terms with j_θ in terms of the magnetic field **B** (with the aid of the equation curl $\mathbf{B}=4\pi c^{-1}\mathbf{j}$), we obtain after simple transformations in place of (A1)

$$\int (\alpha f_0^2+\beta f_0^4)\,dv+\frac{1}{2}\mathbf{H}_0\mathbf{M}+\frac{1}{8\pi}H_0V-\frac{1}{8\pi}H_1^2V_1$$
$$-\frac{1}{8\pi}\int\limits_{V_s}B_z^2\,dv+\frac{\hbar cn}{8e}(H_1-H_0)+\int\limits_{V_s}\frac{2e^2}{mc^2}A_z^2f_0^2\,dv=0. \qquad \text{(A2)}$$

(Obviously, relations (A1) and (A2) are equivalent to the extremum condition on the functional $\delta F=0$ (or $\delta\Phi_s=0$), from which Eqs. (7) and (8) follows.

It is necessary next to use the expressions for the field H_1 and the moment **M** of the hollow cylinder in terms of Bessel functions (these expressions are given in Ref. 7), as well as the expressions for $A_z(r)$ and $A_\theta(r)$ (cf. Ref. 17):

$$A_z(r)=H_1\frac{\delta[I_0(\xi)K_1(\xi_1)+K_0(\xi)I_1(\xi_1)]}{I_1(\xi_1)K_1(\xi_2)-I_1(\xi_2)K_1(\xi_1)},\qquad B_z(r)=\frac{1}{r}\frac{d}{dr}(rA_\theta),$$
$$A_\theta(r)=\frac{\hbar cn}{2e\delta\xi}+\delta\frac{aI_1(\xi)+bK_1(\xi)}{K_0(\xi_1)I_0(\xi_2)-I_0(\xi_1)K_0(\xi_2)}$$
$$a=H_0K_0(\xi_1)-H_1K_0(\xi_2),\qquad b=H_0I_0(\xi_1)-H_1I_0(\xi_2).$$

Here K_n and I_n are Bessel functions of imaginary argument, $\xi=r/\delta$, $\xi_1=r_1/\delta$, $\xi_2=r_2/\delta$, $\delta=\delta_L/\psi$; ψ is the modulus of the order parameter (in relative units, see the text). Expanding the Bessel functions in the small parameter $d/\delta\ll 1$ $(d=r_2-r_1)$ and retaining terms of order $(d/\delta)^3$ we arrive after cumbersome calculations again at relation (11).

[1] Actually the destruction of superconductivity by a current takes place in some finite region of the value of I near I_c, and is due to the appearance of the resistive state, i.e., to the gradual restoration of the normal resistance. This process can not be described within the simple Ginzberg-Landau thermodynamic theory.

[2] Relation (11) can be obtained also directly from (7) and (8) by using the explicit solutions[17] for the potentials $A_-(r)$ and $A_z(r)$. The corresponding calculation is given in the Appendix.

[3] It can be shown that at this point $f'''_\psi>0$, i.e., we have an inflection point. Since it is clear that the inflection point must be located at $f>0$, where $f\sim\Phi_s(H_0,\ I)-\Phi_n(H_0,\ I)$ [see (9)], it follows that this point is obviously in the region of metastability of the superconducting state. It can be verified that it is necessary in this case to put $c_I>0$ in (9).

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Translated by J. G. Adashko

Elastic properties of crystal surfaces

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(Submitted 11 January 1980)
Zh. Eksp. Teor. Fiz. 79, 257–260 (July 1980)

The general properties of the surface stress tensor, describing elastic properties of crystal surfaces, are determined. The boundary conditions are obtained for the bulk stress tensor on the surface of a crystal of arbitrary shape. The elastic interaction between point and line defects on crystal surfaces is considered.

PACS numbers: 68.25. + j, 61.70.Yq

It is well known that the thermodynamic properties of a liquid surface are governed entirely by one quantity which is the work done in reversible changes of the surface area. As pointed out long ago by Gibbs,[1] in the case of a solid we have to distinguish the work done in forming the surface and in deforming it. Thus, in describing the properties of crystal surfaces we have to introduce not only the surface energy but also the surface stress tensor. We shall determine the general properties of this tensor and find the boundary conditions replacing in our case the familiar Laplace formula for the capillary pressure.

