assumed in our calculation.

Owing to the nonexistence at present of experimental papers with $n > 20$, the following fact should be emphasized. Since

$$\gamma = \frac{1}{2n}\frac{\mathscr{E}_{\rm at}}{\mathscr{E}}, \quad \bar{\gamma} = \frac{1}{2n^{3/2}}\frac{\mathscr{E}_{\rm at}}{\mathscr{E}},$$

the conditions $\gamma, \bar{\gamma} \lesssim 1$ for which the ionization probability deviates from the power law are satisfied for

$$n \gtrsim \mathscr{E}_{\rm at}/2\mathscr{E},$$

in a monochromatic field, but for

$$n \gtrsim (\mathscr{E}_{\rm at}/2\mathscr{E})^{2/3},$$

in a random field; that is, for a lower value of n in the latter case. For the experiment of Ref. 15 we have $\mathscr{E}_{\rm at}/\mathscr{E} = 110$; thus the inequality $\gamma \lesssim 1$ gives $n \gtrsim 55$, whereas the inequality $\bar{\gamma} \lesssim 1$ gives $n \gtrsim 15$. Accordingly, appreciable deviation of the ionization probability from the power law, characteristic of ordinary perturbation theory can be achieved more easily in a stochastic field than in a monochromatic field.

In conclusion we can state that the proposed theory contains simple universal one-parameter functions, by using which one can easily calculate probabilities of nonresonance many-photon ionization in a strong stochastic electromagnetic field, and also the corresponding statistical factors.

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Fine structure of the spectrum of depolarized light scattering in liquids

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It is shown that accounting for the correlation of simultaneous fluctuations leads to the appearance of terms with spatial velocity derivatives in the permittivity. As a consequence the permittivity is no longer a scalar and a fine structure appears in the spectrum of the depolarized light scattering.

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INTRODUCTION

The intensity of light scattered by a liquid consists of two essentially different parts (see, e.g., Ref. 1). The first is scattering by macroscopic fluctuations, i.e., the fluctuations of such hydrodynamic quantities as entropy or pressure. The second is scattering primarily by the anisotropy fluctuations and in general by the relative location fluctuations of the particles on microscopic scale. Both types of scattering give comparable contributions to the integrated scattering intensity, but are characterized by essentially different spectral distributions.

The microscopic fluctuations result in the appearance of non-shifted line (wing) the width of which $\Delta\omega_{\rm w}$ is of the order of the characteristic frequency ν of microscopic movements in the liquid, $\Delta\omega_{\rm w} \sim \nu = 1/\tau$ (for simplicity in the estimates below only simple liquids having no slowly relaxing parameters will be kept in mind).

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The fluctuations of macroscopic quantities cause the appearance of a triplet of narrow lines of comparable intensity—a central non-shifted component of the entropic scattering (Rayleigh component) and the Mandel'shtam-Brillouin (MB) doublet. The widths of all three lines are smaller by a factor $(\lambda/a)^2 \sim 1/(ka)^2 \gg 1$ (where λ and k are the wavelength and the wave vector of the light, a is a characteristic length of the order of interatomic distance) than the width of the wing. Correspondingly their spectral intensity in the maxima is $(\lambda/a)^2$ times greater than the spectral intensity of the wing.

An important experimentally most studied characteristic of scattering is the spectrum of depolarized scattering.[1,2] The fluctuations of macroscopic quantities do not change the scalar character of the permittivity and therefore do not contribute to the depolarized scattering. This conclusion, however, is correct only when the spatial correlation of simultaneous fluctuations is neglected. When account is taken of spatial correlation, the macroscopic fluctuations disturb the scalar character of the permittivity and lead to the depolarized scattering. In the spectrum of depolarized scattering, as will be shown below, three narrow lines appear. Two of them are caused by the scattering by acoustic fluctuations, and their position and width of the spectrum coincide with the MB doublet. The third, non-shifted line is caused by the fluctuations of the transverse velocity (viscous waves) in contrast to the central (Rayleigh) scalar-scattering line caused by the entropy fluctuations. Therefore the width of the central line of the depolarized scattering is determined by the viscosity of the liquid and not by the thermal diffusivity as in scalar Rayleigh scattering.

