Fabry-Perot interferometer in the field of a gravitational wave

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The effect of a gravitational wave on the interference pattern in a Fabry-Perot interferometer is investigated in a wide range of frequencies of the gravitational radiation. It is shown that there is a modulation of the intensity of the radiation passing through the interferometer, and the greatest contrast is achieved in the region of low frequencies of the gravitational waves, and also in the case when the wavelength λ of the gravitational wave is related to the interferometer base length L by $L = \lambda p$, where $p = 1/2, 1, 3/2, 2, 5/2, 3\ldots$.

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The possibility of using a Fabry–Perot interferometer, which is an optical system consisting of two plane-parallel mirrors, for detecting gravitational waves was discussed in Refs. 1 and 2. This possibility arises because of the occurrence in such a system of the so-called gravitational–electromagnetic resonance under certain conditions. In the present paper, solving the eikonal equation with appropriate boundary conditions, we obtain an analytic expression for the phase of a light ray propagating between the mirrors with any number of reflections in the presence of a gravitational wave, and on the basis of this expression we investigate the influence of the gravitational wave on the interference pattern in the Fabry–Perot interferometer (thereby indicating a way of measuring the effect).

§1. SOLUTION OF THE EIKONAL EQUATION

We assume that the two plane-parallel mirrors of the Fabry–Perot interferometer are fixed on two free bodies. One of them is at the coordinate origin, while the other is on the z axis at the point with coordinate $z=L$, so that light propagates between the mirrors parallel to the z axis. We assume further that a plane gravitational wave propagates at angle θ to the z axis. The metric of the gravitational wave is usually specified in a coordinate system $x'y'z'$ in which one of the axes coincides with the direction of propagation of the wave. If this is the z' axis, the nonzero components of the metric have the form

$$g_{00}'=-g_{33}'=1,\quad g_{11}'=-1+a',\quad g_{22}'=-1-a',\quad g_{12}'=g_{21}'=b'. \tag{1}$$

Going over to the laboratory coordinate system, for the nonvanishing components of the metric g^{ik} we obtain the expressions

$$g^{00}=1,\ g^{11}=-1-a,\ g^{12}=g^{21}=-b\cos\theta,\ g^{13}=g^{31}=b\sin\theta,$$
$$g^{22}=-1+a\cos^2\theta,\ g^{23}=g^{32}=-{}^1/_2 a\sin 2\theta,\ g^{33}=-1+a\sin^2\theta; \tag{2}$$
$$a=a'\cos 2\varphi+b'\sin 2\varphi,\quad b=-a'\sin 2\varphi+b'\cos 2\varphi. \tag{3}$$

Here, the angle φ is one of the Eulerian angles that determines the orientation of the x' and y' axes in the plane of the front of the gravitational wave. The angle φ distinguishes one of the two possible polarizations of the gravitational wave, and if $\varphi=0$, then $a=a'$, and if $\varphi=\pi/4$, then $a=b'$. In the general case, we have a mixture of two polarizations.

In the laboratory coordinate system fixed by the form of the metrics (2) and (3), the coordinates of the free bodies do not change under the influence of the gravitational wave (see Ref. 3).

We shall describe the phenomenon of interference in the approximation of geometrical optics, representing the light ray in the form of a plane wave (see, for example, Ref. 4). In this case, the light wave is completely determined by the phase ψ, which in the presence of the gravitational field satisfies the eikonal equation[5]

$$g^{ik}\frac{\partial\psi}{\partial x^i}\frac{\partial\psi}{\partial x^k}=0. \tag{4}$$

Since the wave is plane, $\partial\psi/\partial x=\partial\psi/\partial y=0$, and the eikonal equation for a light wave propagating along the z axis has, with allowance for the form of the metric (2), the form

$$\left(\frac{\partial\psi}{\partial\tau}\right)^2-(1-a\sin^2\theta)\left(\frac{\partial\psi}{\partial z}\right)^2=0. \tag{5}$$

It is easy to show that this equation is equivalent to two equations, one of which determines the phase of an electromagnetic wave propagating in the positive direction of the z axis,

$$\frac{\partial\psi}{\partial\tau}+\left(1-\frac{a}{2}\sin^2\theta\right)\frac{\partial\psi}{\partial z}=0, \tag{6}$$

and the other the phase of a wave propagating in the opposite direction:

$$\frac{\partial\psi}{\partial\tau}-\left(1-\frac{a}{2}\sin^2\theta\right)\frac{\partial\psi}{\partial z}=0. \tag{7}$$

