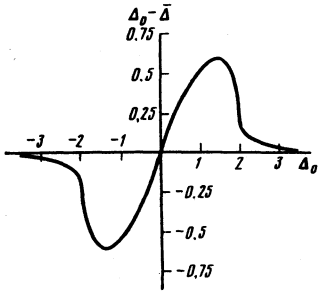

FIG. 3. Dependence of the mean energy $\bar{\sigma}E/E$ (in units of $\gamma^2\Omega/\omega$) transferred by the electron to the field on the detuning, calculated from formulas (10) and (17).

$$\Delta_0-\bar{\Delta}=\Delta_0\left(1-\frac{1}{\ln(1/|\Delta_0|)}\right),\quad |\Delta_0|\ll 1. \tag{19}$$

Formula (19) reflects the fact that in the case of small detunings, a large part of the electrons are trapped by the wave and a small fraction (of the order of Δ_0) of the untrapped electrons have a small [of the order of $1/\ln(1/|\Delta_0|)$] mean velocity relative to the wave.

Figure 3 shows a graph of the dependence of the mean energy $\overline{\delta E}$ transferred to the electromagnetic field on the detuning Δ_0. The maximum in the graph corresponds to $\Delta_0\approx 1.4$ with $\Delta_0-\bar{\Delta}\approx 0.6$, so that the maximum relative change in the energy of the electron in the strong saturation regime is

$$\overline{\delta E}/E=0.6\gamma^2\Omega/\omega. \tag{20}$$

We note that at small values of the saturation parameter $\tau=z/l\ll 1$, the width of the amplification band is decreased with increase in z: $\delta\omega/\omega\sim a/z$ (a is the pitch of the helical magnetic field).[5-7] In the strong saturation regime, the length z in this relation is replaced by the bunching length l (in our notation, this corresponds to $\Delta_0\sim 1$).

The given-field approximation that we have used is valid under the condition that the relative change in the energy of the electromagnetic wave is small: $n\overline{\delta E}\ll\mathscr{E}^2$, where n is the concentration of electrons in the beam. Using formulas (5) and (20), we can rewrite this condition in the form $\mathscr{E}\gg\mathscr{K}^{1/3}(ena)^{2/3}$. If this inequality is satisfied, the bunching length l is small in comparison with the amplification length that enters into the linear theory.[6]

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Angular distributions of resonant gamma-ray scattering by ^{57}Fe nuclei in hydrated sulfates of iron

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The angular distributions of the resonant scattering of 14.4 keV gamma rays by ^{57}Fe nuclei in the polycrystalline iron compounds ($FeSO_4\cdot H_2O$ and $FeSO_4\cdot 7H_2O$) were measured for the individual components of the quadrupole doublets of the hyperfine structure. The gamma quanta scattered with and without recoil were separated by using the method of the "black" absorber placed between the scatterer and the detector. The measured angular distributions of the resonant scattering differ from the "hard core" distribution, a fact attributed to the anisotropy of the Mössbauer-effect probability. The values of the anisotropy ε are -0.20 ± 0.05 and 0.10 ± 0.06 for $FeSO_4\cdot H_2O$ and $FeSO_4\cdot 7H_2$, respectively, at positive values of the electric field gradient.

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INTRODUCTION

It is known[1] that the quadrupole doublets of the Mössbauer transitions $\frac{3}{2}\to\frac{1}{2}$(^{57}Fe, ^{119}Sn) are symmetrical, i.e., both lines have the same intensity, if the Mössbauer-effect probability f' is isotropic. If, however, f' depends on the angle between the directions of the crystal axis and of the emission of the gamma quantum, $f'=f'(\theta_f)$, then an asymmetry of the intensities can appear in polycrystalline samples (the Gol'danskiĭ-

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Karyagin effect).[2] Komissarova *et al.*[3,4] have shown that the anisotropy of the Mössbauer-effect probability leads also to substantial changes in the angular distributions of the resonant scattering.

In the absence of hyperfine interaction, the angular distribution of the resonant scattering is determined by a function of the angular correlation for the transition $I_i(L)I(L)I_{f=i}(L)$, where I is the spin of the ground state of the nucleus, I is the spin of the excited state, and L is the multipolarity of the gamma transition.[5] For $I_{f=i}=\frac{1}{2}, I=\frac{3}{2}, L=1$ we have

$$W(\theta)=1+0.25P_2(\cos\theta). \quad (1)$$

The hyperfine interaction leads to a perturbation of this distribution:

$$W(\theta)=1+0.25G_2P_2(\cos\theta), \quad (2)$$

where $P_2(\cos\theta)$ is a Legendre polynomial, θ is the scattering angle, and G_2 is the perturbation (attenuation) factor, which takes for a static quadrupole interaction the form

$$G_2=0.2\left[1+\frac{4}{1+(\Delta E/\Gamma)^2}\right]; \quad (3)$$

here ΔE is the energy of the quadrupole splitting of the $I=\frac{3}{2}$ level and Γ is its natural width.

