Anisotropy of current fluctuations and of the electron diffusion in a strong electric field in the case of quasielastic scattering

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The dependence of the anisotropy of the diffusion coefficient of the carriers and of the low-frequency spectral density of the current fluctuations in an isotropic medium (in a gas or in an isotropic semiconductor) and in an electric field on the quasielastic-scattering mechanisms is calculated. It is demonstrated that the applicability of the Wannier-Robson approximate relation is limited only to a special class of scattering mechanisms. An alternate approximate formula is proposed. The influence of electron-electron collisions of the degree of anisotropy of the diffusion coefficient and the current fluctuations is investigated.

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1. INTRODUCTION

It is well known that under conditions of thermodynamic equilibrium the fluctuation power radiated by a sample is determined only by its temperature. This fact–the Nyquist theorem or the fluctuation-dissipation theorem-means, in particular, that measurement of electric noise under equilibrium conditions cannot yield new information on the system.

Under nonequilibrium conditions, for example in a system with stationary current, the situation is different, and the investigation of the fluctuation power series, alongside with the study of the response, as a method for the diagnostics of the nonequilibrium state itself. The theory (Ref. 4, Sec. 23, and Ref. 2) predicts not only an increase of the fluctuation power on account of the heating of the carriers under nonequilibrium conditions, but also the onset of noise anisotropy due to the influence of the fluctuations of the average carrier energy on the electric noise along the stationary current.

In fact, under equilibrium the electric noise is determined by the thermal "flicker" of the carrier system as a whole in the direction of interest to us, i.e., by one macroscopic degree of freedom. According to the equipartition law, the kinetic energy share of this motion is $kT_0/2$, the quantity that enters in the fluctuation-dissipation theorem.

Under nonequilibrium conditions, however, the electric noise along the stationary current is influenced by the thermal motion of one more degree of freedom, namely, the fluctuation of the average energy of the carriers. Whereas at equilibrium the fluctuations of the average random velocity (i.e., of the energy or temperature of the carrier system) and the fluctuations of the translational motion of the carrier system are fully independent, under conditions when the average current differs from zero the fluctuation change of the average carrier energy leads (via the dependence of the mobility on the energy) to a change in the "drift" velocity, i.e., to a fluctuation of the current. In directions transverse to the current this phenomenon does not appear, and it is this which causes the anisotropy of the electric noise in a nonequilibrium system. In particular, the energy fluctuations can increase as well as suppress the longitudinal electric noise compared with the transverse one.

Even the first correct calculation[3] of the current fluctuations, on the basis of the fluctuation kinetics, has shown that in typical situations, both in semiconductors and in ionized gases, the contribution of the energy fluctuations to the current noise contains, under nonequilibrium conditions, neither a symbolic nor a numerical small quantity. The concrete calculation[3] was performed for the model of quasielastic carrier scattering by the acoustic phonons in a semiconductor with straight band. It turned out that in the considered model the fluctuations of the energy suppress partially the current fluctuations. In the strong-field limit, the low-frequency spectral density of the longitudinal fluctuations of the current j turned out to be less than the corresponding density of the transverse fluctuations, by a factor of two (more accurately, $\delta j_\parallel^2/\delta j_\perp^2 = 0.49$).

By virtue of the proportionality of the low-frequency spectral density of the current fluctuations and of the current diffusion coefficient[4] (under conditions when the collisions between the carriers can be neglected[2]), the calculation of Ref. 3 yielded equally well also the anisotropy of the diffusion coefficient. Moreover, it yielded by the same token also the coefficient of diffusion of electrons in gases in a strong electric field within the framework of the hard-sphere model of electron-atom collisions. In fact, the equivalence of such a scattering of electrons in gases and scattering by acoustic phonon has been long for a long time.[5]

Inasmuch as for this model the longitudinal ($\tilde{\mu}_\parallel$) and transverse ($\tilde{\mu}_\perp$) differential mobilities differ (in the strong-field limit) by a factor of two, the anisotropy of the diffusion coefficient D (of the spectral density of the current fluctuations) and the anisotropy of the differential mobility turned out to be practically equal. The approximate equality of the ratios $D_\parallel/D_\perp$ and $\tilde{\mu}_\parallel/\tilde{\mu}_\perp$ was observed also in the calculations for other models

