It is known that in a system of interpenetrating opposite ion beams moving at a velocity $V \lesssim C_s$, one can expect an aperiodic $\mathrm{Re}\,\omega = 0$ ion-acoustic instability. In our case this condition is violated already during the formation of a well when its depth becomes $\varphi \gtrsim T_e/e$. Moreover, interpenetration of the opposite beams is not observed experimentally in the $x = 0$ plane, i.e., the waves are not excited by the two-stream instability but by the initial bunching of ions whose flight time to the plane is approximately the same because of the approximately parabolic nature of the potential near the bottom of the well.

As shown above, the nature of the motion of ions trapped in a potential well of an ion beam is largely governed by the space charge of the ions. The phase trajectories of the ions then differ qualitatively from the case of motion of noninteracting trapped particles. In particular, the ion space charge prevents the appearance of multivelocity motion typical of the case of free oscillations of charges in the well field. The phase picture of these motions is disturbed already after the first quarter of the period ω when the trapped ions first collect near the bottom of the well. The rapid accumulation of this space charge results in generation of nonlinear ion waves diverging to the walls at a supersonic velocity. The process of wave creation is characterized by a frequency close to the ion plasma value, which corresponds to the density near the bottom of the potential well. The resultant bunches of the ion density resemble the formation of a sequence of ion-acoustic solitons as a result of decay of a low-frequency large-amplitude perturbation in a plasma.[3]

It follows that the dynamics of filling of a current discontinuity in an ion beam is associated with the excitation of a strongly nonlinear wave process caused by the interaction of the trapped ions.

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Translated by A. Tybulewicz

Pair production by photons in a dense pinch

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The production of electron-positron pairs by photons in the electromagnetic field of a strongly compressed discharge current channel is investigated. The dependence of the cross section on the pinch radius is connected mainly with the need of transfering the momentum to the electromagnetic field. With increasing channel radius, the cross section decreases rapidly because of the impossibility of momentum transfer. For high-energy gamma quanta, the main contribution is made by the region of distances far from the axis. In this case the cross section is determined by the logarithmic asymptotic expression for the field of a current regarded as a charged current carrying filament, and is independent of the radius. It is shown that with the aid of the pair-production process it is possible in principle to resolve the spatial structure of the pinch in the angstrom range, and consequently to determine experimentally whether the strong compression to the linear-atom state, predicted by the theory of equilibrium and radiation of a strong-current plasma, can be realized in pinches.

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1. INTRODUCTION

An analysis of the phenomena that accompany the pinch effect in a high-current diode,[1-5] based on the theory of equilibrium[6] and collisionless radiation[7,8] of a dense high-current plasma, leads to the conclusion that the compression of the current channel can proceed up to degeneracy of the electrons, i.e., up to the density of the condensed state at high temperatures. The presence of such an unusual state of matter, the so-called linear atom,[9] makes it possible to explain consistently an aggregate of phenomena that accompany the pinch efect. Nonetheless, to prove experimentally the presence of linear atoms in the pinches, direct measurements must be made of the channel radius in the angstrom band during the nanosecond durations of the supercompressions.

The traditional methods of plasma study by means of its own radiation do not permit direct measurement of the radius of the current channel in the angstron band. The difficulty of measuring the radius of a pinch compressed to the state of electron degeneracy are similar to those encountered, for example, when attempts are made to determine the radius of the hydrogen atom from its emission. If the high-current compression reaches the state of electron degeneracy, i.e., atomic dimensions, then the electrons in the field of the current become likewise subject to the laws of quantum electrodynamics. In attempts to find a tool capable of spatially resolve the current channel in the angstron range, it is natural to turn to interactions between extraneous particles and the electromagnetic field of the pinch.

We consider in this paper pair production by photons in the electromagnetic field produced by the current-channel plasma. To simplify the formulas we use a system of units with $\hbar = c = 1$.

In the pair-production process, the recoil momentum is transferred to the external field. It is obvious that the momentum q can be transferred to the field provided that the characteristic field-variation scale $r_0 \lesssim 1/q$. The minimum momentum that must be transferred to the external field for pair production by a photon of frequency ω is of the order of

$$q \sim m^2/\omega.$$

It is easily seen that at $\omega \gtrsim 2m$ the momentum transfer is of the order of m. This means that the characteristic size of the reaction region is of the order of $1/m \sim 10^{-11}$ cm and consequently the scale of variation of the external field should be of the same order. On the other hand, to measure a radius $r_0 \sim 10^{-8}$ cm by the pair-production method it is necessary for the gamma quantum to have an ultralativistic energy $\omega \gg m$ such that

$$\lambda \equiv m^2 r_0/\omega \lesssim 1. \quad (1.1)$$

The characteristic time of the process is of the order of the time of flight of the photon through the reaction zone, $\sim 10^{-20}$ sec. The state of the pinch does not change significantly within so short a time, so that the photon "feels" the true instantaneous field produced by the plasma charges. We separate from the true microscopic field $\mathscr{A}_\mu$ its value averaged over the ensemble of charges $A_\mu = \langle \mathscr{A}_\mu \rangle$:

$$\mathscr{A}_\mu = A_\mu + a_\mu, \quad \langle a_\mu \rangle = 0. \quad (1.2)$$

A_μ is the mean field of the collective interaction of the charges, and a_μ describes the field fluctuations, including those of the Coulomb field near the nuclei.

