 1$, and the perturbation theory of the standard Minkowski-space physical vacuum by the gravitational field becomes inapplicable; it is therefore possible that in strong gravitational fields there is a radical rearrangement of the physical vacuum the description of which requires a significant modification of the basic propositions of the theory. Such considerations suggest that the limit of applicability of Einstein's classical theory of gravitation occurs at Compton curvatures.

§4. QUANTUM CORRECTIONS TO THE GRAVITATIONAL EQUATIONS IN STRONG GRAVITATIONAL FIELDS

In §3, after our discussion of the structure of the perturbation series for the radiative corrections, we conjectured that the higher derivatives in this series could reflect a nonlocal nature of the theory of gravitation at Compton curvatures. It is well known that a precedent for such a interpretation was set in quantum electrodynamics (see Ref. 15). However, in quantum electrodynamics an alternative view is nevertheless dominant, namely, the theory can remain local even at very high field intensities, its limits of applicability being established by an estimate of the quantitative contribution of the quantum corrections to the field equations. Clearly in the theory of gravitation we must also consider both possibilities. In this connection, let us suppose that the fundamental principles of quantum field theory are valid at curvatures significantly exceeding the Compton curvatures ($|R_i^k|\gg m^2$), and let us calculate the quantum corrections to the equations of the theory of gravitation in this region.

The formulation of the problem described above concerning the calculation of the quantum corrections to the field equations is due to Zel'dovich and Starobinskii.[1] They established the order of magnitude of the leading quantum terms by an analysis of the effect of vacuum polarization by an anisotropic field. Our aim in the present paper is to obtain covariant expressions *for the* quantum corrections that arise in conformal fields.

Before we present the results of the actual calculations, let us briefly discuss the question of the conformal invariance of physical fields. We support the widely accepted view that all physical fields are of conformal type. The equations of motion and energy-momentum tensors obtained from the Lagrangians (1.2) correspond to such fields; after the transformation

$$g_{ik}=\tilde{g}_{ik}e^{2\sigma},\quad \varphi=\tilde{\varphi},\quad \psi=\tilde{\psi}e^{-3\sigma},\quad \gamma=\tilde{\gamma}^i e^{-\sigma},\quad \psi_i=\tilde{\psi}_i$$

the equations of motion for the field variable $\tilde{\varphi}, \tilde{\psi}, \tilde{\psi}_i$ preserve their form if m is replaced by $\mu=me^{\sigma}$. The tensors have the same property: $T_i^{\,k}=\tilde{T}_i^{\,k}e^{-4\sigma}$, where $\tilde{T}_i^k$ can be expressed in terms of the transformed field variables in the same way as T_i^k in terms of the original variables. It is clear from the above considerations that all corrections from scalar particles, fermions, and the transverse components of a vector field will be proportional to m^2 and negligibly small[1)] when $m^2\ll|R_i^k|$. With regard to the longitudinal component of a vector field, the property of "quasiconformality" (i.e., conformal invariance up to replacement of m by μ) of the vector field does not imply that the quantum gravitational effects in conformally flat space-time are proportional to m^2; in this respect, particles with spin $j=1$ differ from scalar particles and fermions with $j=\frac{1}{2}$. The reason for such a difference is that in the theory of boson fields ($j\geqslant 1$) there is no passage to the limit with respect to the mass; for if $m\equiv 0$, the particles have only two physical (contributing to observable quantities) polarizations, whereas for $m\neq 0$ the number of such polarizations is $2j+1$. Therefore, when $m\neq 0$ the behavior of the new degrees of freedom must be studied by investigating the equations of motion. For a vector field, we can obtain the answer to our problem in general form by investigating the four-identity

$$(\mu^2\tilde{\psi}^i)_{;i}=0,\qquad(4.1)$$

which is contained in the equations of motion of the vector field [see (1.2)]

$$\tilde{\psi}^{ik}{}_{;k}-\mu^2\tilde{\psi}^i=0,\qquad \tilde{\psi}_{ik}=\tilde{\psi}_{k;i}-\tilde{\psi}_{i;k}.$$

