Influence of dynamic screening on photon absorption in Coulomb collisions in a degenerate plasma in a quantizing magnetic field

A. N. Panov

All-Union Research Institute for Electric Welding Equipment
(Submitted 17 November 1979)
Zh. Eksp. Teor. Fiz. **78**, 2251–2258 (June 1980)

The dissipative part of the high-frequency conductivity, which describes the absorption of electromagnetic waves in electron-ion collisions in a degenerate plasma in a quantizing magnetic field $\hbar\omega_B > \varepsilon_F$ (ω_B is the cyclotron frequency and ε_F is the Fermi energy) is obtained. Consistent account is taken of the screening of the Coulomb interaction of the electron and ion. The dependence of the longitudinal effective collision frequency $\nu_\parallel$, in terms of which the absorption coefficient of electromagnetic waves polarized along an external magnetic field B is expressed, on the photon frequency $\omega < \omega_B$ and on the ratio $\hbar\Omega_p/\varepsilon_F$ is investigated (Ω_p is the plasma frequency). The absorption coefficient of the ordinary wave propagating in a direction transverse to **B** is calculated. It is shown that the peak in the frequency dependence of the absorption coefficient, which takes place in samples with small effective carrier mass at $\hbar\omega = \varepsilon_F - \hbar\omega_B/2$ is perfectly observable at $\hbar\Omega_p \lesssim \varepsilon_F - \hbar\Omega_B/2$. The conditions under which this absorption peak can be observed in n-InSb are discussed.

PACS numbers: 72.30. + q

1. Emission and absorption of photons in electron-ion collisions are among the principal interactions between radiation and magnetized plasma. These processes have by now been considered for both nondegenerate[1] and degenerate[2] plasma. In Ref. 2 is considered the case of a quantizing magnetic field **B**, when the distance $\hbar\omega_B$ between the Landau levels exceeds the Fermi energy ε_F. In this case the high-frequency conductivity, which describes the absorption of the electromagnetic waves at different values of $\hbar\omega/\varepsilon_F$ (ω is the frequency of the absorbed radiation) has been calculated and a comparison made with the cases of a degenerate plasma at $B=0$ and a nondegenerate plasma in a quantizing magnetic field. The dissipative properties of the plasma are described by the Hermitian part of the conductivity tensor $\sigma_{\alpha\beta'}$, whose components are determined by the effective electron-ion collision frequencies $\nu_\parallel(\omega)$ and $\nu_\perp(\omega)$ along and across the magnetic field, respectively. An important feature of the results obtained in Ref. 2 is the presence, at $\hbar\omega_B > \varepsilon_F$, of peaks on the $\nu_\parallel(\omega)$ and $\nu_\perp(\omega)$ curves at a frequency $\omega = \varepsilon_F/\hbar$. In contrast to the ordinary peaks at the cyclotron harmonics, which appear much less frequently on the $\nu_\perp(\omega)$ curve, the peak at the frequency $\omega = \varepsilon_F/\hbar$ is more clearly pronounced on the $\nu_\parallel(\omega)$ curve, i.e., for waves polarized along the external magnetic field. We note that the above-mentioned results were obtained in the limiting case

$$\varepsilon_F \gg \hbar\Omega_p, \tag{1}$$

when the screening of the impurity charge can be disregarded. Here $\Omega_p = (4\pi N_e e^2/mk_0)^{1/2}$ is the plasma frequency, N_e is the concentration and m the effective mass of the electrons, and k_0 is the dielectric constant of the crystal.

As will be made clear below, the effects observed in Ref. 2 can be experimentally investigated only for samples with small effective carrier mass. In this case the spin splitting of the Landau levels can be neglected (see Ref. 2),

$$\varepsilon_F = \hbar\omega_B/2 + \varepsilon_B = \hbar\omega_B/2 + \pi^4\hbar^4 N_e^2/2m^3\omega_B^2, \tag{2}$$

and the condition (1) should be replaced with

$$\hbar\Omega_p/\varepsilon_B = (64b^2\,\mathrm{Ry}/\pi^2\varepsilon_B)^{1/4} \ll 1, \tag{3}$$

where $\mathrm{Ry} = me^4/2k_0^2\hbar^2$ is the effective energy of the first Bohr orbit, and $b = \hbar\omega_B/\varepsilon_B$. Even for the crystal n-InSb ($m = 1.3\ 10^{-29}\,r$, $k_0 = 15.9$, $\mathrm{Ry} = 1.1\cdot 10^{-15}$ erg), however, it is impossible to ensure simultaneous satisfaction of the strong inequality (3) and the quantization condition $b > 1$. Thus, for $b = 5$ we get from (3) $\hbar\omega_B \gg 0.8\cdot 10^{-12}$ erg or $B \gg 700$ kG which is unattainable, and for $b = 2$ we have $\hbar\omega_B \gg 0.6\cdot 10^{-13}$ erg or $B \gg 50$ kG. Therefore in experiments on photon absorption in a degenerate semiconductor situated in a quantizing magnetic field it is realistic to attain satisfaction of the condition $\hbar\Omega_p \lesssim \varepsilon_B$, at which it becomes important to take into account the screening of the impurity charge.