In first-order perturbation theory in the external field, the cross section of the process is expressed by a bilinear combination of $\mathscr{A}_\mu$:

$$d\sigma = d\sigma_{\mu\nu} \mathscr{A}_\mu \mathscr{A}_\nu^*.$$

By virtue of (1.2) the cross section averaged over the various charge configurations is represented in the form of a sum of cross sections

$$d\sigma = d\sigma_{\mu\nu}(A_\mu A_\nu + \langle a_\mu a_\nu^* \rangle) \equiv d\sigma_A + d\sigma_a, \quad (1.3)$$

where $d\sigma_A$ is the cross section of the process in the mean field, and $d\sigma_a$ is the cross section for pair production on the field fluctuations, including Coulomb fields near individual charges. If a large number are simultaneously present in the reaction zone and the mean field varies substantially over these distances, the main contribution to the pair production is made by the mean field—the first term in (1.3). In the opposite limiting case of a slowly varying mean field $\lambda \gg 1$, the cross section $d\sigma_A$ decreases rapidly with increasing r_0, and the main contribution to the cross section is made by the second term in (1.3).

We shall assume the current channel to be cylindrically symmetrical and uniform along the current and in azimuth. For the fields produced by the pinch plasma, the nonzero components of the 4-vector of the potential A_μ of the average field of the collective interaction are A_0 and A_z. Near the current axis, A_μ depends quadratically on the radius, while at large distances from the axis it increases logarithmically:

$$A_0(r) = -2\rho \ln r, \; A_z(r) = -2(I/c) \ln r, \; r \gg r_0. \quad (1.4)$$

Here ρ is the linear charge density and I is the pinch current.

In the transition region $r \sim r_0$ (r_0 is the radius of the current channel) the $A_\mu(r)$ dependence is determined by the plasma equilibrium conditions and depends on the concrete parameters of the pinch. In the particular case $e|\rho| = 2T_i$, $|e(\rho - \beta I)| = 2T_\perp (T_\perp = T_e(1-\beta^2)^{1/2}$; T_i and T_e are the respective termperatures of the ions and electrons in the coordinate frames moving with the charges) we get the so-called Bennett distribution[11]

$$A_\mu(r) = \bar{A}_\mu \ln(1 + r^2/r_0^2), \quad \bar{A}_0 = |\rho|, \; \bar{A}_z = |I/c|, \quad (1.5)$$

where $|\rho| < |\beta| I|$, $\beta \approx v_0/v$, and v_0 is the electron drift velocity. The energy of interaction of one charge with the field $A(r)$ is of the order of

$$e\bar{A}_z = mc^2(eI/mc^3). \quad (1.6)$$

Since $mc^3/e = 17$ kA, it follows from (1.6) that the characteristic energy scale of the electrons in the field of the pinch is $\gtrsim mc^2$ if $I \gtrsim 17$ kA.

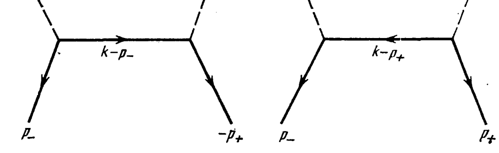
Since specialists engaged in plasma physics hardly ever deal with quantum electrodynamics, we present below a detailed calculation of the cross section of the process of pair production by a photon in the pinch field, using the diagram method of perturbation theory. The condition under which the external field can be regarded as a small perturbation is

$$e|\bar{A}| \ll \omega. \quad (1.7)$$

From this it follows, in particular, that for currents $I \gtrsim 17$ kA the field of the collective interaction can be regarded as a perturbation only in the ultrarelativistic case $\omega \gg m$.

2. PAIR PRODUCTION BY A PHOTON IN A CYLINDRICALLY SYMMETRICAL FIELD

The production of an electron and positron with momenta p_- and p_+ by a photon with 4-momentum k corresponds in second-order perturbation theory (in first order in the external field) to two Feynman diagrams:



The matrix element is written in the form[10]

$$M_{fi}=-e^2\mathscr{A}_\alpha(\mathbf{q})(4\pi)^{1/2}e_\mu{}^*\bar{u}(p_-)\cdot$$
$$\times\left(\gamma_\mu\frac{\hat{k}-\hat{p}_-+m}{(k-p_-)^2-m^2}\gamma_\alpha+\gamma_\alpha\frac{\hat{k}-\hat{p}_++m}{(k-p_+)^2-m^2}\gamma_\mu\right)u(-p_+), \quad (2.1)$$

where $q=p_-+p_+-k$ is the 4-momentum transferred to the external field, e_μ is the photon-polarization 4-vector, $k\equiv k_\mu\gamma_\mu, u$ is the bispinor amplitude of the wave function, $\bar{u}=u^*\gamma_0$, and γ_μ are the Dirac γ matrices.

The differential cross section of the production of an electron in the momentum interval $(\mathbf{p}_-, \mathbf{p}_-+d\mathbf{p}_-)$ and a positron in the momentum interval $\mathbf{p}_++d\mathbf{p}_+$ is of the form[10]

$$d\sigma=|M_{fi}|^2\delta(\omega-\varepsilon_+-\varepsilon_-)\frac{d^3p_-\,d^3p_+}{8\omega\varepsilon_+\varepsilon_-(2\pi)^5}. \quad (2.2)$$

Averaging over the polarization of the primary photon and summation over the polarizations of the secondary fermions yields

$$d\sigma=d\sigma_{\alpha\beta}\mathscr{A}_\alpha\mathscr{A}_\beta{}^*,\quad d\sigma_{\alpha\beta}=\frac{e^4\,d^3p_-\,d^3p_+}{(2\pi)^4\omega^3\varepsilon_+\varepsilon_-}\delta(\omega-\varepsilon_+-\varepsilon_-)S_{\alpha\beta}. \quad (2.3)$$

Here

$$S_{\alpha\beta}=-{}^1/_{32}\,\mathrm{Sp}\{K_{\mu\alpha}(\hat{p}_+-m)\bar{K}_{\beta\mu}(\hat{p}_-+m)\},$$

Sp stands for the operation of calculating the spur of the matrix

$$K_{\mu\alpha}=\varkappa_-^{-1}\gamma_\mu(\hat{k}-\hat{p}_-+m)\gamma_\alpha+\varkappa_+^{-1}\gamma_\alpha(\hat{k}-\hat{p}_++m)\gamma_\mu,$$

$\bar{K}=\gamma_0K^+\gamma_0$, $\varkappa_\pm=(kp_\pm)/\omega=\varepsilon_\pm-\mathbf{n}\mathbf{p}_\pm$, $\varepsilon_\pm$ are the energies of the positron and electron, and $\mathbf{n}=\mathbf{k}/\omega$ is the photon propagation direction.