For $m\neq 0$, the expression (4.1) takes the conformally noninvariant form

$$\tilde{\psi}^i{}_{;i}+2\sigma_{;i}\tilde{\psi}^i=0,\qquad(4.2)$$

a violation of the conformal invariance being contained in terms that do not depend on the rest mass at all. This violation of the conformal invariance does not affect the transverse field components, since for them (4.2) is satisfied identically. Quite different is the situation for the longitudinal component, in which case (4.2) fixes a connection between the longitudinal and time components of the four-vector $\tilde{\psi}_i$. This condition is used essentially in the construction of solutions to the field equations. Therefore longitudinally polarized vector particles lead in the expressions for the radiative corrections to conformally noninvariant terms that asymptotically do not depend on the rest mass. In the case of a strong gravitational field ($|R_i^k| \gg m^2$), it will be these terms that are of the greatest interest.

To obtain the structure of the series of the radiative corrections for $|R_i^k| \gg m^2$, we use a method of calculation that differs somewhat from the one presented in §§1–3. This method does not require a decomposition of the field operator into positive and negative frequency parts. In the Friedmann metric (1.1) and after the conformal transformation $e^\sigma = a(t)$, transition to the (3+1)-dimensional form of expression $\tilde{\psi}_i = (\phi, \psi_\alpha)$, Fourier transformation

$$\phi=\sum_p \phi_p e^{ipx}, \quad \psi_\alpha=\sum_p \psi_{\alpha p} e^{ipx},$$

and expansion of the spatial part of the field operator with respect to a local orthonormal basis in **p** space,

$$\psi_{\alpha p}=\sum_{\sigma=0,\pm1} e_{\alpha\sigma}(\mathbf{p})\psi_{p\sigma}, \quad e_{\alpha\sigma}{}^*(\mathbf{p}) e_{\sigma'}{}^\alpha(\mathbf{p})=\delta_{\sigma\sigma'},$$

$$p^\alpha e_{\alpha(\pm1)}=0, \quad p^\alpha e_{\alpha(0)}=|\mathbf{p}|,$$

the system of field equations for the longitudinal–time component takes the form[2)]

$$\mathscr{H}_i=\frac{1}{a^4}\sum_p \omega_p{}^2\langle u_p{}^+u_p+v_p{}^+v_p\rangle, \quad T_i=\frac{2}{a^4}\sum_p \langle \omega_p{}^2 v_p{}^+v_p-p^2u_p{}^+u_p\rangle,$$
$$\ddot{u}_p+\Omega_p{}^2u_p=0, \quad \omega_p v_p=-i(\mu u_p)^{\cdot}/\mu, \tag{4.3}$$
$$u_p=\frac{\mu\phi_p}{p}, \quad v_p=\frac{\mu\psi_{p(0)}}{\omega_p}, \quad \Omega_p{}^2=p^2+\mu^2+\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}.$$

In (4.3), the averaging is performed with respect to a state vector defined for $t=-\infty$, where space is assumed to be flat. Using the wave equation for u_p, we can readily construct an equation for $G_p=\langle u_p{}^+u_p\rangle$:

$$\dddot{G}_p+4\Omega_p{}^2\dot{G}_p+4\Omega_p\dot{\Omega}_pG_p=0 \tag{4.4}$$

and express the observable quantities in terms of G_p:

$$\mathscr{H}_i=\frac{1}{a^4}\sum_p\left[\left(\omega_p{}^2+\Omega_p{}^2+\frac{\dot{a}^2}{a^2}\right)G_p+\frac{\dot{a}}{a}\dot{G}_p+\frac{1}{2}\ddot{G}_p\right],$$
$$T_i=\frac{2}{a^4}\sum_p\left[\left(\Omega_p{}^2-p^2+\frac{\dot{a}^2}{a^2}\right)G_p+\frac{\dot{a}}{a}\dot{G}_p+\frac{1}{2}\ddot{G}_p\right]. \tag{4.5}$$

One of the solutions of (4.4) is an asymptotically local series[3)] in the parameter $\xi=\dot{\Omega}_p/\Omega_p{}^2$:

$$G_p(t)=\frac{(G_p\Omega_p)_{t=-\infty}}{\Omega_p}\left[1+\frac{1}{4\Omega_p{}^2}\left(\frac{\ddot{\Omega}_p}{\Omega_p}-\frac{3}{2}\frac{\dot{\Omega}_p{}^2}{\Omega_p{}^2}\right)+\ldots\right].$$