Allowance for the static screening reduces usually to replacing in the expression for the collision frequency the Fourier transform of the "bare" Coulomb potential $U_k = -4\pi Ze^2/k^2$ by $U_k = -4\pi Ze^2/(k^2 + q_0^2)$, where $q_0 = \mathrm{const}$ is the reciprocal Debye radius and k is the wave vector.[2] It is known that in a degenerate plasma the static screening differs substantially from the Debye screening. This manifests itself in the fact that q_0 becomes a function of the wave vector **k** (for more details see Ref. 3, p. 156 of the Russian translation). In the case of rapidly alternating processes, such as photon absorption, the screening is dynamic, as a result of which U_k becomes a function of **k** and ω. Photon absorption in a nondegenerate plasma, with dynamic screening taken into account and in the absence of a magnetic field, was investigated by Perel' and Éliashberg.[4] Their allowance for the screening resulted in a small change in the argument of the Coulomb logarithm. As we shall show, in a degenerate magnetized plasma allowance for the dynamic screening leads to a substantial change in the absorption coefficient.

In the present paper we calculate the high-frequency conductivity, which describes the absorption of electromagnetic waves whose polarization vector is parallel to the external magnetic field **B**, in a degenerate plasma at $\hbar\omega_B > \varepsilon_B$, $\nu_\parallel(\omega) \ll \omega < \omega_B$, with account taken of the dynamic screening of the Coulomb field of the impurity. We have investigated the shape of the absorption peak observed in Ref. 2 at the frequency $\omega = \varepsilon_B/\hbar$ for different values of $\hbar\Omega_p/\varepsilon_B$. With the propagation of ordinary waves in a direction perpendicular to the field **B** in n-InSb as an example, we discuss the conditions for the experimental study of the indicated absorption peak.

2. To determine the high-frequency conductivity of a degenerate plasma situated in a quantizing field $\mathbf{B} = (0, 0, B)$, we calculate the electron current δj_z induced by the electric field of the wave $\mathbf{E} = (0, 0, E_z e^{-i\omega t})$, i.e., we confine ourselves to the dipole approximation. We have

$$\delta j_z = i\frac{e^2 N_e}{m\omega} E_z + \delta j_z^{(1)}, \quad \delta j_z^{(1)} = \sum_{\nu_e} \langle \nu_e | \delta\rho_e | \nu_e \rangle p_{\nu_e}, \tag{4}$$

where $\delta\rho_e$ is the high-frequency increment to the electronic density matrix, $\nu_e \equiv (N, y_{0e}, p_{\nu_e})$ is the complete set of quantum numbers of an electron located in the quantizing magnetic field; these numbers correspond to the energy levels

$$E_\nu = (N + 1/2)\hbar\omega_B + p_\nu^2/2m \tag{5}$$

(in this section of the article we neglect solid-state effects, i.e., we assume that $k_0 = 1$).

Silin[5] has solved the equation for the binary collective matrix, and obtained as a result, in the born approximation, the integral $\langle \nu_\alpha | \delta I_\alpha(\delta\rho_\alpha) | \nu'_\alpha \rangle$ of the collisions of the particles of species α with all other particles and with one another. Using this collision integral, we obtain the kinetic equation for the single-particle electron density matrix $\delta\rho_e$:

$$\hbar(\omega - \omega_{\nu_e\nu'_e}) \langle \nu_e | \delta\rho_e | \nu_e' \rangle = [\rho_e^0(\nu_e') - \rho_e^0(\nu_e)] \langle \nu_e | V_e | \nu_e' \rangle + i\hbar \langle \nu_e | \delta I_e(\delta\rho_e) | \nu_e' \rangle. \tag{6}$$

Here $\hbar\omega_{\nu_e'\nu_e} = E_{\nu_e'} - E_{\nu_e}$, $\rho_e^0(\nu_e)$ is the equilibrium density matrix, $V_e = ie(\mathbf{E}\mathbf{p}_e)/m\omega$ is the operator of the energy of the interaction of the electrons with the field **E**, and $\mathbf{p}_e$ is the momentum operator.