Calculation of the spur yields

$$S_{\alpha\beta}=p_{+\alpha}p_{-\beta}\left[\frac{2(p_+p_-)}{\varkappa_+\varkappa_-}-\omega\left(\frac{1}{\varkappa_+}+\frac{1}{\varkappa_-}\right)-m^2\left(\frac{1}{\varkappa_+^2}+\frac{1}{\varkappa_-^2}\right)\right]$$
$$+\frac{k_\alpha p_{+\beta}}{\varkappa_-}\left[\omega+\frac{m^2}{\varkappa_-}-\frac{(p_+p_-)}{\varkappa_+}\right]+\frac{k_\alpha p_{-\beta}}{\varkappa_+}\left[\omega+\frac{m^2}{\varkappa_+}-\frac{(p_+p_-)}{\varkappa_-}\right]$$
$$-k_\alpha k_\beta\frac{m^2}{\varkappa_+\varkappa_-}+p_{+\alpha}p_{+\beta}\frac{\omega}{\varkappa_+}+p_{-\alpha}p_{-\beta}\frac{\omega}{\varkappa_-}+\frac{1}{2}\delta_{\alpha\beta}\left[\left(\frac{m^2q^2}{2}-\omega^2\varkappa_+\varkappa_-\right)\right.$$
$$\left.\times\left(\frac{1}{\varkappa_+^2}+\frac{1}{\varkappa_-^2}\right)-\frac{q^2(p_+p_-)}{\varkappa_+\varkappa_-}\right], \quad (2.4)$$

(p_+p_-) is the scalar product of the 4-vectors.

The component S_{00} of the tensor (2.4) yields the well known result of Bethe and Heitler for the cross section of pair production in the electrostatic field of the nucleus. Pair production by photons in a nuclear field is a process described in detail in the literature. We shall therefore consider in detail the process of pair production by a photon in the mean field A_μ and calculate $d\sigma_A$ in (1.3).

In the case of uniformity along the current channel, the Fourier component of the field $A_\mu(\mathbf{q})$ contains a δ-function of q_z. Separating in $A_\mu(r)$ the amplitude $\bar{A}_\mu$ from the coordinate function $F_\mu(r)$

$$A_\mu(r)=\bar{A}_\mu F_\mu(r).$$

We obtain for the Fourier component $A_\mu(\mathbf{q})$

$$A_\mu(\mathbf{q})=\int e^{i\mathbf{q}\mathbf{r}}A_\mu(r)d^3r=(2\pi)^2\delta(q_z)A_\mu(q_\perp), \quad (2.5)$$
$$A_\mu(q_\perp)=\bar{A}_\mu f_\mu(q_\perp),\quad f_\mu(q_\perp)=\int_0^\infty F_\mu(r)J_0(q_\perp r)r\,dr.$$

Here $\mathbf{q}_\perp$ is the projection of the vector $\mathbf{q}$ on the plane perpendicular to the current direction.

In the calculation of $f_\alpha(q_\perp)$ we encounter a certain difficulty; the integral (2.5) diverges for a potential having an asymptotic form (1.4) at large distances. To eliminate this difficulty we must make use of gauge invariance. We note that addtion of an arbitrary constant C to $A_\alpha(r)$ adds to $A_\alpha(\mathbf{q})$ a term $(2\pi)^3C\delta(\mathbf{q})$ that differs from zero only at $\mathbf{q}=0$. This operation does not manifest itself, consequently, in processes that require the transfer of a finite momentum $\mathbf{q}\neq0$ to the external field. By suitable choice of the constant C we can express $F_\alpha(r)$ in the form

$$F_\alpha(r)=-\int_r^R\frac{dF_\alpha}{dr}dr. \quad (2.6)$$

Letting $R\to\infty$, substituting (2.6) in (2.5), and changing the order of the integration, we get

$$f_\alpha(q_\perp)=\frac{1}{q_\perp}\int_0^\infty\frac{dF_\alpha(r)}{dr}J_1(q_\perp r)r\,dr. \quad (2.7)$$

The integral (2.7) converges for the potential (1.4) which is logarithmic at large distance.

As $R\to\infty$, however, the constant C also becomes infinite. If, however, we recognize that the length l of the current channel is finite, then it becomes clear that the asymptotic form (1.4) holds in the region $r_0\ll r\ll l$, and consequently $C\sim\ln(l/r_0)$.

In the particular case of the Bennett distribution (1.5) we have

$$f_\alpha{}^B(q_\perp)=\frac{2r_0}{q_\perp}K_1(q_\perp r_0),$$
$$K_1(z)=\begin{cases}1/z, & z\ll1,\\ (\pi/2z)^{1/2}e^{-z}, & z\gg1,\end{cases} \quad (2.8)$$

$K_1(z)$ is a Macdonald function. Inasmuch as in the case of a Bennett distribution the coordinate dependence of both components of the 4-vector of the field potential is the same, $f^B(q_\perp)$ does not depend on α.

It is seen from the concrete example of the Bennett distribution that the Fourier component of the field potential in the region $q_\perp r_0\gg1$ decreases exponentially with increasing radius r_0 of the current channel. This is due to the difficulty of transferring the momentum $q_\perp$ to the field in the case when the characteristic dimension r_0 of the region of the field variation is large compared with $1/q_\perp$. Of course, a rapid (but generally speaking not necessarily exponential) decrease of $f_\alpha(q_\perp)$ in the region $q_\perp r_0\gg1$ is a feature of potentials $F_\alpha(r)$ of arbitrary form that go over from a constant value at $r\ll r_0$ to a logarithmic growth ad distances $r\gg r_0$ far from the axis.