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of the interaction of electrons with atoms in gases.[6,7] This has prompted Robson[8] to suggest that in semiconductors, too, the anisotropy of the carrier diffusion coefficient in quasielastic scattering is approximately described by the relation

$$D_{\parallel}/D_{\perp}\approx\tilde{\mu}_{\parallel}/\tilde{\mu}_{\perp}$$

(the Wannier-Robson relation). Since $\tilde{\mu}_{\parallel}/\mu_{\perp}=d\ln j/d\ln E$, it follows that, according to Robson, the anisotropy of the diffusion coefficient could be determined from the slope of the current-voltage characteristic of the semiconductor (j is the current density and E is the external electric field).

Robson's suggestion, however, met with objections.[9,2] The quantity $eD_{\alpha\alpha}/\tilde{\mu}_{\alpha\alpha}$ in the absence of collisions between electrons is none other than the noise temperature in the α directions (see Refs. 4 and 2), so that Robson's assumption is equivalent to assuming isotropy of the noise temperature of the hot carriers under conditions when the scattering is quasielastic. The experimental studies of the noise temperature in semiconductors, however, show it to be strongly anisotropic (see Refs. 10–12). Taken by itself, this circumstance still does not refute Robson's suggestion, since the latter pertains only to quasielastic scattering of the carriers, whereas in experiment the inelastic collisions are always significant to one degree or another.

Thus, to clarify the situation we need theoretical data on the anisotropy of the diffusion coefficient for different quasielastic-scattering mechanisms. A detailed investigation of the anisotropy of the diffusion coefficient (of the spectral density of the current fluctuations) in quasielastic carrier scattering in an isotropic medium in a strong electric field is in fact the subject of the present paper.

2. GENERAL EXPRESSIONS

In quasielastic scattering, the loss or acquisition of energy by a carrier in a collision are small compared with the carrier energy, whereas the particle momentum changes appreciably. Under these conditions, the relaxation of the carrier energy is governed by the Davydov differential operator[5]

$$\varepsilon^{-1/2}\partial_{\varepsilon}[\varepsilon^{3/2}\tau_{en}(\varepsilon)^{-1}(1+kT_0\partial_{\varepsilon})], \tag{1}$$

where $\varepsilon=p^2/2m$ is the carrier energy, T_0 is the temperature of the medium, and $\tau_{en}(\varepsilon)$ is the energy relaxation time. The stationary distribution of the carriers in energy is described by the Davydov distribution function

$$F(\varepsilon)=C\exp\left[-\int_0^{\varepsilon}d\varepsilon'/kT_0(1+E^2\Theta(\varepsilon'))\right], \tag{2}$$

where

$$\Theta(\varepsilon)=(2e^2/3mkT_0)\tau(\varepsilon)\tau_{en}(\varepsilon), \tag{3}$$

$\tau(\varepsilon)$ is the momentum relaxation time.

In an isotropic medium, the ratio $K(D)\equiv D_{\parallel}/D_{\perp}$ is given by [see, e.g., Ref. 2, Eq. (3.78)]

$$K(D)=1+E^2\int_0^{\infty}\frac{\Theta[\varkappa^2-(\tau\varepsilon^{1/2}F)^2]}{(1+E^2\Theta)\tau\varepsilon^{1/2}F}d\varepsilon\Big/\int_0^{\infty}\tau\varepsilon^{1/2}F\,d\varepsilon, \tag{4}$$

where $F=F(\varepsilon)$ is the distribution of the electrons in energy,

$$\varkappa(\varepsilon)=-\int_{\varepsilon}^{\infty}[\tau\varepsilon_1^{1/2}\partial_{\varepsilon_1}F+{}^3/_2\langle\tau\rangle\varepsilon_1^{1/2}F]d\varepsilon_1,$$

$$\langle\tau\rangle=-{}^2/_3\int_0^{\infty}\tau\varepsilon^{1/2}dF\Big/\int_0^{\infty}\varepsilon^{1/2}F\,d\varepsilon.$$