The operator $\delta I_e(\delta\rho_e)$ is a functional of $\delta\rho_e$, and therefore the exact solution of Eq. (6) is a very complicated task. In the high-frequency case, however, δI_e contains a small factor $\sim\nu_\parallel(\omega)/\omega$, and Eq. (6) can be solved by successive approximations. Putting $\delta\rho_e = \delta\rho_e^{(1)} + \delta\rho_e^{(2)}$, where $\delta\rho_e^{(1)}$ is the solution of Eq. (6) with the collision term discarded, we obtain

$$\hbar(\omega - \omega_{\nu_e\nu_e'}) \langle \nu_e | \delta\rho_e^{(1)} | \nu_e' \rangle = [\rho_e^0(\nu_e') - \rho_e^0(\nu_e)] \langle \nu_e | V_e | \nu_e' \rangle. \tag{7}$$

In the considered case $\mathbf{E} \parallel \mathbf{B}$, the operator V_e is diagonal and in accordance with (7) we have $\langle \nu_e | \delta\rho | \nu_e' \rangle = 0$, thus making the calculations much simpler. The kinetic equation (6) then takes the form

$$\hbar(\omega - \omega_{\nu_e\nu_e'}) \langle \nu_e | \delta\rho_e | \nu_e' \rangle = i\hbar \langle \nu_e | \delta I_e(0) | \nu_e' \rangle. \tag{8}$$

Substituting (8) in (4) we obtain the collision increment to the electron current:

$$\delta j_z^{(1)} = \frac{i}{\omega} \sum_{\nu_e} \langle \nu_e | \delta I_e(0) | \nu_e \rangle p_{\nu_e}. \tag{9}$$

Since V_e is diagonal only in the dipole approximation and at $\mathbf{E} \parallel \mathbf{B}$, Eq. (9) is valid under the same conditions. The expression for the collision term $\langle \nu_e | \delta I_e(0) | \nu_e \rangle$ can be obtained by substituting in Eq. (3.11) of Ref. 8 the operator V_e, the values of the matrix elements calculated with the wave functions of the electron in the magnetic field, and the energy levels (5). Cumbersome calculations yield for the collision increment to the electron current:

$$\delta j_z^{(1)} = \frac{4e^2 e_i^2 N_i E_z}{m_e^2 \omega^2} \int_{-\infty}^{\infty} \frac{d\Omega}{\omega + \Omega + i\Delta} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{ik_z^2}{k^2 \varepsilon^+(0, \mathbf{k})} \times \left\{ \mathrm{Im}\left[\frac{1 - \varepsilon^+(\Omega + \omega, -\mathbf{k})}{\varepsilon^+(\Omega, -\mathbf{k})(\Omega + i\Delta)} \right] + \frac{\pi\delta(\Omega)[1 - \varepsilon^+(\omega, -\mathbf{k})]}{\varepsilon^+(\omega, -\mathbf{k})} \right\}. \tag{10}$$

Here $\varepsilon^+(\omega, \mathbf{k})$ is the longitudinal dielectric constant, which depends on the frequency and on the wave vector, and is analytic in the upper ω plane:

$$\varepsilon^+(\omega, \mathbf{k}) = 1 + \sum_{\beta, \nu_\beta, \nu'_\beta} \frac{4\pi e_\beta^2}{\hbar k^2} \frac{N_\beta}{(2\pi)^3} \frac{\rho_\beta^{(0)}(\nu_\beta) - \rho_\beta^{(0)}(\nu_\beta')}{\omega + \omega_{\nu'_\beta\nu_\beta} + i\Delta} \times |\langle \nu_\beta' | \exp(-i\mathbf{k}\mathbf{r}_\beta) | \nu_\beta \rangle|^2, \tag{11}$$

N_β is the concentration of the particles of species β $(\beta = e, i)$.

The imaginary part of (10) determines the small collision correction to the anti-Hermitian part of the conductivity tensor, which we shall not calculate. The real part of (10) determines the Hermitian part of the conductivity tensor σ'_{zz}, and in accordance with Ref. 2 also the effective longitudinal frequency of the electron-ion collisions $\nu_\parallel(\omega)$:

$$\mathrm{Re}\, \delta j_z^{(1)} = \sigma'_{zz} E_z = \frac{\Omega_p^2}{4\pi\omega^2} \nu_\parallel(\omega) E_z. \tag{12}$$

Formula (12) is valid for frequencies much higher than $\nu_\parallel(\omega)$, and also not too close to the first cyclotron resonance $|\omega - \omega_B| \gg \nu_\parallel(\omega)$. According to (10) and (12) we have

$$\nu_\parallel(\omega) = \frac{4\pi Z e^3}{m\omega} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{k_z^2}{k^2} \frac{\mathrm{Im}\, \varepsilon^+(\omega, \mathbf{k})}{|\varepsilon^+(\omega, \mathbf{k})|^2}. \tag{13}$$

Here $m = m_e$ and Ze is the ion charge. We note that (13) is valid in the presence of an arbitrary external field in which the z-component of the electron momentum is conserved. In particular, in the absence of an external field (i.e., after making the substitution $k_z^2/k^2 \to \frac{1}{3}$), expression (13) coincides with the known result obtained from the Vlasov equation (see Ref. 6, p. 182 of the translation).