For $t=-\infty$, there corresponds to flat space

$$(G_p\Omega_p)_{t=-\infty}={}^1/_2(N_p(-\infty)+\bar{N}_{-p}(-\infty)+1),$$

where $N_p(-\infty)$ and $\bar{N}_{-p}(-\infty)$ are the numbers of particles and antiparticles at $t=-\infty$. Separating from $G_p(t)$ the polarization part $G_p^{(pol)}(t)$, to which there corresponds $(G_p\Omega_p)^{(vac)}_{t=-\infty}=\frac{1}{2}$, substituting $G_p^{(pol)}(t)$ in (4.5), and using formula (2.7), we obtain

$$\mathscr{L}_{1(pol)}=\frac{1}{a^4}\sum_p\left[\frac{1}{2\Omega_p}\left(\frac{\dot{a}^2}{a^2}-m_1{}^2a^2\right)+\frac{1}{2\Omega_p{}^3}\left(-\frac{1}{8}m_1{}^4a^4\right.\right.$$
$$\left.\left.+m_1{}^2\left(\frac{3}{4}\dot{a}^2+\frac{1}{2}a\ddot{a}\right)-\frac{1}{4}\frac{\ddot{a}\dot{a}^2}{a^3}+\frac{1}{8}\left(\frac{\ddot{a}}{a}\right)^2\right)\right].$$

After integration over the p space and regularization of the power divergences, the expression for $\mathfrak{L}_{1(pol)}$ takes the form

$$\mathscr{L}_{1(pol)}=\frac{1}{2\pi^2a^4}\left[\left(\frac{1}{16}\left(\frac{\ddot{a}}{a}\right)^2+\frac{3}{8}m_1{}^2a\ddot{a}\right)\ln\left(p_0{}^2\Big/\left|\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}\right|\right)\right.$$
$$\left.-\frac{1}{16}m_1{}^4a^4\ln\left(\left|\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}\right|\Big/m_1a^2\right)\right]. \tag{4.6}$$

Representing the result (4.6) in covariant form and taking into account the logarithmically diverging contribution to the Lagrangian from the fermions [the results of §2 for the terms containing $\ln(p_0/ma)$ are also valid for the case of a strong field], we obtain the Lagrangian of the radiative corrections in the case of a strong field, $|R_i^k| \gg m^2$:

$$\mathscr{L}^{(phys)}_{(pol)}=\mathscr{L}'+\mathscr{L}_{(q)},$$

where

$$\mathscr{L}'=\frac{1}{2\pi^2}\left[R\sum_{jl}m_{jl}{}^2q_j^{(1)}\ln\frac{P_0}{m_{jl}}+R^2\sum_{jl}q_j^{(2)}\ln\frac{P_0}{m_{jl}}\right], \tag{4.7}$$

$$\mathscr{L}_{(q)}=-\frac{1}{2\pi^2}\sum_l^{N_1}\frac{1}{4}[q_1{}^{(2)}R^2+q_1{}^{(1)}m_{1l}{}^2R+q_1{}^{(0)}m_{1l}{}^4]\ln\frac{4R_i^kR_k^i-R^2}{m_{1l}{}^4}, \tag{4.8}$$

in which $\mathfrak{L}'$ is the polynomial divergent part of the Lagrangian of the radiative corrections, $\mathfrak{L}_{(q)}$ is the non-polynomial quantum correction, P_0 is the physical limiting momentum, the numerical coefficients $q_j^{(1)}$ and $q_j^{(2)}$ are determined by formula (2.9), and $q_1^{(0)}=1/8$.

Our calculation shows that $\mathfrak{L}'$, the logarithmically divergent part of the Lagrangian of the radiative corrections, is equal to the divergent part of the Lagrangian of the radiative corrections calculated for the weak field case.[4)] The regularization of this part of the Lagrangian of the radiative corrections, which leads to a Lagrangian of the classical gravitational field of the form[5)]

$$\mathscr{L}_{(c)}=-\frac{1}{2\varkappa}R-\frac{1}{2\nu}R^2, \tag{4.9}$$

has already been discussed in §2 [see (2.14)–(2.16)]. The total Lagrangian of a strong gravitational field ($|R_i^k| \gg m^2$) is the sum of the classical Lagrangian (4.9) and the quantum radiative correction (4.8):

$$\mathscr{L}_{(g)}=\mathscr{L}_{(c)}+\mathscr{L}_{(q)}. \tag{4.10}$$

In accordance with (4.9), one of the problems of the theory of a conformal field consists of establishing the value of the dimensionless coefficient ν^{-1} in front of the quadratic invariant in the classical gravitational Lagrangian.

