Fig. 2, of $G_2(\Delta)|_{V=0}$ on the detuning from resonance at various temperatures but at a constant saturation level $z=10$.

In the limit $\omega_0 \gg \Theta$ and $\omega_e \ll \Theta$ we get for $G_2(\Delta)|_{V=0}$ the simple expression

$$G_2(\Delta)|_{V=0} \approx \pi e K\{[1-4J_a F(\omega_0/2\Theta)] +2[1-2J_a(F(0)+F(\omega_0/2\Theta))][1-(\rho_{11}-\rho_{22})]\}, \tag{23}$$

$$[1-(\rho_{11}-\rho_{22})] \approx \frac{\omega_1^2\tau_1\tau_2}{1+\omega_1^2\tau_1\tau_2+\Delta^2\tau_2^2},$$

which corresponds to the standard magnetic-resonance line shape under saturation conditions. The width $\delta$ of this line is determined by the alternating-field amplitude and by the relaxation times:

$$\delta=\omega_1(\tau_1/\tau_2)^{1/2}. \tag{24}$$

Comparison of the approximate result (23) with the conductance $G_2(\Delta)|_{V=0}$ calculated from (19) and (20) shows that the width of the real resonance curve practically coincides with (24).

Thus, an investigation of the function $G_2(\Delta)|_{V=0}$ at sufficiently low temperatures ($\tanh(\omega_0/2\Theta) \sim 1$) makes it possible in principle to study the resonance characteristics of spins localized inside a tunnel junction. The sensitivity of the method proposed for detecting the magnetic resonance may turn out to be very high. As already noted, the resonance signal is comparable with the tunnel-conductivity peaks, which are reliably observed at a rather small number of impurity moments. For example, it follows from the data of Bermon *et al.*[6] that the presence of $10^{11}$ iron ions in the oxide layer of an $Al-Al_2O-Al$ junction leads to a considerable amplitude of the conductivity anomalies. Under the experimental conditions of Ref. 6, the iron ions are concentrated in a narrow layer, and this makes it necessary to take into account the interaction between the impurity spins, something not done in our model. Allowance for the spin-spin interaction, however, does not alter the result qualitatively, and furthermore the number $10^{11}$ is apparently not the lowest one at which anomalies in the conductivity can be observed. At the same time, for typical parameters, for modern EPR installations $5\times10^{10}-10^{11}$ spins is the limit.

The authors thank E. G. Kharakhash'yan for discussions that stimulated the work.

[1]E. L. Wolf, Solid State Physics, ed. by H. Ehrenreich, F. Seitz, and D. Turnbull, Academic, 1975, p. 1.
[2]J. Appelbaum, Phys. Rev. Lett. **17**, 91 (1966); Phys. Rev. **154**, 633 (1967). P. W. Anderson, Phys. Rev. Lett. **17**, 95 (1966).
[3]R. H. Wallis and A. F. G. Wyatt, J. Phys. **C7**, 1293 (1974).
[4]I. V. Aleksandrov, Teoriya magnitnoĭ relaksatsii. Relaksatsiya v zhidkikh i tverdykh nemetallicheskikh paramagnetikakh (Theory of Magnetic Relaxation. Relaxation in Liquid and Solid Nonmetallic Paramagnets), Nauka, 1975, Chap. I, § 1.
[5]M. Goldman, Spin Temperature and Nuclear Magnetic Resonance in Solids, Oxford, 1970.
[6]S. Bermon, D. E. Paraskevopoulos, and P. M. Tedrow, Phys. Rev. **17B**, 2110 (1978).

Translated by J. G. Adashko

# Surface absorption of electromagnetic waves in metals by random boundary inhomogeneities

É. A. Kaner, A. A. Krokhin, N. M. Makarov, and V. A. Yampol'skiĭ

*Institute of Radiophysics and Electronics, Academy of Sciences, Ukrainian SSR*

(Submitted 7 April 1980)
Zh. Eksp. Teor. Fiz. **79**, 1858–1870 (November 1980)

Absorption of an electromagnetic wave by scattering of grazing electrons by random inhomogeneities of the surface of a metal, in a parallel magnetic field, is investigated theoretically. On the basis of diffraction theory, the effective electronic diffusivity coefficient of a slightly rough boundary is found as a function of its statistical characteristics. Additivity is established for the contributions of volume and surface collisions to the electromagnetic absorption, and the possibility is demonstrated of introducting a surface scattering frequency $\nu_\alpha^{(s)}$ of the grazing electrons. The dependence of the surface impedance, the diffusivity coefficient, and the frequency $\nu_\alpha^{(s)}$ on the mean height and length of the irregularities, the constant magnetic field, and the skin thickness, frequency, and polarization of the external electromagnetic field is determined and analyzed. It is shown that there exists a quite broad range of values of the parameters within which surface scattering of electrons dominates over volume scattering.

PACS numbers: 78.20.Ls, 78.70. – g, 72.10.Fk

## 1. INTRODUCTION

Interaction of electrons with the specimen surface exerts a substantial influence on the high-frequency properties of metals in a magnetic field **H**. As an example, we mention a number of papers[1-7] in which the role of reflection of electrons from a metal boundary in the phenomenon of cyclotron resonance was investigated. When the reflection is nearly specular, the character of the resonance changes because of the appearance in the

  

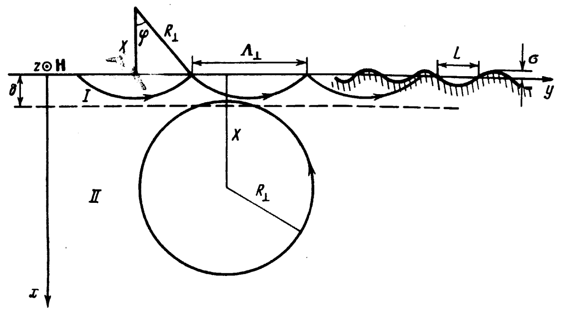

FIG. 1. Trajectories of the electrons: I, trajectory of grazing electrons, with characteristic grazing angles $\varphi \sim (\delta/R)^{1/2}$; II, trajectory of volume electrons ($X > R_\perp$).

metal of grazing electrons, which drift along the surface by virtue of multiple collisions with it. Their trajectories consist of small arcs of Larmor circles (Fig. 1) and do not emerge beyond the limits of the skin layer. Therefore the grazing electrons interact most effectively with the electromagnetic wave and make the principal contribution to the conductivity.[4]