3. We proceed now to investigate the influence of dynamic screening on the dissipative properties of the plasma. We confine ourselves to degenerate semiconductors placed in a quantizing magnetic field $\hbar\omega_B > \varepsilon_B$ at electromagnetic-field frequencies $\omega < \omega_B$. In this case the quantities m, ω_B, b, etc. in the relations obtained above must be taken to mean their effective values [we note that the effective mass of the electron in the expression for ω_B does not necessarily coincide with the effective mass in expression (2) for ω_B]. In the right-hand side of (11) it is necessary to replace unity by k_0 and take N_e to mean the concentration of the free car-

riers in the conduction band, which generally speaking depends on the magnetic field. However, according to Ref. 7, for n-InSb at $N_e \gg 2\cdot 10^{16}$ cm^{-3} the concentration of the free electrons ceases to depend on the magnetic field and is equal to the donor concentration. The effective energy Ry of the first Bohr orbit increases with increasing **B**. Thus, according to Ref. 8, for n-InSb we have $\mathrm{Ry}(\mathbf{B}=150\ \mathrm{kG})\approx 3\,\mathrm{Ry}(\mathbf{B}=0)$, but at $B>150$ kG the dependence of Ry on B has a tendency to saturate.

We introduce, following Ref. 2, the dimensionless variables $x=\hbar\omega/\varepsilon_B$, $\omega=k_z^2R^2/2$, $u=k_\perp^2R^2/2$, where $R=(\hbar c/eB)^{1/2}$ is the magnetic length. We then obtain after simple calculations ($x<b$, $b>1$)

$$\mathrm{Re}\,\varepsilon^+(u,w)=k_0[u+w+\varphi(u,w)]/(u+w),$$

$$\varphi(u,w)=\frac{1}{\pi}\left(\frac{\mathrm{Ry}}{\varepsilon_B bw}\right)^{1/2}\sum_{n=0}^{\infty}\frac{u^n e^{-u}}{n!}\ln\left|\frac{[n+(4w/b)^{1/2}+w]^2-x^2/b^2}{[n-(4w/b)^{1/2}+w]^2-x^2/b^2}\right|, \quad (14)$$

and

$$\mathrm{Im}\,\varepsilon^+(u,w)=k_0\psi(u,w)/(u+w), \quad (15)$$

$$\psi(u,w)=\begin{cases}(\mathrm{Ry}/\varepsilon_B bw)^{1/2}e^{-u} & \text{at}\quad w_1<w<w_2,\quad w_3<w<w_4\ (x\leqslant 1)\\ (\mathrm{Ry}/\varepsilon_B bw)^{1/2}e^{-u} & \text{at}\quad w_1<w<w_4\quad (x>1).\\ 0 & \text{outside these intervals}\end{cases}$$

Here

$$w_{1,4}=[1\mp(1+x)^{1/2}]^2/b,\quad w_{2,3}=[1\mp(1-x)^{1/2}]^2/b. \quad (16)$$

Formulas (14) and (15) were obtained with account taken of the relation

$$\Omega_p^2/\omega_B^2=8(\mathrm{Ry}/\varepsilon_B)^{1/2}/\pi b,$$

from which it is seen that under the conditions when the born approximation is valid and at $b>1$ we have $\Omega_p<\omega_B$. Substituting (14) and (15) in (13) we obtain

$$\nu_\parallel(x)=\frac{\nu_0}{x}\int_0^\infty du\int dw\frac{e^{-u}}{[u+w+\varphi(u,w)]^2+[\psi(u,w)]^2}, \quad (17)$$

where the intervals of integration with respect to w are determined by relation (15). Here

$$\nu_0=\pi Z^2e^4N_i/k_0^2\varepsilon_B(2m\varepsilon_B)^{1/2}.$$

It is easy to verify that the term with $n=0$ in the sum (14) has logarithmic singularities at the points w_1, w_2, w_3, and w_4, inasmuch as at these points either the numerator or the denominator of the argument of the logarithm vanishes. The remaining terms of the sum have no singularities. In the limiting case $b\gg1$, $x\ll b$ it is possible to carry out the summation in (14):