In the opposite limiting case of a thin charged current-carrying filament $q_\perp r_0\ll1$ the main contribution to the integral (2.7) is made by the region of distances $r\sim1/q_\perp\gg r_0$ far from the axis. In this case the Fourier component of the potential ceases to depend on r_0:

$$f_\alpha(q_\perp)=2/q_\perp^2,\quad q_\perp r_0\ll1. \quad (2.9)$$

This property is likewise not necessarily due to the Bennett distribution (1.5), and is only the consequence of the logarithmic behavior of the potential (1.7) at large distances from the current axis.

The independence of the field $A_\mu(r)$ of time and the uniformity along the current direction lead to conservation of the energy and of the momentum projection on the current direction, as is manifest by the appearance of $\delta(q_z)$ in (2.5). Assuming, as usual, the square of the δ function to mean

$$[\delta(q_z)]^2=(l/2\pi)\delta(q_z)$$

(l is the length of the current channel), we reduce the cross section $d\sigma_A$ (1.3) to the form

$$d\sigma_A=\frac{l}{2\pi}\frac{e^4 d^3p_- d^3p_+}{\omega^3\varepsilon_+\varepsilon_-}S_{\alpha\beta}A_\alpha(q_\perp)A_\beta(q_\perp)\delta(\omega-\varepsilon_+-\varepsilon_-)\delta(k_z-p_{+z}-p_{-z}). \tag{2.10}$$

Recognizing that the only nonzero components are A_0 and A_z, we can simplify somewhat the expression for $S_{\alpha\beta}$ (1.4) with the aid of the δ functions in (2.10). The final formula for the cross section of pair production in a cylindrically symmetrical collective-interaction mean field is

$$d\sigma_A=\frac{le^4 d^3p_- d^3p_+}{2\pi\omega^3\varepsilon_+\varepsilon_-}\left\{\frac{\mathbf{q}^2}{2\varkappa_+\varkappa_-}[(p_+A)^2+(p_-A)^2]+m^2\left[\frac{(p_+A)}{\varkappa_-}-\frac{(p_-A)}{\varkappa_+}\right]^2 -\frac{1}{2}A^2\left[\left(\omega^2\varkappa_+\varkappa_-+\frac{m^2\mathbf{q}^2}{2}\right)\left(\frac{1}{\varkappa_+^2}+\frac{1}{\varkappa_-^2}\right)-\frac{\mathbf{q}^2(p_+p_-)}{\varkappa_+\varkappa_-}\right]\right\}\delta(\omega-\varepsilon_+-\varepsilon_-)\delta(k_z-p_{+z}-p_{-z}). \tag{2.11}$$

Here
$A\equiv A_\mu(q_\perp)$, $\mathbf{q}^2=q_\perp^2=2[\omega(\varkappa_++\varkappa_-)-m^2-(p_+p_-)]$.

For further investigation it is convenient to introduce a coordinate system defined in the following manner. The z axis is directed along the current. The x axis is chosen such that the wave vector $\mathbf{k}$ of the photon lies in the xz plane and makes an angle α with the x axis (Fig. 1). Let $\mathbf{p}_\perp$ be the projection of the vector $\mathbf{p}$ on the xz plane, θ the angle between $\mathbf{p}$ and $\mathbf{p}_\perp$, φ the angle between $\mathbf{p}$ and the x axis, and $\zeta=\varphi-\alpha$ the angle between $\mathbf{p}_\perp$ and $\mathbf{k}$. Thus,

$$p_y=|\mathbf{p}|\sin\theta,\quad p_x=|\mathbf{p}|\cos\theta\cos\varphi,\quad p_z=|\mathbf{p}|\cos\theta\sin\varphi,\qquad d^3p=|\mathbf{p}|\varepsilon d\varepsilon\cos\theta d\theta d\zeta. \tag{2.12}$$

The momentum transferred to the field is expressed in terms of the coordinates $p=|\mathbf{p}|$, θ, and ζ in the form

$$-q^2=\mathbf{q}^2=2\{\omega(\omega-p_+\cos\theta_+\cos\zeta_+-p_-\cos\theta_-\cos\zeta_-)-m^2-\varepsilon_+\varepsilon_-+p_+p_-[\sin\theta_+\sin\theta_-+\cos\theta_+\cos\theta_-\cos(\zeta_+-\zeta_-)]\}. \tag{2.13}$$

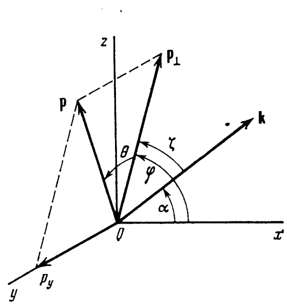


FIG. 1

3. ULTRARELATIVISTIC CASE

Since the Fourier component of the field is a rapidly decreasing function of the momentum transferred to the field, the main contribution to the cross section is made by the region of the minimum possible values of $q_\perp$. In the region of not too hard quanta $\omega\gtrsim 2m$, the momentum transferred to the field upon formation of the pair (2.13) is $q_\perp\sim m$, corresponding to a spatial scale of the order of 10^{-8} cm. To change to the scale of order 10^{-8} cm, which is of interest to us from the point of view of observing a linear atom in a pinch, it is necessary to turn to the relativistic case:

$$\omega\gg m. \tag{3.1}$$

It is known that under the condition (3.1) the electron and positron propagate in the laboratory frame in a narrow cone in the direction of the initial photon with an apex angle on the order of $m/\omega\ll 1$. Expression (2.13) for the momentum transferred to the field must be expanded in powers of the small angles and of m/ω up to terms of fourth order. We obtain

$$\frac{\mathbf{q}^2}{m^2}=(\vartheta_++\vartheta_-)^2+(\tau_++\tau_-)^2+\frac{m^2\omega^2}{4\varepsilon_+^2\varepsilon_-^2}(1+\tau_+^2+\vartheta_+^2)^2. \tag{3.2}$$