1. Earlier,[4] the problem of the anomalous skin effect was solved for a metal with an ideal surface, when the field **H** is parallel to the specimen boundary. It was found that an electromagnetic field of frequency $\omega$ decays in the interior of the metal over a distance $\delta_\alpha = |k_\alpha|^{-1}$. Here

$$k_\alpha = \left(\frac{3\pi\sqrt{2}}{5\Gamma^2(1/4)} C_\alpha \frac{\omega\omega_0^2}{(\nu - i\omega)c^2R^{1/2}}\right)^{2/5} \tag{1.1}$$

is the complex wave number, $\omega_0$ is the plasma frequency, $c$ is the velocity of light, $R$ is the maximum radius of revolution of an electron in the magnetic field, and $\nu$ is the frequency of collision of the electrons with volume scatterers. The constant $C_\alpha$ depends on the polarization of the electric field **E** with respect to the vector $\mathbf{H} \| z$: if $\mathbf{E} \| \mathbf{H}$, then $C_z = 1$, for transverse polarization ($\mathbf{E} \perp \mathbf{H}$, $\alpha = y$), the constant $C_y = \frac{3}{2}$. According to Ref. 4, the surface impedance is described by the following formula:

$$Z_\alpha = \frac{4\pi^2\Gamma^{-2}(2/5)}{(4\pi/5)^{1/5}\sin(2\pi/5)} \frac{\omega\delta_\alpha}{c^2} \exp\left(-\frac{\pi i}{2} + \frac{2i}{5}\,\mathrm{arctg}\,\frac{\nu}{\omega}\right). \tag{1.2}$$

Formulas (1.1) and (1.2) are a consequence of the fact that the electrodynamic properties of metals are determined by grazing electrons, with characteristic grazing angles $\varphi \sim (\delta_\alpha/R)^{1/2}$ (Fig. 1). They are valid under the conditions of the anomalous skin effect: the skin depth $\delta_\alpha$ is small in comparison with the radius $R$ and the effective path length $l^* = v/|\nu - i\omega|$ ($v$ is the Fermi velocity of an electron). In addition, the characteristic path of a grazing electron between two successive encounters with the surface, $2R\varphi \sim (R\delta_\alpha)^{1/2}$, must be much smaller than $l^*$. Consequently,

$$R/\delta_\alpha \gg 1 + |\gamma|^2, \quad \gamma = (\nu - i\omega)/\Omega, \tag{1.3}$$

where $\Omega$ is the cyclotron frequency.

It follows from the expression (1.2) that at low temperatures or at high frequencies $\omega$, the electromagnetic absorption is determined solely by the frequency of collision of electrons with volume scatterers, $\nu$. In fact, the real part of the impedance for $\omega \gg \nu$,

$$\mathrm{Re}\, Z_\alpha = \frac{2\nu}{5\omega}|Z_\alpha| \tag{1.4}$$

is independent of $\omega$, and for $\nu \to 0$ the absorption disappears. The absence of absorption at $\nu = 0$ is due to the fact that in formula (1.2) only the volume relaxation of electrons has been taken into account, and their reflection from the metal boundary is assumed to be strictly specular. But in a real situation, the surface of a metal is not ideal, and consequently the reflection of the electrons will not be strictly specular. From physical considerations, it is natural to expect that the electromagnetic absorption, along with (1.4), should contain an additional component, due to scattering of electrons by random inhomogeneities of the boundary. Then the surface absorption may be competitive with the volume, if $\nu \ll \omega$.

2. In order to explain the role of surface scattering of electrons, we shall calculate and analyze electromagnetic absorption in the simplest and often used model of an effective reflection coefficient $\rho$. This model, proposed in his time by Fuchs,[8] describes the interaction of electrons with the specimen boundary by a single parameter $\rho$, which is the probability of specular reflection ($0 \leq \rho \leq 1$). In order to find the surface absorption, it is necessary to know the Fourier component of the surface current density $j_\alpha^{(s)}(k)$, which is proportional to a small power of the diffusivity $1 - \rho$. It can be extracted from the exact formulas for the current given in Ref. 4. If the inequality

$$1 - \rho \ll |\gamma| (\delta_\alpha/R)^{1/2}, \tag{1.5}$$

is satisfied, then in the main approximation with respect to $1 - \rho$, the asymptotic behavior of the current density is described by the expression (3.7) of Ref. 4 [see (2.10) of the present article], and the impedance corresponding to it is given by formula (1.2). The correction of interest to us, $j_\alpha^{(s)}(k)$, is obtained in the form

$$j_\alpha^{(s)}(k) = -\frac{\omega_0^2(1-\rho)}{16\pi^2\Omega\gamma^2} B_\alpha \int_0^\infty \frac{dk'}{(kk')^{1/2}} \mathscr{E}_\alpha(k') \ln \frac{k^{1/2} + k'^{1/2}}{|k - k'|^{1/2}}, \tag{1.6}$$

where $B_y = 2$, $B_z = 1$, and $\mathscr{E}_\alpha(k)$ is the Fourier transform of the $\alpha$ component of the electric field $E(x)$:

$$\mathscr{E}_\alpha(k) = 2\int_0^\infty dx \cos(kx) E_\alpha(x).$$

The impedance $Z_\alpha^{(s)}$ due to the current (1.6) is calculated by perturbation theory. As a result we have

$$Z_\alpha^{(s)} = a_\alpha Z_\alpha (1-\rho) \frac{(k_\alpha R)^{1/2}}{\gamma} \exp\left(-\frac{\pi i}{10}\right). \tag{1.7}$$

Here

$$a_\alpha = \frac{B_\alpha}{C_\alpha} \frac{(50\pi^3)^{1/5}}{24\pi^4\sqrt{2}} \sin\left(\frac{2\pi}{5}\right) \Gamma^2\left(\frac{1}{4}\right) \int_0^\infty \mathrm{th}(\pi x)$$

$$\times \left|\Gamma\left(\frac{2}{5} + i\frac{2x}{5}\right) \Gamma\left(\frac{4}{5} + i\frac{2x}{5}\right)\right|^2 \frac{dx}{x}, \quad a_\alpha \approx 1.91 B_\alpha/C_\alpha.$$