Here, $\tau=ct$, and a is a function of the variables τ and $\mathbf{r}$,

$$a=a(k\tau-\mathbf{kr})=a(\tau-z\cos\theta),$$

where $\mathbf{k}$ is the wave vector and $k=\omega/c=2\pi/\lambda$ is the wave number of the gravitational wave. To be specific, we shall restrict ourselves to polarization corresponding to the angle $\varphi=0$, and we specify the gravitational wave in the form

$$a=a'=a_0\cos k(\tau-z\cos\theta). \tag{8}$$

In this case, the general solution of Eq. (6) [and also

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(7)] can be readily found. It can be expressed in terms of an arbitrary function

$$\psi^+=\psi^+\left(\tau-z-\frac{a_0}{k}\cos^2\frac{\theta}{2}\sin k(\tau-z\cos\theta)\right). \tag{9}$$

In the absence of a gravitational wave, the expression (9) must go over into the well-known expression for the phase of a plane electromagnetic wave:

$$\psi_0{}^+=k_{em}(\tau-z)+C^+, \tag{10}$$

where C^+ is a constant. Requiring that $\psi^+\to\psi_0^+$ as $a_0\to 0$, we conclude that the solution (9) must have the form

$$\psi^+=k_{em}(\tau-z)+C^+-a_0\frac{k_{em}}{k}\cos^2\frac{\theta}{2}\sin k(\tau-z\cos\theta)+a_0f^+(\tau-z), \tag{11}$$

where f^+ is an arbitrary function.

For the electromagnetic wave propagating in the negative direction of the z axis, we find similarly

$$\psi^-=k_{em}(\tau+z)+C^--a_0\frac{k_{em}}{k}\sin^2\frac{\theta}{2}\sin k(\tau-z\cos\theta)+a_0f^-(\tau+z). \tag{12}$$

Here, C^- is an arbitrary constant, and f^- is an arbitrary function.

§2. FABRY-PEROT INTERFEROMETER

Figure 1 shows the light rays in the interferometer. The light ray, which is incident on the plane-parallel plates of the interferometer with base L at angle φ_0, gives rise to the system of rays 1′, 2′, 3′..., which pass through the interferometer, and the system of rays 1″, 2″, 3″..., which are reflected by it. In all that follows, we shall consider a plane-parallel beam of light which enters the interferometer at angle $\varphi_0=0$, so that the trajectories of the transmitted and reflected waves coincide. Using the solutions (11) and (12), we calculate the phase of each transmitted and reflected ray. For this, we require that at the entrance to the interferometer the following condition is satisfied for the ray:

$$z=0,\ \psi^+(t,0)=\omega t=k_{em}\tau. \tag{13}$$

This condition uniquely determines the phase of the first ray $\psi_1^+(\tau,z)$ propagating in the space between the mirrors in the positive direction of the z axis; for it, we obtain

$$\psi_1{}^+(\tau,z)=k_{em}(\tau-z)-a_0\frac{k_{em}}{k}\cos^2\frac{\theta}{2}\times\{\sin k(\tau-z\cos\theta)-\sin k(\tau-z)\}. \tag{14}$$

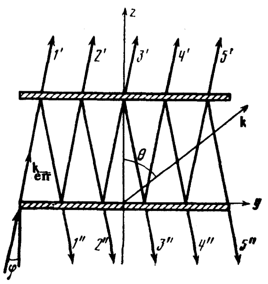

FIG. 1.

To determine the phase of the ray propagating in the opposite direction, it is necessary in the general case to take into account the fact that the phase changes on reflection by π (see, for example, Ref. 4). However, since the total change in phase during one cycle of reflections is 2π, this will not affect the interference pattern, and we can therefore assume that the phase is continuous at a reflection

$$\psi^-(\tau,z)\,|_{z=0,L}=\psi^+(\tau,z)\,|_{z=0,L}.$$

Using this condition, we find

$$\psi_1{}^-=k_{em}(\tau+z-2L)-a_0\frac{k_{em}}{k}\Big\{\sin^2\frac{\theta}{2}[\sin k(\tau-z\cos\theta)+\sin k(\tau+z-L(1+\cos\theta))]+\cos^2\frac{\theta}{2}[\sin k(\tau+z-L(1+\cos\theta))+\sin k(\tau+z-2L)]\Big\}. \tag{15}$$