It is interesting to note that in the strong field equations there occurs the term Λ, this depending weakly (logarithmically) on the curvature:

$$\Lambda(R_i^k)=\frac{1}{\pi^2}\sum_l^{N_1}q_1{}^{(0)}m_{1l}{}^4\ln\frac{4R_i^kR_k^i-R^2}{m^4}. \tag{4.11}$$

For the terms in the quantum corrections that are linear and quadratic in the curvature one can propose the following interpretation: they lead to a finite renormalization of the gravitational constant and the constant ν:

$$\varkappa(R_i^k)=\varkappa\left[1+\frac{\varkappa}{4\pi^2}\sum_l^{N_1}q_1{}^{(1)}m_{1l}{}^2\ln\frac{4R_i^kR_k^i-R^2}{m_{1l}{}^4}\right]^{-1}, \tag{4.12}$$

$$\nu(R_i^k)=\nu\left[1+\frac{\nu}{4\pi^2}q_i^{(2)}\sum_i^{N_i}\ln\frac{4R_iR_k-R^2}{m_{1i}^4}\right]^{-1}. \qquad (4.13)$$

Using (4.11)–(4.13), we can represent the Lagrangian (4.10) of a strong gravitational field in the form

$$\mathscr{L}_{(g)}=-\frac{1}{2\varkappa(R_i^k)}R-\frac{1}{2\nu(R_i^k)}R^2-\frac{\Lambda(R_i^k)}{2}. \qquad (4.14)$$

It must however be emphasized that the dependences $\varkappa(R_i^k)$, $\nu(R_i^k)$, $\Lambda(R_i^k)$, noted above hold only in the asymptotic region $|R_i^k|\gg m^2$; in the case of a weak field, $|R_i^k|\ll m^2$, it follows from the results of §§2 and 3 that $\varkappa$, $\nu=\text{const}$ and $\Lambda=0$. Indeed, the calculations show that the term $\Lambda(R_i^k)$ arises from a divergent correction of the form

$$\frac{1}{32\pi^2}m_i^4\ln\left\{p_0^2\left|m_i^2a^2+\frac{\ddot a}{a}-2\frac{\dot a^2}{a^2}\right|^{-1}\right\}. \qquad (4.15)$$

It is readily seen that after regularization of the divergences by the subtractional procedure the term (4.15) gives in a weak field ($|R_i^k|\ll m^2$) finite power corrections in the curvature, which are taken into account in $\alpha^{(1)}$ and $\alpha^{(2)}$ [see (2.8)], while in a strong field ($|R_i^k|\gg m^2$) it gives the term $\Lambda(R_i^k)$.

The equations obtained from (4.11)–(4.14) can be used to analyze the early stages in the evolution of an isotropic universe with allowance for quantum effects. (Having this in mind, we give in the Appendix the energy–momentum tensor of matter with allowance for the production of particles and their interaction with self-consistent conformal fields.)

In §3, we have noted the alternative approach to the problems of the theory based on the conjecture that the gravitational interaction is nonlocal. In such an approach, it is at present impossible to draw any quantitative conclusions except that the effects of nonlocality, if they really do exist, must be mainfested at Compton curvatures.

We are grateful to A. A. Starobinskiĭ for valuable comments.