We note that when $\omega \gg \nu$, the impedance $Z_\alpha^{(s)}$ is actually a real quantity. Although it is also small in comparison with $|Z_\alpha|$, nevertheless, since $Z_\alpha^{(s)}$ describes an additional mechanism of electromagnetic absorption, it must be taken into account along with $\mathrm{Re}\, Z_\alpha$. Thus the total absorption consists of two independent terms:

$$\mathrm{Re}(Z_\alpha+Z_\alpha^{(s)})=\left[\frac{2\nu}{5\omega}+a_\alpha(1-\rho)\left(\frac{R}{\delta_\alpha}\right)^{1/2}\frac{\Omega}{\omega}\right]|Z_\alpha|, \qquad (1.8)$$

the first of these is due to volume collisions, the second to surface scattering of grazing electrons. It is evident from (1.8) that there is a quite broad range of frequencies $\omega$, magnetic fields, and temperature within which the surface absorption is of the order of or even exceeds the volume. For this, it is necessary that

$$\frac{\nu}{\Omega}\left(\frac{\delta_\alpha}{R}\right)^{1/2}\leqslant 1-\rho \ll \frac{\omega}{\Omega}\left(\frac{\delta_\alpha}{R}\right)^{1/2}. \qquad (1.9)$$

The left inequality in (1.9) means that the probability $1-\rho$ of diffuse scattering by the surface is larger than, or of the order of, the probability $\nu(\delta/R)^{1/2}/\Omega$ of scattering in the volume after the time between two consecutive collisions with the boundary. The right-hand inequality in (1.9) coincides with (1.5).

We point out that $\mathrm{Re}\, Z_\alpha^{(s)}$ depends on the magnetic field $H$ but is independent of the temperature (of $\nu$) in the range $\omega \gg \nu$. At the same time, the volume absorption is a function of the temperature but is insensitive to a change of $H$. Therefore the surface electromagnetic absorption can be separated from the volume even when the left-hand condition in (1.9) is not satisfied.

3. In investigation of the kinetic properties of metals, the information about the interaction of electrons with the surface is contained in the boundary condition to Boltzmann's kinetic equation for the electronic distribution function. The specularity-parameter model,[8] in terms of which formulas (1.6) and (1.7) were obtained, was until recently widely used as a phenomenological boundary condition. It has a number of definite merits, of which the principal ones are mathematical simplicity and the transparency of the physical interpretation of the results. Along with this, this model also has important shortcomings. The principal one of these is that the model actually ignores the existence of a characteristic curve of scattering of the electrons by the surface; that is, it fails to take into account the finite probability of reflection in nonspecular directions. At the same time, it is the form of this characteristic curve that contains the specific mechanism of surface scattering. That the Fuchs model[8] is unrealistic is also manifest in the fact that the parameter $\rho$ does not depend on the geometrical characteristics of the surface or on the angle of incidence of the electrons on the boundary. The contemporary state of the theory of kinetic phenomena in bounded specimens, which requires a more accurate and heuristic formulation of the boundary condition, has been covered quite fully in reviews of Andreev[9] and of Okulov and Ustinov.[10] In a number of papers,[11-14] there have been obtained various versions of the boundary conditions that describe the scattering of quasiparticles by random potential surfaces of one kind or another.

In the present paper, the surface impedance is calculated for a bulk metal placed in a parallel magnetic field, under the conditions (1.3) of the anomalous skin effect, with allowance for scattering by random inhomogeneities of the surface. As a boundary condition to the kinetic equation, an integral relation is used, which was first obtained by Fal'kovskiĭ[13] and which describes the scattering of electrons by a statistically irregular boundary of a metal. The final formulas are obtained in two limiting cases: sufficiently large grazing angles $\varphi$, and small angles, when it is possible to justify the introduction of a specularity parameter $\rho$ dependent on the angle of encounter of the electrons with the surface. Comparison of the results obtained by this method with expression (1.7) provides a possibility of analyzing the dependence of the effective reflection coefficient $\rho_{\mathrm{eff}}$ on the magnetic field $H$ and on the microscopic characteristics of the specimen boundary (height and length of the irregularities). The results of this paper, in conjunction with experimental investigations of surface absorption, open possibilities of studying the statistical properties of metallic surfaces.

## 2. FORMULATION OF THE PROBLEM. CURRENT DENSITY

1. We consider a metal bounded by a rough surface. We suppose that the irregularities of the boundary are random and statistically homogeneous and that the constant magnetic field H is parallel to the mean surface of the specimen, $x=0$ ($yz$ plane). The $x$ axis is directed into the interior of the metal, the $z$ axis along the vector $\mathbf{H}$ (Fig. 1). A plane monochromatic wave is incident on the interface in the direction of the $x$ axis. The electric field vector $\mathbf{E}=\mathbf{E}(x)\exp(-i\omega t)$ inside the metal depends only on the coordinate $x$.

In order to calculate the surface impedance, it is first necessary to find the nonequilibrium correction

$$-\chi\delta(\varepsilon-\varepsilon_F)e^{-i\omega t}$$

to the electronic Fermi distribution function ($\varepsilon$ is the energy and $\varepsilon_F$ the Fermi energy of an electron), averaged over the random irregularities of the boundary. The kinetic equation for $\chi$ has the usual form

$$v_x\frac{\partial\chi}{\partial x}+\Omega\frac{\partial\chi}{\partial\tau}+(\nu-i\omega)\chi=e\mathbf{v}\mathbf{E}(x). \qquad (2.1)$$

Here $\tau$ is dimensionless time in the motion of an electron along an orbit in the magnetic field $\mathbf{H}$, $\mathbf{v}(t)$ is the velocity, and $e$ is the absolute value of the charge of the electron.

Equation (2.1) must be supplemented by a boundary condition, which is formulated at the mean surface $x=0$. For simplicity, we restrict ourselves to an isotropic and quadratic dispersion law for the electrons. In this case, the boundary condition for the correction $\chi$ averaged over the irregularities has the form[13]

$$\chi(p_x,\mathbf{p})=\chi(-p_x,\mathbf{p})-\frac{4\sigma^2}{\hbar^2}p_x\int\limits_{p'<p_F}\frac{d^2p'}{(2\pi\hbar)^2}p_x'W(\mathbf{p}-\mathbf{p}')\times[\chi(-p_x,\mathbf{p})-\chi(-p_x',\mathbf{p}')],\quad x=0. \qquad (2.2)$$