Generally speaking, the influence of spatial correlation on light scattering is small and is determined by the parameter $a/\lambda \ll 1$. Therefore the intensity of the narrow lines in the depolarized scattering is much less than in the scalar scattering. It turns out that the ratio of intensities at the maxima of the narrow lines of the depolarized and scalar scattering is of the order of $(a/\lambda)^2$, i.e., of the same order as the ratio of the intensities at the maxima of wing and of the narrow lines of the scalar scattering. Thus the intensity in the maxima of the narrow lines of depolarized light is of the same order of magnitude as the intensity in the wing maximum. The spectral width of the narrow lines, as before, is $(\lambda/a)^2$ times less than the wing width. Therefore the contribution of the narrow lines to the integrated intensity of depolarized scattering is negligibly small.

The spectrum of depolarized light scattering has been investigated theoretically in many works.[3-6] The main attention was paid to the nature of a wing, i.e., of the spectrum of scattering in a region of large frequency shifts $\Delta\omega \sim \Delta\omega_w \approx 1/\tau$, which is caused by microscopic fluctuations. In this connection the temporal dispersion of liquid properties, neglecting the effect of spatial correlation of fluctuations, have been taken into account by Leontovich[3] and later more completely by Rytov.[5,6] This approach is justified in the region of sufficiently large frequency shifts, where $\Delta\omega\tau \sim 1$ and the quantity $ka \sim a/\lambda \ll 1$ is always small. The formal use of the formulas obtained by Leontovich[3] in the region of the fine structure of scattering leads to singularities of both at the MB frequencies $\Delta\omega = \pm\omega_{\text{MB}}$ and at the non-shifted frequency $\Delta\omega = \omega - \omega_0 = 0$, except that dips appear in place of the maxima of the intensity. It should be noticed that the use of the above mentioned formulas[3] as well as of the corresponding expressions from the works of Rytov[6] in the region of small frequency shifts $\Delta\omega \leqslant \omega_{\text{MB}}$ is not valid since $\omega_{\text{MB}} \propto k$ and therefore the expansion in the parameter $ka \sim a/\lambda$, i.e., the spatial correlation, must be taken into account along with the expansion in small $\Delta\omega\tau$. Moreover, at small $\Delta\omega\tau \leqslant \omega_{\text{MB}}/\Delta\omega_{\text{K}} \ll 1$ only the spatial correlations must actually be taken into account. The point is that the expansion in the parameter $\Delta\omega\tau$ means allowance for the dependence of the permittivity on time derivatives of quantities that determine the state of liquid. However for small $\Delta\omega\tau \ll 1$ the hydrodynamic approximation is valid and the hydrodynamic equations express the time derivatives of all the hydrodynamic quantities in terms spatial derivatives (but not conversely).

1. NONLOCAL CORRECTIONS TO THE PERMITTIVITY

Light scattering is determined by fluctuations of the permittivity tensor $\delta\varepsilon_{ik}(\mathbf{r}, t)$. In the macroscopic description, an arbitrary nonequilibrium liquid state is determined completely by specifying the hydrodynamic quantities, such as the pressure $P(\mathbf{r}, t)$, temperature $T(\mathbf{r}, t)$, and liquid velocity $\mathbf{v}(\mathbf{r}, t)$. In this approximation the fluctuations of the permittivity must be expressed through these hydrodynamic quantities.

In the zeroth approximation in a/λ this dependence is local in space and in time, i.e., $\delta\varepsilon_{ik}(\mathbf{r}, t)$ is determined by the values of the hydrodynamic quantities at the same point in space and at the same moment of time. Because of the scalar character of pressure and temperature, the fluctuations of these parameters in the zeroth approximation do not violate the scalar character of the permittivity and therefore do not contribute to the depolarized scattering of interest to us. As regards the velocity, in the local approximation the scattering by velocity fluctuations is connected, in principle, with the appearance of additional terms in the material electrodynamic equations which connect **D** and **B** with **E** and **H** (see Ref. 7, p. 310):

$$D_i = \varepsilon_{ik} E_k + \frac{\varepsilon_0 - 1}{c} [\mathbf{v} \times \mathbf{H}]_i, \quad B_i = H_i - \frac{\varepsilon_0 - 1}{c} [\mathbf{v} \times \mathbf{E}]_i, \tag{1}$$

where ε_0 is the equilibrium value of the permittivity, $\varepsilon_{ik} = \varepsilon_0 \delta_{ik} + \delta\varepsilon_{ik}$ the permittivity in the non-equilibrium state at $\mathbf{v} = 0$. As is seen from Eq. (1), the relative values of these additional terms are of the order of u/c. Their relative contribution to the light scattering is of the order of $(u/c)^2$, where u is the characteristic velocity (sound velocity) in the liquid. The order of magnitude of the ratio u/c equals

$$\frac{\omega}{c} \frac{u}{\Delta\omega_{\text{w}}} \frac{\Delta\omega_{\text{w}}}{\omega} \sim ka \frac{\Delta\omega_{\text{w}}}{\omega} \sim ka \cdot 10^{-3} \ll ka;$$

here ω and k are the frequency and the wave vector of light, and $\Delta\omega_w$ is the wing width. Thus, the additional terms in (1) can be neglected since the allowance for the non-locality, as will be shown below, gives a much larger contribution, of relative order of $(ka)^2$, to the scattering.