Expression (4) was obtained from the definition of the diffusion coefficient

$$D_{\alpha\beta}=\sum_p v_{\alpha}I_p^{-1}(v_{\beta}-V_{\beta})\bar{F}_p, \tag{5}$$

given by Wannier[13] (see also Refs. 2 and 14), by consistent use of the smallness of the parameter $\tau/\tau_{en}\ll 1$ (see Ref. 1, Secs. 21–23). The operator I_p in (5) is the kinetic-equation operator

$$I_p\equiv eE\partial_p+I_p^{th} \tag{6}$$

I_p^{th} is the operator of scattering by the medium (by the thermostat, $\bar{F}_p$ is the stationary distribution function, V is the carrier drift velocity, and v is the carrier velocity.

As already mentioned in the Introduction, at low carrier densities, when the collisions between can be neglected, the tensor of the diffusion coefficients and the tensor of the low-frequency ($\omega\tau_{en}\ll 1$) of the spectral densities of the current fluctuations are proportional to each other, so that

$$K(D)\equiv\frac{D_{\parallel}}{D_{\perp}}=\frac{(\delta j_{\parallel}^2)_{\omega}}{(\delta j_{\perp}^2)_{\omega}}\equiv K(\delta j^2), \tag{7}$$

i.e., in quasielastic scattering $K(D)$ and $K(\delta j^2)$ are described by the same expression (4).

The anisotropy of the differential mobility $\tilde{\mu}_{\alpha\beta}$,

$$\tilde{\mu}_{\alpha\beta}\equiv\partial_{E_{\alpha}}V_{\beta}=e\sum_p v_{\alpha}I_p^{-1}\partial_{p_{\beta}}\bar{F}_p, \tag{8}$$

is given by [see Ref. 2, Eq. (3.77)]

$$K(\tilde{\mu})\equiv\frac{\tilde{\mu}_{\parallel}}{\tilde{\mu}_{\perp}}=1-2E^2\int_0^{\infty}\frac{\Theta(\varkappa-\tau\varepsilon^{1/2}F)d\varepsilon}{kT_0(1+E^2\Theta)^2}\Big/\int_0^{\infty}\tau\varepsilon^{1/2}dF. \tag{9}$$

In this paper we confine ourselves to investigation of the power-law scattering mechanisms, i.e., we assume that $\tau(\varepsilon)$ and $\tau_{en}(\varepsilon)$ are power-law functions of the energy:

$$\tau(\varepsilon)\sim\varepsilon^{s},\ \tau_{en}(\varepsilon)\sim\varepsilon^{t}. \tag{10}$$

3. THE STRONG FIELD LIMIT

In the strong-field limit, when

$$E^2\Theta(\varepsilon)\gg 1, \tag{11}$$

we get from (4), (9), and (10)

$$K(D)=\int_0^{\infty}x^{(1-s-t)/(1-s-t)}\psi^2(x)e^x\,dx/\Gamma\left(\frac{s+{}^5/_2}{1-s-t}\right), \tag{12}$$

$$K(\tilde{\mu})=1+\frac{2s}{1-s-t}, \tag{13}$$

where

$$\psi(x)=\frac{\Gamma(({}^5/_2-t)/(1-s-t))}{\Gamma(3/2(1-s-t))}\gamma'\left(\frac{3}{2(1-s-t)},x\right)-\gamma\left(\frac{{}^5/_2-t}{1-s-t},x\right), \tag{14}$$

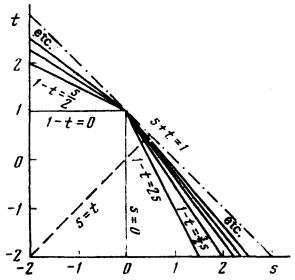

FIG. 1. Analytic investigation of the value of $K(D)$ along the solid straight lines on the (s,t) plane in the strong-field limit.

$\gamma(\alpha,x)$ is the incomplete gamma function. To avoid the runaway effect, we confine ourselves to values of s and t satisfying the condition $s+t<1$.