$$\sum_{n=1}^{\infty}\frac{u^n e^{-u}}{n!}\ln\left|\frac{[n+(4w/b)^{1/2}+w]^2-x^2/b^2}{[n-(4w/b)^{1/2}+w]^2-x^2/b^2}\right|\approx 8(w/b)^{1/2}e^{-u}[\mathrm{Ei}^*(u)-\ln u-C]$$
$$\approx 8(w/b)^{1/2}\cdot\begin{cases}u & \text{at}\quad u\ll1\\ 1/u & \text{at}\quad u\gg1\end{cases}, \quad (18)$$

where $\mathrm{Ei}^*(u)$ is the integral exponential function and $C\approx0.577$ is the Euler constant. It follows from (18) that if the born approximation is valid the terms of the sum with $n\geqslant1$ in (14) are small compared with u, and can therefore be neglected in (17).

It is obvious that at

$$w<|\varphi(u,w)|\ll1,\quad w<\psi(u,w)\ll1 \quad (19)$$

the main contribution to the integral with respect to u in (17) is made by the region $u\ll1$ and in expressions (14) and (15) we can put $e^{-u}=1$. At $w\gg|\varphi|$ and $w\gg\psi$ the functions φ and ψ have no effect at all on the value of $\nu_\parallel(x)$, and in this case we can also put $e^{-u}=1$ in (14) and (15). The function $\psi(u,w)\sim(\mathrm{Ry}/\hbar\omega_B w)^{1/2}$. The logarithm in (14) can noticeably exceed unity only in the immediate vicinity of the singular points w_1, w_2, w_3, w_4, so that in practically the entire range of variation of w we have $\varphi(u,w)\sim(\mathrm{Ry}/\hbar\omega_B w)^{1/2}$.

When solving the kinetic equation (6) we have assumed satisfaction of the condition $\omega\gg\nu_\parallel$, which takes according to (17) the form

$$\nu_\parallel/\omega\sim\hbar\nu_0/\varepsilon_B x^2=2\,\mathrm{Ry}\,b/\pi\varepsilon_B x^2\ll1, \quad (20)$$

from which we get at $x\ll1$

$$w\geqslant w_1\approx x^2/4b\gg\mathrm{Ry}/2\pi\varepsilon_B,$$
$$|\varphi|,\ \psi\sim(\mathrm{Ry}/\varepsilon_B bw)^{1/2}\ll(2\pi/b)^{1/2}\sim1. \quad (21)$$

Consequently, in expression (17) we can assume that $\varphi(u,w)$ and $\psi(u,w)$ do not depend on u. We have

$$\varphi(u,w)\approx\varphi(w)=\frac{1}{\pi}\left(\frac{\mathrm{Ry}}{\hbar\omega_B w}\right)^{1/2}\ln\left|\frac{[(4w/b)^{1/2}+w]^2-x^2/b^2}{[(4w/b)^{1/2}-w]^2-x^2/b^2}\right| \quad (22)$$

and $\psi(u,w)\approx\psi(w)$ is given by the formulas in (15), in which we put $e^{-u}=1$. In this case we can obtain an analytic expression; for $\nu_\parallel(x)$ in the limit $x\ll1$, and the numerical calculations at $x\sim1$ can be greatly simplified.

Inasmuch as at $x\ll1$ we have $w_4-w_3\gg w_2-w_1$, and the denominator of the integrand in (17) does not tend to zero as $u\to0$ and $w\to0$, the main contribution to $\nu_\parallel(x)$ is made by the integration interval $w_3\leqslant w\leqslant w_4$. It is easy to verify that the last statement is valid at $\gamma=(\mathrm{Ry}/\varepsilon_B)^{1/2}/2\pi>x^3/4b$.

We introduce a new integration variable in accordance with the formula

$$w=[1+(1+\alpha x)^{1/2}]^2/b,\quad -1\leqslant\alpha\leqslant1.$$

Then

$$\nu_\parallel(x)\approx\frac{4}{b}\int_0^\infty e^{-u}du\int_0^1 d\alpha[u+\beta-\gamma\ln(1-\alpha)]^{-2}, \quad (23)$$

where $\beta=4/b+\gamma\ln(64/x^2)$. In the derivation of (23) we took into account the fact that $\varphi(w)\gg\psi(w)$ at $\ln(64/x^2)>\pi$. Integration of (23) with respect to α is carried out with the aid of formula (4.212) of Ref. 9, and with respect to u with the aid of formula (14) on p. 219 of Ref. 10 (Russian translation). We have ($x\ll1$):

$$\nu_\parallel(x)=\nu_0[(4/b\gamma)\Phi_0(\beta/\gamma)-(4/b)\Phi_0(\beta)],$$
$$\Phi_0(\beta)=-e^\beta\,\mathrm{Ei}\,(-\beta). \quad (24)$$

At $\gamma\ln(64/x^2)\ll4b\ll1$ we obtain from (24)

$$\nu_\parallel(x)\approx\nu_0. \quad (25)$$

The value in (25) is half the corresponding value in Ref. 2, since we have considered the limit $\gamma>x^3/4b$ ($x\ll1$) and in the derivation of (23) we have neglected the integral over the interval $w_1<w<w_2$.