In the next approximation the non-locality of the relationship between the permittivity $\delta\varepsilon_{ik}$ and the hydrodynamic quantities, i.e., the dependence on spatial and temporal derivatives, must be taken into account. In so doing it must be kept in mind that the time derivatives are expressed by the hydrodynamic equations through the spatial derivatives, so that actually only the spatial derivatives should be taken into account. Because of the pressure and the temperature are scalars, the contribution of their fluctuations to $\delta\varepsilon_{ik}$ is proportional to second derivatives of the type $\partial^2 P/\partial x_i \partial x_k$, i.e., it appears in the second-order approximation in $a/\lambda \sim ka$. In the first approximation $\delta\varepsilon_{ik}$ contains only terms with spatial derivatives of the velocity. In the general case these terms are of the form[7]

$$\delta\varepsilon_{ik}=M_0\frac{\partial v_l}{\partial x_l}\delta_{ik}+M_1\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}-\frac{2}{3}\frac{\partial v_l}{\partial x_l}\delta_{ik}\right)+iM_2\left(\frac{\partial v_i}{\partial x_k}-\frac{\partial v_k}{\partial x_i}\right), \tag{2}$$

where M_0, M_1, M_2 are some constants (more accurately, they are functions of the light frequency ω).

Since in the future we will be interested only in the nonscalar part of $\delta\varepsilon_{ik}$, we can use in place of (2) the relation

$$\delta\varepsilon_{ik}=M_1\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)+iM_2\left(\frac{\partial v_i}{\partial x_k}-\frac{\partial v_k}{\partial x_i}\right). \tag{3}$$

The quantities M_1 and M_2 are the known dynamo-optic Maxwell constants[1,8] (more accurately, the combination of the quantities M_1 and M_2 is called the Maxwell constant M, namely $M=(M_1^2+M_2^2)^{1/2}$ [Ref. (8)]. In a nonabsorbing medium the constants M_1 and M_2 are real.

2. FINE STRUCTURE OF DEPOLARIZED SCATTERING

One can represent an arbitrary velocity field $\mathbf{v}(\mathbf{r}, t)$ in the form of the sum $\mathbf{v}_l+\mathbf{v}_t$ of a longitudinal (curl $\mathbf{v}_l=0$) and a transverse (div $\mathbf{v}_t=0$) part. The longitudinal velocity fluctuations are accompanied by the isentropic pressure fluctuations and therefore represent acoustical waves. The transverse velocity fluctuations are strongly damped viscous waves described by the hydrodynamic equation $\dot{\mathbf{v}}_t=\nu\Delta\mathbf{v}_t$, where $\nu=\eta/\rho$ is the kinematic viscosity, and η and ρ are the shear viscosity and the density of the liquid. The natural frequency of these waves $i\nu q^2$($q=|\mathbf{k}'-\mathbf{k}|$, $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident ($\mathbf{k}$) and the scattered ($\mathbf{k}'$) light waves) is pure imaginary, therefore the scattering by the transverse velocity fluctuations results in the appearance of a nonshifted line, the width of which is

$$\nu q^2=4\nu\frac{\omega^2}{c^2}\varepsilon_0\sin^2\frac{\theta}{2};$$

here θ is the scattering angle (the angle between $\mathbf{k}$ and $\mathbf{k}'$).

We will be interested in the spectrum of depolarized scattering, i.e., in the spectral intensity of the light scattered at the angle $\theta=90°$ and polarized along the x axis, which coincides with the direction of propagation of the incident light. It is clear from the above that scattering by the transverse velocity fluctuations gives a non-shifted Lorentz-form line

$$I_x^{(t)}(\omega)=\frac{I_0^{(t)}}{\pi}\frac{\nu q^2}{(\omega-\omega_0)^2+(\nu q^2)^2}. \tag{4}$$

Here ω_0 is the incident light frequency, and $I_0^{(t)}$ is the integrated intensity of the depolarized (i.e., polarized along the x axis) scattering by transverse velocity fluctuations. Therefore the problem reduced to the calculation of the intensity $I_0^{(t)}$ of the scattered light caused by the permittivity variations described by Eq. (3).