Setting $z=L$ in Eq. (14), we obtain the phase of ray 1′ passing through the interferometer. Equation (15) for $z=0$ gives the phase of the reflected ray 1″. By lengthy but straightforward calculations we can find the phases of any transmitted or reflected rays. By mathematical induction, we prove the validity of the following expression for the n-th transmitted ray:

$$\psi_n{}^+(L,\tau)=k_{em}\{\tau-(2n-1)L\}-2a_0\frac{k_{em}}{k}\Big\{\sin^2\frac{\theta}{2}\frac{\sin k(n-1)L}{\sin kL}\sin\frac{1+\cos\theta}{2}kL+\cos^2\frac{\theta}{2}\frac{\sin knL}{\sin kL}\sin\frac{1-\cos\theta}{2}kL\Big\}\cos k\Big\{\tau-\Big(n-\frac{1-\cos\theta}{2}\Big)L\Big\}. \tag{16}$$

§3. RESONANCE EFFECT

In the absence of a gravitational wave ($a_0=0$), Eq. (16) gives the well-known expression in the theory of the Fabry–Perot interferometer for the change in the phase ψ between two successive transmitted rays: $\delta_0=2k_{em}L$. In this case ($a_0=0$), δ_0, which specifies the position of the working point on the instrumental function of the interferometer, determines the intensity of the light transmitted through it. In the general case, the phase difference for an arbitrary pair of successive transmitted rays has in accordance with Eq. (16) the form

$$\delta=\psi_n{}^+-\psi_{n-1}^+=\delta_0+\delta_{gr}(n), \tag{17}$$

where $\delta_{gr}(n)$ is the additional advance of the phase due to the gravitational wave. If the greatest contrast in the interference pattern is to be achieved, $\delta_{gr}(n)$ must not depend on the number n. Applied to Eq. (16), this means that $\psi_n^+(L,t)$ must depend linearly on n. This is the case if

$$kL=2\pi p, \tag{18}$$

where p is an integral or half-integral number: $p=\frac{1}{2}$, 1, 3/2, 2,.... For if p is an integer, then the oscillating factor $\cos k\{\tau-[n-(1-\cos\theta)/2]L\}$ does not depend on n by virtue of the condition (18). The factors of the form $\sin knL/\sin kL$ in this case are indeterminate forms of the type 0/0 which can be readily evaluated. Namely,

$$\lim_{kL\to 2\pi p} (\sin knL/\sin kL) = n.$$

As a result, for δ_{gr} we obtain the following n-independent expression:

$$\delta_{gr} = (-1)^p 2a_0 \frac{k_{em}}{k} \cos\theta \sin(\pi p \cos\theta) \cos\left[k\left(\tau + \frac{1-\cos\theta}{2} L\right)\right]. \tag{19}$$

If p is a half-integral number, $p=(2m+1)/2$, where $m=0,1,2,\ldots$, then for δ_{gr} we also obtain an n-independent expression:

$$\delta_{gr,m} = (-1)^{m+1} 2a_0 \frac{k_{em}}{k} \cos\theta \cos\left[\frac{\pi}{2}(2m+1)\cos\theta\right] \cos\left[k\left(\tau + \frac{1-\cos\theta}{2} L\right)\right]. \tag{20}$$

Since $\delta_{gr} \ll \delta_0$, using Airy's well-known formula for the intensity of the light transmitted through the interferometer (see, for example, Ref. 4), we can readily find the change in this intensity due to the effect of the gravitational wave. We have

$$\Delta I/I = 2Rk_{em}La_0 \sin\delta_0 F(\theta) \cos k(\tau+\tau_0)/[(1-R)^2 + 4R\sin^2(\delta_0/2)]\pi p, \tag{21}$$

where R is the energy reflection coefficient of the mirrors, and $F(\theta)$ is the directivity pattern

$$F(\theta) = \cos\theta \sin(\pi p \cos\theta), \quad p=1,2,3\ldots,$$
$$F(\theta) = \cos\theta \cos(\tfrac{1}{2}\pi(2m+1)\cos\theta), \quad p=\tfrac{1}{2}(2m+1) = \tfrac{1}{2}, \tfrac{3}{2}\ldots. \tag{22}$$

