Fiz. 27, 252 (1978) [JETP Lett. 27, 236 (1978)].
[6]D. L. Talwar, B. K. Ghosh, B. K. Agarwal, Phys. Rev. B18, 1762 (1978).

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Parametric amplified echo

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Amplification of a nuclear induction signal was obtained by exciting a spin system with a parametric pumping pulse. The experiments were performed on Mn^{55} nuclei in $CsMnF_3$ under conditions of coupled nuclear-electron precession. A theory is constructed for the formation of the parametric amplified echo excited by a high-power parametric pumping pulse.

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The spin-echo method makes it possible in a number of cases to obtain an echo signal of higher intensity than one of the exciting radiofrequency (RF) pulses. One can then speak of an RF amplifier of sorts, which produces also a time delay. This effect has been named "amplified" echo and was first observed in a ferromagnet for a system of long-wave spin waves.[1] It was observed for the same system[2] that replacement of the second resonant rf pulse by an RF pulse at double the frequency greatly enhances the amplification effect. The dynamics of the oscillations of the long-wave spin waves and their interaction with RF fields have a number of nonlinear features that hinder both the use of the effect and the development of a quantitative theory. The amplified-echo effect can be obtained in principle in systems with dynamic frequency shift, in which the spin echo is formed with the aid of a frequency-modulation (FM) mechanism (concerning the FM echo mechanism see, e.g., the review[3]). So far, however, this effect has not been obtained experimentally via the FM signal-formation mechanism.

Another echo-formation mechanism, in which echo amplification is possible, is parametric echo. We report here experimental observation of the amplified-echo effect via the parametric-echo mechanism. The parametric echo is produced in systems of oscillators on which it is possible to act directly both in resonant and in parametric fashions. Among the spin systems, these include electron spin systems in many magnetically ordered substances, as well as a system of nuclear spins under conditions of coupled nuclear-electron precession.

Parametric echo was first observed on Mn^{55} nuclei in a number of antiferromagnets.[4] To produce this echo, an RF pulse was applied to the spin system and was followed, after a time delay t_{12}, by an RF pulse having double the frequency and a magnetic field polarized along the constant magnetic field. This pulse excited the spin system parametrically. The specific nature of the direct parametric excitation of spin system makes the signal echo produced at the instant $2t_{12}$ an effect of first order in the RF pulse amplitude (and not of second order as in the Hahn or in the FM echo).

One of us and Gladkov[4,5] investigated theoretically and experimentally the mechanism of parametric echo formation at low amplitudes of the exciting pulses. With increasing amplitude of the amplitude of the RF parametric pumping pulse, we succeeded in observing the amplified echo effect. Experiments aimed at observing parametrically amplified echo were carried out on the system of Mn^{55} nuclei in the antiferrmagnet $CsMnF_3$ at 1.5 K and an NMR frequency 500 MHz. The experiments were performed with the parametric-echo spectrometer described in Ref. 5. To increase the parametric pumping power, a pulsed voltage up to 2 kV was applied to the oscillator and amplifier tubes of the double-frequency oscillator (G4-37A), so that the pulse RF power could be raised to 10 W.

The direct effect of the amplified echo is that the perpendicular magnetization of the sample, which forms the spin-echo signal, turns out to be larger than the perpendicular magnetization induced by the first pulse. In the experiment this reduces to a higher intensity of the echo signal than the intensity of the induction signal after the first pulse. The possibility of obtaining an echo signal exceeding in intensity the RF field of the resonant pulse is determined by the coupling between the RF field and the spin system. In the case of nuclear spin systems with low susceptibility, this is an extremely difficult task and is not considered in the present article.

Unfortunately, direct observation of the induction signal intensity following the resonant RF pulse is strongly hindered by the "dead" time of the receiving system, which amounts to 5 μsec. Therefore the intensity of the induction signal was estimated by the three-pulse procedure described in Chapter 5 of Ref. 5. At a maximum parametric buildup pulse, we succeeded in obtaining a parametric-echo signal of intensity four times higher than that of the induction signal.