Owing to the known relation between the incomplete gamma functions

$$\frac{\gamma(\alpha+n,x)}{\Gamma(\alpha+n)}-\frac{\gamma(\alpha,x)}{\Gamma(\alpha)}=-e^{-x}\sum_{i=0}^{n-1}\frac{x^{\alpha+i}}{\Gamma(\alpha+1+i)}, \tag{15}$$

it is possible to obtain the values of $K(D)$ analytically at certain values of s and t. To this end, $(1-t)/(1-s-t)$ must be an integer. We obtain, for example,

$$1-t=s/2,\ K(\tilde{\mu})=-3,\ K(D)=(5+s)(5+2s)/(3+s)^2;$$
$$1-t=0,\ K(\tilde{\mu})=-1,\ K(D)=0;$$
$$s=0,\ K(\tilde{\mu})=1,\ K(D)=1;$$
$$1-t=2s,\quad K(\tilde{\mu})=3,\quad K(D)=1+\frac{(3+2s)(13-2s)}{5(5-2s)};$$
$$1-t=3s/2,\ K(\tilde{\mu})=5,$$
$$K(D)=1+\frac{2s+3}{5(5+s)}\left[4s+19+\frac{(2s+3)(3+s)(13-3s)}{(5-s)(5-2s)}\right].$$

The anisotropy of the diffusion coefficient can be investigated analytically in this way if the current-voltage characteristic is strongly superlinear (at $\tilde{\mu}_\parallel/\tilde{\mu}_\perp=3,5,7$, etc.), i.e., in the region close to runaway [see Fig. 1; in this region the density of the lines along which $K(D)$ can be calculated analytically increases]. It is of interest to note that the longitudinal diffusion coefficient $D_\parallel$ at $t=1$ (i.e., for $\tau_{en}\sim\varepsilon$) tends to zero in the strong field limit (in this case $\tilde{\mu}_\parallel=-\tilde{\mu}_\perp$).

For values of s and t that do not lie on the indicated straight lines in the s,t plane, the value $K(D)$ must be calculated numerically. The results of such a calculation are shown in the table and in Fig. 2 (solid curves) and demonstrate the wide range in which the anisotropy

TABLE I. Numerical values of $K(D)=K(\delta j^2)$ in the strong-field limit.

t \ s	−2	−7/4	−3/2	−5/4	−1	−3/4	−1/2	−1/4	0	1/4	1/2	3/4	1	5/4
2	3	—	—	—	—	—	—	—	—	—	—	—	—	—
7/4	—	1	—	12.6	—	—	—	—	—	—	—	—	—	—
3/2	0.08	—	0.4	—	3	—	—	—	—	—	—	—	—	—
5/4	—	0.02	—	—	—	0.49	—	—	—	—	—	—	—	—
1	0	—	0	—	0	—	0	—	—	—	—	—	—	—
3/4	—	0.01	—	0.02	—	0.06	—	0.29	—	—	—	—	—	—
1/2	—	—	0.04	—	0.1	—	0.26	—	1	—	—	—	—	—
1/4	—	—	—	0.1	—	0.25	—	0.57	—	2.03	—	—	—	—
0	—	—	—	—	0.2	—	0.42	—	1	—	3.4	—	—	—
−1/4	—	—	—	—	—	0.3	—	0.67	—	1.57	—	5.2	—	—
−1/2	—	—	—	—	—	—	0.49	—	1	—	2.33	—	7.61	—
−3/4	—	—	—	—	—	0.37	—	0.7	—	1.43	—	3.33	—	11.03
−1	—	—	—	—	0.28	—	—	—	1	—	2	—	4.66	—
−5/4	—	—	—	0.2	—	—	—	—	—	1.36	—	2.72	—	6.52
−3/2	—	—	0.12	—	—	—	—	—	—	—	1.8	—	3.7	—
−2	0.05	—	—	—	—	—	—	—	—	—	—	—	—	—