To study the influence of dynamic screening of the impurity charge on the shape of the absorption peak at $x\sim1$, we expand the integrand in (17) in partial fractions and, taking into account the dependences of φ and ψ on u, integrated with respect to u. We obtain

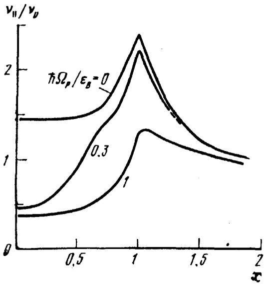

FIG. 1. Dependence of $\nu_\parallel$ in a degenerate plasma ($T = 0$) on the dimensionless radiation frequency $x=\hbar\omega/\varepsilon_B$, $b=\hbar\omega_B/\varepsilon_B=5$ at different values of the ratio $\hbar\Omega_p/\varepsilon_B$. The ordinates are the ratios of the collisions frequency to ν_0, see Eq. (17).

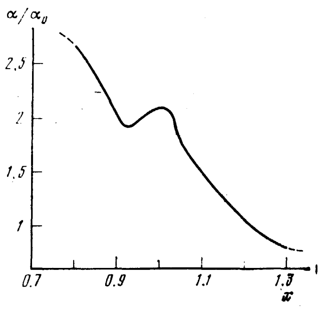

FIG. 2. Dependence of the absorption coefficient α of an ordinary wave polarized along the external magnetic field in a degenerate plasma ($T = 0$) on the dimensionless radiation frequency $x=\hbar\omega/\varepsilon_B$, $b=\hbar\omega_B/\varepsilon_B=2$, $\hbar\Omega_p/\varepsilon_B=0.7$. The ordinates are the ratios of the absorption coefficients α and α_0, see (29).

$$\nu_\parallel(x)=\frac{\nu_0}{x}\int\frac{dw}{\psi(w)}\,\mathrm{Im}\{\exp[w+\varphi(w)+i\psi(w)]\,\mathrm{Ei}[-w-\varphi(w)-i\psi(w)]\}. \quad (26)$$

The frequency dependence of $\nu_\parallel(x)$ at $b=5$ and at different values of $\hbar\Omega_p/\varepsilon_B=(16b\gamma)^{1/2}$, obtained as a result of numerical integration in accordance with (26), is shown in Fig. 1. It is seen that the decrease of the effective collision frequency, due to allowance for the dynamic screening of the impurity charge, is much more pronounced at $x<1$ than at $x>1$. Therefore when the point $x=1$ is approached from the low frequency size, the slope of the peak increases in comparison with the case $\hbar\Omega_p/\varepsilon_B=0$, and decreases when approached from the high frequency side. A shift of the maximum of the curve towards higher frequencies is simultaneously observed. The peak remains perfectly distinguishable up to $\hbar\Omega_p/\varepsilon_B=1.5$, but at $\hbar\Omega_p/\varepsilon_B>2$ it practically disappears.

4. We consider now collision absorption of an ordinary wave whose electric field intensity vector is parallel to the external magnetic field **B**. In the case of propagation perpendicular to **B**, the dispersion equation for such a wave is of the form (see Ref. 11, p. 109)

$$\tilde{n}^2=k_0[1-\Omega_p^2/\omega^2+i\Omega_p^2\nu_\parallel(\omega)/\omega^3], \quad (27)$$

where $\tilde{n}$ is the complex refractive index. It follows from (27) that at

$$\nu_\parallel(\omega)\ll\omega,\quad (1-\Omega_p^2/\omega^2)\gg\Omega_p^2\nu_\parallel(\omega)/\omega^3$$

the frequency is

$$\omega=(\Omega_p^2+k^2c^2/k_0)^{1/2}, \quad (28)$$

where **k** is the wave vector, and the absorption coefficient is

$$\alpha=\frac{2\omega\,\mathrm{Im}\,\tilde{n}}{c}=\frac{\alpha_0}{x^2(1-\Omega_p^2/\omega^2)^{1/2}}\frac{\nu_\parallel(x)}{\nu_0} \quad (29)$$

$$\alpha_0=2k_0^{1/2}b\,\mathrm{Ry}\,\hbar^2\Omega_p^2/\pi\hbar c\varepsilon_B^2\quad (Z=1).$$

In the considered case $\mathbf{k}\perp\mathbf{B}$ we can neglect the spatial dispersion if $kR\ll1$ (R is the magnetic length). At $\Omega_p<\omega\sim\varepsilon_B/\hbar$ we have from (28)

$$kR\sim(k_0\varepsilon_B^2/mc^2\hbar\omega_B)^{1/2}\ll1,$$

i.e., the dipole approximation used to derive (17) is valid.