For a quantitative investigation of the effect of the amplified parametric echo, we used as the amplified signal a spin-echo FM signal produced by two resonant

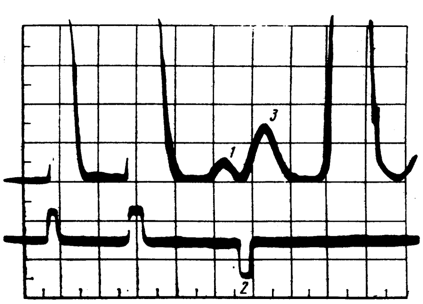

FIG. 1. Oscillogram of amplified parametric echo signal (3) produced by the FM echo signal (1), following the action of a high-power parametric pumping pulse (2). The lower trace of the oscillogram shows the positions of the RF pulses.

RF pulses (Fig. 1). At a time t_{12} after this echo signal we applied to the system a high power parametric buildup pulse of 1 μsec duration. At a time t_{12}, after turning off the parametric buildup pulse, a parametric echo signal was observed whose intensity exceeded that of the amplified echo signal. Figure 2 shows a plot of the gain K of the echo signal against the amplitude of the RF parametric pumping field. In the calculation of the gain we took into account the relaxation effect, whose time constant under the experimental conditions was $T_2 = 35$ μsec ($t_{12} = 5$ μsec). As a result

$$K = \frac{I_a}{I_{in}} e^{t/T_2} = 1.15\frac{I_a}{I_{in}},$$

where I_a and I_{in} are the intensities of the amplified and initial echoes (in units of the RF field amplitude). The relative amplitude of the parametric pumping RF field was plotted using the signal induced in the frequency meter. The RF field was calibrated against the theorectical formula given at the end of the article and shown by the continuous curve in Fig. 2.

A linear theory of the mechanism of formation of parametric echo was developed in Ref. 1 and describes well the echo at low amplitudes of the RF pulses. To construct a theory of the amplified parametric echo, we use a method proposed by Kurkin.[6] In his notation, the motion of the nuclear magnetization, averaged over

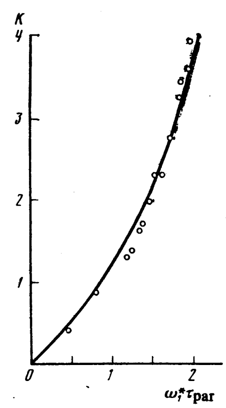

FIG. 2. Dependence of the echo-signal gain on the RF field amplitude in the parametric pumping pulse: the solid curve corresponds to $K = \sinh(\omega_1^*\tau_{par})$, and the points to $K = 1.15 I_a / I_{FM}$.

the distances m_i of the Suhl-Nakamura interaction during the time of the parametric buildup, is described by the system of equations

$$\begin{aligned} \dot m_i^x &= \Delta\omega_i m_i^y + \omega_1^* m_i^z m_i^y / m_i^0, \\ \dot m_i^y &= -\Delta\omega_i m_i^x + \omega_1^* m_i^z m_i^x / m_i^0, \\ \dot m_i^z &= -2\omega_1^* m_i^x m_i^y / m_i^0. \end{aligned} \quad (1)$$

The motion is considered in a coordinate frame that rotates at half the frequency of the parametric pump. $\Delta\omega_i$ is the deviation of the nuclear-magnetization precession frequency, $\omega_1^* = \gamma\eta_\parallel h_{RF}$ is the effective amplitude of the RF field of the parametric pump in frequency units, and $\eta_\parallel$ is a coefficient that determines the efficiency of the parametric pumping process. Using Kurkin's method, we change to a coordinate frame rotated through $\pi/4$:

$$m_i^+ = m_i^x + m_i^y, \quad m_i^- = m_i^x - m_i^y,$$

and, neglecting effects connected with changes of m_i^z (the angle of inclination of the magnetization is small, so that $m^z \approx m^0$), we have

$$\begin{aligned} \dot m_i^+ &= -\Delta\omega_i m_i^- + \omega_1^* m_i^+, \\ \dot m_i^- &= \Delta\omega_i m_i^+ - \omega_1^* m_i^-. \end{aligned} \quad (2)$$