The projection of the scattered-wave field on the x axis is determined by the known formula[7]

$$E_x'=\frac{e^{ikR_0}\omega_0^2}{4\pi R_0 c^2}G_x, \tag{5}$$

where

$$G_x=E_{0y}\int\delta\varepsilon_{xy}e^{-i\mathbf{q}\mathbf{r}}\,dV+E_{0z}\int\delta\varepsilon_{xz}e^{-i\mathbf{q}\mathbf{r}}\,dV. \tag{6}$$

Here $\mathbf{R}_0$ is the radius vector from some point inside the scattering volume V to the point of the field observation; $\mathbf{E}=\mathbf{E}_0e^{i\mathbf{k}\cdot\mathbf{r}}$ is the electric field of the incident wave, and $\mathbf{q}=\mathbf{k}'-\mathbf{k}$. The x axis is chosen along the vector $\mathbf{k}$ and the y axis along $\mathbf{k}'$.

Substituting the expression (3) for $\delta\varepsilon_{ik}$ in (6) and integrating by parts (after which we can put, as usual, $\mathbf{q}=0$ in the exponent), we obtain

$$\begin{aligned}G_x={}&VikE_{0y}\{(M_1+iM_2)\bar{v}_x-(M_1-iM_2)\bar{v}_y\}\\&-VikE_{0z}(M_1-iM_2)\bar{v}_z,\end{aligned} \tag{7}$$

where $\bar{\mathbf{v}}=V^{-1}\int\mathbf{v}dV$ is the velocity averaged over the volume.

The transverse acoustic wave can have two independent directions of polarization in the plane perpendicular to $\mathbf{q}=\mathbf{k}'-\mathbf{k}$. It is convenient to choose the z axis and perpendicular to it the ξ axis as the directions of polarization, so that $v_x=v_y=v_\xi/2^{1/2}$. The intensity of the scattered light is determined by the square averaged over fluctuations, or the modulus of the quantity

$$\langle|E_x'|^2\rangle=\frac{\omega^4\langle|G_x|^2\rangle}{16\pi^2R_0^2c^4}.$$

Substituting the value of G_x from Eq. (7) and using the equations[9]

$$\langle\bar{v}_z^2\rangle=\langle\bar{v}_\xi^2\rangle=T/\rho V,\quad\langle\bar{v}_z\bar{v}_\xi\rangle=0,$$

where T and ρ are the equilibrium values of temperature and density of the liquid, we will get for the integrated intensity of depolarized scattering

$$I_0^{(t)}=I_0V\frac{\omega^4Tk^2}{16\pi^2c^4R_0^2\rho}\{2M_2^2\cos^2\varphi+(M_1^2+M_2^2)\sin^2\varphi\}, \tag{8}$$

where I_0 is the incident light intensity, and φ is the angle between the direction of polarization of the incident light $\mathbf{E}_0$ and the vector $\mathbf{k}'$.

The second possible geometry of an experiment in which there is no scalar scattering at the observation angle $\theta = 90°$ is as follows: the incident wave is propagated along the x axis and is polarized along the y axis, i.e., $\mathbf{E}_0 = \mathbf{E}_{0y}$; as previously, the scattering is observed along the y-axis. Then the scattered-wave-field projection on the z axis does not have a scalar component and can be easily calculated from Eqs. (3) and (6). In this case the final expression for the integrated intensity $I_{0z}^{(t)}$ of the depolarized scattering is of the form:

$$I_{0z}^{(t)} = I_0 V \frac{\omega^4 T k^2}{16\pi^2 c^4 R_0^2 \rho} (M_1^2 + M_2^2). \tag{8'}$$

The spectral dependence of the scattering is described, naturally, by Eq. (4) as before.

Let us consider now scattering by fluctuations of the longitudinal velocity $\mathbf{v}_l$. Because of above-mentioned acoustic character of these fluctuations, they produce in the depolarized scattering doublet lines whose position and form do not differ from the ordinary MB doublet in scalar scattering. The scattering spectrum is described by a Lorentz curve

$$I_x^{(l)}(\omega) = \frac{I_0^{(l)}}{\pi} \frac{\gamma q^2}{(\omega - \omega_0 \pm uq)^2 + (\gamma q^2)^2}, \tag{9}$$

$$\gamma = \frac{1}{2\rho}\left[\frac{4}{3}\eta + \zeta + \varkappa\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right], \tag{9'}$$

and $\eta, \zeta, \varkappa$ are respectively the first and the second viscosity coefficients and the thermal conductivity coefficient; C_v and C_p are the heat capacities.