We indicate now the conditions under which the absorption peak at the frequency $\omega\sim\varepsilon_B/\hbar$ can be observed in n-InSb. It is known (see Ref. 12, Chap. 5) that the absorption in n-InSb can be measured at a free-space wavelength $\lambda<40\ \mu$m or $\hbar\omega>0.45\cdot10^{-13}$ erg. We choose $\varepsilon_B=1.2\cdot10^{-13}$ erg; to observe the peak we must vary the frequency in the range $0.6\cdot10^{-13}<\hbar\omega<2.2\cdot10^{-13}$ erg. The width of the forbidden band is 2.9×10^{-13} erg, therefore the interband transitions make no contribution to the absorption. As seen from (29), at $\omega\to\Omega_p$ the refractive index increases, and the absorption increases sharply. The measurements must be made in the region $\omega>\Omega_p$, in which this effect does not manifest itself very strongly. As we have shown, satisfaction of the conditions $\hbar\omega_B>\varepsilon_B>\hbar\Omega_p$ is realistic at not too large b. We choose $b=2$, then $\hbar\omega_B\sim2.4\cdot10^{-13}$ erg and $B\approx200$ kG. In this case [see (2)] we have

$$N_e=5.6\cdot10^{17}\ \mathrm{cm}^{-3},\quad \alpha_0=100\ \mathrm{cm}^{-1},\quad \hbar\Omega_p/\varepsilon_B=0.7,$$
$$\hbar\nu_0/\varepsilon_B=2b\,\mathrm{Ry}/\pi\varepsilon_B=1.2\cdot10^{-2}$$

i.e., the condition of applicability of the high-frequency approximation is well satisfied.

The results of the calculations of the absorption coefficient from formulas (26) and (29) at the indicated values of the parameters are shown in Fig. 2. At $x>0.8$ we have $|\varphi|$, $\psi\lesssim(\mathrm{Ry}/\hbar\omega_Bw_1)^{1/2}\sim0.27$ in practically the entire range of variation of w, and consequently replacement of e^{-u} by unity in (14) and (15) is justified, and expression (26) is a good approximation for $\nu_\parallel(x)$. It is shown in Ref. 2 that the temperature smearing of the peak is negligible at $T<0.1\varepsilon_B$ (T is the temperature in energy units), i.e., in our case the experiment should be carried out at $T\lesssim80$ K.

The author thanks L. É. Gurevich, G. G. Pavlov, and D. G. Yakovlev for helpful discussions.

[1]G. G. Pavlov and A. N. Panov, Zh. Eksp. Teor. Fiz. **71**, 572 (1976) [Sov. Phys. JETP **44**, 300 (1976)].

[2]G. G. Pavlov, A. N. Panov, and D. G. Yakovlev, Zh. Eksp. Teor. Fiz. **74**, 1356 (1978) [Sov. Phys. JETP **47**, 710 (1978)].

[3]H. N. March, S. Sampanthar, and W. H. Young, The Many-Body Problem in Quantum Mechanics, Cambridge Univ. Press, 1967.

[4]V. I. Perel' and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **41**, 886 (1961) [Sov. Phys. JETP **14**, 633 (1961)].

[5]V. P. Silin, Fiz. Met. Metalloved. **13**, 180 (1962).

[6]G. Bekefi, ed., Raditation Processes in Plasmas, Wiley, 1966,

[7]R. W. Keyes and R. J. Sladek, J. Phys. Chem. Solids **1**, 143 (1956).

[8]O. Beckman, E. Hanamura, and L. J. Neuringer, Phys. Rev. Lett. 18, 773 (1967).
[9]I. S. Gradshtein and I. M. Ryzhik, Tablitsy integralov, summ, ryadov, i proizvedenii (Tables of Integrals, Sums, Series and Products), Nauka, 1971 [Academic, 1965].
[10]A. Erdelyi, ed. Tables of Integral Transforms, Vol. 2, McGraw, 1953.
[11]V. L. Ginzburg and A. A. Rukhadze, Volny v magnitoaktivnoĭ plazme (Waves in a Magnetoactive Plasma), Nauka, 1970.
[12]In: Optical Properties of III-V Semiconductors, ed. by R. K. Willardson and A. S. Beer, Academic, 1967.