Here $p_x=(p_F^2-p^2)^{1/2}$ is the absolute value of the component of the momentum of the electron normal to the metal boundary at the surface $x=0$ ($p_x'=(p_F^2-p'^2)^{1/2}$), and $\mathbf{p}$ is the two-dimensional momentum in the $yz$ plane. Its components $p_y$ and $p_z$ are canonically conjugate to the coordinates $y$ and $z$ respectively. We note that $p_z$ is simultaneously the $z$ projection of the kinematic momentum of the electron, whereas its $y$ component coincides with $p_y$ only at the boundary $x=0$. In the boundary condition (2.2), the interaction of electrons with the rough surface is characterized by the root-mean-square height $\sigma$ of the irregularities and by the Fourier transform $W(\mathbf{p})$ of their binary correlation function, which is normalized to unity:

$$\int_{-\infty}^{\infty} \frac{d^2p}{(2\pi\hbar)^2} W(\mathbf{p})=1.$$

Here $W(\mathbf{p})$ is an even, real, and positive definite function of $\mathbf{p}$; it vanishes for $p\to\infty$. The characteristic distance at which $W(\mathbf{p})$ decreases significantly is $2\pi\hbar/L$, where $L$ is the correlation radius of the irregularities, *i.e.*, the mean horizontal dimension of the irregularities (Fig. 1). The integration in (2.2) extends over the region $|\mathbf{p}'|\leqslant p_F$ bounded by the Fermi momentum $p_F$. The second term in the right side of equation (2.2) takes account of the diffuse character of the reflection from the statistically irregular boundary and replaces the expression $(1-\rho)\chi(-p_x, \mathbf{p})$ in the Fuchs model.[8]

The relation (2.2) has a rigorous microscopic (quantum) basis, if the following conditions are satisfied. In the derivation of (2.2), the averaging over an ensemble of realizations of random inclinations to the plane $x=0$ was carried out on the supposition of a single scattering of electrons by the surface. But in a parallel magnetic field H, an electron collides with the specimen surface repeatedly. This means that in the present problem, the boundary condition (2.2) is applicable when successive scattering events may be considered independent. For this it is necessary and sufficient that the distance $\Lambda_\perp$ in the $xy$ plane between two adjacent collisions of an electron with the boundary (Fig. 1) significantly exceed the correlation length $L$; that is,

$$L\ll\Lambda_\perp=2Rp_x/p_F. \tag{2.3}$$

Furthermore, the second ("diffusive") term in the right side of (2.2), proportional to $\sigma^2$, has been written in the simplest (Born) approximation. A more accurate calculation of the diffusivity leads to the appearance in (2.2) of a term proportional to $\sigma^4$. This term (and terms succeeding it) may be discarded if the inequalities

$$(p_x\sigma/\hbar)^2\ll1,\ (\sigma/L)^2\ll1,\ p_F\sigma^2/\hbar L\ll1. \tag{2.4}$$

are satisified.

2. The equation of motion of an electron along the $x$ axis is described by the formula

$$x=X+R_\perp\cos\tau,$$

where $X=-cp_y/eH$ is the projection of the center of the orbit on the $x$ axis, and where $R_\perp$ is the radius of revolution in the $xy$ plane, perpendicular to H. Depending on the value of $X$, the electrons in the metal divide themselves into two independent groups. One of these is composed of the volume electrons, for which $X>R_\perp$, i.e. $x>R_\perp(1+\cos\tau)$. The volume electrons are located in the interior of the metal and do not interact with its boundary. The second group consists of the surface electrons, which collide with the specimen boundary (Fig. 1). For them $-R_\perp<X<R_\perp$, or $0\leqslant x<R_\perp(1+\cos\tau)$. It is only electrons of the second group that satisfy the boundary condition for the distribution function. For volume electrons, it is replaced by the condition of periodicity in $\tau$. The electromagnetic properties of metals with an almost specular boundary are primarily determined by the surface electrons. The surface absorption effect of interest to us is also due to this group. Therefore we shall find the distribution and current density of the surface electrons alone.

The solution of the kinetic equation (2.1) with the boundary condition (2.2) can be written as the sum of two terms:

$$\chi(x,\tau,\theta)=\chi_0(x,\tau,\theta)-e^{-\gamma\tau}F(\varphi,\theta)/2\,\mathrm{sh}(\gamma\varphi). \tag{2.5}$$

The first term in (2.5) is the distribution function of the surface electrons for specular reflection from the metal boundary:

$$\chi_0(x,\tau,\theta)=\frac{eR}{2\,\mathrm{sh}(\gamma\varphi)}\left\{e^{\gamma\varphi}\int_{-\varphi}d\lambda+e^{-\gamma\varphi}\int_{\tau}^{\varphi}d\lambda\right\}$$
$$\times n_\alpha(\theta,\lambda)e^{\gamma(\lambda-\tau)}E_\alpha(x+R_\perp(\cos\lambda-\cos\tau)). \tag{2.6}$$

Here $n_\alpha(\theta,\tau)$ is a component of the unit vector along the electron velocity,

$$n_x=-\sin\theta\sin\tau,\ n_y=\sin\theta\cos\tau,\ n_z=\cos\theta,$$

$\theta$ is the polar angle with polar axis $z$, $R_\perp=R\sin\theta$, $\varphi=\arccos(-X/R_\perp)$ is the grazing angle of a surface electron at the instant of its collision with the boundary $x=0$ (Fig. 1). In formulas (2.5) and (2.6), the angle of encounter $\varphi$ must be considered a function of $x$ and $\tau$, which is determined from the equation of motion $x=R_\perp(\cos\tau-\cos\varphi)$.

The second term in the expression (2.5) is the solution of the homogeneous equation (2.1). The form of the function $F(\varphi,\theta)$ can be determined from the boundary condition (2.2). Here we replace $\chi$ by $\chi_0$ in the integral term, since the reflection of the electrons is nearly specular. Hence we get

$$F(\varphi,\theta)=\frac{4\sigma^2}{\hbar^2}p_x\int_{p'\leqslant p_F}\frac{d^2p'}{(2\pi\hbar)^2}p_x'W(\mathbf{p}-\mathbf{p}')\,[\chi_0(0,\varphi,\theta)-\chi_0(0,\varphi',\theta')]. \tag{2.7}$$

The projections p and $p_x$ of the canonical momentum are connected with the angles $\theta$ and $\varphi$ by the relations

$$p_x=p_F\sin\theta\sin\varphi,\ p_y=p_F\sin\theta\cos\varphi,\ p_z=p_F\cos\theta.$$

3. The Fourier component of the surface-electron current density is expressed in terms of the nonequilibrium correction $\chi(x,\tau,\theta)$ by means of the well-known formula

$$j_\alpha^{\mathrm{surf}}(k)=\frac{3Ne}{2\pi p_F}\int_0^\pi d\theta\sin\theta\int_{-\pi}^{\pi}d\tau\,n_\alpha(\theta,\tau)\int_0^{x_0(\theta,\tau)}dx\cos kx\,\chi(x,\tau,\theta), \tag{2.8}$$

where $N$ is the density of electrons in the metal, and where $x_0(\theta,\tau)=R_\perp(1+\cos\tau)$.