APPENDIX

Production of real particles by a strong gravitational field; energy-momentum tensor of an ultrarelativeistic medium

The problem of the production of real particles reduces to finding the function $N_{p\lambda}(t)$, which is related to the propagator by

$$D_{p\lambda}(t)\pm1=(N_{p\lambda}(t)\pm1)F_{p\lambda}(t).$$

[The solution (1.7) corresponds to $N_{p\lambda}(t)=0$.] From Eq. (1.3), we can obtain an integral equation for $N_{p\lambda}(t)$ (Ref. 3):

$$dN_{p\lambda}(t)/dt=W_{p\lambda}(t)\,⨍_{-\infty}^{t} W_{p\lambda}(t')L_{p\lambda}(t,t')dt', \qquad (A.1)$$

where $L_{p\lambda}(t,t')$ is the function of the reaction of the physical vacuum and the particles to the external gravitational field,

$$L_{p\lambda}(t,t')=L_{p\lambda}^{vac}(t,t')+L_{p\lambda}^{mat}(t,t'),$$
$$L_{p\lambda}^{vac}(t,t')=2F_{p\lambda}(t')\cos\left(2\int_{t'}^{t}\omega_p(\xi)d\xi\right), \qquad (A.2)$$
$$L_{p\lambda}^{mat}(t,t')=4N_{p\lambda}(t')F_{p\lambda}(t')\cos\left(2\int_{t'}^{t}\omega_p(\xi)d\xi\right).$$

The symbol $⨍$ indicates that all local terms which cancel at the end of the calculations[6)] are eliminated from the integral. As follows from (1) and (2), the production of real particles depends strongly on the type of statistics. For Bose particles, the spontaneous effect is added to the stimulated effect, while in the case of Fermi particles the negative sign in front of $L_{p\lambda}^{mat}$ demonstrates the effect of the Pauli exclusion principle.

Investigation of Eq. (A.1) in the weak field case does not give physically interesting results, since in this region the rate of production of the total number of real massive particles is exponentially small.[16] In a strong gravitational field, $|R_i^k|\gg m^2$, substituting (1.7) and (A.2) in (A.1) and restricting ourselves to the calculation of the terms that do not depend on the rest mass and the terms proportional to m^2, we obtain for the density of the total number of particles and antiparticles,

$$N=\sum_\lambda\int\frac{d^3\mathbf{p}}{(2\pi)^3}N_{p\lambda},$$

the result

$$\left(\frac{dN}{dt}\right)_{vac}=\frac{36q^{(2)}}{\pi}\left(\frac{\ddot a}{a}\right)^2+\frac{6m^2q^{(1)}}{\pi}\ddot a^2. \qquad (A.3)$$

The numerical coefficients $q^{(1)}$ and $q^{(2)}$ for the considered particle species are determined by (2.9). To represent (A.3) in four-dimensional form, we introduce the four-vector of the current of the particles

$$j^i=(N/a^4,0,0,0)$$

and an integral characteristic of the effect,

$$\Delta N_{vac}=\int(-g)^{1/2}j^i_{;i}\,d^4x,$$

which is the total number of particles produced in a volume $V=1$ of the three-dimensional space during the entire time of evolution of the universe. For ΔN_{vac}, we have in accordance with (A.3)

$$\Delta N_{vac}=\int\frac{1}{\pi}(q^{(2)}R^2+m^2q^{(1)}R)(-g)^{1/2}d^4x. \qquad (A.4)$$

The integrand in (A.4) is the covariant expression for the rate of spontaneous production of real particles in the strong gravitational field. Note that up to a common factor $(1/2\pi)\ln(P_0/m)$ this rate is equal to the expression for the divergent part of the radiative corrections to the gravitational Lagrangian [see (2.12) and (4.7)].

Suppose that in the early stage of the cosmological evolution there is a state of local thermodynamic equilibrium, this leading at each instant of time to equilibrium distributions of the particles of all species. In this case, the production of real particles reduces to a growth of the entropy and temperature of the medium. Therefore, the problem of particle production under conditions of local thermodynamic equilibrium reduces to the formulation of the law of increase of the entropy and the finding of the dependence of the temperature on the time.