Translated by J. G. Adashko

Spatial development of the instability of a dense beam of negative ions in a rarefied gas

D. G. Dzhabbarov and A. Naĭda

Institute of Physics, Ukrainian Academy of Sciences
(Submitted 12 December 1979)
Zh. Eksp. Teor. Fiz. 78, 2259–2265 (June 1980)

The mechanism of the appearance of the decompensation effect of a dense (~ 50 mA/cm^2) beam of negative ions in a rarefied gas is elucidated. The effect is due to singularities in the development in the resultant ion-ion plasma, of the instability of oscillations that are almost perpendicular to the beam velocity. It is shown that the spatial characteristics of the instability depend in turn on the depth of the stationary negative potential well that determines the mean transverse energy of the positive ions. It is established experimentally for the first time that the longitudinal phase velocity of the excited oscillations is much smaller than the beam velocity.

PACS numbers: 51.50. + v

Upon propagation of a beam of negatively charged particles in a gas and its ionization as a result of the accumulation of positive ions in the potential well of the beam and of the rapid departure from it of electrons at a sufficiently low gas pressure, a two-beam plasma is formed.

Interest in the study of the stability of such beams relative to transverse perturbations is stimulated first by applications, such, for example, as the acceleration of positive ions by means of electron rings,[1-3] electron-beam welding,[4,5] and, in recent times, the contemplated use of beams of negative ions in fast-arom injectors for thermonuclear devices.[6] At the beam current densities (~50 mA/cm^2) and low gas pressure required for injectors, as was discovered in Ref. 7, effect of strong decompensation of the beam arises as a result of the instability of the transverse oscillations of the ion-ion plasma; this hinders the effective beam transport.

In the present paper we study those characteristic features of the spatial development of the instability in such a plasma which make clear the mechanism that produces the effect.

Before proceeding to the esposition of the experimental results, we consider the buildup of oscillations in an unbounded plasma, consisting of immobile positive ions with mass M_+ and a beam of negative ions (or electrons) with mass m_-, propagating with velocity v_b in the z direction. It is known that the natural oscillations of the charge density of each of the components, in the frame of reference in which it is at rest, take place with the frequencies $\omega_p=(4\pi e^2 n_+/M_+)^{1/2}$ and $\omega_b=(4\pi e^2 n_b/m_-)$, where $n_+=n_b=n$ are the densities of the components. In the case of resonance of these oscillations, i. e., $|\omega_p-k_z v_b|\approx\omega_b$, their buildup occurs; here $k_z=2\pi/\lambda_z$ is the longitudinal wave number of the oscillations.

An important consequence then follows, namely, that in the case $\omega_p/\omega_b=(m_-/M_+)^{1/2}\ll 1$,

$$k_z\approx\omega_b/v_b, \tag{1}$$

$$\omega/k_z\approx(m_-/M_+)^{1/2}v_b\ll v_b. \tag{2}$$

The growth rates of the buildup can be obtained from the dispersion equation. Usually the temporal problem is solved in the approximation of a cold plasma,[1-4] the analysis of which was first given by Buneman.[8] However, this approximation, as is well known, is inapplicable in finding the spatial growth rate. Account of thermal motion of the positive ions[9] leads to a more complicated equation:

$$1+\frac{1}{k^2d_+^2}\left[1+i\pi^{1/2}\frac{\omega}{|k|v_T}W\left(\frac{\omega}{|k|v_T}\right)\right]-\frac{\omega_b^2}{(\omega-k_z'v_b)^2}=0, \tag{3}$$

where $v_T=(2T_+/M_+)^{1/2}$, $d_+=(T_+/4\pi e^2 n)^{1/2}$, $W(\omega/|k|v_T)$ is a Kramp function, $k^2=|k_z'|^2+k_\perp^2$, and $k_z'=k_z+i\varkappa$ must be determined.

The problem simplifies slightly at $\omega/|k|v_T\gg 1$; then

$$1-\frac{\omega_p^2}{\omega^2}\left(1+\frac{3}{2}\frac{k^2v_T^2}{\omega^2}\right)+\frac{i\pi^{1/2}}{k^2d_+^2}\frac{\omega}{|k|v_T}\exp\left(-\frac{\omega^2}{k^2v_T^2}\right)-\frac{\omega_b^2}{(\omega-k_z'v_b)^2}=0. \tag{4}$$

Upon satisfaction of the condition $k_z\ll k_\perp$, the maximum value of the spatial growth rate $\varkappa$ and its corresponding longitudinal wave number k_z and resonance frequency ω_{res} are connected by the following relations, as follows from the solution of (4):

$$\omega_{res}=\omega_p\left(1+\frac{3}{2}\frac{k^2v_T^2}{\omega_{res}^2}\right)^{1/2},\quad -\varkappa+k_z\approx\frac{\omega_{res}}{v_b},$$