The calculation of the integrated intensity $I_0^{(l)}$ is similar to that of $I_0^{(t)}$ with the only difference that in this case $\mathbf{v}_l$ is directed along the vector $\mathbf{q}$. Finally, for the quantity $I_0^{(l)}$ we obtain

$$I_0^{(l)} = \frac{I_0 V \omega^4 T k^2}{16\pi^2 c^4 R_0^2 \rho} 2M_1 \cos^2\varphi; \tag{10}$$

here, as usual I_0, is the incident-light intensity and φ is the angle between $\mathbf{E}_0$ and $\mathbf{k}'$. It is easy to see that in the second considered variant of the experimental geometry there is no scattering by longitudinal velocity fluctuations, since the polarization direction of the scattered wave (along the z axis) is perpendicular to the vector q and therefore to the longitudinal velocity vector $\mathbf{v}_l$ which is directed along q.

3. DISCUSSION

Thus, the addition, described by Eq. (3), to the permittivity of terms with spatial derivatives of velocity produces in the depolarized part of the scattering a triplet of lines whose integrated intensities are determined by Eqs. (8) and (10) and whose spectrum is given by Eqs. (4) and (9) respectively.

For comparison with the experimental data it is more convenient to use the value of the scattering coefficient[1] at the scattering angle $\theta = 90°$

$$R_{90°} = R_0^2 I / V I_0.$$

There are data in the literature[1,8] on the birefringence constant M in a radiant flux, given with allowance for M_2 by[8] $M = (M_1^2 + M_2^2)^{1/2}$. By way of estimate let us, assuming $\varphi = 0$, consider the scattering coefficient determined by the Eq. (10) multiplied by the factor R_0^2/VI_0. Substituting in Eq. (10) the values of $M \approx M_1$ for benzene, toluene, and carbon tetrachloride from Ref. 8, we obtain the value of $R_{90°}(10)$ and the value of the coefficient $R_{90°}^{\text{an}}$ of scattering by anisotropy fluctuations for these substances from Ref. 1; for their ratios we obtain $\alpha(\lambda = 435.8\text{ nm}) = R_{90°}(10)/R_{90°}^{\text{an}} \approx 10^{-4}$ for benzene and toluene and $\alpha(\lambda = 435.8\text{ nm}) \approx 2.5 \times 10^{-5}$ for carbon tetrachloride. From this it is seen that the integrated intensity of the depolarized scattering by velocity-gradient fluctuations is a small part ($\sim 10^{-4} - 10^{-5}$) of the integrated intensity of the entire depolarized wing. However, the intensities at the maxima of the depolarized MB doublet and the wing intensities at these frequencies can already be of the same order of magnitude: 0.01–1 since the ratio of the widths of the MB components to the depolarized-wing width is usually[1] $\sim 10^{-2} - 10^{-4}$. (For example, this ratio is $\sim 10^{-2}$ for benzene and $\sim 10^{-3}$ for carbon bisulfide.)

As to the depolarized component (4) at the non-shifted frequency ω_0, its integrated intensity (8) has the same order of magnitude as (10), and its width is less than that of the MB components in the ratio η/ξ, which for the majority of polyatomic liquids has the order of magnitude of 0.1 – 10 (Ref. 1). Therefore in this case one can expect even better ratio (0.1 – 10) of the scattering intensities determined by Eq. (8) to the depolarized-wing intensity in vicinity of $\omega \approx \omega_0$. The width of this component, determined by the ratio η/ρ, is then of the same order or even less than the width of the scalar central component.

The depolarized scattering spectrum described by Eqs. (4), (8) and (9), (10) contains two real constants: M_1 and M_2. It is possible to measure both quantities by varying the angle φ between the polarization direction of the incident wave $\mathbf{E}_0$ and the scattering direction $\mathbf{k}'$. We note that, as mentioned above, the constants M_1 and M_2 in the combination $M = (M_1^2 + M_2^2)^{1/2}$ are determined in experiments on the birefringence in a radiant flux, and M_2 can be obtained independently in the following way. Upon uniform rotation of the liquid as a whole $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$ ($\mathbf{\Omega}$ is the angular velocity of rotation), the symmetric term in (3) turns into zero and only the antisymmetric term $\delta\varepsilon_{ik} = -2iM_2 e_{ikl}\Omega_l$ remains, where e_{ikl} is the unit antisymmetric pseudotensor. It is known[7] that a medium with such a permittivity is gyrotropic with a gyration vector $\mathbf{g} = -2M_2\mathbf{\Omega}$. When plane polarized light propagates in such a medium the polarization plane rotates and the angle of rotation per unit path length equal