In accordance with (2.5), the current density is a sum of two terms:

$$j_\alpha^{\mathrm{surf}}(k)=j_\alpha^{(0)}(k)+j_\alpha^{(s)}(k). \tag{2.9}$$

The first term $j_\alpha^{(0)}(k)$ is the current density of surface electrons for specular reflection. It is determined from (2.8) with substitution of $\chi_0(x,\tau,\theta)$ for $\chi(x,\tau,\theta)$. An exact expression for $j_\alpha^{(0)}(k)$ was given in Ref. 4. In the same place, the asymptotic behavior of $j_\alpha^{(0)}(k)$ was calculated under the conditions of the anomalous skin effect (1.3):

$$j_\alpha^{(0)}(k)=\frac{c^2k_\alpha^{1/3}}{4\pi\omega}\int_0^\infty dk'\mathscr{E}_\alpha(k')\frac{|k-k'|^{-1/3}-(k+k')^{-1/3}}{(kk')^{1/3}}. \tag{2.10}$$

This asymptotic behavior is due to grazing electrons with arrival angles $\varphi \lesssim (\delta_\alpha/R)^{1/2}$.

The second term in (2.9) is due to diffuse scattering of electrons by the metal boundary:

$$j_\alpha^{(s)}(k) = -\frac{3Ne}{2\pi m\Omega}\int_0^\pi d\theta \sin^2\theta \int_0^\pi \frac{d\varphi \sin\varphi}{\mathrm{sh}(\gamma\varphi)} F(\varphi,\theta)$$
$$\times \int_0^\varphi d\tau\, n_\alpha(\theta,\tau)\,\mathrm{ch}(\gamma\tau)\cos[kR_\perp(\cos\varphi-\cos\tau)], \quad (2.11)$$

where $\alpha = y, z$; $m$ is the mass of the electron. It is obvious that the second term $j_\alpha^{(s)}(k)$ is much smaller in modulus than the first; that is,

$$|j_\alpha^{(s)}(k)/j_\alpha^{(0)}(k)| \ll 1. \quad (2.12)$$

## 3. SURFACE IMPEDANCE

By virtue of the inequality (2.12), we can find the impedance of the metal by perturbation theory, using as zeroth approximation the expression (1.2), which is determined by the current density $j_\alpha^{(0)}(k)$. In the approximation linear in $j_\alpha^{(s)}(k)$, the correction to the impedance is,

$$Z_\alpha^{(s)} = \frac{8\pi\omega^2}{c^4 E_\alpha'^2(0)}\int_0^\infty dk\, \mathscr{E}_\alpha(k) j_\alpha^{(s)}(k), \quad (3.1)$$

where the prime denotes differentiation with respect to $x$. In the expression (3.1), the Fourier component $\mathscr{E}_\alpha(k)$ is the solution of Maxwell's equation with current density $j_\alpha^{(0)}(k)$. According to Ref. 4, the function $\mathscr{E}_\alpha(k)$ can be written as a Mellin contour integral:

$$\mathscr{E}_\alpha(k) = -\frac{2E_\alpha'(0)}{k_\alpha^2}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dz\, M(z)\,(k/k_\alpha)^z, \quad -2<c=\mathrm{Re}\, z<1/2. \quad (3.2)$$

The Mellin transform $M(z)$ is determined by the following formula:

$$M(z) = \exp\left\{\frac{z+2}{5}\left[i\pi + \ln\frac{(2/5)^4}{2\pi}\right]\right\}\cos\left(\frac{\pi z}{2}\right)\frac{\Gamma(z+1)}{\Gamma(7/5)}$$
$$\times\Gamma\left(\frac{1}{5}-\frac{2z}{5}\right)\Gamma\left(\frac{3}{5}-\frac{2z}{5}\right). \quad (3.3)$$

The only singularities of $M(z)$ are simple poles located on the real axis.

The current density $j_\alpha^{(s)}(k)$ permits simple asymptotic expansions if the explicit dependence of the function $F(\varphi, \theta)$ on its arguments is known. Formula (2.7) contains, under the integral sign, the product of two "sharp" functions: $W$ and $\chi_0$. The correlation function $W(\mathbf{p}-\mathbf{p}')$ has a maximum at the point $\mathbf{p}' = \mathbf{p}$, with characteristic width $\Delta p' \sim 2\pi\hbar/L$. Under the conditions (1.3) of the anomalous skin effect, the distribution function $\chi_0(0, \varphi', \theta')$ is a maximum at $\varphi' = 0$ (grazing electrons), with width $\Delta\varphi' \sim \varphi \sim (\delta R)^{1/2}$. In the variables $\mathbf{p}'$, this corresponds to a maximum at the point $|\mathbf{p}'| = p_F$ with width $\Delta p' \sim p_F\varphi^2 \sim p_F\delta/R$. The limiting cases in which $j_\alpha^{(s)}(k)$ has simple asymptotic behaviors are determined by the relation between the characteristic momenta $2\pi\hbar/L$ and $p_F\varphi^2$. Let the irregularities of the boundary be statistically isotropic, i.e., let the characteristic scattering curve $W(\mathbf{p})$ depend only on the modulus $p \equiv |\mathbf{p}|$ of the two-dimensional momentum $\mathbf{p}$: and let the correlation radius $L$ be a constant quantity.

1. We first consider the case of "low-angle" incidence of the electrons on the specimen surface:

$$\varphi^2 \ll 2\pi\hbar/p_F L. \quad (3.4)$$

According to (3.4), in the formula for $F(\varphi, \theta)$ the actual range of integration in the arrival term is considerably smaller than in the departure term, and therefore the second term in (2.7) may be neglected in comparison with the first. Then

$$F(\varphi,\theta) = \frac{4\sigma^2}{\hbar^2} p_x \chi_0(0,\varphi,\theta)\int_{p'<p_F}\frac{d^2p'}{(2\pi\hbar)^2} p_x' W(|\mathbf{p}-\mathbf{p}'|). \quad (3.5)$$

The small value of the arrival term in this case means that the boundary condition (2.2) is essentially equivalent to a condition with a specularity parameter $\rho$ that depends on the angle of encounter of the electrons with the metal boundary:

$$\rho(p_x) = 1 - \frac{4\sigma^2}{\hbar^2} p_x \int_{p'<p_F}\frac{d^2p'}{(2\pi\hbar)^2} p_x' W(|\mathbf{p}-\mathbf{p}'|). \quad (3.6)$$