In accordance with Eq. (21), the effect of the gravitational wave on the interferometer leads to a modulation of the intensity of the transmitted light with the frequency of the gravitational wave. The modulation depth depends strongly on the choice of the phase δ_0. For optimal adjustment of the interferometer (corresponding to the choice of δ_0 on the section of maximal steepness of the Airy function), Eq. (21) is transformed to

$$\Delta I/I = (Q/\pi p) a_0 F(\theta) \cos k(\tau+\tau_0), \tag{23}$$

where $Q = k_{em}L/(I-R)$ is the Q of the Fabry–Perot interferometer. Comparing (21) and (23), we readily see that if R is near unity the difference between the magnitudes of the effect for optimal and nonoptimal adjustment of the interferometer can be very great. For example, for $R=0.998$ the difference between the effects may reach three orders of magnitude.

In Fig. 2, we show the directivity patterns $F(\theta)$ for $p=1$ (Fig. 2a, continuous curve) and $p=1/2$ (Fig 2a, broken curve), and also for $p=2$ (Fig. 2b, continuous curve) and $p=3/2$ (Fig. 2b, broken curve). With increasing p, the number of petals in the pattern increases. Common to all values of p—both integer and half-integer—is the absence of an effect for coincident ($\theta=0$) and mutually perpendicular ($\theta=\pi/2$) directions of propagation of the gravitational and electromagnetic waves. This result is in complete agreement with the predictions of Refs. 1 and 2. The direction of maximum sensitivity for $p=1$ and 1/2 corresponds to the angles $\theta_0 \approx 50°$ and $\theta_0 \approx 57°$, respectively, and for $p=2$ and 3/2 to the angles $\theta_1 \approx 37°$, 44° and $\theta_2 = 73°$, 80°. We recall that these results correspond to the polarization defined by the angle $\varphi=0$ [see Eq. (5)]. In the general

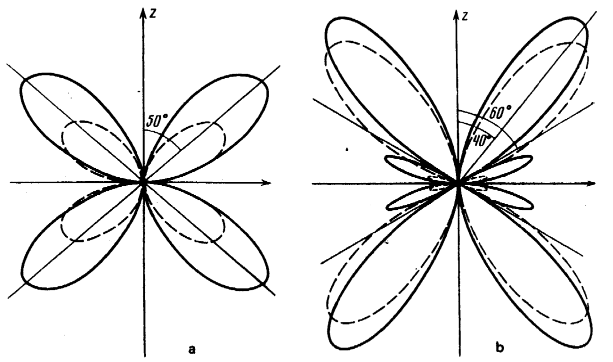

FIG. 2.

case, the pattern depends on the angle φ:

$$F(\theta,\varphi) = \cos\theta \sin(\pi p \cos\theta) \cos 2(\varphi-\varphi_0), \quad p=1,2,3\ldots,$$
$$F(\theta,\varphi) = \cos\theta \cos(\tfrac{1}{2}\pi(2m+1)\cos\theta) \cos 2(\varphi-\varphi_0), \quad p=\tfrac{1}{2}(2m+1) = \tfrac{1}{2}, \tfrac{3}{2}\ldots. \tag{24}$$

In accordance with the relation $\tan 2\varphi_0 = b'/a'$, the phase φ_0 in this formula determines the polarization of the gravitational wave, namely, $\varphi_0=0$ corresponds to the polarization a' and $\varphi_0=\pi/4$ to the polarization b'; in the remaining cases, there is a mixture of polarizations.

The condition (18), which is an additional condition for the interference of light waves passing through the interferometer (and also reflected from it) in the field of the gravitational wave, is equivalent to the condition of gravitational–electromagnetic resonance calculated by Braginskiĭ and Menskiĭ in Ref. 6 for an electromagnetic wave in an annular wave tube (see also Refs. 1 and 2). In the case of the Fabry–Perot interferometer, this condition distinguishes an infinite series of resonance frequencies which are multiples of the fundamental frequency ($p=1$), for which the gravitational wavelength is equal to the base length of the interferometer and for which, in accordance with Eqs. (21) and (23), the effect is maximal.

Let us consider in more detail the interpretation of the effect. For optimal adjustment of the interferometer, the magnitude of the effect is, in accordance with Eq. (23), proportional to the combination $a_0L/\lambda(1-R)$. Now $1/(1-R)$ is proportional to the number of double passages of the optical ray between the mirrors of the interferometer: $1/(1-R) \approx 2n$. Hence

$$\Delta I/I \approx 2\pi(L2n)a_0/\lambda\pi.$$

Therefore, the effect can be interpreted as due to a change in the refractive index of the medium by an amount equal to a_0/π.