In the case of a short powerful parametric pulse, we can neglect the term with $\Delta\omega$ ($\Delta\omega\tau \ll 1$). Then the motion of the nuclear-magnetization component during the time of the parametric pumping pulse is of the form

$$m_i^+(t) = m_i^+(0) e^{\omega_1^* t}, \quad m_i^-(t) = m_i^-(0) e^{-\omega_1^* t}. \quad (3)$$

We proceed now to a description of the mechanism of formation of the parametric echo. At the initial instant the resonant RF pulse rotates the nuclear magnetization through an angle α. By the instant the parametric pulse is applied, the nuclear-magnetization vectors of the different regions of the crystals are dephased:

$$m_i^+ = m_i^0 \alpha \cos(\Delta\omega_i t_{12}), \quad m_i^- = m_i^0 \alpha \sin(\Delta\omega_i t_{12}). \quad (4)$$

(It is assumed here for simplicity that the nuclear magnetization was deflected by a resonant pulse along the [1, 1] axis.) The action of the short parametric pulse alters the distribution of the magnetization in the following manner:

$$m_i^+ = m_i^0 \alpha \cos(\Delta\omega_i t_{12}) e^{\omega_1^* \tau_p}, \quad m_i^- = m_i^0 \alpha \sin(\Delta\omega_i t_{12}) e^{-\omega_1^* \tau_p}. \quad (5)$$

The magnetizations then continue to precess at a frequency $\Delta\omega_i$ and take at the instant t after the parametric pulse the form

$$\begin{aligned} m_i^+ &= m_i^0 \alpha \cos(\Delta\omega_i t_{12}) \exp(\omega_1^* \tau_p) \cos(\Delta\omega_i t) \\ &\quad - m_i^0 \alpha \sin(\Delta\omega_i t_{12}) \exp(-\omega_1^* \tau_p) \sin(\Delta\omega_i t), \\ m_i^- &= m_i^0 \alpha \sin(\Delta\omega_i t_{12}) \exp(-\omega_1^* \tau_p) \cos(\Delta\omega_i t) \\ &\quad + m_i^0 \alpha \cos(\Delta\omega_i t_{12}) \exp(\omega_1^* \tau_p) \sin(\Delta\omega_i t). \end{aligned} \quad (6)$$

Summing the components m^+ and m^- over the entire frequency spread, we have at the instant of time $t = t_{12}$

$$M^+ = M^0 \alpha \operatorname{sh} \omega_1^* \tau_p. \quad (7)$$

Thus, the intensity of the signal of the amplified parametric echo is ($\sinh \omega_1^* \tau_i$) times larger than the intensity of the induction signal after the resonant pulse.

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[1]D. E. Kaplan, R. M. Hill, and G. F. Herrmann, J. Appl. Phys. **40**, 1164 (1969). G. F. Herrmann, D. E. Kaplan, and R. M. Hill, Phys. Rev. **181**, 829 (1969). G. F. Herrmann, R. M. Hill, and D. E. Kaplan, J. Appl. Phys. **41**, 925 (1979).
[2]V. V. Danilov, V. I. Sugakov, and A. V. Tychinskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 520 (1972) [JETP Lett. **15**, 368 (1972)]. V. V. Danilov, A. V. Tychinskiĭ, and V. I. Sugakov, Izv. vyssh. ucheb. zav. No. 3, 37 (1973).
[3]Yu. M. Bun'kov and B. S. Dumesh, in: Problemy magnitnogo rezonansa (Problems in Magnetic Resonance), Nauka, 1978, p. 310.
[4]Yu. M. Bun'kov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 271 (1976) [JETP Lett. **23**, 244 (1976)].
[5]Yu. M. Bun'kov and S. O. Gladkov, Zh. Eksp. Teor. Fiz. **73**, 2181 (1977) [Sov. Phys. JETP **46**, 1141 (1977)].
[6]E. A. Turov and M. I. Kurkin, op. cit.[3], p. 271.

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