The degree of diffuseness $1-\rho(p_x)$ is found to be related to the mean coefficient of reflection $V$ of a plane wave, with wave vector $(p_x/\hbar, \mathbf{p}/\hbar)$, from a statistically irregular surface, first obtained by Bass[15]:

$$1-\rho(p_x) = 2\,\mathrm{Re}\,(1+V). \quad (3.7)$$

The asymptotic behavior of the current density $j_\alpha^{(s)}(k)$ under the conditions (1.3) and (3.4) is expressed in terms of the asymptotic behavior of $j_\alpha^{(0)}(k)$ as follows:

$$j_\alpha^{(s)}(k) = -\frac{5}{2}a_\alpha(1-\rho_{\mathrm{eff}})_\alpha\frac{(k_\alpha R)^{1/5}}{\gamma} j_\alpha^{(0)}(k)\exp\left(-\frac{\pi i}{10}\right). \quad (3.8)$$

Here the complex quantity $(1-\rho_{\mathrm{eff}})_\alpha$ is an effective coefficient of diffuseness, introduced so that the impedance $Z_\alpha^{(s)}$ may be described by formula (1.7). The function $1-\rho(p_x)$, like the reflection coefficient $V$, has, within the framework of (3.4), different asymptotic behaviors at large and at small values of $2\pi\hbar/p_F L$.[15] Accordingly, the value of $(1-\rho_{\mathrm{eff}})_\alpha$ will also be different, depending on the value of the parameter $2\pi\hbar/p_F L$.

For fine-scale irregularities we have

$$(1-\rho_{\mathrm{eff}})_\alpha = \frac{A_\alpha\Gamma^4(1/4)}{756\,a_\alpha}\frac{\sigma^2 p_F}{\hbar L}\left(\frac{p_F L}{\pi\hbar}\right)^3\frac{\exp(\pi i/10)}{(k_\alpha R)^{1/5}}w(0) \quad (3.9)$$
$$\text{at } 2\pi\hbar/p_F L \gg 1.$$

In the case of coarse-scale irregularities and small-angle incidence (3.4), it is necessary to take as $(1-\rho_{\mathrm{eff}})_\alpha$

$$(1-\rho_{\mathrm{eff}})_\alpha = \frac{A_\alpha\Gamma^2(1/4)}{63\pi^2 a_\alpha}\frac{\sigma^2 p_F}{\hbar L}\left(\frac{p_F L}{\pi\hbar}\right)^{1/2}\frac{\exp(\pi i/10)}{(k_\alpha R)^{1/5}}\int_0^\infty x^{3/2} w(x)\,dx \quad (3.10)$$
$$\text{at } \delta_\alpha/R \ll 2\pi\hbar/p_F L \ll 1.$$

In formulas (3.9) and (3.10), the constants are $A_y = 5$, $A_z = 3$. Furthermore, we have introduced here the dimensionless correlation coefficient $w(\mathbf{x}) \equiv W(\hbar\mathbf{x}/L)/L^2$, which is independent of $L$ and decreases significantly over a distance $\Delta x \sim 1$. We note that in typical metals with $p_F/\hbar \sim 10^8\,\mathrm{cm}^{-1}$, the case (3.10) is realized. But for small electron clusters, the case (3.9) may occur.

In order to demonstrate that the correction $Z_\alpha^{(s)}$ is described by formula (1.7) with $1-\rho$ replaced by $(1-\rho_{\mathrm{eff}})_\alpha$, it is not obligatory to substitute the asymptotic behavior (3.8) in (3.1) and to carry out the calculations indicated in (3.1). It is sufficient to notice the fact that the current $j_\alpha^{(s)}(k)$ under the condition (3.4) is proportional to

$j_\alpha^{(0)}(k)$. This means that the total current $j_\alpha^{\mathrm{surf}}(k)$ has the form (2.10), but with a renormalized wave number $k_\alpha$: instead of it, there now occurs in (2.10)

$$k_\alpha\left[1-\frac{5}{2}a_\alpha(1-\rho_{\mathrm{eff}})_\alpha\frac{(k_\alpha R)^{1/2}}{\gamma}\exp\left(-\frac{\pi i}{10}\right)\right]^{2/5}.$$

Therefore the total impedance of the metal is given by the expression (1.2) with the indicated renormalization. On using the inequality (1.5), we arrive at formula (1.7) for $Z_\alpha^{(s)}$.

2. In metals with large-scale irregularities of the boundary ($p_F L/2\pi\hbar\gg 1$), the case inverse to (3.4) may occur:

$$2\pi\hbar/p_F L\ll\varphi^2\ll 1, \tag{3.11}$$

when the width of the characteristic scattering curve is considerably smaller than the characteristic arrival angles of the grazing electrons, $\varphi\sim(\delta/R)^{1/2}$. Then in the integral (2.7) for $F(\varphi,\theta)$, the function $W(|\mathbf{p}-\mathbf{p}'|)$ will be sharpest, and consequently the departure and arrival terms are quantities of the same order. Furthermore, the condition (3.11) actually means that the transfer of momentum during scattering is small; that is, the collision integral (2.7) of electrons with the boundary may be written in the Fokker-Planck approximation. As usual, expanding the difference $p_x'[\chi_0(0,\varphi,\theta)-\chi_0(0,\varphi',\theta')]$ in the neighborhood of the point $\mathbf{p}'=\mathbf{p}(\varphi'=\varphi,\theta'=\theta)$ through terms quadratic in $\mathbf{p}-\mathbf{p}'$, we get

$$F(\varphi,\theta)=-\frac{\sigma^2}{2\pi L^2}\left(\frac{d^2\chi_0}{d\varphi^2}+\operatorname{ctg}\varphi\frac{d\chi_0}{d\varphi}\right)\int_0^\infty x^3 w(x)\,dx. \tag{3.12}$$

Thus in the case of "steep" incidence (3.11), the boundary condition (2.2) for the distribution function does not reduce to a specularity parameter. The reason for this is obvious: the state of an electron reflected by the metal surface with momentum $(p_x,\mathbf{p})$ is entered into by incident particles with all possible momenta, and not only those that are incident at the specular direction, with momentum $(-p_x,\mathbf{p})$.