It is however necessary to take into account one further channel of particle production due to bulk deformations of the field of the matter velocities. Allowance for this channel leads to the appearance in the equations of the coefficient of bulk viscosity. In a state of local thermodynamic equilibrium, the entropy S and the temperature Θ in the ultrarelativistic limit are connected by

$$S=k_s\Theta^3,\quad k_s=\frac{2\pi^2}{45}\left(\sum_j^B g_j+\frac{7}{8}\sum_j^F g_j\right)$$

where g_j is the number of independent polarizations of the particle of species j. Introducing the four-vector of the entropy flux density σ^i, the four-tensor of the bulk deformations d^k, and the four-scalar for the physical temperature T,

$$\sigma^i=(S/a^4,0,0,0),\quad d_i^k=(\delta_i^k-u_iu^k)u^l{}_{;l},$$

$$\left(d_0^0=0,\ d_\alpha^\beta=3\delta_\alpha^\beta\frac{\dot a}{a}\right),\quad T=\frac{\Theta}{a},$$

we write the law of increase of the entropy of the ultrarelativistic medium in the form

$$\sigma^i{}_{;i}=\xi_{vac}R^2+\frac{\xi_{mat}}{9T}d^2. \tag{A.5}$$

In accordance with (3) and (4),

$$\xi_{vac}=\beta_1N_1,\quad \beta_1=\pi/0.122\cdot 12\,960,$$

where N_1 is the number of species of vector bosons ($m_1\neq 0$).

The coefficient of bulk viscosity ξ_{mat} can be calculated only for known laws of interaction of the elementary particles. Here, we shall restrict ourselves to a single remark. The conformal invariance of the Lagrangian of the interaction in conjunction with dimensional considerations suggests that in the ultrarelativistic limit

$$\xi_{mat}=\chi T^3,$$

where χ is a dimensionless numerical factor whose magnitude is determined by the coupling constants, the species of the particles, and the dynamical properties of their internal degrees of freedom.

The energy–momentum tensor of a locally equilibrium medium can be obtained from (1.4) after separation of the vacuum part, substitution as propagator of the distribution function of the real particles interacting with the external gravitational field

$$D_{p\lambda}^{real}(t)=N_{p\lambda}(t)F_{p\lambda}(t),$$

and averaging over the statistical ensemble. In the ultrarelativistic limit,

$$(ma)^2\ll\frac{\dot a^2}{a^2},\qquad \left|\frac{\ddot a}{a}\right|\ll\Theta^2$$

direct calculation leads to the result

$$\bar{\mathscr{H}}_{(real)}=\frac{3}{4}k_s\Theta^4+\frac{N_1\Theta^2}{24}\frac{\dot a^2}{a^2},$$

$$\bar T_{(real)}=\frac{N_1\Theta^2}{12}\left(\frac{\ddot a}{a}-\frac{\dot a^2}{a^2}\right)+3k_s\Theta^3\left(1+\frac{N_1}{36k_s\Theta^2}\frac{\dot a^2}{a^2}\right)\frac{\dot\Theta}{\dot a/a}.$$

Using the law of increase (A.5) of the entropy, we can represent the last expression in the covariant form

$$\bar T_{i(real)}^{k}=\frac{k_s}{4}T^4(4u_iu^k-\delta_i^k)+\frac{N_1}{324}\left[\frac{1}{6}(4u_iu^k-\delta_i^k)d_m^ld_l^m\right.$$

$$\left.+(d_i^ku^l{}_{;l}+3u^ld_{i;l}^k)T^2\right]+\xi d_i^k,$$

$$\xi=\left(\frac{9T}{d^2}\xi_{vac}R^2+\xi_{mat}\right)\left(1+\frac{N_1}{324k_s}\frac{d^2}{9T^2}\right),$$

where the bar denotes averaging over the statistical ensemble in the state of local thermodynamic equilibrium.

[1)]Generally speaking, the spinor particles make a contribution to the Lagrangian proportional to $m^2R\ln(P_0/m)$.

[2)]In the considered type of space, the system of field equations decouples into the systems of equations for the transverse components and the longitudinal—time component.

[3)]The two other solutions to Eq. (4.4) are essentially nonlocal and do not contribute to the considered effect.

[4)]Up to finite terms, $\mathscr{L}'$ preserves its form in the region of Compton curvatures $|R_l^k|\sim m^2$ as well.

[5)]As we have already noted, the obtained covariant expressions do not take into account terms which vanish in conformally flat spaces (such as, for example, the second invariant of the Weyl tensor).

[6)]All the local terms of the asymptotic expansion of the propagator are taken into account by the solution (1.7), and therefore $N_{\mathbf{p}\lambda}(t)$ is essentially nonlocal.

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