To simplify the notation, we have changed from the angles $\theta_\pm$ and $\zeta_\pm$ to the quantities

$$\vartheta_\pm=\varepsilon_\pm\theta_\pm/m,\quad \tau_\pm=\varepsilon_\pm\zeta_\pm/m.$$

It is seen from (3.2) that in the region $\vartheta_\pm\sim 1$ and $\tau_\pm\sim 1$ the quantity $\mathbf{q}^2$ is of the order of m^2. The last term of (3.2) is a small quantity of order $m^2/\omega^2\ll 1$. This term, however, is significant in the narrow region

$$(\vartheta_++\vartheta_-)^2+(\tau_++\tau_-)^2\sim m^2/\omega^2\ll 1, \tag{3.3}$$

in which the momentum transferred to the field becomes small: $\mathbf{q}^2\sim m^4/\omega^2\ll m^2$. In the region (3.3), accurate to $m^2/\omega^2\ll 1$, the angles ϑ_+ and ϑ_-, as well as τ_+ and τ_-, are equal in absolute value but are of opposite sign. However, ϑ_+ and τ_+ themselves are of the order of unity. We note that in the derivation of (3.2) the last term was obtained by using the condition (3.3).

The region (3.3) corresponds to the smallest momentum that must be transferred to the field, and makes the principal contribution to the pair-production section in the ultrarelativistic case. The characteristic spatial scale is in this case of the order of ω/m^2 and increases with the photon energy. In the ultrarelativistic case, depending on the photon energy, the parameter λ (1.1) can be larger as well as smaller than unity.

Relation (3.3) shows that the produced electron and positron are emitted at almost equal angles to the initial photon propagation direction, but on opposite sides. This is a general property of pair production by a photon in the ultrarelativistic case. It is connected only with the expansion (3.2) of the momentum transfer near its minimum value, and does not depend on the nature of the external field.

The specifics of the cylindrical symmetry of the field manifests itself in conservation of the momentum projection on the z axis. The corresponding δ function in (2.11) reduces in terms of the coordinates (2.12) to the form

$$\delta(k_z-p_{+z}-p_{-z})=\frac{1}{m\cos\alpha}\delta\left(\tau_++\tau_--\frac{m\omega}{2\varepsilon_+\varepsilon_-}(1+\vartheta_+^2)\operatorname{tg}\alpha\right),\quad \omega\gg m. \tag{3.4}$$

When (3.4) is taken into account, the angular dependence of the momentum (3.2) transferred to the field takes the form

$$\frac{\mathbf{q}^2}{m^2}=(\vartheta_++\vartheta_-)^2+\frac{m^2\omega^2}{4\varepsilon_+^2\varepsilon_-^2}[(1+\vartheta_+^2+\tau_+^2)^2+(1+\vartheta_+^2)^2\operatorname{tg}^2\alpha]. \tag{3.5}$$

In the ultrarelativistic limit (3.1) we have

$$\varkappa_\pm=(m^2/2\varepsilon_\pm)(1+\vartheta_\pm^2+\tau_\pm^2),$$
$$d^3p_-d^3p_+=m^4(p_-p_+/\varepsilon_-\varepsilon_+)\,d\varepsilon_-d\vartheta_-d\tau_-d\varepsilon_+d\vartheta_+d\tau_+.$$

Integration with respect to ε_- and τ_- is with the aid of δ functions. From (2.11) we get

$$\frac{d\sigma_A}{d\varepsilon_+\,d\vartheta_+\,d\tau_+\,d\vartheta_-}=\frac{le^4m^3}{2\pi\varepsilon_+\varepsilon_-\cos\alpha\,\omega^3}\left\{\frac{\mathbf{q}^2}{2\varkappa_+\varkappa_-}[(p_+A)^2+(p_-A)^2]\right.$$
$$\left.+m^2\left[\frac{(p_+A)}{\varkappa_-}-\frac{(p_-A)}{\varkappa_+}\right]^2-\frac{\omega}{2}\left(\frac{\varepsilon_+}{\varepsilon_-}+\frac{\varepsilon_-}{\varepsilon_+}\right)A^2\right\},\quad\omega\gg m. \tag{3.6}$$

Here $\omega=\varepsilon_++\varepsilon_-$.

In the particular case of the Bennett distribution (1.5) the Fourier component takes the form (2.8), and the cross section is equal to

$$\frac{d\sigma_A}{d\varepsilon_+\,d\vartheta_+\,d\tau_+\,d\vartheta_-}=\frac{2le^4m^3r_0^2K_1^2(q_\perp r_0)}{\pi\omega^3\varepsilon_+\varepsilon_-\cos\alpha\,\mathbf{q}^2}\left\{\frac{\mathbf{q}^2(\varepsilon_+^2+\varepsilon_-^2)}{2\varkappa_+\varkappa_-}(\bar{A}_0-\bar{A}_z\sin\alpha)^2\right.$$
$$\left.+m^2\left[\frac{(p_+\bar{A})}{\varkappa_-}-\frac{(p_-\bar{A})}{\varkappa_+}\right]^2-\frac{\omega^2}{2}\left(\frac{\varepsilon_+}{\varepsilon_-}+\frac{\varepsilon_-}{\varepsilon_+}\right)(\bar{A}_0^2-\bar{A}_z^2)\right\},\quad\omega\gg m. \tag{3.7}$$

Here $q_\perp=|\mathbf{q}|$ is given by (3.5).

We continue the analysis of the cross section of the process for the limiting cases of small and large radius r_0, using the Bennett distribution as the example. As noted in Sec. 2, the general properties of the pair-production process in the limiting cases $\lambda\ll1$ and $\lambda\gg1$ are not connected with the concrete dependence of the potential $A_\mu(r)$ in the region $r\sim r_0$.