After substitution of (3.12) in (2.11), the calculation of the asymptotic behavior of $j_\alpha^{(s)}(k)$ is carried out by the standard method. The formula for the current takes the following form:

$$j_\alpha^{(s)}(k)=\frac{5\Gamma^2(1/4)B_\alpha}{16\pi^4 b_\alpha\sqrt{2}}(1-\rho_{\mathrm{eff}})_\alpha\frac{(k_\alpha R)^{1/2}}{\gamma}\frac{c^2k_\alpha}{4\omega}e^{\pi i/5}$$
$$\times\left(k\frac{d}{dk}\right)\int_0^\infty dk'\,\mathscr{E}_\alpha(k')\int_0^1 dx\int_0^1 dy\left[1-\frac{k^2}{k'^2}\left(\frac{1-x^2}{1-y^2}\right)^2\right]^{-1}. \tag{3.13}$$

Although the boundary condition (2.2) in this case contains nonlocal terms of the type (3.12), it is nevertheless convenient to introduce an effective diffusivity coefficient $(1-\rho_{\mathrm{eff}})_\alpha$ in order that the impedance $Z_{\alpha s}$ may have the customary form (1.7) [after substitution of (3.13) in (3.1) and performance of the calculations indicated in (3.1)]. Then we get

$$(1-\rho_{\mathrm{eff}})_\alpha=b_\alpha\frac{\sigma^2}{L^2}k_\alpha Re^{-\pi i/5}\int_0^\infty x^3w(x)\,dx\qquad\text{at }2\pi\hbar/p_FL\ll\delta_\alpha/R\ll 1,$$
$$b_\alpha=\frac{B_\alpha}{a_\alpha}\frac{(5/4\pi)^{1/5}}{16\pi^2\sqrt{2}}\sin\left(\frac{2\pi}{5}\right)\Gamma^2\left(\frac{1}{4}\right)\int_0^\infty\frac{x^2\,dx}{4x^2+1}\operatorname{cth}(\pi x)\cdot\frac{\left|\Gamma\left(\frac{3}{5}-i\,2x/5\right)\right|^2}{\operatorname{sh}(2\pi x/5)},\qquad b_\alpha\approx 0.118\,C_\alpha. \tag{3.14}$$

We note that the formulas for $(1-\rho_{\mathrm{eff}})_\alpha$ and for the relative correction to the impedance $Z_\alpha^{(s)}/Z_\alpha$ are in qualitative agreement with the results of Fal'kovskiĭ[13] and of Okulov and Ustinov,[16] who investigated the role of surface scattering in the anomalous skin effect in the absence of a magnetic field. One can show this easily by expressing the coefficient of diffusivity $(1-\rho_{\mathrm{eff}})_\alpha$ in terms of the characteristic angle of grazing $\varphi\sim(\delta_\alpha/R)^{1/2}$ and taking into account that when $H=0$, the role of grazing electrons is played by those particles that move along straightline trajectories with $\varphi\sim\delta/l^*$.

## 4. DISCUSSION OF RESULTS

We shall discuss the results obtained. The expressions given above for the effective diffuseness parameter $(1-\rho_{\mathrm{eff}})_\alpha$ are functions of the magnetic field $H$, of the statistical characteristics $\sigma$ and $L$ of the irregular boundary of the metal, and also of the skin thickness $\delta_\alpha$. It is interesting to note that even for a Fermi sphere, the degree of diffuseness $(1-\rho_{\mathrm{eff}})_\alpha$ of the boundary depends on the polarization of the external electromagnetic wave. This dependence is due to the anisotropy of the electrodynamic properties of the metal in a parallel magnetic field, which manifests itself in the scattering of electrons by even a slightly rough surface. A criterion for applicability of the results is the inequality (1.5) with $1-\rho$ replaced by $(1-\rho_{\mathrm{eff}})_\alpha$. This inequality is a necessary and sufficient condition for use of the method of successive approximations for solution of the kinetic equation (2.1) with the boundary condition (2.2).

For an arbitrary relation between $\omega$ and $\nu$, the effective coefficient of diffuseness is a complex quantity, in consequence of the fact that it actually depends on the complex angle of encounter of the electrons with the surface,

$$\varphi_\alpha=(k_\alpha R)^{-1/2}e^{\pi i/10}. \tag{4.1}$$

At high frequencies $\omega\gg\nu$, however, the angle $\varphi_\alpha$ is real, $\varphi_\alpha=(\delta_\alpha/R)^{1/2}$, since in the main approximation the impedance (1.2) becomes purely imaginary and the external wave undergoes total reflection. Therefore the degree of diffuseness $(1-\rho_{\mathrm{eff}})_\alpha$ also becomes a real quantity.

The additivity of the contributions of volume absorption and of surface absorption by grazing electrons, which for $\omega\gg\nu$ is expressed by formula (1.8), enables us to introduce an effective frequency of surface scattering of them, $\nu_\alpha^{(s)}$. Its relation to $(1-\rho_{\mathrm{eff}})_\alpha$ is determined by the formula

$$\nu_\alpha^{(s)}=\frac{5}{2}a_\alpha(1-\rho_{\mathrm{eff}})_\alpha\frac{\Omega}{\varphi_\alpha}. \tag{4.2}$$

This formula has a lucid physical meaning: $\nu_\alpha^{(s)}$ is the product of the "probability" of diffuse scattering $(1-\rho_{\mathrm{eff}})_\alpha$ by the characteristic "frequency" $\pi\Omega/\varphi_\alpha$ of the periodic motion of the grazing electrons along the $x$ axis. It is noteworthy that the surface electromagnetic absorption caused by scattering of grazing electrons by random inhomogeneities of the boundary can be obtained directly from the expression (1.2) for the impedance $\mathcal{Z}_\alpha(\nu)$ of a metal with a specular boundary, by replacing the quantity $\nu$ by the sum $\nu+\nu_\alpha^{(s)}$. Here the relation between $\omega$ and $\nu+\nu_\alpha^{(s)}$ in unimportant.

We shall give formulas for the frequency $\nu_\alpha^{(s)}$ in the three limiting cases considered above; for simplicity, we shall assume that the correlation function of the irregularities is Gaussian $[w(x)=\pi\exp(-x^2/4)]$. For fine-scale irregularities (3.9),

$$\nu_\alpha^{(s)}=\frac{5\pi A_\alpha}{189}\Gamma^4\left(\frac{1}{4}\right)\frac{\sigma^2 p_F}{\hbar L}\left(\frac{p_F L}{2\pi\hbar}\right)^3\Omega\propto H \quad \text{at} \quad 1\ll\frac{2\pi\hbar}{p_F L}. \tag{4.3}$$

For coarse-scale irregularities and for low-angle incidence, we find from (3.10) and (4.2)

$$\nu_\alpha^{(s)}=\frac{5A_\alpha}{63}\Gamma^2\left(\frac{1}{4}\right)\frac{\sigma^2}{L^2}\left(\frac{p_F L}{2\pi\hbar}\right)^{1/2}\Omega\propto H \quad \text{at} \quad |\varphi_\alpha|^2\ll\frac{2\pi\hbar}{p_F L}\ll 1. \tag{4.4}$$

