Phase transitions in a superfluid neutron liquid

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It is shown that the cooling of neutron matter may be accompanied by a phase transition from one anisotropic superfluid state to another with very different physical properties. Such phase transitions must have a significant influence on the evolution of neutron stars.

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In this note, I wish to draw attention to a curious effect that may be observed in neutron matter at densities $\rho \gtrsim 1.5\times10^{14}$ g/cm³ and temperatures $T\sim10^9$ °K, which are characteristic for the interior regions of sufficiently young ($\tau < 1000$ years) neutron stars.[1] I am referring to a phase transition from one anisotropic superfluid state to another, in which the gap in the excitation spectrum has a qualitatively different angular dependence. Since the form of Δ as a function of the direction of the momentum affects the thermodynamics of neutron matter, the intensity of the emitted neutrino flux,[2] and so forth, such a phase transition must influence the evolution of the neutron star and, in particular, lead to a change in its cooling rate.

It is known that a neutron liquid with density slightly greater than the nuclear can be in one of three different superfluid states corresponding to triplet pairing.[2] Because of the presence of strong spin-orbit interaction, the spin and orbital angular momenta of the Cooper pairs in these states are parallel, so that the total angular momentum of a pair is always 2. In this case, the order parameter is isomorphic to a symmetric fourth-rank spinor, which can be represented in the form of a symmetric traceless 3×3 matrix $\hat{A}$. The complex numbers A_{ij} transform like the components of a vector under rotations of both the spin space (first index) and the configuration space (second index). The Landau expansion for the free-energy density of the neutron liquid has the form[2]

$$F=\frac{\alpha}{2}A_{ij}A_{ij}^{*}+\frac{\beta_1}{4}A_{ij}A_{ij}A_{kl}^{*}A_{kl}^{*}+\frac{\beta_2}{4}A_{ij}A_{ij}^{*}A_{kl}A_{kl}^{*}+\frac{\beta_3}{4}A_{ij}A_{jk}A_{kl}^{*}A_{li}^{*}. \tag{1}$$

Since we are dealing with a strongly coupled system, the calculation of the coefficients in the expansion (1) from first principles can hardly lead to reliable results. Therefore, the only sensible alternative is to study the thermodynamics of neutron matter in the general case, i.e., without prior particularization of the values of β_1, β_2, and β_3. Such an analysis was made by Sauls and Serene[2] without allowance for critical fluctuations. Noting the analogy between the present problem and that of phase transition to the superfluid state with d pairing, which had been solved earlier by Mermin,[3] Sauls and Serene[2] determined the structure of the order parameter for the three regions of values of the constants β_ν that correspond to the different superfluid phases. These regions are shown in Fig. 1 (in the coordinates $x_0=\beta_1/\beta_2$ and $y_0=\beta_3/\beta_2$). The matrices $\hat{A}$ corresponding to them have the form

$$\hat{A}_{\rm I}=\frac{\Delta_0}{2\sqrt{2}}\begin{pmatrix}1 & i & 0\\ i & -1 & 0\\ 0 & 0 & 0\end{pmatrix},\quad \hat{A}_{\rm II}=\frac{\Delta_0}{2}\begin{pmatrix}1 & i & 0\\ i & 1 & 0\\ 0 & 0 & -2\end{pmatrix},$$
$$\hat{A}_{\rm III}=\Delta_0\begin{pmatrix}1 & 0 & 0\\ 0 & b & 0\\ 0 & 0 & -1-b\end{pmatrix}, \tag{2}$$

where Δ_0 is the amplitude of the gap and b is the degeneracy factor. The specific degeneracy of the order parameter in region III[1)] when allowance is made for the sixth-order invariants in the expansion (1). In this case, a minimum of F corresponds to $b=-\frac{1}{2}$ (for more details, see Ref. 2), so that the angular dependence of the gap in the excitation spectrum has here the quite definite form

$$|\Delta_{\rm III}(\theta,\varphi)|^2 \sim 1+3\cos^2\theta. \tag{3}$$

For regions I and II, the corresponding expressions are

$$|\Delta_{\rm I}(\theta,\varphi)|^2\sim\sin^2\theta,$$
$$|\Delta_{\rm II}(\theta,\varphi)|^2 \sim\sin^2\theta\cos^2\varphi+\cos^2\theta. \tag{4}$$

A basic difference between the last two functions and (3) is that they have nodes, whereas $\Delta_{\rm III}$ vanishes nowhere. Therefore, the thermodynamic and various other characteristics of a superfluid neutron liquid will depend strongly on the position—inside region III (see Fig. 1) or outside it—of the point representing the system.