In the second section we shall consider the elastic interaction of surface defects over distances which are large with the atomic separations. As in the case of bulk defects, the problem can be solved without postulating any models about the microscopic structure of defects.

1. SURFACE STRESS TENSOR

We shall consider a plane boundary of a crystal. A small elastic deformation u_i alters the surface energy by

$$-\int f_\mu u_\mu \, dS_0, \quad \mu=1,2 \tag{1}$$

(integration is carried out over an undeformed surface); f_μ are the surface forces which have only tangential ($\mu=1$ or 2) components because the normal surface forces vanish simply as a consequence of the third law of mechanics. As in the case of the bulk forces (see, for example, §2 in Ref. 2), the surface force f_μ can be represented by a divergence of a certain symmetric vector:

$$f_\mu=\partial\beta_{\mu\nu}/\partial x_\nu, \quad \mu,\ \nu=1,2.$$

Substituting in Eq. (1) and integrating by parts, we obtain

$$\int \beta_{\mu\nu} u_{\mu\nu} \, dS_0, \tag{2}$$

where $u_{\mu\nu}$ are the tangential components of the usual strain tensor.

In the general case of a crystal of arbitrary shape the correction to the surface energy is also of the form given by Eq. (2), where the indices μ and ν for each point on the surface correspond to a coordinate system in a tangential plane. Thus, the surface energy of a deformed crystal is given by

$$\int (\alpha+\beta_{\mu\nu}u_{\mu\nu})\, dS_0; \tag{3}$$

here, α is the surface energy density of an undeformed crystal. It is natural to call $\beta_{\mu\nu}$ the surface stress tensor. It is important to note that, in general, all the components of this tensor differ from zero and have the same order of magnitude as α. In the case of a surface which have a symmetry axis higher than twofold, we have $\beta_{\mu\nu}=\beta\delta_{\mu\nu}$, where β is the surface tensor coefficient. In the case of a liquid, the quantity β is identical with the surface energy α (Ref. 1) and Eq. (3) has the usual form

$$\int \alpha(1+u_{\mu\mu})\, dS_0 = \int \alpha \, dS.$$

We shall regard as undeformed such a homogeneous state of a crystal which corresponds to an external pressure p in the absence of the capillary effects. Thus, the strains in a crystal are governed entirely by the surface stress tensor. The conditions for a mechanical equilibrium on the surface reduce to vanishing of the sum of the bulk and surface forces:

$$\begin{gathered}\sigma_{nn}+\beta_{11}/R_1+\beta_{22}/R_2+p=0,\\ \sigma_{\mu n}=\frac{1}{R_1}\frac{\partial\beta_{\mu 1}}{\partial\varphi_1}+\frac{1}{R_2}\frac{\partial\beta_{\mu 2}}{\partial\varphi_2},\end{gathered} \tag{4}$$

where σ_{ik} is the stress tensor; n is the index of the normal to the surface; R_1 and R_2 are the principal radii of curvature; φ_1 and φ_2 are the angles measured in the planes of the principal normal cross sections. It is interesting to note that the pressure in an isotropic solid can be higher or lower than the external pressure.

It follows from the system (4) that there are always inhomogeneous stresses in a crystal which is in mechanical equilibrium with a liquid. This is also true of a phase equilibrium. The corresponding condition[1] (see also Ref. 3) is now

$$F_0+p+\left(\alpha+\frac{\partial^2\alpha}{\partial\varphi_1^2}\right)\frac{1}{R_1}+\left(\alpha+\frac{\partial^2\alpha}{\partial\varphi_2^2}\right)\frac{1}{R_2}=\frac{\mu}{v_0}, \tag{5}$$

where F_0 is the free energy per unit volume of an undeformed crystal; v_0 is the atomic volume of such a crystal; μ is the chemical potential of the liquid. The stresses given by Eq. (4) should strictly occur also in the condition for a phase equilibrium but only in the next approximation with respect to $1/R$. In this connection it is essential to stress that all the relationships given above represent essentially only the principal terms of an expansion in a small parameter a/R, where a is the interatomic distance.