Finally, in the case of steep incidence and coarse-scale irregularities (3.14), we get

$$\nu_\alpha^{(s)}=20\pi a_\alpha b_\alpha\frac{\sigma^2}{L^2}\frac{\Omega}{\varphi_\alpha^3}\propto H^{-1/2} \quad \text{at} \quad \frac{2\pi\hbar}{p_F L}\ll|\varphi_\alpha|^2\ll 1. \tag{4.5}$$

It is evident from the expressions (4.3) and (4.4) that the frequency of surface relaxation $\nu_\alpha^{(s)}$ is a real quantity, whatever the relation between $\omega$ and $\nu$, since in these cases $\nu_\alpha^{(s)}$ is independent of the electrodynamic characteristics of the metal. The reason is that scattering of electrons by boundary inhomogeneities, at low-angle incidence (3.4), has a local character. Then the degree of diffuseness $(1-\rho_{eff})_\alpha$ is proportional to $\varphi_\alpha$, but $\nu_\alpha^{(s)}$ is unrelated to the grazing angle $\varphi_\alpha$ and to the frequencies $\omega$ and $\nu$. It is proportional to the field $H$ and is determined by the relation between the de Broglie wavelength and the mean parameters of the rough boundary. The dependence of $\nu_\alpha^{(s)}$ on the polarization in cases (4.3) and (4.4) is of geometrical nature and is due to the anisotropy of the surface scattering in a magnetic field.

With increase of $H$, there occurs on increase of $|\varphi_\alpha|$ a transition from the case of small-angle incidence (4.4) to the case of steep incidence (4.5), during which the dependence of the coefficient of diffuseness on $\varphi_\alpha$ changes: instead of a direct proportionality, $(1-\rho_{eff})_\alpha$ begins to decrease in inverse proportion to $\varphi_\alpha^2$. This explains the complexity of the value (4.5) of $\nu_\alpha^{(s)}$ in a strong field (vanishing for $\omega\gg\nu$), and also the different dependence on $H$ and the sensitivity to the skin depth $\delta_\alpha$. From the nature of the asymptotic behaviors (4.4) and (4.5) it is quite obvious that the frequency of surface scattering $\nu_\alpha^{(s)}$ must have a maximum at magnetic fields for which $|\varphi_\alpha|^2\approx 2\pi\hbar/p_F L$.

Collisions of electrons affect the absorption of electromagnetic waves in metals when there is no mechanism of collisionless attenuation. This is the case under the conditions of the normal skin effect at low frequencies ($\omega\ll\nu$) and in the infrared frequency range ($\nu, v\omega_0/c<\omega<\omega_0$). In the latter case, the impedance of the metal is almost imaginary, and the electromagnetic absorption is caused by both volume and by surface scattering of electrons.[17,18] Under the conditions of the anomalous skin effect, the collision mechanism of absorption as a rule plays no role in comparison with the collisionless mechanism. A special situation arises in a magnetic field, when the vector **H** is perpendicular to the direction of propagation of the wave and collisionless attenuation is absent in consequence of the cyclotron revolution of the electrons. Then the electromagnetic absorption is caused entirely by electronic collisions, both in the volume and with the surface. It is this situation that has been analyzed in the present paper.

In conclusion, we point out that in electromagnetic absorption in a metal, a contribution is made not only by grazing electrons but also by volume electrons, which do not collide with the specimen boundary. But far from cyclotron resonance, the absorption from volume electrons, by virtue of the condition (1.3), is found to be negligibly small in comparison with the volume absorption of grazing electrons.


[1]M. Ya. Azbel' and E. A. Kaner, Zh. Eksp. Teor. Fiz. **32**, 896 (1956) [Sov. Phys. JETP **5**, 730 (1957)].
[2]É. A. Kaner, Zh. Eksp. Teor. Fiz. **33**, 1472 (1957) [Sov. Phys. JETP **6**, 1135 (1958)],
[3]L. E. Hartmann and J. M. Luttinger, Phys. Rev. **151**, 430 (1966).
[4]É. A. Kaner and N. M. Makarov, Zh. Eksp. Teor. Fiz. **57**, 1435 (1969) [Sov. Phys. JETP **30**, 777 (1970)],
[5]B. É. Meierovich, Zh. Eksp. Teor. Fiz. **58**, 324 (1970) [Sov. Phys. JETP **31**, 175 (1970)].
[6]D. É. Zherebchevskiĭ and É. A. Kaner, Zh. Eksp. Teor. Fiz. **63**, 1858 (1972) [Sov. Phys. JETP **36**, 983 (1973)].
[7]É. A. Kaner, N. M. Makarov, V. L. Fal'ko, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **73**, 1400 (1977) [Sov. Phys. JETP **46**, 737 (1977)].
[8]K. Fuchs, Proc. Cambridge Philos. Soc. **34**, 100 (1938).
[9]A. F. Andreev, Usp. Fiz. Nauk **105**, 113 (1971) [Sov. Phys. Usp. **14**, 609 (1972)].
[10]V. I. Okulov and V. V. Ustinov, Fiz. Nizk. Temp. **5**, 213 (1979) [Sov. J. Low Temp. Phys. **5**, 101 (1979)].
[11]É. M. Baskin and M. V. Éntin, Zh. Eksp. Teor. Fiz. **57**, 460 (1969) [Sov. Phys. JETP **30**, 252 (1970)].
[12]I. N. Adamenko and I. M. Fuks, Zh. Eksp. Teor. Fiz. **58**, 360 (1970) [Sov. Phys. JETP **31**, 193 (1970)].
[13]L. A. Fal'kovskiĭ Pis'ma Zh. Eksp. Teor. Fiz. **11**, 222 (1970) [JETP Lett. **11**, 138 (1970)]; Zh. Eksp. Teor. Fiz. **58**, 1830 (1970) [Sov. Phys. JETP **31**, 981 (1970)].
[14]V. I. Okulov and V. V. Ustinov, Zh. Eksp. Teor. Fiz. **67**, 1176 (1974) [Sov. Phys. JETP **40**, 584 (1974)].
[15]F. G. Bass, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **4**, 476 (1961).
[16]V. I. Okulov and V. V. Ustinov, Fiz. Met. Metalloved. **41**, 231 (1976) [Phys. Met. Metallogr. (USSR) **41**, No. 2, 1 (1976)].
[17]R. B. Dingle, Physica **19**, 311 348 and 729 (1953).
[18]V. V. Ustinov, Fiz. Tverd. Tela **20**, 2121 (1978) [Sov. Phys. Solid State **20**, 1225 (1978)].


Translated by W. F. Brown, Jr.