§4. LOW-FREQUENCY REGION

Let us continue the analysis of Eq. (16). We require fulfillment of the condition

$$knL \ll 1. \tag{25}$$

In other words, we shall consider the low-frequency part of the spectrum of gravitational waves in the frequency range

$$\nu=0-\nu_{max}. \tag{26}$$

The upper limit of this range is determined by the condition (25). If we set $knL=0.1$, $n=10^3$, and $L=1$ m, then $\nu_{max}=5\times10^3$ Hz. After some manipulations and simplifications of the expression (16), we obtain

$$\psi_n^+=k_{em}\{\tau-2(n-1)L\}-(n+1)a_0k_{em}L\sin^2\theta\cos k\tau. \tag{27}$$

In terms of the metric of the gravitational wave, this expression can be written in the form

$$\psi_n^+=k_{em}\{\tau-2(n-1)L\}-(n+1)k_{em}Lh_{33}, \tag{28}$$

where h_{33} is related to the metric component g_{33} by $g_{33}=-1+h_{33}$. In Eq. (28), h_{33} is taken at the point $z=0$: $h_{33}=a_0\sin^2\theta\cos k\tau$. As can be seen from (27), the gravitational part of the phase ψ_n^+ is proportional to n and, therefore, the condition (18) for obtaining a sharp interference pattern is satisfied for the range of frequencies (26). We can then readily find the gravitational advance of the phase over one double passage of the optical ray:

$$\delta_{gr}=-k_{em}Lh_{33}=-2a_0\frac{\pi L}{\lambda_{em}}\sin^2\theta\cos k\tau. \tag{29}$$

Further, as in Sec. 3, we can use Airy's formula to find the modulation of the intensity of the transmitted light. For optimal adjustment of the interferometer, we have

$$\Delta I/I=Qa_0\sin^2\theta\cos k\tau. \tag{30}$$

Comparing Eqs. (30) and (23), we see that in the low-frequency region determined by the relations (25) and (26) the Fabry–Perot interferometer has as a gravitational detector a somewhat greater (by π times) sensitivity (in the amplitude a_0) than in the region of high-frequency resonances.

The directivity pattern of the Fabry–Perot interferometer in the low-frequency region is determined by the function

$$F(\theta,\varphi)=\sin^2\theta, \tag{31}$$

and, with allowance for arbitrary polarization, by the function

$$F(\theta,\varphi)=\sin^2\theta\cos 2(\varphi-\varphi_0). \tag{32}$$

The maximum is at the angle $\theta=\pi/2$, when the symmetry axis of the interferometer and the direction of the propagation of the gravitational wave are mutually perpendicular (see Fig. 3).

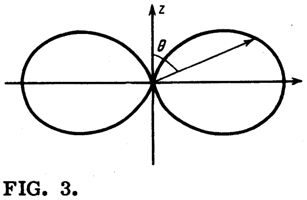

FIG. 3.

The condition (25) enables us to find a solution to the problem in a simpler and more perspicuous manner. We introduce a local Lorentz coordinate system $\bar{x}$ attached to the body (mirror) at the coordinate origin. Along the worldline of this mirror, the metric will have the form of the Minkowski metric. We require that in the neighborhood of this worldline the metric take the form

$$\bar{g}_{ik}=\eta_{ik}+O(hx/\lambda). \tag{33}$$

It is easy to find a transformation satisfying these conditions:

$$\begin{aligned}\bar{x}^0&=x^0,\quad \bar{x}^1=(1-{}^1/_2h_{11})x^1,\\ \bar{x}^2&=-h_{12}x^1+(1-{}^1/_2h_{22})x^2,\\ \bar{x}^3&=-h_{13}x^1-h_{23}x^2+(1-{}^1/_2h_{33})x^3.\end{aligned} \tag{34}$$

The values of the metric $h_{\mu\nu}$ are taken at the point $x^3=0$.

In the neighborhood of the coordinate origin $\bar{x}\ll\lambda$, this neighborhood including the second mirror by virtue of the condition (25); the deviations from the Minkowski metric have the order $\sim hx/\lambda\ll h$ and, therefore, the contribution of the gravitational wave to the eikonal equation vanishes but the coordinate $\bar{x}^3$ of the second mirror undergoes periodic oscillations in accordance with (34):

$$\bar{x}^3=(1-{}^1/_2h_{33})L. \tag{35}$$

These displacements of the second mirror with respect to the first give rise to an additional advance of the phase equal to $-\frac{1}{2}h_{33}Lk_{em}$. From this we obtain for the gravitational advance of the phase over one double passage of the optical ray the previously obtained expression (29), and then from it the results (30)–(32).