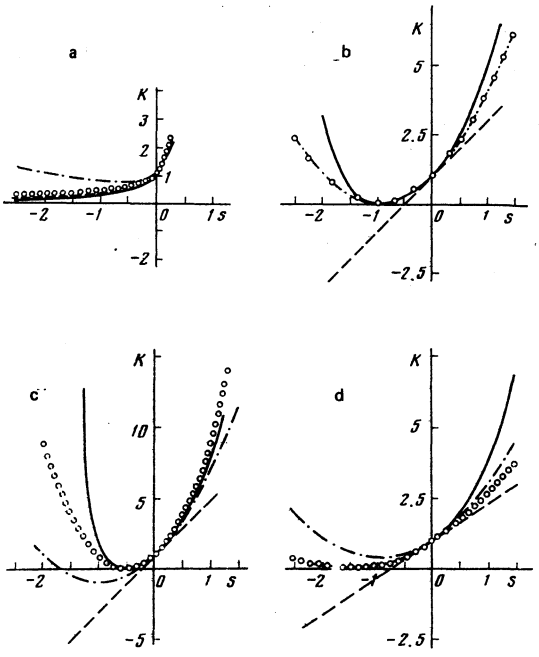

FIG. 2. Dependence of the anisotropy of the coefficients of diffusion $K(D)$, of the current fluctuations $K(\delta j^2)$, and of the differential mobility $K(\tilde{\mu})$ in the strong field limit on s at various t: a—$s=t$, b—$s+t=0$, c—$s+t=1/2$, d—$s+t=-1/2$. $K(D)=K(\delta j^2)$—solid curves, $K(D_{ee})$—dash-dot, $K(\delta j_{ee}^2)$—points, $K(\tilde{\mu})=K(\tilde{\mu}_{ee})$—dashed.

of the diffusion coefficient and of the low-frequency spectral density of the current fluctuations varies with variation of the exponents s and t.

In earlier studies (see Refs. 3, 6, 7, 15, 16) principal attention was paid to the case of equal exponents $s=t$, because this case is important in the theory of electron diffusion in gases.[1)] Our results for $K(D)$ in this case only extend the curve obtained in Ref. 7 into the region of lower values of s. As noted by Robson,[15] this curve is very close to the $K(\tilde{\mu})\equiv\tilde{\mu}_\parallel/\tilde{\mu}_\perp$ curve (dashed in Fig. 2a), and it was this which prompted him to propose that $K(\tilde{\mu})$ and $K(D)$ be regarded approximately equal. However, as shown by a comparison of Fig. 2a with Figs. 2b, 2c, and 2d, the near equality of the $K(D)$ and $K(\tilde{\mu})$ curves as $s=t$ is more readily an exception: at $s\neq t$ these curves diverge noticeably, and their behavior is quite different at values of s not close to zero. Nor was Robson right in suggesting[15] that the discrepancy between the $K(D)$ and $K(\tilde{\mu})$ curves is due to the deviation of the stationary electron-energy distribution function from Maxwellian. Under the condition $s+t=0$ the quantity $\Theta\sim\tau(\varepsilon)\tau_{en}(\varepsilon)$ does not depend on the energy, so that the stationary distribution in energy is Maxwellian also in a strong field.

$$F(\varepsilon)\sim\exp\left(-\frac{\varepsilon}{kT}\right),\quad T=T_0+\frac{2e^2E^2\tau(\varepsilon)\tau_{en}(\varepsilon)}{3km}.$$

Therefore, according to Robson's logic, one should expect in this case the $K(D)$ and $K(\tilde{\mu})$ curves to coincide, in fact, however, they diverge to no less a degree that at $s+t\neq0$ (see Fig. 3b and Figs. 2c and 2d). Thus, Robson's assumption $K(D)\approx K(\tilde{\mu})$ is valid at equal or close values of s and t, i.e.,

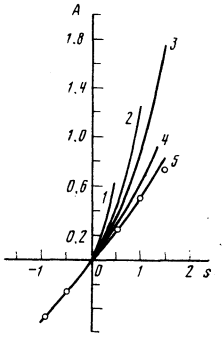

FIG. 3. Dependence of the quantity $A=[K(D)-1]/[K(\tilde{\mu})-1]-1$ in the region of weak electric fields on s at various t: 1—$s+t=-1$, 2—$s+t=-1/2$, 3—$s+t=0$, 4—$s+t=1/2$, 5—$s+t=1$. Curves 1, 2, 3, 4, and 5 practically coincide with curve 3 in the region $s<0$. The points correspond to the case of frequent electron-electron collisions.

in the vicinity of the vicinity of the dashed curve on Fig. 1 (although the energy distribution can in this case be quite far from Maxwellian!), and is not valid in the remaining part of the s,t plane.