4. CASE OF LARGE RADIUS $\lambda\gg1$

In the limiting case $\lambda=m^2r_0/\omega\gg1$ the Fourier component of the field is exponentially small (2.8), and this leads to exponential smallness of the pair-production cross section. If the function (2.8) is exponential, the condition $\lambda\gg1$ narrows down the range of angles that make the principal contribution to the cross section:

$$u\equiv\vartheta_++\vartheta_-\sim m/\omega\lambda^{1/2}\ll m/\omega,\quad\vartheta_+\sim\lambda^{-1/2}\ll1,\quad\tau_+\sim\lambda^{-1/2}\ll1.$$

This means that we can put $u=\vartheta_+=\tau_+=0$ in the pre-exponential factor. The dependence of the cross sections on the angles is determined by the exponential factor.

The energy distribution of the produced particles has a sharp maximum near the value $\varepsilon=\omega/2$ under the condition

1. We have:

$$\frac{d\sigma}{d\varepsilon_+\,d\vartheta_+\,d\tau_+\,du}=\frac{lr_0}{2}\frac{e^4(\bar{A}_0\sin\alpha-\bar{A}_z)^2}{m^3}\exp(-2q_\perp r_0),\quad\lambda\gg1, \tag{4.1}$$

where

$$q_\perp r_0=\frac{2\lambda}{\cos\alpha}+\frac{1}{4}\omega r_0\cos\alpha u^2+\frac{8\lambda}{\cos\alpha}\left(\frac{\varepsilon_+}{\omega}-\frac{1}{2}\right)^2+\frac{2\lambda}{\cos\alpha}\vartheta_+^2+2\lambda\cos\alpha\tau_+^2.$$

Integration of (4.1) with respect to u yields the distribution with respect to the positron parameters

$$\frac{d\sigma_A}{d\varepsilon_+\,d\vartheta_+\,d\tau_+}=\frac{lr_0}{2}\frac{e^4}{m^3}(\bar{A}_0\sin\alpha-\bar{A}_z)^2$$
$$\times\left(\frac{2\pi}{\omega r_0\cos\alpha}\right)^{1/2}\exp\left[-\frac{4\lambda}{\cos\alpha}-\frac{16\lambda}{\cos\alpha}\left(\frac{\varepsilon_+}{\omega}-\frac{1}{2}\right)^2\right.$$
$$\left.-\frac{4\lambda}{\cos\alpha}\vartheta_+^2-4\lambda\tau_+^2\cos\alpha\right],\quad\lambda\gg1. \tag{4.2}$$

Integrating (4.2) with respect to the angles, we obtain the energy distribution of the positrons:

$$\frac{d\sigma_A}{d\varepsilon_+}=\frac{\pi^{1/2}}{2^{5/2}}\frac{lr_0e^4(\bar{A}_0\sin\alpha-\bar{A}_z)^2\omega^{1/2}}{m^5r_0^{1/2}\cos^{1/2}\alpha}$$
$$\times\exp\left[-\frac{4\lambda}{\cos\alpha}-\frac{16\lambda}{\cos\alpha}\left(\frac{\varepsilon_+}{\omega}-\frac{1}{2}\right)^2\right]. \tag{4.3}$$

The total cross section of the process is

$$\sigma_A=\frac{\pi^2}{2^{9/2}}lr_0\frac{e^4(\bar{A}_0\sin\alpha-\bar{A}_z)^2}{m^2}\frac{1}{\lambda^2}\exp\left(-\frac{4\lambda}{\cos\alpha}\right),\quad\lambda=\frac{m^2r_0}{\omega}\gg1. \tag{4.4}$$

The cross section of the process of pair production by a photon in a cylindrically symmetrical field, as a function of the angle α between the photon emission direction and the plane perpendicular to the current direction, has (at $\lambda\gg1$) a sharp maximum at $\alpha=0$.

5. REGION OF HARD GAMMA QUANTA

We consider now the opposite limiting case, that of a small radius and (or) hard gamma quanta:

$$\lambda=m^2r_0/\omega\ll1. \tag{5.1}$$

In this case the Fourier component of the field (2.9), and hence also the pair-production cross section, does not depend on the radius r_0 of the current channel. This is due to the fact that under the condition (5.1) the main contribution is made by the region of the logarithmic $A_\mu(r)$ dependence, a region far from the axis. The concrete structure of the pinch in the region $r\sim r_0$ does not affect in this case the pair-production cross section.

The general formula for the differential cross section of the reaction in the region (5.1) is of the form

$$\frac{d\sigma_A}{d\varepsilon_+\,d\vartheta_+\,d\tau_+\,du}=\frac{le^4}{\pi m^2\varepsilon_+\varepsilon_-\cos\alpha}\left\{\frac{(\varepsilon_+^2+\varepsilon_-^2)(\bar{A}_0-\bar{A}_z\sin\alpha)^2}{\varkappa_+\varkappa_-(u^2+B^2)}\right.$$
$$-\frac{\omega^2(\varepsilon_+^2+\varepsilon_-^2)(\bar{A}_0^2-\bar{A}_z^2)}{m^2\varepsilon_+\varepsilon_-(u^2+B^2)^2}+2\left[\frac{4\varepsilon_+\varepsilon_-}{m^2}(\bar{A}_0-\bar{A}_z\sin\alpha)\vartheta_+u\right.$$
$$\left.+\frac{2\omega}{m}\left(\bar{A}_0(1+\vartheta_+^2)\operatorname{tg}\alpha-\frac{\bar{A}_z}{\cos\alpha}(1+\vartheta_+^2+\tau_+^2\cos^2\alpha)\right)\tau_+\right]^2$$
$$\left.\times[(1+\vartheta_+^2+\tau_+^2)^4(u^2+B^2)^2]^{-1}\right\}. \tag{5.2}$$

Here $u=\vartheta_++\vartheta_-$, $\omega=\varepsilon_++\varepsilon_-$, and

$$B^2=\frac{m^2\omega^2}{4\varepsilon_+^2\varepsilon_-^2}[(1+\vartheta_+^2+\tau_+^2)^2+(1+\vartheta_+^2)^2\operatorname{tg}^2\alpha]. \tag{5.3}$$

We do not write out the cumbersome formulas and confine ourselves to the limiting cases $\alpha=0$ and $\alpha\to\pi/2$, when the photon propagates perpendicular and parallel to the current direction.