§5. SENSITIVITY OF THE GRAVITATIONAL DETECTOR

It is now regarded as realistic to construct a Fabry–Perot interferometer with base $L=10$ m and reflection coefficient $R=0.998$ with a laser light source. The Q of such an interferometer for a helium–neon laser with wavelength $\lambda=6.3\times10^{-5}$ cm will be $Q=5\times10^{10}$. If the limit to the resolution of the modulation depth is determined by the photon noise, as was the cause in the experiment using a Michelson interferometer,[7] then

$$\min(\Delta I/I)=N^{-1/2},$$

where N is the number of photons. Let us take a measurement time of $\Delta t=10^{-2}$ sec. Then for a helium–neon laser of power $W=10$ W,

$$N=W\Delta t\lambda_{em}/hc=3.2\cdot10^{17},\quad \min(\Delta I/I)=1.8\cdot10^{-9}.$$

In accordance with (30), we find the resolution limit for the gravitational-wave amplitude:

$$\min a_0\approx0.4\cdot10^{-19}.$$

This value is somewhat lower than the so-called optimistic estimate for the amplitude of gravitational radiation expected from an event such as a supernova explosion (see Ref. 8).

In conclusion, we note that the results obtained in this paper, in particular, Eq. (16), are valid for the analysis of multipassage interferometers, i.e., interferometers with two interfering rays but with multiple reflection in a system of two or more mirrors used to increase the optical length of the interferometer.

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Relativistic corrections and corrections for the electromagnetic structure of the nuclei to the energy levels of μ-mesic molecules of hydrogen isotopes

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An effective Hamiltonian is constructed for a three-body system with allowance for the electromagnetic structure of the particles and relativistic effects of order α^2 that do not depend on the spin orientation of the particles. This Hamiltonian and the nonrelativistic wave functions of a system of three particles with Coulomb interaction found in the adiabatic representation are used in a perturbation-theory calculation to accuracy $\sim 5\times 10^{-3}$ eV of the relativistic corrections and the corrections for the electromagnetic structure of the nuclei to the energy levels of the μ-mesic molecules of hydrogen isotopes.

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§1. INTRODUCTION

The recent interest in the physical characteristics of μ-mesic molecules of hydrogen isotopes such as the energy levels and their hyperfine structure arises from a number of new high-precision experiments on μ^- capture by light nuclei[1] and, above all, investigation of muon catalysis of the synthesis of the nuclei of the heavy isotopes of hydrogen.[2] The coupling in the μ-mesic molecules is due entirely to the electromagnetic interaction; this makes it possible to describe their stationary states with high accuracy,[3,4] which, in its turn, increases the value of the experimental results and the reliability of their interpretation. At the same time, because the masses of the μ^- meson and the nuclei are comparable, the relative contribution of the corrections to the energy levels of the mesic molecules due to the relativistic dynamics is about two orders of magnitude greater than in ordinary molecules. To describe many processes with spin dependence[1,3,4] (such as μ^- capture) and especially the resonance formation of mesic molecules, the nonrelativistic approximation is inadequate, and relativistic effects make a contribution at the level of the accuracy required in these cases in the calculation of the energy levels of the μ-mesic molecules, namely, $\sim 10^{-3}$ eV.

In the present paper, mesic molecules are treated as systems of three spin particles with electromagnetic interaction, and their dynamics is described by the Schrödinger equation with the approximate (accurate to terms of order α^2) relativistic Hamiltonian obtained in the framework of the formalism of Foldy and Krajcik.[5] The operators of the two-particle relativistic interaction are constructed in the framework of Todorov's quasipotential approach.[6] The relativistic effects in the Hamiltonian correspond to additive terms of two types: diagonal and nondiagonal with respect to the spin variables.

The interactions associated with the latter generate a hyperfine splitting of the energy levels; they have been considered earlier.[7,8] The present paper is devoted to a study of the relativistic effects that do not depend on the spin orientation of the particles and lead only to shifts of the nonrelativistic energy levels. We consider in general form systems of three particles with spins not exceeding 1, and we take into account their electro-

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