4. COMPARISON WITH THE CASE OF FREQUENT INTERELECTRON COLLISIONS

It is of interest to compare the value of $K(D)=K(\delta j^2)$ calculated by us for rare collisions between carriers with the anisotropy of the diffusion coefficient and the anisotropy of the current fluctuations under the conditions of frequeny electron-electron collisions, when

$$\tau \ll \tau^{ee} \ll \tau_{\varepsilon u}. \tag{17}$$

where $1/\tau^{ee}$ is the characteristic frequency of the electron-electron collisions. It is known that in this case the carrier energy distribution is Maxwellian, and analytic expressions were obtained for the diffusion coefficient, just as for the spectral density of the current fluctuations (see, e.g., Ref. 2, pp. 63–67). The diffusion-coefficient tensor $D^{ee}_{\alpha\beta}$ is then no longer proportional to the tensor $(\delta j_\alpha \delta j_\beta)^{ee}_\omega$ of the current fluctuations, owing to the additional correction introduced by the collisions between the carriers (see Ref. 1, §20, or Ref. 2), which affects the current fluctuations. Thus, we now have

$$K(D_{ee})\equiv \frac{D_\parallel^{ee}}{D_\perp^{ee}} \neq \frac{(\delta j_\parallel^2)_\omega^{ee}}{(\delta j_\perp^2)_\omega^{ee}} \equiv K(\delta j_{ee}^2). \tag{18}$$

The influence of the electron-electron collisions on the anisotropy of the diffusion and current-fluctuation coefficients is illustrated by the corresponding curves in Fig. 2, plotted in accordance with Eqs. (3.153) and (3.155) of Ref. 2:

$$K(D_{ee})=(1+s-t+s^2)/(1-s-t), \tag{19}$$

$$K(\delta j_{ee}^2)=(1-t)^2/(1-s-t). \tag{20}$$

Attention is called to the possibility of a negative longitudinal-diffusion coefficient $D_\perp^{ee}$ (Fig. 2c). Also worthy of a more detailed discussion is the case $s+t=0$ (Fig. 2b). In this case the stationary distribution in energy becomes Maxwellian regardless of the intensity of the electron-electron scattering, no additional correlation is produced by the electron-electron collisions

$$K(D_{ee})=K(\delta j_{ee}^2),\ s+t=0. \tag{21}$$

In this case, as seen from Fig. 2b, we nevertheless have $K(\delta j_{ee}^2)\neq K(\delta j^2)$, i.e., without affecting the stationary distribution function, the electron-electron collisions still affect the fluctuations and the diffusion. This interesting circumstance is due to the fact that the frequency electron-electron collisions determine the form of the fluctuations in energy space, converting them essentially into fluctuations of the electron temperature (any distribution becomes Maxwellian after a time τ^{ee}). On the other hand if the electron-electron collisions are so infrequent that they can be neglected, then only a distribution stationary at $s+t=0$ is Maxwellian.

As seen from Fig. 2, on the whole the $K(\delta j^2)$ and $K(\delta j_{ee}^2)$ do not differ greatly, i.e., the influence of the electron-electron collisions on the anisotropy of the current fluctuations is not too significant.

We note that for $K(\delta j_{ee}^2)$ there exists a simple analytic expression obtained by Shul'man[17]:

$$K(\delta j_{ee}^2)=1/4(\tilde{\mu}_\parallel/\tilde{\mu}_\perp+1)^2. \tag{22}$$

Bearing in mind the already noted similarity of the $K(\delta j^2)$ and $K(\delta j_{ee}^2)$ curves, we can roughly estimate the anisotropies of the diffusion coefficient and of the current fluctuations for infrequent electron-electron collisions in the strong-field case by using expression (22), i.e., we can assume $K(D)=K(\delta j^2)\approx K(\delta j_{ee}^2)$. If we do not confine ourselves to the case of an energy-independent ratio of energy and momentum relaxation times, this estimate is more realistic than Robson's estimate $K(D)\approx K(\tilde{\mu})$.