At $\alpha=0$ the distribution of the cross section with respect to the positron parameters is given by

$$\frac{d\sigma_A}{dE\,d\vartheta\,d\tau}=\frac{4le^4\omega}{m^4}\left\{E(1-E)[E^2+(1-E)^2]\right.$$
$$\left.\times\frac{\bar{A}_0^2+\bar{A}_z^2}{(1+\vartheta^2+\tau^2)^3}+8E^2(1-E)^2\frac{\bar{A}_0^2\vartheta^2+\bar{A}_z^2\tau^2}{(1+\vartheta^2+\tau^2)^5}\right\}.$$

Here $\vartheta=\vartheta_+$, $\tau=\tau_+$ and $E=\varepsilon_+/\omega$.

Integrating with respect to the angles, we obtain the distribution of the positron in energy:

$$\frac{d\sigma_A}{dE}=\frac{2\pi l e^4\omega}{m^4}(\bar{A}_0^2+\bar{A}_z^2)\left\{E(1-E)[E^2+(1-E)^2]+\frac{2}{3}E^2(1-E)^2\right\}. \quad (5.4)$$

The total cross section of the process at $\alpha=0$ and $\lambda\ll 1$ is

$$\sigma_A=\frac{11\pi}{45}\alpha\frac{l}{m}\frac{e^2(\bar{A}_0^2+\bar{A}_z^2)}{m^2}\frac{\omega}{m}, \quad (5.5)$$

where $\alpha=e^2=1/137$ is the fine-structure constant.

We now consider the case $\alpha\to\pi/2$, or more accurately

$$m/\omega\ll\pi/2-\alpha\ll 1. \quad (5.6)$$

In this case $\tan\alpha\gg 1$, but on account of the first inequality of (5.6) the value of B^2 (5.3) is small as before: $B^2\sim(m/\omega)^2\,\mathrm{tg}^2\alpha\ll 1$. The angular and energy distribution of the produced positrons is of the form

$$\frac{d\sigma_A}{dE\,d\vartheta\,d\tau}=\frac{8le^4\omega(\bar{A}_0-\bar{A}_z)^2}{m^4}\left\{\frac{E(1-E)[E^2+(1-E)^2]}{(1+\vartheta^2+\tau^2)^2(1+\vartheta^2)}+\frac{4E^2(1-E)^2(\vartheta^2+\tau^2)}{(1+\vartheta^2+\tau^2)^4(1+\vartheta^2)}\right\},\quad \alpha\to\frac{\pi}{2}. \quad (5.7)$$

The cross section of the reaction as a function of the positron energy is of the form

$$\frac{d\sigma_A}{dE}=\frac{16\pi}{3}\frac{le^4\omega(\bar{A}_0-\bar{A}_z)^2}{m^4}\left\{E(1-E)[E^2+(1-E)^2]+\frac{24}{35}E^2(1-E)^2\right\},\quad \alpha\to\frac{\pi}{2}. \quad (5.8)$$

Finally, integrating (5.8) with respect to E from zero to unity we obtain the total cross section

$$\sigma_A=\frac{344\pi}{525}\frac{le^4(\bar{A}_0-\bar{A}_z)^2\omega}{m^4},\quad \alpha\to\frac{\pi}{2},\quad \lambda\ll 1,\quad \omega\gg m. \quad (5.9)$$

In the case of a cylindrically symmetrical field, the cross section for pair production by a photon is proportional to ω/m and not to $\ln(\omega/m)$ as in a centrosymmetric field, and consequently increases much more strongly with increasing photon energy.

In the intermediate region $\lambda\sim 1$ expressions (4.4), (5.5), and (5.9) yield a value of the same order:

$$\sigma_A\sim\alpha l r_0(e\bar{A}_z/m)^2,\quad \lambda\sim 1,\quad \alpha={}^1/_{137}. \quad (5.10)$$

By virtue of (1.6), with increasing current I the cross section (5.10) increases in proportion to I^2 and in the region $\alpha(eI/mc^3)^2\gtrsim 1$, i.e., at $I\gtrsim 200$ kA, the cross section for pair production by a photon in the mean field of the collective interaction of the pinch charges becomes larger than the geometric cross section lr_0 of the current channel.

To trace the dependence on the angle α between the photon propagation angle and the plane perpendicular to the current, we present the total cross section of the process at $\lambda\ll 1$ in the case of an electrically neutral ($\rho=0$ in (1.4), and hence $\bar{A}_0=0$) current channel:

$$\sigma_A=\frac{11\pi}{45}\frac{le^4\bar{A}_z^2\omega}{m^4}f(\alpha),\quad \bar{A}_0=0,\quad \lambda\ll 1. \quad (5.11)$$

The function $f(\alpha)$ is given by

$$f(\alpha)=\frac{24}{11\pi}\int_{-\infty}^{+\infty}\frac{dx}{[(1+x^2)^2\cos^2\alpha+\sin^2\alpha]^{1/2}}\times\left\{\frac{2\sin^2\alpha}{(1+x^2)^2}+\frac{\cos^2\alpha}{(1+x^2)^2\cos^2\alpha+\sin^2\alpha}+\frac{32}{15}\frac{1}{(1+x^2)^4}\left[\frac{\sin^2\alpha}{7}+\frac{x^2(1+x^2\cos^2\alpha)^2}{(1+x^2)^2\cos^2\alpha+\sin^2\alpha}\right]\right\},$$

$$f(0)=1,\quad f\left(\frac{\pi}{2}\right)=\frac{1032}{385} \quad (5.12)$$

and is plotted in Fig. 2. The integral in (5.12) is expressed in terms of elliptic integrals, but is simpler to find the sought dependence numerically with a computer. We point out that, in contrast to the case $\lambda\gg 1$, the cross section increases with increasing angle α in the region (5.1).