The use of the Mössbauer effect makes it possible to observe the angular distributions of resonantly scattered gamma quanta when the hyperfine-structure components are separately excited, if $\Delta E \gg \Gamma$. If f' is isotropic, then in the case of a polycrystalline scatter the angular distributions of both quadrupole-doublet components are identical with the "hard core" distribution (i.e., $G_2(\min)=0.2$), and (2) goes over into

$$W(\theta)=1+0.05P_2(\cos\theta). \quad (4)$$

Anisotropy of the Mössbauer effect probability leads to deviations from this formula. This situation was experimentally demonstrated[3,4] for a number of tin compounds characterized by resolvable quadrupole doublets for ^{119}Sn. In the same references were derived and calculated the corresponding coefficients, under the assumption that $f'(\theta_f)$ can be expressed by the simple formula

$$f'(\theta_f)=f_0+f_2P_2(\cos\theta_f), \quad (5)$$

i.e., the anisotropy of the Mössbauer-effect probability can be characterized by a single parameter

$$\varepsilon=\frac{f'(\pi)-f'(\pi/2)}{f'(\pi)+f'(\pi/2)}. \quad (6)$$

If f' is anisotropic and the individual components of the quadrupole doublets are excited, the angular-distribution functions can be expressed in the form

$$W_{|m|}(\theta)=1+C_2(\varepsilon,|m|,\beta)P_2(\cos\theta), \quad (7)$$

where the coefficients C_2 depend on the magnitude and the sign of ε, on the absolute value of the projection $|m|$ of I on the gradient axis (i.e., on the sign of the quadrupole interaction), and on the angle β between the crystal axis and the principal axis of the electric-field gradient (EFG). As shown in Refs. 3 and 4, the coefficients C_2 are particularly sensitive to these parameters if one detects not the total scattered radiation, but only that emitted by the scatterer without recoil. This was demonstrated in Refs. 3 and 4 using a resonant detector of ^{119}Sn radiation.

It should be noted that the use of a resonant detector frequently entails methodological difficulties, inasmuch as selection of a definite resonance makes it necessary to set in motion either the scatterer (if the source and detector are at resonance) or the source and detector, using independent motors. This circumstance has prevented the authors of Refs. 3 and 4 from measuring, with a resonant detector, the angular distributions for both components of the quadrupole doublet in the compound $(C_4H_9)_2\cdot SnO$.

In the present study the investigations of the dependences of the angular distributions of the resonant scattering on the anisotropy of the probability f' were expanded to include iron compounds. We have measured the angular distributions of the resonant scattering of 14.4-keV gamma quanta by ^{57}Fe nuclei upon excitation of the individual components of the quadrupole doublets in the compounds $FeSO_4\cdot H_2O$ and $FeSO_4\cdot 7H_2O$. The doublet splittings for these compounds are respectively 2.65 and 3.20 mm/sec, and the intensities of the components were unequal. The $FeSO_4\cdot 7H_2O$ absorption spectrum is shown in Fig. 1a.

To separate the radiation scattered with and without recoil, we used a "black" absorber (Fig. 1b). When it was placed in the path of the scattered beam ahead of the detector, only the recoil-free scattered γ quanta were registered. Without the absorber, the gamma quanta emitted by the scatterer both with and without recoil were recorded. The difference effect pertains entirely to recoil-free scattering. The "black" absorber can be made of the scatterer material. Its use therefore does not call for the aforementioned complication of the setup and permits in this case measurements to be made for all the hyperfine structure components. If the absorber is thin with respect to the nonresonant gamma quanta, then its use does not lower

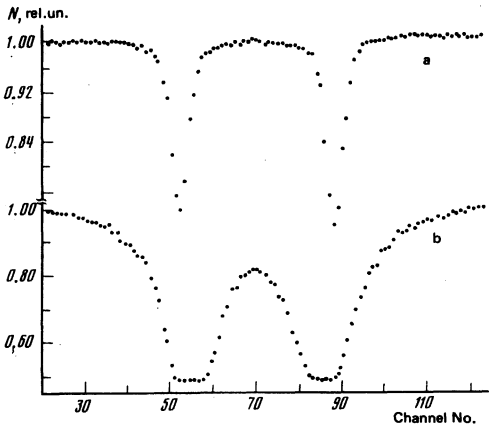

FIG. 1. Mössbauer spectera of hydrated iron sulfates: a—absorption spectrum of $FeSO_4\cdot 7H_2O$, b—spectrum of "black" absorber $^{57}FeSO_4\cdot H_2O$.