5. HOT ELECTRONS

We consider, under conditions of infrequent electron-electron collisions, the region of sufficiently weak electric field, when it suffices to retain the terms of order E^2 in the expressions for the coefficient of diffusion, the spectral density of the current fluctuations and the mobility. The opinion was advanced (Ref. 18; cf. also Ref. 19, Chap, 4, §8) that the applicability of the Wannier-Robson approximation in the region of weak field can be regarded as established, since Robson[15] presented a derivation of the relation $D_\parallel/D_\perp\approx\tilde{\mu}_\parallel/\tilde{\mu}_\perp$ on the basis of nonequilibrium thermodynamics. However, the applicability of the methods of nonequilibrium thermodynamics to determine the corrections of order E^2 to the kinetic coefficients in a system with stationary current is doubtful. In fact, a kinetic analysis shows that the coefficients of E^2 in the expressions for $D_\parallel/D_\perp$ and $\tilde{\mu}_\parallel/\tilde{\mu}_\perp$ are far from equal. For scattering by acoustic phonons (hard spheres), the terms of order E^2 in the longitudinal and transverse noise temperatures were calculated earlier (Ref. 3; see also Ref. 20). The coefficient of E^2 in the expansion of $D_\parallel/\tilde{\mu}_\parallel$ turned out to be 40% larger than the corresponding coefficient in the expansion of $D_\perp/\tilde{\mu}_\perp$. Thus, even where the Robson relation works well in a strong field (in this case—accurate to 2%), the first term of the expansion in the field in no way satisfy this relation.

To obtain a better idea of the degree of difference between $K(D)=K(\delta j^2)$ and $K(\tilde{\mu})$ in the weak-field region, we calculated the quantity (terms of order E^2 were taken into account)

$$A=\frac{K(D)-1}{K(\tilde{\mu})-1}-1 \tag{23}$$

for different values of the exponents s and t (if Robson's relation were to be satisfied we would have $A=0$). The results of the calculations are shown in Fig. 3. We see that, except for the trivial case $s=0$, the terms of order E^2 in $K(D)$ and $K(\tilde{\mu})$ differ significantly. In particular, owing to the recurrence relation (15), we have $A=-\frac{1}{2}$ at $s=-1$ and $A=\frac{5}{4}(t+\frac{3}{2})$ at $s=1$.

We note that in the case of frequency electron-electron collisions

$$K(\delta j_{ee}^2)=1+s(2+s)\Delta T/T_0, \tag{24}$$

$$K(\tilde{\mu}_{ee})=1+2s\Delta T/T_0, \tag{25}$$

[see Ref. 2, Eqs. (3.151) and (3.118a)], where $\Delta T\sim E^2$ is the excess of the electron temperature above the ambient temperature. From (24) and (25) we get

$$A^{ee}=\frac{K(\delta j_{ee}^2)-1}{K(\tilde{\mu}_{ee})-1}-1=\frac{s}{2} \tag{26}$$

(the points in Fig. 3). It is seen from Fig. 3 that in the case of infrequent electron-electron collisions, too, we have with good accuracy $A=s/2$ in the region $s<0$.

6. CONCLUSIONS

The calculation of the dependence of the anisotropies of the carrier diffusion coefficient and of the low-frequency spectral density of the current fluctuations in an isotropic medium in an electric field on the quasielastic scattering has demonstrated the limited applicability of the Wannier-Robson relation. The latter is valid only in the strong-field limit for a special class of scattering mechanisms, when the ratio of the energy and momentum relaxation times do not depend on the energy. Comparison of the degree of anisotropy of the diffusion coefficient and of the current fluctuations for infrequent and for frequent electron-electron collisions has shown that the effect of the latter on the anisotropy of the current fluctuations is not too large. This allows us to recommend the use of an analytic formula, that describes the anisotropy of the spectral density of the current fluctuations in frequent electron-electron collisions, for an estimate of the degree of anisotropy of the diffusion coefficient and of the current fluctuations in the case of infrequent electron-electron collisions.

[1)]We note incidentally that in gases, too, one cannot exclude the possibility of a situation wherein the ratio of the energy and momentum relaxation times in quasi-elastic scattering depend on the energy—for example if rotational degrees of freedom are excited when the electron collides with a gas molecule (see Ref. 7).

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