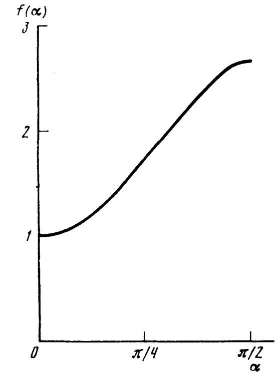


FIG. 2.

6. DISCUSSION

We compare now the cross section σ_A for pair production in the mean field of the collective excitation with the the cross section σ_a (1.3) of this reaction on field fluctuations near individual charges. Noting that the current I is connected with the number N_e of electrons per unit length of the discharge channel by the relation $I=eN_ev_0$, assuming that the electron drift velocity is $v_0\sim c$, and assuming the number of ions to be $N_i\sim N_e$, we find that the number of charges over the length l is $\mathfrak{N}\sim N_il\sim Il/ec$. The cross section for pair production by a photon in the field of a nucleus is well known[10]:

$$\sigma_{\rm nuc}={}^{28}/_9Z^2\alpha r_e^2[\ln(2\omega/m)-{}^{109}/_{42}],\quad \omega\gg m, \quad (6.1)$$

where Z is the charge of the nucleus, $r_e=e^2/mc^2$ is the classical radius of the electron. The value of σ_a can be estimated by multiplying the cross section (6.1) by the number of charges $\mathfrak{N}$

$$\sigma_a\sim Z^2\alpha l r_e\Lambda(eI/mc^3),\quad \Lambda=\ln(2\omega/m). \quad (6.2)$$

The ratio σ_A/σ_a in the characteristic region $\lambda\sim 1$ is thus of the order of

$$\frac{\sigma_A}{\sigma_a}\sim\frac{r_0}{r_eZ^2\Lambda}\frac{eI}{mc^3}. \quad (6.3)$$

Since $r_e\sim 10^{-13}$ cm and $r_0/r_e\sim 10^4$, it follows that at currents $I\gtrsim 17$ kA, in the case of singly charged ions the ratio (6.3) is of the order of 10^3-10^4. Consequently, for a strong current $I\gtrsim 17$ kA, the main contribution to the cross section of pair production by a photon in a pinch, in the characteristic region $\lambda=m^2r_0/\omega\sim 1$ is made by the mean field of the collective interaction of the charges. The specific dependence of the cross section (4.4) of the process in the region $\lambda>1$ gives grounds for hoping to use pair production by photons in the pinch field to resolve the spatial structure of the current channel in the angstrom region. The spectral and angular distributions of the positrons carry information on the spatial structure of the current and can reveal, in principle, the presence of linear atoms in pinches, and can consequently ascertain whether the compression under the influence of the forces of collective interaction and radiation collapse[9,12-15] can reach the stage of electron degeneracy.

It must be noted, however, that despite the large cross section of the process ($\sim 10^{-13}$ cm^2), the characteristic time of pinch evolution in the state of maximum compression is estimated at $\lesssim 10^{-9}$ sec, so that realization of the proposed experiment is a major problem in high-energy experimental physics.

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Nonstationary Josephson effect in quasi-two-dimensional superconductors

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Equations are obtained for the Green's function describing the nonequilibrium phenomena in quasi-two-dimensional superconductors. The flow of a current in a direction perpendicular to the conducting layers is investigated by means of these equations in dirty quasi-two-dimensional superconductors ($\tau\varepsilon_1 \ll 1$, where τ is the mean free path along the layer, ε_1 is the width of the energy band corresponding to motion across the layers). For finite voltages V applied to the sample, and sufficiently weak coupling between the layers ($\tau\varepsilon_1^2 \ll \Delta$), Josephson oscillations occur in the system with a frequency $2eV/N$, where N is the number of layers in the system. In contrast to tunnel junctions, in which the electric field is localized in the dielectric and does not enter the superconductor, the field in a quasi-two-dimensional superconductor does not vanish at any point within the crystal. The energy distribution of the quasiparticles is not an equilibrium one and this results in an increase in the energy gap of the superconductor. The transverse conductivity of the system in the normal state has the form $\sigma_\perp \sim e^2 m d\tau\varepsilon_1^2$.

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As is well known, a number of layered compounds become superconductors at helium temperatures. The most studied of these layered compounds are the dichalcogenides of the transition metals, for example, $NbSe_2$, TaS_2, $TaSe_2$. The properties of such materials are described, for example in the review of Bulaevskiĭ.[1] These highly anisotropic crystals consist of layers, within which the binding between the atoms is covalent, while the layers are bound to one another by weak van der Waals interaction. The anisotropy of these compounds can be significantly increased artificially by intercalation—the introduction of other atoms or molecules in the space between the layers. Thus, for example, the ratio of the longitudinal and transverse conductivities of $2H$-TaS_2 increases from 28 to 10^5 by intercalation of pyridine. The critical temperature of superconductors of such type is determined basically by the interaction within the layer and changes in intercalation only to the degree that the characteristics of the conducting layer change upon change in the distance between the layers. Thus, for example, in the intercalation of $2H$-TaS_2 the temperature T_c increases from 0.8–2 K to 2–4.5 K, and in the case of $2H$-$NbSe_2$ it decreases from 7 to 4 K.

There is interest in the problem of the transverse conductivity of the quasi-two-dimensional superconductors. If the anisotropy is sufficiently strong that the characteristic energy connected with motion between the layers is less than the value of the gap energy, then the superconductivity has a quasi-two-dimensional character.[1)] In this case, in the flow of near-

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