In the framework of Landau's theory, the position of the point is strictly fixed, since the values of the parameters β_ν are uniquely determined by the values of the corresponding coupling constants of the neutron-neutron interaction. In this situation, it is obvious that

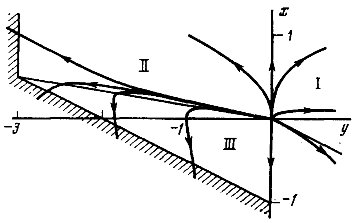

FIG. 1. Phase trajectories of the renormalization-group equations (5). The Roman numerals label the regions of the x—y diagram corresponding to different anisotropic superfluid states. The hatched line bounds the region in which the fourth-order form in the expansion (1) of the free energy is positive definite.

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as the temperature decreases the neutron liquid must go over into one of the three possible superfluid states and remain in it for all $T < T_c$. But Landau's theory holds only at temperatures sufficiently far from T_c, since it completely ignores the critical fluctuations of the order parameter. We must therefore consider whether allowance for these fluctuations changes the behavior of our system in the critical region, and if so, then how.

To answer this question, we must establish how the effective coupling constants γ_ν vary as $T \to T_c$; the dressed charges γ_ν play the same part in the region of strong fluctuations as the constants β_ν in the region of applicability of Landau's theory. The evolution of the charges is described by the renormalization-group equations. The Gell-Mann-Low functions which occur in these equations were calculated recently in the lowest approximation in γ_ν by Bailin, Green, and Love.[4] However, we shall not consider here the renormalization-group equations for the charges themselves, since in the present problem it is only their ratios $x = \gamma_1/\gamma_2$ and $y = \gamma_3/\gamma_2$ which are important. The "equations of motion" for x and y can be readily obtained from the results of Ref. 4. These equations are

$$\begin{aligned} \partial x/\partial t &= \gamma_2 (8x^3 + 8x^2 y + {}^{49}/_{18} xy^2 - 2x^2 - {}^{10}/_3 xy - {}^{59}/_{36} y^2 + 6x), \\ \partial y/\partial t &= \gamma_2 y (8x^2 + 8xy + {}^{49}/_{18} y^2 + 16x + {}^{51}/_6 y + 6), \quad t = c/\varkappa, \end{aligned} \tag{5}$$

where $\varkappa$ is the reciprocal correlation radius and c is a positive constant. The phase trajectories of Eqs. (5) are shown in Fig. 1. It is readily seen that there are among them lines which leave the stability zone of the Hamiltonian from a region of the x–y diagram which is not the same as the one in which they begin (these are the trajectories which intersect the boundary between regions II and III). In such a situation, we know that the system can go over into the low-temperature state predicted by Landau's theory not only directly but also by passing through an intermediate phase, which is thermodynamically stable in a certain range of temperatures solely on account of the interaction of the critical fluctuations.[5,6] The intermediate phase in this case is the superfluid phase with order parameter $\hat{A}_{III}$ and angular dependence of the gap of the form (3). As a result, a phase transition is possible in the neutron liquid from one anisotropic superfluid state to another with physical properties very different from the first.

Examining Fig. 1, we conclude that the probability of this phenomenon is small; for only a small part of region II has the property that the phase trajectories which begin in it enter region III. However, the real situation may be more favorable. The point is that the renormalization-group equations (5), which we have taken as basis, are themselves approximate; they are derived in the lowest order in the charges, which are not small in the asymptotic region; in addition, the derivation of these equations ignores the anisotropy of the fluctuation spectrum. Therefore, the true phase trajectories may be significantly different from the ones shown in Fig. 1. Accordingly, in the exact theory the "phase space" of the trajectories which intersect the boundary of regions corresponding to different superfluid phases may be greater.

[1)]Note that in all three regions there is a trivial degeneracy of the order parameter due to the presence of a gauge group and the rotation group.

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