$$\beta = -\frac{\omega}{cn} M_2 |\mathbf{\Omega}| \cos\gamma, \tag{11}$$

where $n = \varepsilon^{1/2}$ is the index of refraction of the medium and γ is the angle between the vectors $\mathbf{k}$ and $\mathbf{\Omega}$.

Let us estimate the value of β, assuming[8] $M_2 \sim M_1 \sim 10^{-(12-13)}$ sec, $\cos\gamma = 1$, $\lambda = 5 \times 10^{-5}$ cm, $\Omega = 100$ Hz, and $n = 1.5$. Substituting these values in (11) we obtain $\beta \approx 10^{-(5-6)}$ rad/cm; this value can be measured experimentally.

Thus, even if M_2 is 10–100 times less than M_1 it can be determined in principle by measuring the rotation of the polarization plane of the light in a uniformly rotating liquid.

Relatively recently the rotation of the polarization plane of a light wave propagating through a rotating transparent medium (Pockels-glass SF-57) was detected experimentally.[10] In the experiment of Jones[10] $\Omega = 100$ Hz, $n = 1.84$, the light path-length in the medium $l = 40$ cm, the rotation of polarization plane $\varphi \sim 10^{-6}$ rad, which corresponds to $\beta \approx 2.5 \times 10^{-8}$ rad/cm. It is seen from this that in this case M_2 is of the order of $M_2 \approx 10^{-(2-3)} M_1 \sim 10^{-(14-16)}$ sec, where the values of M_1 values were taken for simple liquids in the transparency region.[8]

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The role of solitons in strong turbulence

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We show that in those cases where it is difficult to subdivide the scale of the turbulence, equilibrium turbulence is possible. Stratification into phases is typical for such turbulence; the role of liquid drops is played by solitons. We find the parameters for the weak-turbulence spectrum which is in equilibrium with a soliton of a given amplitude. We consider the example of a system which allows soliton solutions, but which in not stratified into phases.

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INTRODUCTION

We clarify in the present paper the reason why in strong turbulence the particular solutions in the form of solitons play such an important role. We show that the solitons, though particular solutions, are important solutions, since in a number of situations the typical behavior of strong turbulence consists in a decomposition into two phases-weakly non-linear spectrum and solitons.

The dispersion of the waves for which soliton solutions exist is usually such that the energy transfer to the short-wavelength region is difficult, and the wave system is able to evolve for a long time without damping. This fact makes it possible to arrive at a state of thermodynamic equilibrium, characterized by a collective temperature, and one can obtain exactly the shape of the spectrum in equilibrium with the solitons. The solitons play here the role of the liquid phase droplets, while the binding energy of the waves in the soliton plays the role of the vaporization heat. The reason why the soliton solution can be singled out is that the soliton guarantees the largest binding energy and, hence, also the largest entropy of the weakly nonlinear spectrum. We stress that in such considerations we are dealing with thermodynamic equilibrium only in the wave degrees of freedom; there is no equilibrium between the waves and the medium.

The picture painted above is illustrated by the example of waves close to sound waves. In those cases where a subdivision of the scales is allowed and thermodynamic equilibrium has no meaning, solitons are not necessarily attractive solutions, as is illustrated by the example of turbulence in the Rudakov-Tsytovich equation.[1]

EQUILIBRIUM OF WAVES WITH SOLITONS

We shall assume that the waves have a frequency $\omega_k = c_s|\mathbf{k}| + \delta\omega_k$, where c_s is the wave velocity when we neglect dispersion, while the dispersion correction $\delta\omega_k$ is small ($\delta\omega_k \ll \omega_k$). Initially we consider weakly non-linear spectra for which the non-linear corrections are smaller than the dispersive ones. When the dispersion is almost that of sound, only almost collinear triads of waves interact, so that there appears apart from the energy and momentum conservation laws a new conservation law—we shall call it the law of conservation of "longitudinal momentum"

$$p_{\parallel} = \sum |\mathbf{k}| n_k,$$

where n_k is the occupation number. When multiplied by c_s, the longitudinal momentum is nearly the same as the energy