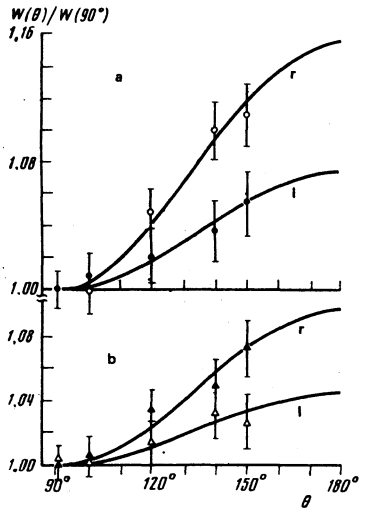

FIG. 2. Angular distribution of resonantly scattered 14.4-keV gamma quanta by ^{57}Fe nuclei in $FeSO_4 \cdot H_2O$. The curves were calculated by least squares: a—for pure recoil-free scattering, b—for the entire scattered radiation; the curves "r" and "l" refer to the right and left components of the doublet.

the counting efficiency. The latter can on the whole be much higher than when resonant detectors are used, since the efficiency of the latter for resonant gamma quanta is low, 10 to 15% according to the data of Ref. 6.

EXPERIMENT

The measurements were performed with a setup similar in geometry to that described in Refs. 3 and 7. The scatterer dimensions were 80×80 mm. The scattering angle was varied by simultaneously rotating the source and the detector. A ^{57}Co source in Cr, with activity 100 mCi, was set in motion with constant acceleration by an electrodynamic vibrator. The scattered radiation was registered with a scintillation spectrometer with an NaI(Tl) crystal.

For each scattering angle from 90 to 150° we measured the velocity spectra of the scattered radiation. The scatterers were made of very finely ground powders of the iron sulfates $FeSO_4 \cdot H_2O$ and $FeSO_4 \cdot 7H_2O$ (with natural content of ^{57}Fe). The powder was spread on an aluminum substrate (0.10–0.12 mm) in a plastic frame and was covered on the outside by two layers of gauze. The effective scatterer thickness was chosen to be $t_a = \sigma_0 n_a f' \approx 12$, where σ_0 is the maximum resonant-absorption cross section and n_a is the number of resonant nuclei per square centimeter.

The "black" absorber used was $FeSO_4 \cdot H_2O$ made of iron enriched with ^{57}Fe. Its thickness was $t_a \approx 30$. It remained thin to nonresonant 14.4-keV gamma quanta.

As a control, we measured first the angular distribution of the resonant scattering by the compound $K_4Fe(CN)_6 \cdot 3H_2O$, for which the energy of the hyperfine interaction is equal to zero and the angular distribution should be unperturbed and take the form (1). In the experiment, the coefficient of $P_2(\cos\theta)$ was found to be $C_2 = 0.253 \pm 0.010$, in good agreement with the calculation.

The measurement results for $FeSO_4 \cdot H_2O$ are shown in Fig. 2 for the case of recoil-free scattering (a) and for the entire (with and without recoil) scattered radiation. For the first case these distributions are approximated by the expressions

$$\begin{aligned} W_r(0) &= 1+(0.107\pm0.012)P_2(\cos\theta), \\ W_l(\theta) &= 1+(0.045\pm0.015)P_2(\cos\theta), \end{aligned} \tag{8}$$

where the subscripts "r" and "l" refer to the right and left components of the doublet.

In the second case

$$\begin{aligned} W_r(\theta) &= 1+(0.062\pm0.015)P_2(\cos\theta), \\ W_l(\theta) &= 1+(0.030\pm0.015)P_2(\cos\theta). \end{aligned} \tag{9}$$

For the scatterer $FeSO_4 \cdot 7H_2O$ we obtained the coefficients $C_{2l} = 0.035 \pm 0.015$, $C_{2l} = 0.082 \pm 0.015$ for scattering without recoil, and $C_{2l} = 0.032 \pm 0.015$, $C_{2r} = 0.055 \pm 0.015$ for the entire scattered radiation.