2. ELASTIC INTERACTION OF SURFACE DEFECTS

It is known (see, for example, Ref. 4) that the field of elastic strains far from a bulk point defect (which may be a vacancy, an interstice, or an impurity) can be calculated by introducing the point distribution of forces of the type

$$F_i=A_{ik}\frac{\partial}{\partial x_k}\delta(\mathbf{r}), \quad i,k=1,2,3;$$

where $\mathbf{r}$ is a three-dimensional radius vector (it is assumed that the defect is located at the coordinate origin); A_{ik} is some symmetric tensor; the total force and the moment of forces of such a distribution vanish. It is clear that similar defects on a crystal surface should be described by the point distribution of the surface forces:

$$f_\mu=A_{\mu\nu}\frac{\partial}{\partial x_\nu}\delta(\rho), \quad \mu,\nu=1,2, \tag{6}$$

where ρ is a two-dimensional radius vector in the plane of the boundary; the defect is located at a point $\rho=0$; $A_{\mu\nu}=A_{\nu\mu}$. If we know the field of elastic strains caused by the forces of Eq. (6) (see §8 in Ref. 2), we can easily calculate the elastic interaction energy of such defects. It is found that in the case of an isotropic solid the "isotropic" defects ($A_{\mu\nu}=A\delta_{\mu\nu}$) repel in accordance with the law

$$U(\rho)=\frac{1-\sigma^2}{\pi E}\frac{A^2}{\rho^3},$$

where E is the Young modulus and σ is the Poisson ratio. The same result was obtained by Lau and Kohn[5] in a microscopic theory. In the general case of an arbitrary symmetric tensor $A_{\mu\nu}$ we can expect repulsion or attraction governed by the same law $U\propto\rho^{-3}$.

Surface defects need not have zero total moment. We shall consider an elementary step (Fig. 1). We shall select a certain large-radius region around the step. At the points 1 and 2 this region is acted upon by capillary forces equal to, per unit length of the step, to the surface tension coefficient β (for simplicity, we shall assume that $\beta_{\mu\nu}=\beta\delta_{\mu\nu}$). These forces create a moment βa directed along the line of the step. The quantity a is

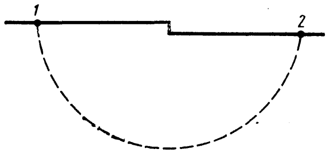

FIG. 1.

the interatomic distance in the direction along the normal to the surface, whereas the height of the step at large distances is equal to a irrespective of its short-range structure and of the structure of the surface itself.

Internal stresses should compensate this moment. We can describe them macroscopically by introducing the following linear distribution of the force normal to the surface:

$$f_n=\beta a\frac{\partial}{\partial x}\delta(x). \tag{7}$$

The x axis lies in the plane of the boundary and it is directed at right-angles to the line of the step. In addition to the distribution (7), the step–like any other line defect–has also certain linear distribution of forces of the (6) type with zero total moment:

$$f_x=f\frac{\partial}{\partial x}\delta(x). \tag{7a}$$

Using (7) and (7a), we find that the interaction of two identical steps should be repulsion in accordance with the law

$$U(x)=\frac{2(1-\sigma^2)}{\pi E}[f^2+(\beta a)^2]\frac{1}{x^2}$$

(per unit length of the step), where x is the distance between the steps. Steps of different sign differ by the direction of the moment and, therefore, the energy of the interaction between them is

$$U(x)=\frac{2(1-\sigma^2)}{\pi E}[f^2-(\beta a)^2]\frac{1}{x^2},$$

i.e., we can have attraction or repulsion.

Kinks in steps are point defects which have to be described by introducing a moment, in addition to the distribution (6). In this case the moment has two components in a plane perpendicular to the line of the step. The tangential component is of the same origin as the moment of the step itself, whereas the component normal to the surface is created by the forces of linear tension. The interaction energy of the kinks depends on the distance in accordance with the law ρ^{-3}, and we can have both attraction and repulsion. This should be allowed for investigations of the equilibrium shape of a crystal because attraction between identical kinks makes the surface unstable in a certain range of its orientation (compare with Ref. 6).

The authors are grateful to A. F. Andreev and A. A. Chernov for valuable discussions.

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Translated by A. Tybulewicz