DISCUSSION OF RESULTS

In the present study we have demonstrated for the first time ever, following the reports[3,4] of the investigations with ^{119}Sn, the influence of the anisotropy of f' on the angular distributions of resonant Mössbauer scattering for ^{57}Fe nuclei in hydrated iron sulfates. In addition, we demonstrated the effectiveness of the method of the "black" absorber when it comes to distinguishing between gamma quanta scattered with an without energy loss to recoil. We have shown that the angular distribution functions for the components of the quadrupole doublet differ from the "hard core" distribution for the components of the quadrupole doublet for both samples. The aggregate of the obtained data obtained for $FeSO_4 \cdot H_2O$ can be explained with the aid of the calculated curves of Ref. 4 by putting $\varepsilon = 0.20 \pm 0.05$, $f'(\pi)/f'(\pi/2) = 0.7$ and $\beta \approx 90°$, and by assuming that the left component corresponds to the transition $\pm\frac{1}{2} \to \pm\frac{1}{2}$ and the right to $\pm\frac{3}{2} \to \pm\frac{1}{2}$, i.e., that the sign of the EFG is positive. The latter agrees with the prediction of Ref. 8.

For the $FeSO_4$ $7H_2O$ sample, assuming a positive EFG, the results agree with the calculation at the parameter values $\varepsilon = +0.10 \pm 0.06$, $\beta \lesssim 20°$ and $f'(\pi)/f'(\pi/2) = 1.2$. Chandra and Puri[9] determined the value of the EFG and its asymmetry parameter in single crystals of this compound and indicated that the anisotropy of f' is negligible, in agreement with our results.

It is of interest to note that the described measurements indicate that a change in the number of molecules of the crystalization water leads to rotation of the EFG axis relative to the crystal axis ($\beta \lesssim 20°$ for $FeSO_4 \cdot 7H_2O$ and $\beta \approx 90°$ for $FeSO_4 \cdot H_2O$).

It is noted in Ref. 10 that incorrect allowance for the contribution of Rayleigh scattering can distort the angular distribution of the resonant scattering. Allowance for this contribution, in accord with the formulas of

Refs. 10 and 11, has shown that, first, in our range of angles 90–150° the Rayleigh background is almost constant, and second, its value is less than 1% of the resonant effect, in agreement with the data of Refs. 12–14. Therefore the usual procedure of taking the nonresonant background into account by using the counting rate at $v=\infty$ introduces no noticeable corrections in our experimental angular distributions.

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Vortex formation in small superconducting samples

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An expression is obtained for the critical magnetic field of a cylinder and of a cylindrical cavity inside a superconducting matrix. Possible states of a superconducting cylinder with radius $R \sim \xi(T)$, when the number of vortices in it is small, are investigated.

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1. INTRODUCTION

The interaction of vortices with the surface increases the critical field for nucleation[1] and produces a threshold for the penetration of the vortices into the sample. For bulky samples, a detailed investigation of this phenomenon raises considerable difficulties because of the large number of degrees of freedom. It is of interest therefore to consider small samples, in which the number of vortices is small and it is much simpler to obtain physical results. At the same time, bulky samples always contain defects whose interaction with the vortex lattice determines the dynamics of the current state. Therefore the investigation of various types of inclusions in superconducting materials is of particular interest.

We investigate below the oscillatory dependence of the critical magnetic vortex-nucleation field for a cylinder and a cylindrical pore in a superconducting matrix, when their dimension is of the order of the correlation length $\xi(T)$. We investigate the penetration of one and two vortices into a cylinder and show that a first-order transition between states of different type is realized in this case. An expression is also obtained for the critical field of nucleation on a defect in the form of a hollow small-radius sphere in a superconducting matrix.

The process of penetration of vortices into a cylindrical sample of radius R of the order of the penetration depth λ, at a large value of the Ginzburg-Landau parameter $\varkappa$, was considered both experimentally and theoretically.[2-5] In the case considered by us, that of a strong magnetic field, the surface effects are large and lead to a qualitative change of the vortex distribution in the sample.

2. CRITICAL FIELD OF FORMATION OF A SUPERCONDUCTING NUCLEUS ON A CYLINDRICAL CHANNEL

The presence of a surface increases the critical field for the formation of the superconducting nucleus.[1] At the same time, when the radius of the pore is changed, a discrete change takes place in the type of the solution that describes the superconducting nucleus. For a pore of radius on the order of $\xi(T)$ it is therefore necessary to expect an oscillatory dependence of the critical magnetic field on the radius of the pore. We confine ourselves below to temperatures close to critical:

$$\tau = 1 - T/T_c \ll 1. \tag{1}$$

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